Mock Test 2

Time : 3 hrs. **Max. Marks : 300**

INSTRUCTIONS

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. There are three parts in the question paper consisting of Physics (Q.no.1 to 30), Chemistry (Q.no.31 to 60) and Mathematics (Q. no.61 to 90). Each part is divided into two sections, Section A consists of 20 multiple choice questions & Section B consists of 10 Numerical value answer Questions.
   In Section B, candidates have to attempt **only 5 questions out of 10**.
5. There will be only one correct choice in the given four choices in Section A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for unattempted question. For Section B 4 marks will be awarded for correct answer and zero for marked for each review / unattempted/incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations / written work should be done in the rough sheet provided.

PHYSICS

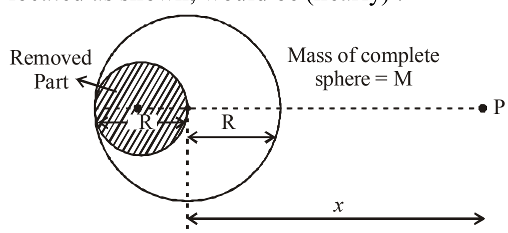
Section - A

1. N divisions on the main scale of a vernier callipers coincide with N + 1 divisions on the vernier scale. If each division on the main scale is of ‘a’ units, the least count of the instrument is

(1) $\frac{a}{N+1}$ (2) $\frac{a}{N-1}$

(3) $\frac{2a}{N-1}$ (4) $\frac{2a}{N+1}$

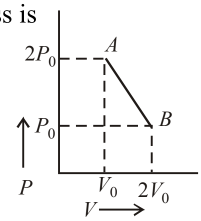
2. The gravitational field, due to the 'left over part' of a uniform sphere (from which a part as shown, has been 'removed out'), at a very far off point, P, located as shown, would be (nearly) :



(1) $\frac{5}{6}\frac{GM}{x^2}$ (2) $\frac{8}{9}\frac{GM}{x^2}$

(3) $\frac{7}{8}\frac{GM}{x^2}$ (4) $\frac{6}{7}\frac{GM}{x^2}$

3. n moles of an ideal gas undergo a process $A \rightarrow B$ as shown in the figure. Maximum temperature of the gas during the process is



(1) $\frac{9P_0V_0}{nR}$ (2) $\frac{3P_0V_0}{2nR}$

(3) $\frac{9P_0V_0}{2nR}$ (4) $\frac{9P_0V_0}{4nR}$

4. Which of these nuclear reactions is possible –

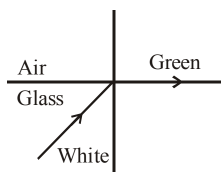
(1) ${}^{23}_{11}Na + {}^{1}_{1}H \longrightarrow {}^{20}_{10}Ne + {}^{4}_{2}He$

(2) ${}^{10}_{5}B + {}^{4}_{2}He \longrightarrow {}^{13}_{7}N + {}^{1}_{1}H$

(3) ${}^{10}_{5}B + {}^{1}_{0}n \longrightarrow {}^{11}_{5}B + \beta^{-} + \overline{\nu}$

(4) ${}^{14}_{7}N + {}^{1}_{1}H \longrightarrow {}^{12}_{6}C + \beta^{-} + \overline{\nu}$

5. White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected then the emerging ray in air contains



(1) yellow, orange, red

(2) violet, indigo, blue

(3) All colours

(4) All colours except green

6. What is the conductivity of a semiconductor sample having electron concentration of 5×10^{18} m^{-3}, hole concentration of 5×10^{19} m^{-3}, electron mobility of 2.0 $m^2 V^{-1} s^{-1}$ and hole mobility of 0.01 $m^2 V^{-1} s^{-1}$?

(Take charge of electron as 1.6×10^{-19} C)

(1) 1.68 $(\Omega - m)^{-1}$ (2) 1.83 $(\Omega - m)^{-1}$

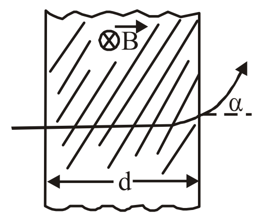
(3) 0.59 $(\Omega - m)^{-1}$ (4) 1.20 $(\Omega - m)^{-1}$

7. A body is executing simple harmonic motion. At a displacement x from mean position, its potential energy is E_1 = 2J and at a displacement y from mean position, its potential energy is E_2 = 8J. The potential energy E at a displacement (x + y) from mean position is

(1) 10J (2) 14J

(3) 18J (4) 4J

8. A proton (mass m) accelerated by a potential difference V flies through a uniform transverse magnetic field B. The field occupies a region of space by width 'd'. If α be the angle of deviation of proton from initial direction of motion (see figure), the value of $sin\ \alpha$ will be :



(1) $qV\sqrt{\frac{Bd}{2m}}$

(2) $\frac{B}{2}\sqrt{\frac{qd}{mV}}$

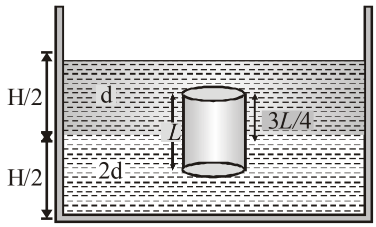
(3) $\frac{B}{d}\sqrt{\frac{q}{2mV}}$

(4) $Bd\sqrt{\frac{q}{2mV}}$

9. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(1) $4F$ (2) $6F$ (3) $9F$ (4) F

10. A homogeneous solid cylinder of length L (L < H/2). Cross-sectional area A/5 is immersed such that it floats with its axis vertical at the liquid-liquid interface with length L/4 in the denser liquid as shown in the fig. The lower density liquid is open to atmosphere having pressure P_0. Then density of solid is given by



(1) $\frac{5}{4}d$

(2) $\frac{4}{5}d$

(3) d

(4) $\frac{d}{5}$

11. In a Young's double slit experiment with light of wavelength λ, fringe pattern on the screen has fringe width β. When two thin transparent glass (refractive index μ) plates of thickness t_1 and t_2 ($t_1 > t_2$) are placed in the path of the two beams respectively, the fringe pattern will shift by a distance

(1) $\frac{\beta(\mu-1)}{\lambda}\left(\frac{t_1}{t_2}\right)$ (2) $\frac{\mu\beta}{\lambda}\frac{t_1}{t_2}$

(3) $\frac{\beta(\mu-1)}{\lambda}(t_1-t_2)$ (4) $(\mu-1)\frac{\lambda}{\beta}(t_1+t_2)$

12. A dipole consisting of two charges +q and –q joined by a massless rod of length ℓ, is seen oscillating with a small amplitude in a uniform electric field of magnitude E. The period of oscillation

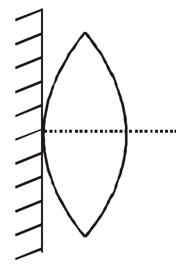
(1) is proportional to E

(2) is proportional to 1/E

(3) is $\pi\sqrt{\frac{m\ell}{3qE}}$

(4) is proportional to $\frac{1}{\sqrt{E}}$ but ≠ $\pi\sqrt{\frac{m\ell}{3qE}}$

13. A thin convex lens of focal length 'f' is put on a plane mirror as shown in the figure. When an object is kept at a distance 'a' from the lens - mirror combination, its image is formed at a distance $\frac{a}{3}$ in front of the combination. The value of 'a' is :



(1) 3f

(2) $\frac{3}{2}$f

(3) f

(4) 2f

14. A sinusoidal wave is propagating in negative x-direction in a string stretched along x-axis. A particle of string at x = 2m is found at its mean position and it is moving in positive y-direction at t = 1 sec. The amplitude of the wave, the wavelength and the angular frequency of the wave are 0.1 meter, π/2 meter and 2π rad/sec respectively.

The equation of the wave is

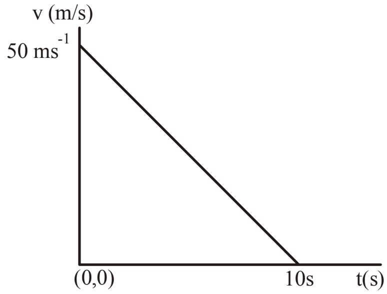
(1) y = 0.1 sin (4π (t – 1) + 8 (x – 2))

(2) y = 0.1 sin ((t – 1) – (x – 2))

(3) y = 0.1 sin (2π (t – 1) + 4 (x – 2))

(4) None of these

15. Velocity–time graph for a body of mass 10 kg is shown in figure. Work–done on the body in first two seconds of the motion is :



(1) – 9300 J (2) 12000 J
(3) –4500 J (4) –12000 J

16. A charged particle of mass m and having a charge Q is placed in an electric field E which varies with time as $E = E_0 \sin \omega t$. What is the amplitude of the S.H.M. executed by the particle?

