Chapter 13

Surface Areas and Volumes

Exercise 13.3

Q. 1 A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder

Answer:

To find: Height of the cylinder (h)

Given:

Radius (r_1) of sphere = 4.2 cm

Radius (r_2) of cylinder = 6 cm

Let the height of the cylinder be h.

The object formed by recasting the sphere will be the same in volume.

Volume of sphere = Volume of cylinder

$= \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$

$= \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2 = \frac{22}{7} \times 6 \times 6 \times h$