(1) $\dfrac{QE_0}{m\omega^2}$ (2) $\dfrac{1}{2}\dfrac{QE_0}{m\omega^2}$

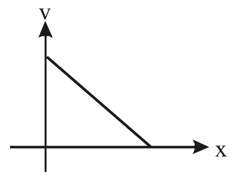
(3) $\dfrac{2QE_0}{m\omega^2}$ (4) None of these

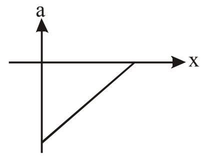
17. In a photoelectric experiment, with light of wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will become –

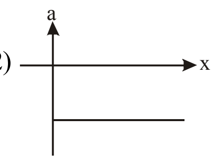
(1) $v\sqrt{\dfrac{3}{4}}$ (2) $v\sqrt{\dfrac{4}{3}}$

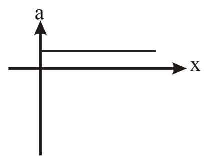
(3) less than $v\sqrt{\dfrac{4}{3}}$ (4) greater than $v\sqrt{\dfrac{4}{3}}$

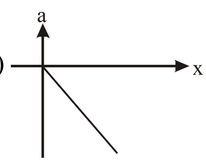
18. A particle moves along x-axis with initial position x = 0. Its velocity varies with x-coordinate as shown in graph. The acceleration 'a' of this particle varies with x as –



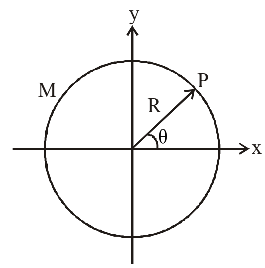
(1) 

(2) 

(3) 

(4) 

19. A ring of mass M and radius R lies in x-y plane with its centre at origin as shown. The mass distribution of ring is non-uniform such that at any point P on the ring, the mass per unit length is given by $\lambda = \lambda_0 \cos^2\theta$ (where λ_0 is a positive constant). Then the moment of inertia of the ring about z-axis is –



(1) MR^2

(2) $\dfrac{1}{2}MR^2$

(3) $\dfrac{1}{2}\dfrac{M}{\lambda_0}R$

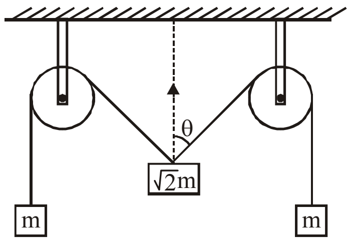
(4) $\dfrac{1}{\pi}\dfrac{M}{\lambda_0}R$

20. The electric potential V is given as a function of distance x by $V = (5x^2 + 10x - 9)$ volt. Value of electric field at x = 1m is

(1) –20 V/m (2) 6 V/m
(3) 11 V/m (4) 20 V/m

Section - B

21. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be _____ degree.



22. A current of 4A produces a deflection of 30° in the galvanometer. The figure of merit is _____ A/rad.

23. A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}\right)L$ from A, then a and b are related as b = xa. Find the value of x.

24. A transmitting antenna at the top of a tower has height 32 m and height of the receiving antenna is 50 m. What is the maximum distance (in km) between them for satisfactory communication in line of sight (LOS) mode?

25. A ball of mass 160 g is thrown up at an angle of 60° to the horizontal at a speed of 10 ms^{-1}. The angular momentum of the ball at the highest point of the trajectory with respect to the point from which the ball is thrown is _____ kg m^2/s. (g = 10 ms^{-2})

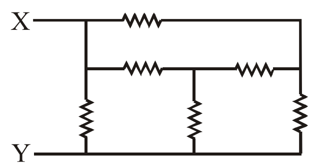
26. A series LR circuit is connected to an ac source of frequency ω and the inductive reactance is equal to 2R. A capacitance of capacitive reactance equal to R is added in series with L and R. The ratio of the new power factor to the old one is $\sqrt{\frac{x}{2}}$. Find the value of x.

27. A mass of 50g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from 30°C to 25°C. A mass of 100g of another liquid in an identical vessel with identical surroundings takes the same time to cool from 30° C to 25° C. The specific heat of the liquid is _____ kcal/kg.
(The water equivalent of the vessel is 30g.)

28. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is _____ V/m.

29. An ideal monatomic gas with pressure P, volume V and temperature T is expanded isothermally to a volume $2V$ and a final pressure P_i. If the same gas is expanded adiabatically to a volume $2V$, the final pressure is P_a. The ratio $\frac{P_a}{P_i}$ is _____.

30. Six resistors of 10Ω each are connected as shown. The equivalent resistance between points X and Y is _____.



CHEMISTRY

Section - A

31. Consider the following reactions

(i) $Cd^{2+}(aq) + 2e^- \longrightarrow Cd(s)$, $E° = -0.40\,V$

(ii) $Ag^+(aq) + e^- \longrightarrow Ag(s)$, $E° = 0.80\,V$

For the galvanic cell involving the above reactions. Which of the following is not correct?

(1) Molar concentration of the cation in the cathodic compartment changes faster than that of the cation in the anodic compartment.

(2) E_{cell} increase when Cd^{2+} solution is diluted.

(3) Twice as many electrons pass through the cadmium electrode as through silver electrode.

(4) E_{cell} decreases when Ag^+ solution is diluted.

32. A container contains certain gas of mass 'm' of high pressure. Some of the gas has been allowed to escape from the container and after some time the pressure of the gas becomes half and its absolute temperature 2/3 rd. The amount of the gas escaped is

(1) 2/3 m (2) 1/2 m

(3) 1/4 m (4) 1/6 m

33. The hybrid state of S in SO_3 is similar to that of

(1) C in C_2H_2 (2) C in C_2H_4

(3) C in CH_4 (4) C in CO_2

34. Choose the correct options –

(1) Tranquillizers are called psychotherapeutic drugs.

(2) Vitamin C, Vitamin E and β-carotene are antioxidants.

(3) Sodium or potassium salt of a long chain fatty acid is called soap.

(4) All of these.

35. Electrode potential of the half cell Pt (s) | Hg (l) | Hg_2Cl_2(s) | Cl^- (aq) can be increased by :

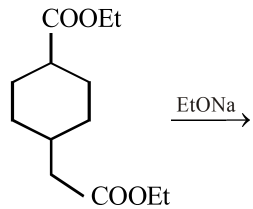
(1) Increasing $[Cl^-]$

(2) Decreasing $[Cl^-]$

(3) Increasing Hg_2Cl_2 (s)

(4) Decreasing Hg (l)

36. What is the name of the following reaction :



(1) Knoevenagel reaction

(2) Perkin reaction

(3) Reformatsky reaction

(4) Dieckmann's condensation.

37. Choose the correct options –

(1) The change of atmospheric temperature with altitude is called the lapse rate.

(2) The gases responsible for greenhouse effects are CO_2, water vapour, CH_4 and ozone.

(3) The order of contribution to the acid rain is $H_2SO_4 < HNO_3 < HCl$

(4) Both (1) and (2)

38. Optically active isomer (A) of (C_5H_9Cl) on treatment with one mole of H_2 gives an optically inactive compound (B) compound (A) will be:

(1) $CH_3-CH(CH_2Cl)-CH=CH_2$

(2) $Cl-CH(CH_3)-CH=CH-CH_3$

(3) $CH_3-CH(Cl)-CH_2-CH=CH_2$

(4) $CH_3-CH_2-CH(Cl)-CH=CH_2$

39. Which statement is/are correct about ICl_3 molecule ?
(1) All I – Cl bonds are equivalent.
(2) Molecule is polar and non-planar.
(3) All adjacent bond angles are equal.
(4) All hybrid orbitals of central atom having equal *s*-character.

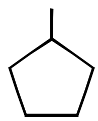
40. Isopropyl alcohol is obtained by reacting which of the following alkenes with concentrated H_2SO_4 followed by boiling with H_2O?
(1) Ethylene (2) Propylene
(3) 2-Methylpropene (4) Isoprene

41. In electro-refining of metal the impure metal is used to make the anode and a strip of pure metal as the cathode, during the electrolysis of an aqueous solution of a complex metal salt. This method cannot be used for refining of
(1) silver (2) copper
(3) aluminium (4) sodium

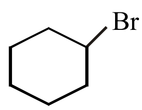
42. Which of the following compounds does not give nitrogen gas on heating?
(1) NH_4NO_2 (2) $(NH_4)_2SO_4$
(3) NH_4ClO_4 (4) $(NH_4)_2Cr_2O_7$

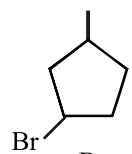
43. Flocculation value of $BaCl_2$ is much less than that of KCl for sol A and flocculation value of Na_2SO_4 is much less than that of NaBr for sol B. The correct statement among the following is :
(1) Both the sols A and B are negatively charged.
(2) Sol A is positively charged and Sol B is negatively charged.
(3) Both the sols A and B are positively charged.
(4) Sol A is negatively charged and sol B is positively charged.

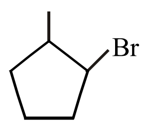
44. $HC\equiv CH \xrightarrow[H_2SO_4]{HgSO_4} (A) \xrightarrow[5\,°C]{dil.\ OH^-} (B)$
Give the IUPAC name of "B" is –
(1) 2-butanal
(2) 3-hydroxy butanal
(3) 3-formyl-2-propanol
(4) 4-oxo-2-propanol

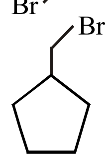
45.  $\xrightarrow{Br_2/h\nu}$ Major (X) $\xrightarrow[KOH/\Delta]{Alcoholic}$ Major (Y) $\xrightarrow[Peroxide]{H-Br}$ Major (Z)

Major final product (Z) is –

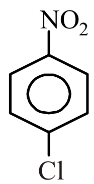
(1) 

(2) 

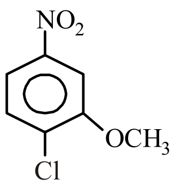
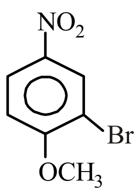
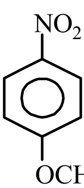
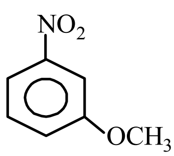
(3) 

(4) 

46. Choose the incorrect option –
(1) When a dilute solution of an acid is added to a dilute solution of a base, neutralization reaction takes place.
(2) In acid-base titrations, at the end point, the amount of acid becomes chemically equivalent to the amount of base present.
(3) In the case of a strong acid and a strong base titration, at the end point the solution becomes acidic.
(4) In acid-base titrations the end point is determined by the hydrogen ion concentration of the solution.

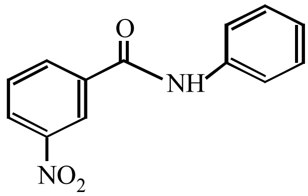
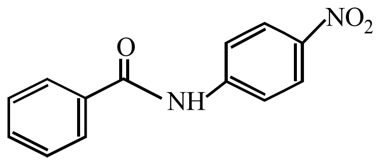
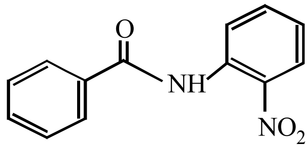
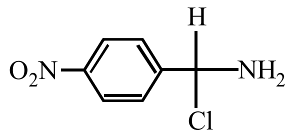
47.  $\xrightarrow[Fe,\ \Delta]{Br_2} X \xrightarrow[CH_3OH,\ \Delta]{CH_3ONa} Y$

Product (Y) of this reaction is –

(1) 
(2) 
(3) 
(4) 

48. $A \xrightarrow[(ii)\ H^{\oplus}]{(i)\ NaOI} B \xrightarrow{SOCl_2} C \xrightarrow[NaOH]{C_6H_5NH_2}$

$D \xrightarrow[\text{Conc. } H_2SO_4 \text{ (1 equivalent)}]{\text{Conc. } HNO_3} E$ (Major) E can be –

(1) 
(2) 
(3) 
(4) 

49. (X) $(C_5H_{10}O)$ $\xrightarrow{H_3O^+}$ Y + Z (Y and Z both give the Iodoform test). The compound X is –
(1) $CH_3-CH=CH-O-CH_2-CH_3$
(2) $CH_3-C(H)(CH_3)-O-CH_2-CH_3$
(3) $CH_3-C(=CH_2)-O-CH_2-CH_3$
(4) Both (1) and (3)

50. If the elevation in boiling point of a solution of non-volatile, non-electrolytic and non-associating solute in a solvent ($K_b = x$ K kg mol^{-1}) is y K, then the depression in freezing point of solution of same concentration would be (K_f of the solvent = z K kg mol^{-1})

(1) $\frac{2xz}{y}$K (2) $\frac{yz}{x}$K

(3) $\frac{xz}{y}$K (4) $\frac{yz}{2x}$K

Section - B

51. The density of CaF_2 (fluorite structure) is 3.18 g/cm^3. Find the length of the side of the unit cell is _____.

52. When the following aldohexose exists in its D-configuration, the total number of stereoisomers in its pyranose form is _____.

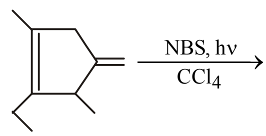
CHO — CH_2 — CHOH — CHOH — CHOH — CH_2OH

53. A 25.0 mm × 40.0 mm piece of gold foil is 0.25 mm thick. The density of gold is 19.32 g/cm^3. How many gold atoms are in the sheet in terms of $x \times 10^{22}$? (Atomic weight : Au = 197.0)

54. In a chemical reaction *A* is converted into *B*. The rates of reaction, starting with initial concentrations of *A* as 2×10^{-3} M and 1×10^{-3} M, are equal to 2.40×10^{-4} Ms^{-1} and 0.60×10^{-4} Ms^{-1} respectively. Find the order of reaction with respect to reactant *A*.

55. Rate constant k = 2.303 min^{-1} for a particular reaction. The initial concentration of the reactant is 1 mol/litre then calculatle the rate of reaction after 1 minute.

56. How many free radicals can be produced during following reaction (ignoring resonating structure) ?



57. An electron has a speed of 30,000 cm sec^{-1} accurate upto 0.001%. What is the uncertainty (in cm) in locating it's position?

58. The standard enthalpy of formation of NH_3 is – 46.0 kJ/mol. If the enthalpy of formation of H_2 from its atoms is – 436 kJ/mol and that of N_2 is – 712 kJ/mol, calculate the average bond enthalpy of N - H bond in NH_3.

59. The rate coefficient (k) for a particular reaction is 1.3×10^{-4} M^{-1} s^{-1} at 100 °C and 1.3×10^{-3} M^{-1} s^{-1} at 150 °C. What is the energy of activation (E_A) (in kJ/mol) for this reaction? (R = molar gas constant = 8.314 JK^{-1} mol^{-1})

60. The e.m.f. of the cell Zn | Zn^{2+} (0.01M) | | Fe^{2+} (0.001M) | Fe at 298 K is 0.2905 then the value of equilibrium constant for the cell reaction 10^x. Find x.

MATHEMATICS

Section - A

61. Given $f(x)=\begin{cases}\sqrt{10-x^2} & \text{if } -3<x<3\\ 2-e^{x-3} & \text{if } x\geq 3\end{cases}$

The graph of $f(x)$ is –

(1) continuous and differentiable at $x = 3$

(2) continuous but not differentiable at $x = 3$

(3) differentiable but not continuous at $x = 3$

(4) neither differentiable nor continuous at $x = 3$

62. The greatest and the least absolute value of $z+1$, where $|z+4| \leq 3$ are respectively

(1) 6 and 0 (2) 10 and 6

(3) 4 and 3 (4) None of these

63. The equation $(5x-1)^2+(5y-2)^2=(\lambda^2-4\lambda+4)(3x+4y-1)^2$ represents an ellipse if $\lambda \in$

(1) $(0,1]$ (2) $(-1,2)$

(3) $(2,3)$ (4) $(-1,0)$

64. If $(-4, 5)$ is one vertex and $7x-y+8=0$ is one diagonal of a square, then the equation of second diagonal is

(1) $x+3y=21$ (2) $2x-3y=7$

(3) $x+7y=31$ (4) $2x+3y=21$

65. Which of the following is always true?

(1) $(\sim p \Rightarrow q) = \sim q \Rightarrow \sim p$

(2) $(\sim p \vee q) \equiv \vee p \vee \sim q$

(3) $\sim(p \Rightarrow q) \equiv p \wedge \sim q$

(4) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

66. Find $\int e^{\sin x}\left(\dfrac{x\cos^3 x-\sin x}{\cos^2 x}\right)dx$

(1) $x\,e^{\sin x}-e^{\sin x}\sec x+C$

(2) $x\,e^{\cos x}-e^{\sin x}\sec x+C$

(3) $x^2\,e^{\sin x}+e^{\sin x}\sec x+C$

(4) $2x\,e^{\sin x}-e^{\sin x}\tan x+C$

67. The function $f:[2,\infty)\rightarrow(0,\infty)$ defined by $f(x)=x^2-4x+a$, then the set of values of 'a' for which f(x) becomes onto is

(1) $(4,\infty)$ (2) $[4,\infty)$

(3) $\{4\}$ (4) ϕ

68. If α and β are the real roots of the equation

$x^2-(k-2)x+(k^2+3k+5)=0\ (k\in R)$.

Find the maximum and minimum values of $(\alpha^2+\beta^2)$.

(1) 18, 50/9 (2) 18, 25/9

(3) 27, 50/9 (4) None of these

69. The sum of the coefficient of all the terms in the expansion of $(2x-y+z)^{20}$ in which y do not appear at all while x appears in even powers and z appears in odd powers is –

(1) 0 (2) $\dfrac{2^{20}-1}{2}$

(3) 2^{19} (4) $\dfrac{3^{20}-1}{2}$

70. All the five digit numbers in which each successive digit exceeds is predecessor are arranged in the increasing order. The $(105)^{th}$ number does not contain the digit

(1) 1 (2) 2
(3) 6 (4) All of these

71. If $\hat{a}, \hat{b}, \hat{c}$ are non-coplanar unit vectors such that $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{\sqrt{2}}(\hat{b} + \hat{c})$ then the angle between the vectors $\hat{a}, \hat{b}$ is

(1) $3\pi/4$ (2) $\pi/4$
(3) $\pi/8$ (4) $\pi/2$

72. The greatest and the least values of $|z_1 + z_2|$ if $z_1 = 24 + 7i$ and $|z_2| = 6$ respectively are

(1) 31, 19 (2) 19, 31
(3) –19, –31 (4) –31, –19

73. Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is

(1) $\frac{\sqrt{3}}{2}x + y = 0$ (2) $x + \sqrt{3y} = 0$
(3) $\sqrt{3}x + y = 0$ (4) $x + \frac{\sqrt{3}}{2}y = 0.$

74. If $\int \sqrt{2}\sqrt{1+\sin x}\,dx = -4\cos(ax+b) + C$ then the value of (a, b) is :

(1) $\frac{1}{2}, \frac{\pi}{4}$ (2) $1, \frac{\pi}{2}$
(3) 1, 1 (4) None of these

75. If the tangent at any point on the curve $x^4 + y^4 = a^4$ cuts off intercepts p and q on the co-ordinate axes then the value of $p^{-4/3} + q^{-4/3}$ is

(1) $a^{-4/3}$ (2) $a^{-1/2}$
(3) $a^{1/2}$ (4) None

76. The area bounded by the curve $y^2(2a - x) = x^3$ and the line $x = 2a$ is

(1) $3\pi a^2$ sq. unit (2) $\frac{3\pi a^2}{2}$ sq. unit
(3) $\frac{3\pi a^2}{4}$ sq. unit (4) $\frac{6\pi a^2}{5}$ sq. unit

77. The expression satisfying the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = 1$ is

(1) $x^2y - xy^2 = c$
(2) $(y^2 - 1)x = y + c$
(3) $(x^2 - 1)y = x + c$
(4) None of these

78. Let $f: R \to R$ and $f_n(x) = f(f_{n-1}(x)) \; \forall \; n \geq 2, n \in N$, the roots of equation $f_3(x)\,f_2(x)\,f(x) - 25 f_2(x)\,f(x) + 175\,f(x) = 375$, which also satisfy equation $f(x) = x$ will be

(1) 5 (2) 15
(3) 10 (4) Both (1) and (2)

79. The value of

$$\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\ dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\ dt \text{ is}$$

(1) π (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{4}$ (4) 1

80. If a is real and $\sqrt{2}ax + \sin By + \cos Bz = 0$, $x + \cos By + \sin Bz = 0$, $-x + \sin By - \cos Bz = 0$, then the set of all values of a for which the system of linear equations has a non-trivial solution, is–

(1) $[1, 2]$ (2) $[-1, 1]$

(3) $[1, \infty]$ (4) $[2^{-1/2}, 2^{1/2}]$

Section - B

81. If two vertical poles 20 m and 80 m high stand apart on a horizontal plane, then the height (in m) of the point of intersection of the lines joining the top of each pole to the foot of other is _____.

82. If $x + iy = \dfrac{3}{\cos\theta + i\sin\theta + 2}$ then the value of $4x - x^2 - y^2$ is _____.

83. Box contains 2 one rupee, 2 five rupee, 2 ten rupee and 2 twenty rupee coin. Two coins are drawn at random simultaneously. The probability that their sum is Rs. 20 or more, is _____.

84. The value of the definite integral,

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{1+\tan\theta} = \frac{501\pi}{K} \text{ where}$$

$\theta_2 = \dfrac{1003\pi}{2008}$ and $\theta_1 = \dfrac{\pi}{2008}$. The value of K is _____.

85. The expansion of $(1 + x)^n$ has 3 consecutive terms with coefficients in the ratio 1 : 2 : 3 and can be written in the form ${}^nC_k : {}^nC_{k+1} : {}^nC_{k+2}$. The sum of all possible values of (n + k) is _____.

86. The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, the new standard deviation of the resulting observations is _____.

87. The value of $\lim_{x\to 0} \dfrac{\tan x\sqrt{\tan x} - \sin x\sqrt{\sin x}}{x^3 . \sqrt{x}}$ is _____.

88. Three people each flip two fair coins. The probability that exactly two of the people flipped one head and one tail, is _____.

89. If *P*(*S*) denotes the set of all subsets of a given set *S*, then the number of one-to-one functions from the set $S = \{1, 2, 3\}$ to the set *P*(*S*) is _____.

90. A triangle ABC satisfies the relation $2\sec 4C + \sin^2 2A + \sqrt{\sin B} = 0$ and a point P is taken on the longest side of the triangle such that it divides the side in the ratio 1 : 3. Let Q and R be the circumcentre and orthocentre of ΔABC. If PQ : QR : RP = 1 : α : β, then the value of $\alpha^2 + \beta^2$ is _____.

Mock Test-2

ANSWER KEY											
1	(1)	**16**	(1)	**31**	(3)	**46**	(3)	**61**	(2)	**76**	(2)
2	(3)	**17**	(4)	**32**	(3)	**47**	(2)	**62**	(1)	**77**	(3)
3	(4)	**18**	(1)	**33**	(2)	**48**	(2)	**63**	(3)	**78**	(4)
4	(1)	**19**	(1)	**34**	(4)	**49**	(3)	**64**	(3)	**79**	(3)
5	(1)	**20**	(1)	**35**	(1)	**50**	(2)	**65**	(3)	**80**	(2)
6	(1)	**21**	(45°)	**36**	(4)	**51**	(546)	**66**	(1)	**81**	(16)
7	(3)	**22**	(7.6)	**37**	(4)	**52**	(8)	**67**	(1)	**82**	(3)
8	(4)	**23**	(2)	**38**	(4)	**53**	(1.47)	**68**	(1)	**83**	(0.5)
9	(3)	**24**	(45.5)	**39**	(3)	**54**	(2)	**69**	(1)	**84**	(2008)
10	(1)	**25**	(3)	**40**	(2)	**55**	(0.2303)	**70**	(1)	**85**	(18)
11	(3)	**26**	(5)	**41**	(4)	**56**	(4)	**71**	(1)	**86**	(12)
12	(3)	**27**	(0.5)	**42**	(2)	**57**	(2)	**72**	(1)	**87**	(0.75)
13	(4)	**28**	(6)	**43**	(4)	**58**	(964)	**73**	(3)	**88**	(0.38)
14	(3)	**29**	($2^{\frac{-2}{3}}$)	**44**	(2)	**59**	(60)	**74**	(1)	**89**	(336.00)
15	(3)	**30**	(5)	**45**	(3)	**60**	(10.85)	**75**	(1)	**90**	(9)

Solutions

PHYSICS

1. (1) Least count of vernier callipers
= value of one division of main scale
– value of one division of vernier scale
Now, $N \times a = (N + 1) a'$
(a' = value of one division of vernier scale)
$$a' = \frac{N}{N+1}a$$
$\therefore$ Least count = $a - a' = \frac{a}{N+1}$

2. (3) Let mass of smaller sphere (which has to be removed) is m
$$\text{Radius} = \frac{R}{2} \text{ (from figure)}$$
$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$$
$$\Rightarrow m = \frac{M}{8}$$
Mass of the left over part of the sphere
$$M' = M - \frac{M}{8} = \frac{7}{8}M$$
Therefore gravitational field due to the left over part of the sphere
$$= \frac{GM'}{x^2} = \frac{7}{8}\frac{GM}{x^2}$$

3. (4)

4. (1) Charge conservation is violated in [2, 3, 4], nucleon conservation is violated in (4), (1) works.

5. (1) $C = \sin^{-1}\left(\frac{1}{\mu}\right)$ and $\mu \propto \frac{1}{\lambda}$
Yellow, orange and red have higher wavelengths than green, so μ will be less for these rays, consequently critical angle for these rays will be high, hence if green is just totally internally reflected then yellow, orange and red rays will emerge out.

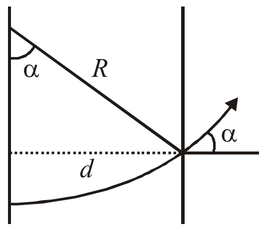
6. (1) The conductivity of semiconductor
$$\sigma = e(\eta_e\mu_e + \eta_h\mu_h)$$
$$= 1.6 \times 10^{-19}(5 \times 10^{18} \times 2 + 5 \times 10^{19} \times 0.01)$$
$$= 1.6 \times 1.05 = 1.68$$

7. (3) $E_1 = \frac{1}{2}kx^2$, $E_2 = \frac{1}{2}ky^2$,

$$E = \frac{1}{2}k(x+y)^2 = E_1 + E_2 + 2\sqrt{E_1E_2}$$

$$= 2 + 8 + 2\sqrt{16} = 18J$$

8. (4) From figure, $\sin\alpha = d/R$



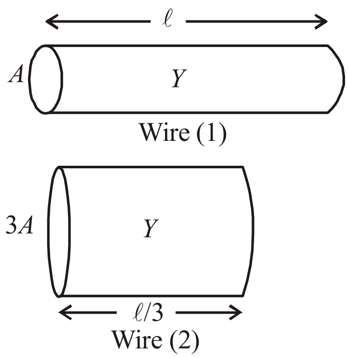
And we know, $\frac{mv^2}{R} = qvB$

$\Rightarrow R = \frac{mv}{qB}$

$\therefore \sin\alpha = \frac{dqB}{mv}$

$\sin\alpha = Bd\sqrt{\frac{q}{2mV}}$ $\left[\because qV = \frac{1}{2}mv^2\right]$

9. (3)



As shown in the figure, the wires will have the same Young's modulus (same material) and the length of the wire of area of cross-section $3A$ will be $\ell/3$ (same volume as wire 1).

For wire 1, $Y = \frac{F/A}{\Delta x/\ell}$ ...(i)

For wire 2, $Y = \frac{F'/3A}{\Delta x/(\ell/3)}$ ...(ii)

From (i) and (ii), $\frac{F}{A}\times\frac{\ell}{\Delta x} = \frac{F'}{3A}\times\frac{\ell}{3\Delta x}$

$\Rightarrow F' = 9F$

10. (1) Weight of cylinder = upthrust due to both liquids

$$V\times D\times g = \left(\frac{A}{5}\times\frac{3}{4}L\right)\times d\times g + \left(\frac{A}{5}\times\frac{L}{4}\right)\times 2d\times g$$

$$\Rightarrow \left(\frac{A}{5}\times L\right)\times D\times g = \frac{A\times L\times d\times g}{4} \Rightarrow \frac{D}{5} = \frac{d}{4}$$

$$\therefore D = \frac{5}{4}d$$

11. (3) Shift $= \frac{\beta(\mu-1)}{\lambda}t_1 - \frac{\beta(\mu-1)}{\lambda}t_2$

$$= \frac{\beta(\mu-1)}{\lambda}(t_1 - t_2)$$

12. (3) $I = \frac{m\ell^2}{12}$;

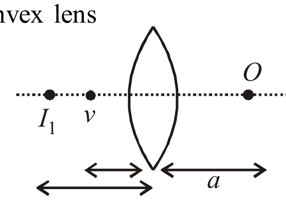
Restoring torque

$$= qE\ell\sin\theta \approx qE\ell\theta = \frac{d^2\theta}{dt^2}I$$

$$\therefore \omega = \sqrt{\frac{qE\ell}{I}} = \sqrt{\frac{qE\ell\times 12}{m\ell^2}} = \frac{2\pi}{T}$$

$$\therefore T = \pi\sqrt{\frac{m\ell}{3qE}}$$

13. (4) When object is kept at a distance 'a' from thin convex lens

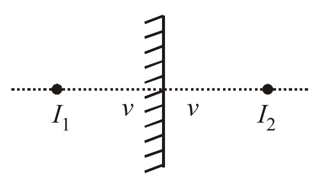


By lens formula : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

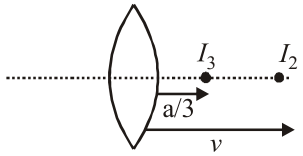
$$\frac{1}{V} - \frac{1}{(-a)} = \frac{1}{f}$$

or, $\frac{1}{v} = \frac{1}{f} - \frac{1}{a}$ ...(i)

Mirror forms image at equal distance from mirror



Now, again from lens formula



$$\frac{3}{a}-\frac{1}{V}=\frac{1}{f}$$

$$\frac{3}{a}-\frac{1}{f}+\frac{1}{a}=\frac{1}{f} \quad \text{[From eqn. (i)]}$$

Hence, $a = 2f$

14. (3) The equation of wave moving in negative x-direction, assuming origin of position at x = 2 and origin of time (i.e. initial time) at t = 1 sec.

$$y = 0.1 \sin(2\pi t + 4x)$$

Shifting the origin of position to left by 2m, that is, to x = 0. Also shifting the origin of time backwards by 1 sec, that is to t = 0 sec.

$$y = 0.1 \sin(2\pi(t-1) + 4(x-2))$$

15. (3) Acceleration (a) $=\frac{v-u}{t}$

$$=\frac{(0-50)}{(10-0)}=-5 \text{ m/s}^2$$

u = 50 m/s

$\therefore v = u + at = 50 - 5t$

Veocity in first two seconds t = 2

$$v_{(\text{at } t=2)} = 40 \text{ m/s}$$

From work-energy theorem,

$$\Delta K.E. = W = \frac{1}{2}(40^2-50^2)\times 10 = -4500 \text{ J}$$

16. (1) For a particle undergoing SHM with an amplitude A and angular frequency ω, the maximum acceleration = $\omega^2 A$

Here the maximum force on the particle = QE_0

$\therefore$ maximum acceleration $=\frac{QE_0}{m}=\omega^2 A$

$$\therefore A=\frac{QE_0}{m\omega^2}$$

17. (4) $\frac{1}{2}mv^2=\frac{hc}{\lambda}-\phi$

$$\frac{1}{2}mv'^2=\frac{hc}{(3\lambda/4)}-\phi=\frac{4hc}{3\lambda}-\phi.$$

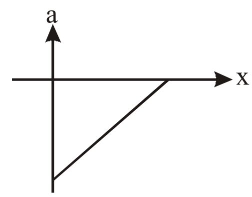
Clearly, $v' > \sqrt{\frac{4}{3}}v$

18. (1) The linear relationship between v and x is $v = -mx + C$ where m and C are positive constants.

$\therefore$ Acceleration, $a = v\frac{dv}{dx} = -m(-mx + C)$

$\therefore a = m^2x - mC$

Hence the graph relating a to x is



19. (1) Divide the ring into infinitely small lengths of mass dm_1. Even though mass distribution is non-uniform, each mass dm_1 is at same distance R from origin.

$\therefore$ MI of ring about z-axis is

$= dm_1R^2 + dm_2R^2 + \ldots\ldots\ldots + dm_nR^2 = MR^2$

20. (1) $E=\frac{-dV}{dx}$ $V = 5x^2 + 10x - 9$,

$$E=\frac{-d}{dx}(5x^2+10x-9)=-(10x+10)$$

On putting value of x in it we get,

$$E = -(10\times 1 + 10) = -20 \text{ V/m}$$

21. (45°) If T is the tension i n the string , Then

$T = mg$ (For outer masses)

$2T\cos\theta = \sqrt{2}mg$ (For inner mass)

Eliminating T, we get $2(mg)\cos\theta = \sqrt{2}mg$

or $\cos\theta = 1/\sqrt{2} \Rightarrow \theta = 45°$

22. (7.6) Here I = 4A

$$\theta = 30° = \left(\frac{30\times\pi}{180}\right)=\frac{\pi}{6}$$

$$\text{Now, } k=\frac{I}{\theta}=\frac{4}{\left(\frac{\pi}{6}\right)}=\frac{4\times6\times7}{22}=\frac{2\times6\times7}{11}$$

$$=\frac{84}{11}=7.6 \text{ A/rad}$$

23. (2) Centre of mass of the rod is given by:

$$x_{cm}=\frac{\int_0^L (ax+\frac{bx^2}{L})dx}{\int_0^L (a+\frac{bx}{L})dx}=\frac{\frac{aL^2}{2}+\frac{bL^2}{3}}{aL+\frac{bL}{2}}=\frac{L\left(\frac{a}{2}+\frac{b}{3}\right)}{a+\frac{b}{2}}$$

Now $\frac{7L}{12} = \frac{\frac{a}{2}+\frac{b}{3}}{a+\frac{b}{2}}$

On solving we get, $b = 2a$

24. (45.5) Given : $h_R = 50$ m

$h_T = 32$ m

Maximum distance, $d_M = ?$

Applying, $d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$= \sqrt{2\times6.4\times10^6\times32} + \sqrt{2\times6.4\times10^6\times50}$

$= 45.5$ km

25. (3) Given : m = 0.160 kg

$\theta = 60°$; v = 10 m/s

Angular momentum $L = \vec{r}\times m\vec{v}$

$= H\, mv\cos\theta$

$= mv\frac{v^2\sin^2\theta}{2g}\cos\theta \quad \left[H = \frac{v^2\sin^2\theta}{2g}\right]$

$= \frac{10^2\times\sin^2 60°\times\cos 60°\times0.160\times10}{2\times10}$

$= 3$ kg m^2/s

26. (5) Power factor $_{(old)}$

$= \frac{R}{\sqrt{R^2+X_L^2}} = \frac{R}{\sqrt{R^2+(2R)^2}} = \frac{R}{\sqrt{5}R}$

Power factor$_{(new)}$

$= \frac{R}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{R}{\sqrt{R^2+(2R-R)^2}}$

$= \frac{R}{\sqrt{2}R}$

$\therefore \frac{\text{New power factor}}{\text{Old power factor}} = \frac{\frac{R}{\sqrt{2}R}}{\frac{R}{\sqrt{5}R}} = \sqrt{\frac{5}{2}}$

27. (0.5) As the surrounding is identical, vessel is identical time taken to cool both water and liquid (from 30°C to 25°C) is same 2 minutes, therefore

$\left(\frac{dQ}{dt}\right)_{water} = \left(\frac{dQ}{dt}\right)_{liquid}$

or, $\frac{(m_wC_w+W)\Delta T}{t} = \frac{(m_\ell C_\ell+W)\Delta T}{t}$

(W = water equivalent of the vessel)

or, $m_wC_w = m_\ell C_\ell$

$\therefore$ Specific heat of liquid, $C_\ell = \frac{m_WC_W}{m_\ell}$

$= \frac{50\times1}{100} = 0.5$ kcal / kg

28. (6) From question,

$B_0 = 20$ nT $= 20\times10^{-9}$T

$\vec{E}_0 = \vec{B}_0\times\vec{C}$

$|\vec{E}_0| = |\vec{B}_0|.|\vec{C}| = 20\times10^{-9}\times3\times10^8 = 6$V/m.

($\because$ velocity of light in vacuum C = 3×10^8 ms^{-1})

29. ($2^{\frac{-2}{3}}$) For isothermal process :

$PV = P_i.2V$

$P = 2P_i$ ...(i)

For adiabatic process

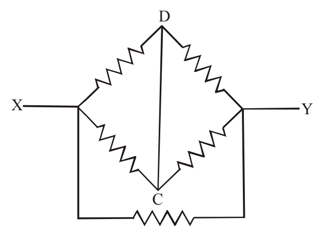
$PV^\gamma = P_a(2V)^\gamma$

($\because$ for monatomic gas $\gamma = 5/3$)

or, $2P_iV^{\frac{5}{3}} = P_a(2V)^{\frac{5}{3}}$ [From (i)]

$\Rightarrow \frac{P_a}{P_i} = \frac{2}{2^{\frac{5}{3}}} \Rightarrow \frac{P_a}{P_i} = 2^{\frac{-2}{3}}$

30. (5)



Equivalent resistance = 5Ω

CHEMISTRY

31. (3) $Cd(s) + 2Ag^+(aq) \longrightarrow$

$Cd^{2+}(aq) + Ag(s)$,

$E_{cell} = E^\circ_{cell} - \frac{0.059}{2}\log\frac{[Cd^{2+}]}{[Ag^+]^2}$

32. (3) Gas equation is $PV = \frac{m}{M}RT$ ...(i)

Again $\frac{P}{2}V = \frac{m_1}{M}R.\frac{2}{3}T$ ...(ii)

Divide (i) by (ii)

$$2 = \frac{m}{m_1} \times \frac{3}{2}$$

$\therefore m_1 = \frac{3}{4}m$. Gas escaped is then $= \frac{1}{4}m$

33. (2)

Molecule	Hybridization
SO_3	sp^2
C_2H_2	sp
C_2H_4	sp^2
CH_4	sp^3
CO_2	sp

34. (4) (1) Tranquillizers are the substances used for the treatment of mental diseases. These act on higher centres of the central nervous system. These are also called psychotherapeutic drugs.

(2) Vitamin C, Vitamin E and β-carotene are antioxidants.

(substances that act against oxidants).

(3) Sodium or potassium salt of a long chain fatty acid is called soap.

35. (1)

$$E = E^\circ - \frac{0.0591}{2}\log\frac{1}{[Cl^-]^2}; \text{as } [Cl^-]\uparrow, E\uparrow$$

36. (4) Ester of dicarboxylic acids undergo an intramolecular version of the claisen condensation when a five or six membered ring can be formed. This reaction is an example of a Dieckmann's condensation

37. (4) (1) The change of atmospheric temperature with altitude is called the lapse rate.

(2) The gases responsible for greenhouse effects are CO_2, water vapour, CH_4 and ozone.

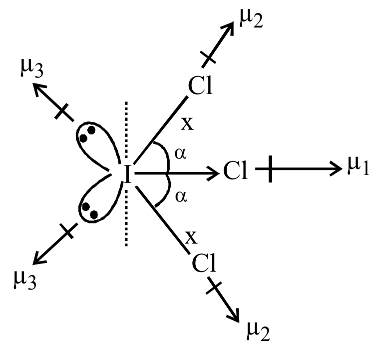
(3) The order of contribution to the acid rain is

$H_2SO_4 > HNO_3 > HCl$

38. (4) $CH_3-CH_2-\underset{\underset{Cl}{|}}{CH}-CH=CH_2 \xrightarrow[Pt]{H_2}$

$Et-\underset{\underset{Cl}{|}}{CH}-Et$ optically inactive.

39. (3)



$(\mu_D \neq 0)$

Molecule is polar and planar.

Both ∠Cl I Cl are equal

Equatorial I – Cl bond has more *s*-character than axial I–Cl bond.

40. (2) Since the compound is formed by hydration of an alkene, to get the structure of alkene remove a molecule of water from the alcohol.

$$\underset{\text{Isopropyl alcohol}}{CH_3\underset{\underset{OH}{|}}{C}HCH_3} \xrightarrow{-H_2O} \underset{\text{Propylene}}{CH_2 = CHCH_3}$$

41. (4) Na reacts vigorously with water (exothermic process).

42. (2) (1) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

(2) $(NH_4)_2SO_4 \xrightarrow{\Delta} NH_3 + H_2SO_4$

(3) $2NH_4ClO_4 \xrightarrow{\Delta} N_2 + Cl_2 + 2O_2 + 4H_2O$

(4) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

43. (4) In first case the given compounds have same anion but different cations having different charge hence they will precipitate negatively charged sol i.e. 'A'.

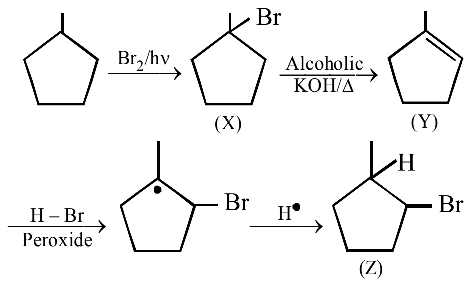
In second case the given compounds have similar cation but different anion with different charge. Hence they will precipitate positively charged sol. i.e. 'B'.

44. (2) $HC \equiv CH \xrightarrow[H_2SO_4]{HgSO_4}$

$CH_3-CHO \xrightarrow{\text{aldol reaction}}$

$$\underset{\text{3-hydroxy butanal}}{\underset{3}{CH_3-\overset{\overset{OH}{|}}{CH}}-\underset{2}{CH_2}-\underset{1}{CHO}}$$

45. (3)



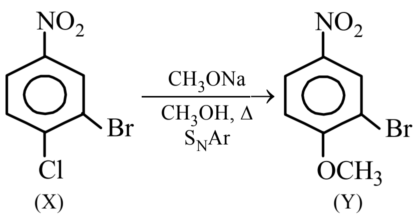
46. (3) (1) When a dilute solution of an acid is added to a dilute solution of a base, neutralization reaction takes place.

(2) In acid-base titrations, at the end point, the amount of acid becomes chemically equilvalent to the amount of base present.

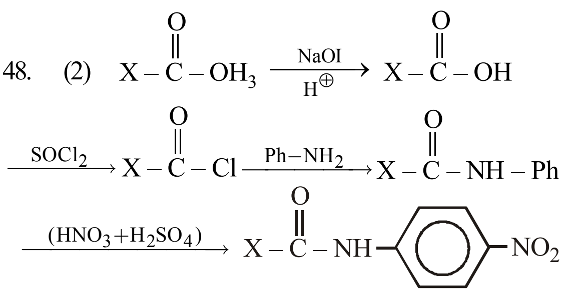
(3) In the case of a strong acid and a strong base titration, at the end point the solution becomes neutral (i.e., pH = 7)

(4) In acid-base titrations the end point is determined by the hydrogen ion concentration of the solution.

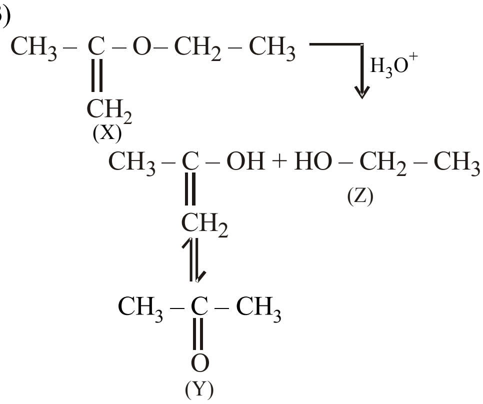
47. (2)



48. (2)



49. (3)



50. (2) Given $k_b = x$ K kg mol^{-1}

$\Delta T_b = k_b \times m$

$\therefore$ $y = x \times m$

$$m = \frac{y}{x}\frac{K}{K\ Kg\ mol^{-1}} = \frac{y}{x} mol\ Kg^{-1}$$

We know

$\Delta T_f = k_f \times m$

On substituting value of m,

$$\Delta T_t = \frac{yz}{x} K$$

51. (546) Use $d = \frac{z M}{N_A\ a^3}$

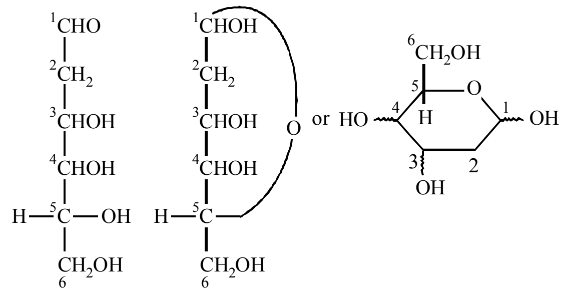
$z = 4; M = 40 + 2 \times 19 = 78$

$$a^3 = \frac{z M}{N_A\ d}$$

$$= \frac{4 \times 78}{6.022 \times 10^{23} \times 3.18}$$

$a = 546$ pm

52. (8)



D- Aldohexose D- Aldohexopyranose

Thus, total number of stereoisomers in pyranose form of D-configuration = $2^3 = 8$

53. (1.47) Volume of gold foil = $25 \times 40 \times 0.25$ mm^3

= 250×10^{-3} cm^3

Mass of gold foil = $19.32 \times 250 \times 10^{-3}$ g

= 4.83 g

No. of gold atoms = $\frac{4.83}{197} \times N_A = 1.47 \times 10^{22}$

54. (2) $A \longrightarrow B$

Initial concentration	Rate of reaction
2×10^{-3} M	2.40×10^{-4} Ms^{-1}
1×10^{-3} M	0.60×10^{-4} Ms^{-1}

rate of reaction $r = k[A]^x$

where x = order of reaction

hence

$2.40 \times 10^{-4} = k\,[2 \times 10^{-3}]^x$ ......(i)

$0.60 \times 10^{-4} = k\,[1 \times 10^{-3}]^x$ ......(ii)

On dividing eqn.(i) from eqn. (ii) we get

$4 = (2)^x$

$\therefore\ x = 2$

i.e. order of reaction = 2

55. (0.2303) $k = \dfrac{2.303}{t}\log\dfrac{[A_0]}{[A]}$;

$$2.303 \times 1 = 2.303 \log\frac{[A_0]}{[A]}$$

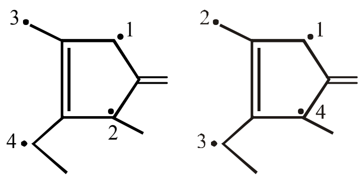
$$\frac{[A_0]}{[A]} = 10$$

$$\therefore\ [A] \Rightarrow \frac{1}{10} \Rightarrow 0.1$$

$\therefore$ rate after 1 min $r_1 = k \cdot [A]$

$\Rightarrow\ 2.303 \times 0.1 \Rightarrow 0.2303\ \text{M min}^{-1}$

56. (4)



57. (2) $\Delta v = \dfrac{0.001}{100} \times 30{,}000 = 0.3\ \text{cm sec}^{-1}$

According to uncertainty principle,

$$\Delta x \,.\, \Delta p \approx \frac{h}{4\pi};\ \Delta x.\Delta v \approx \frac{h}{4\pi m}$$

$$\Delta x \times 9.1 \times 10^{-28} \times 0.3 \approx \frac{6.625 \times 10^{27} \times 7}{4 \times 22}$$

$\Delta x \approx 1.93$ cm. ≈ 2

58. (964) Given $\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightleftharpoons NH_3$;

$$\Delta H_f = -46.0\ \text{kJ/mol}$$

$$H + H \rightleftharpoons H_2;\ \Delta H_f = -436\ \text{kJ/mol}$$

$$N + N \rightleftharpoons N_2;\ \Delta H_f = -712\ \text{kJ/mol}$$

$$\Delta H_f(NH_3) = \frac{1}{2}\Delta H_{N-N} + \frac{3}{2}\Delta H_{H-H} - \Delta H_{N-H}$$

$$-46 = \frac{1}{2}(-712) + \frac{3}{2}(-436) - \Delta H_{N-H}$$

On calculation

$$\Delta H_{N-H} = -964\ \text{kJ/mol}$$

59. (60) According to Arrhenius equation

$$\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\log\frac{1.3 \times 10^{-3}}{1.3 \times 10^{-4}} = \frac{E_a}{2.303 \times 8.314}\left[\frac{1}{373} - \frac{1}{423}\right]$$

$$1 = \frac{E_a}{2.303 \times 8.314}\left[\frac{1}{373} - \frac{1}{423}\right]$$

$$E_a = 60\ \text{kJ/mol}$$

60. (10.85) For this cell, reaction is; $Zn + Fe^{2+} \rightarrow Zn^{2+} + Fe$

$$E = E^\circ - \frac{0.0591}{n}\log\frac{c_1}{c_2};\ E^\circ = E + \frac{0.0591}{n}\log\frac{c_1}{c_2}$$

$$E^\circ = 0.2905 + \frac{0.0591}{2}\log\frac{10^{-2}}{10^{-3}} = 0.32\ \text{V}.$$

$$E^\circ = \frac{0.0591}{2}\log K_{eq}$$

$$\log K_{eq} = \frac{0.32 \times 2}{0.0591} = \frac{0.32}{0.0295}$$

$$\therefore K_{eq} = 10^{\frac{0.32}{0.0295}}$$

Comparing the value of 10^x,

$$x = \frac{0.32}{0.0295} = 10.847 \approx 10.85$$

MATHEMATICS

61. (2) $f'(3^+) = \lim\limits_{h \to 0}\dfrac{f(3+h) - f(3)}{h}$

$$= \lim_{h \to 0}\frac{(2 - e^h) - 1}{h} = -\lim_{h \to 0}\left(\frac{e^h - 1}{h}\right) = -1$$

$$f'(3^-) = \lim_{h \to 0}\frac{f(3-h) - f(3)}{-h}$$

$$= \lim_{h \to 0}\frac{\sqrt{10 - (3-h)^2} - 1}{-h} = -\lim_{h \to 0}\frac{\sqrt{1 + (6h - h^2)} - 1}{-h}$$

$$= \lim_{h \to 0}\frac{6h - h^2}{-h(\sqrt{1 + 6h - h^2} + 1)} = \frac{-6}{2} = -3$$

Hence, $f'(3^+) \neq f(3^-)$

62. (1) We have $|z+1| = |z+4-3|$ ...(i)

Now $|z+4-3| \le |z+4| + |-3| \le 3+3 = 6$

[Given $|z+4| \le 3$ & $|-3| = 3$] $\therefore |z+1| \le 6$

Again $|z+1| \ge 0$ [modulus is always non-negative]

$\therefore$ Least value of $|z+1|$ may be zero, which occurs when $z = -1$, For $z = -1$, $|z+4| = |-1+4| = 3$

Which satisfies the given condition that $|z+4| \ge 3$

Hence, the least and the greatest values of $|z+1|$ are 0 and 6.

63. (3)

$$\left(x-\frac{1}{5}\right)^2 + \left(y-\frac{2}{5}\right)^2 = (\lambda^2 - 4\lambda + 4)\left(\frac{3x+4y-1}{5}\right)^2$$

i.e., $$\sqrt{\left(x-\frac{1}{5}\right)^2 + \left(y-\frac{2}{5}\right)^2} = |\lambda - 2|\left|\frac{3x+4y-1}{\sqrt{5}}\right|$$

is an ellipse.

If $0 < |\lambda - 2| < 1$ i.e., $\lambda \in (1, 2) \cup (2, 3)$

64. (3) One vertex of square is $(-4, 5)$ and equation of one diagonal is $7x - y + 8 = 0$

Diagonal of a square are perpendicular and bisect each other

Let the equation of the other diagonal be $y = mx + c$ where m is the slope of the line and c is the y-intercept.

Since this line passes through $(-4, 5)$

$\therefore$ $5 = -4m + c$ ...(i)

Since this line is at right angle to the line $7x - y + 8 = 0$ or $y = 7x + 8$, having slope $= 7$,

$\therefore$ $7 \times m = -1$ or $m = \frac{-1}{7}$

Putting this value of m in equation (i) we get

$C = \frac{31}{7}$

Hence equation of the other diagonal is

$y = -\frac{1}{7}x + \frac{31}{7}$ or $x + 7y = 31$.

65. (3) $p \Rightarrow q \equiv \sim p \vee q$ $\therefore \sim(p \Rightarrow q) \equiv p \wedge \sim q$.

66. (1) $$\int e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right) dx$$

$= \int e^{\sin x}\ x\cos x\, dx - \int e^{\sin x}\ \tan x \sec x\, dx$

$= \int x\, d\left(e^{\sin x}\right) - \int e^{\sin x}\ d(\sec x)$

$= \left\{x e^{\sin x} - \int e^{\sin x}\, dx\right\}$

$\quad - \left\{e^{\sin x} \sec x - \int e^{\sin x} \sec x \cos x\, dx\right\}$

$= x\, e^{\sin x} - e^{\sin x} \sec x + C$

67. (1) $f(x) = x^2 - 4x + a$

$= x^2 - 4x + 4 - 4 + a$

$= (x-2)^2 + (a-4)$

So, $a \in (4, \infty)$ for domain $[2, \infty)$ and range $(0, \infty)$

68. (1) For real roots $D \ge 0$

$(k-2)^2 - 4(k^2 + 3k + 5) \ge 0$

$(k^2 + 4 - 4k) - 4k^2 - 12k - 20 \ge 0$

$-3k^2 - 16k - 16 \ge 0$; $3k^2 + 16k + 16 \le 0$

$\left(k + \frac{4}{3}\right)(k+4) \le 0$

Now $E = \alpha^2 + \beta^2$; $E = (\alpha+\beta)^2 - 2\alpha\beta$

$E = (k-2)^2 - 2(k^2 + 3k + 5) = -k^2 - 10k - 6$

$E = -(k^2 + 10k + 6) = -[(k+5)^2 - 19]$

$= 19 - (k+5)^2$

$\therefore$ E_{min} occurs when $k = -4/3$

$\therefore$ $E_{min} = 19 - \frac{121}{9} = \frac{171-121}{9} = \frac{50}{9}$

E_{max} occurs when $k = -4$

$E_{max} = 19 - 1 = 18$

69. (1) $\frac{20!}{p!q!r!}(2x)^p(-y)^q(z)^r = \frac{20!}{p!q!r!}2^p(-1)^q\, x^p y^q z^r$

$p + q + r = 20,\ q = 0$

$p + r = 20$ (p is even and r is odd).

even + odd = even (never possible)

Coefficient of such power never occur

$\therefore$ coefficient is zero

70. (1)

Starting with 1	1					23456789

$= {}^8C_4 = 70$

Starting with 2	2					3456789

$= {}^7C_4 = 35$

Total = 105

$(105)^{th}$ number 26789

71. (1) $(\hat{a}.\hat{c})\hat{b} - (\hat{a}.\hat{b})\hat{c} = \frac{1}{\sqrt{2}}\hat{b} + \frac{1}{\sqrt{2}}\hat{c}$

$\therefore$ $\hat{a}.\hat{c} = \frac{1}{\sqrt{2}}$ and $\hat{a}.\hat{b} = -\frac{1}{\sqrt{2}}$

$\Rightarrow$ angle between $\hat{a}$ and $\hat{c} = \frac{\pi}{4}$ and angle between $\hat{a}$ and $\hat{b} = \frac{3\pi}{4}$

72. (1) $|z_1+z_2| \le |z_1|+|z_2| = |24+7i|+6 = 25+6 = 31$

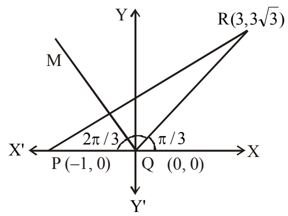
Also, $|z_1+z_2| = |z_1-(-z_2)| \ge ||z_1|-|z_2||$

$\Rightarrow |z_1+z_2| \ge |25-6| = 19$

Hence the least value of $|z_1 + z_2|$ is 19 and the greatest value is 31.

73. (3) The coordinates of points P, Q, R are $(-1, 0), (0, 0), (3, 3\sqrt{3})$ respectively.

Slope of QR = $\sqrt{3}$



$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \frac{\pi}{3}$

$\Rightarrow \angle RQX = \frac{\pi}{3}$

$\therefore \angle RQP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Let QM bisects the $\angle PQR$,

$\therefore$ Slope of the line QM = $\tan\frac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y-0) = -\sqrt{3}(x-0)$

$\Rightarrow y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

74. (1) Given $\int \sqrt{2}\sqrt{1+\sin x}\,dx = -4\cos(ax+b)+C$

$$\Rightarrow \int \sqrt{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)dx = -4\cos(ax+b)+C$$

$$\Rightarrow \int \sqrt{2}.\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\frac{x}{2}+\frac{1}{\sqrt{2}}\cos\frac{x}{2}\right)dx = -4\cos(ax+b)+C$$

$$\Rightarrow \int 2\left(\cos\frac{\pi}{4}\sin\frac{x}{2}+\sin\frac{\pi}{4}\cos\frac{x}{2}\right)dx = -4\cos(ax+b)+C$$

$$\Rightarrow \int 2\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = -4\cos(ax+b)+C$$

$$\Rightarrow -4\cos\left(\frac{x}{2}+\frac{\pi}{4}\right) = -4\cos(ax+b)+C$$

$$\Rightarrow a = \frac{1}{2},\ b = \frac{\pi}{4}$$

75. (1) $4x^3 + 4y^3\,(dy/dx) = 0 \Rightarrow dy/dx = -x^3/y^3$

Equation of tangent, $Y - y = -x^3/y^3\,(X-x)$

$\Rightarrow y^3Y + x^3X = x^4 + y^4 = a^4$

$$\Rightarrow \frac{X}{a^4/x^3} + \frac{Y}{a^4/y^3} = 1$$

Here, $p = a^4/x^3$, $q = a^4/y^3$

$$\Rightarrow p^{-4/3} + q^{-4/3} = \frac{a^{-16/3}}{x^{-4}} + \frac{a^{-16/3}}{y^{-4}} = a^{-16/3}\left(x^4+y^4\right)$$

$$= a^{-16/3}(a^4) = a^{-4/3}$$

76. (2) Let the equation of curve

$y^2(2a-x) = x^3$ ...(i)

and equation of line $x = 2a$ ...(ii)

The given curve is symmetrical about x-axis and passes through origin.

From (i) we have, $y^2 = \frac{x^3}{2a-x}$

But $\frac{x^3}{2a-x} < 0$ for $x > 2a$ and $x < 0$

So, curve does not lie in the portion $x > 2a$ and $x < 0$, therefore curve lies in $0 \le x \le 2a$.

$\therefore$ Area bounded by the curve and line

$$= \int_0^{2a} y\,dx = \int_0^{2a} \frac{x^{3/2}}{\sqrt{2a-x}}dx$$

Put $x = 2a\sin^2\theta$ and $dx = 4a\sin\theta\cos\theta\,d\theta$

$$\therefore\ I = \int_0^{\pi/2} 8a^2\sin^4\theta\,d\theta = 8a^2\left[\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}\right]$$

$$\left(\text{using} \int_0^{\pi/2} \sin^m x\cos^n x\,dx = \frac{\Gamma\frac{m+1}{2}\Gamma\frac{n+1}{2}}{2\Gamma\frac{m+n+2}{2}}\right)$$

$$= \frac{3\pi a^2}{2}\ \text{sq. unit}$$

77. (3) Rewrite the given differential equation as follows :

$\frac{dy}{dx} + \frac{2x}{x^2-1}y = \frac{1}{x^2-1}$, which is a linear form

The integrating factor I.F.

$$= e^{\int\frac{2x}{x^2-1}dx} = e^{\ell n(x^2-1)} = x^2-1$$

Thus multiplying the given equation by (x^2-1), we get $(x^2-1)\frac{dy}{dx}+2xy=1$

$$\Rightarrow \frac{d}{dx}[y(x^2-1)]=1$$

On integrating we get $y(x^2-1)=x+c$

78. (4) $f_2(x)=f(f(x))=f(x)=x$

$f_3(x)=f(f_2(x))=f(x)=x$

$\Rightarrow x^3-25x^2+175x-375=0$

$(x-5)(x^2-20x+75)=0$

$(x-5)^2(x-15)=0 \Rightarrow x=5, 15$

79. (3) Let $I_1=\int_0^{\sin^2 x}\sin^{-1}\sqrt{t}\,dt$

Put $t=\sin^2 u \Rightarrow dt=2\sin u\cos u\,du$

$\Rightarrow dt=\sin 2u\,du$

$\therefore I_1=\int_0^x u\sin 2u\,du$

Let $I_2=\int_0^{\cos^2 x}\cos^{-1}\sqrt{t}\,dt$

Put $t=\cos^2 v \Rightarrow dt=-2\cos v\sin v\,dv$

$\Rightarrow dt=-\sin 2v\,dv$

$\therefore I_2=\int_{\frac{\pi}{2}}^{x} v(-\sin 2v)dv=-\int_{\frac{\pi}{2}}^{x} v\sin 2v\,dv$

$=-\int_{\frac{\pi}{2}}^{x} u\sin 2u\,du$ [change of variable]

$\therefore I=I_1+I_2$

$=\int_0^x u\sin 2u\,du-\int_{\frac{\pi}{2}}^{x} u\sin 2u\,du$

$$=\int_0^{\frac{\pi}{2}} u\sin 2u\,du+\int_{\frac{\pi}{2}}^{x} u\sin 2u\,du-\int_{\frac{\pi}{2}}^{x} u\sin 2u\,du$$

$$=\int_0^{\frac{\pi}{2}} u\sin 2u\,du=\frac{\pi}{4} \quad \text{[Integrate by parts]}$$

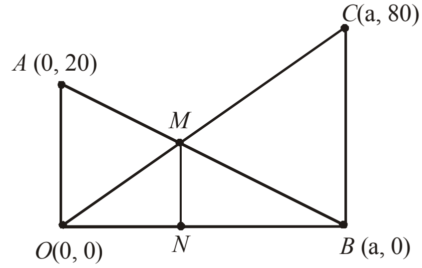
80. (2) For non-trivial solution,

$$\Delta=\begin{vmatrix}\sqrt{2}a & \sin B & \cos B\\ 1 & \cos B & \sin B\\ -1 & \sin B & -\cos B\end{vmatrix}=0$$

$$\Rightarrow a\sqrt{2}[-\cos^2 B-\sin^2 B]-\sin B[-\cos B+\sin B]+\cos B[\sin B+\cos B]=0$$

$$\Rightarrow -a\sqrt{2}+\sin 2B+\cos 2B=0 \Rightarrow a\in[-1,1]$$

81. (16.00)



We put one pole at origin.

$BC=80\,\text{m}, OA=20\,\text{m}$

Line OC and AB intersect at M.

<u>To find</u>: Length of MN.

Eqn of OC: $y=\left(\frac{80-0}{a-0}\right)x$

$\Rightarrow y=\frac{80}{a}x$ ...(1)

Eqn of AB: $y=\left(\frac{20-0}{0-a}\right)(x-a)$

$\Rightarrow y=\frac{-20}{a}(x-a)$ ...(2)

At M: (1) = (2)

$\Rightarrow \frac{80}{a}x=\frac{-20}{a}(x-a)$

$\Rightarrow \frac{80}{a}x=\frac{-20}{a}x+20 \Rightarrow x=\frac{a}{5}$

$\therefore \quad y=\frac{80}{a}\times\frac{a}{5}=16$

82. (3.00) $x+iy=\dfrac{3}{\cos\theta+i\sin\theta+2}$

$$\Rightarrow \frac{1}{x+iy}=\frac{\cos\theta+i\sin\theta+2}{3}$$

$$\Rightarrow \frac{x-iy}{(x+iy)(x-iy)}=\frac{1}{3}[(\cos\theta+2)+i\sin\theta]$$

$$\Rightarrow \frac{x}{x^2+y^2}=\frac{1}{3}(\cos\theta+2)$$

$$\Rightarrow \frac{x}{x^2+y^2}-\frac{2}{3}=\frac{1}{3}\cos\theta$$

and $-\dfrac{y}{x^2+y^2}=\dfrac{1}{3}\sin\theta$

Squaring and adding, we get

$$\left(\frac{x}{x^2+y^2}-\frac{2}{3}\right)^2+\left(\frac{-y}{x^2+y^2}\right)^2=\frac{1}{9}$$

$$\Rightarrow \frac{1}{x^2+y^2}(3-4x)+1=0$$

$$\Rightarrow 3-4x=-x^2-y^2$$

$$\Rightarrow 4x-x^2-y^2=3$$

83. (0.50) Let A be the event such that sum is Rs. 20 or more

$\therefore$ P(1) = 1 – P(Total value is < 20)

$$=1-\frac{{}^6C_2-{}^2C_2}{{}^8C_2}=1-\frac{14}{28}=1-\frac{1}{2}=\frac{1}{2}$$

8 → 1,1; 5,5; 10,10; 20,20

84. (2008.00) $\theta_1+\theta_2=\dfrac{\pi}{2}$

$$\therefore\ I=\int_{\theta_1}^{\theta_2}\frac{d\theta}{1+\tan\left(\frac{\pi}{2}-\theta\right)}=\int_{\theta_1}^{\theta_2}\frac{\tan\theta\,d\theta}{1+\tan\theta}$$

and also $I=\displaystyle\int_{\theta_1}^{\theta_2}\frac{d\theta}{1+\tan\theta}$

$$\therefore\ 2I=\int_{\theta_1}^{\theta_2}d\theta=\theta_2-\theta_1=\frac{1002\pi}{2008}\Rightarrow I=\frac{501\pi}{2008}$$

Hence, K = 2008.

85. (18.00) $\dfrac{{}^nC_k}{{}^nC_{k+1}}=\dfrac{1}{2}$

$$\Rightarrow \frac{n!}{k!(n-k)!}\cdot\frac{(k+1)!(n-k-1)!}{n!}=\frac{1}{2}$$

or $\dfrac{k+1}{n-k}=\dfrac{1}{2}$

$2k+2=n-k$

$n-3k=2$ ...... (1)

Similarly, $\dfrac{{}^nC_{k+1}}{{}^nC_{k+2}}=\dfrac{2}{3}$

$$\frac{n!}{(k+1)!(n-k-1)!}\cdot\frac{(k+2)!(n-k-2)!}{n!}=\frac{2}{3}$$

$\dfrac{k+2}{n-k-1}=\dfrac{2}{3}$

$3k+6=2n-2k-2$

$2n-5k=8$ ...... (2)

From (1) and (2)

$n=14$ and $k=4$

$\therefore\ n+k=18$

86. (12.00) Let the observations be x_1, x_2, x_3, x_4, x_5 and x_6, so

their mean $\bar{x}=\dfrac{\sum_{i=1}^{6}x_i}{6}=8\Rightarrow\sum_{i=1}^{6}x_i=48$

On multiplying each observation by 3, we get the new observations as $3x_1, 3x_2, 3x_3, 3x_4, 3x_5$ and $3x_6$.

Now, their mean

$$=\bar{x}=\frac{\sum_{i=1}^{6}3x_i}{6}=\frac{3\times 48}{6}=24$$

Variance of new observations

$$=\frac{\sum_{i=1}^{6}(3x_i-24)^2}{6}=\frac{3^2\sum_{i=1}^{6}(x_i-8)^2}{6}$$

= 9 × Variance of old observations

$=9\times 4^2=144$

Thus, standard deviation of new observations

$=\sqrt{\text{Variance}}=\sqrt{144}=12$

87. (0.75) $\lim_{x\to 0} \frac{(\tan x)^{3/2}[1-(\cos x)^{3/2}]}{x^{3/2}.x^2}$.

$= 1 \times \lim_{x\to 0} \frac{1-\cos^3 x}{x^2} . \frac{1}{1+(\cos x)^{3/2}}$

$= \frac{1}{2}.\frac{1}{2}(1+\cos x+\cos^2 x) = \frac{3}{4} = 0.25$

88. (0.38) n = 3, P (success) = P (HT or TH) = 1/2

$\Rightarrow \quad p = q = \frac{1}{2}$ and $r = 2$

$P(r=2) = {}^3C_2 \left(\frac{1}{2}\right)^2 . \frac{1}{2} = \frac{3}{8} = 0.38$

89. (336.00) Let $S = \{1, 2, 3\} \Rightarrow n(S) = 3$

Now, $P(S)$ = set of all subsets of S

total no. of subsets = $2^3 = 8$

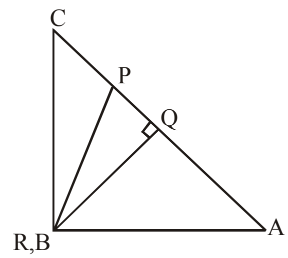
$\therefore n[P(S)] = 8$

Now, number of one-to-one functions from $S \to$

$P(S)$ is ${}^8P_3 = \frac{8!}{5!} = 8 \times 7 \times 6 = 336$.

90. (9.00) $2\sec 4C + \sin^2 2A + \sqrt{\sin B} = 0$

$A = 45°, B = 90°$ and $C = 45°$



Let AQ = a, then $BP = \frac{a}{2}$,

$PQ = \frac{a}{2}$ and QR = a

$\therefore PR = \sqrt{a^2 + \frac{a^2}{4}} = \frac{\sqrt{5}a}{2}$

$\therefore 1 : \alpha : \beta = \frac{a}{2} : a : \frac{\sqrt{5}a}{2} = 1 : 2 : \sqrt{5}$

$\therefore \alpha = 2$ and $\beta = \sqrt{5} \therefore \alpha^2 + \beta^2 = 9$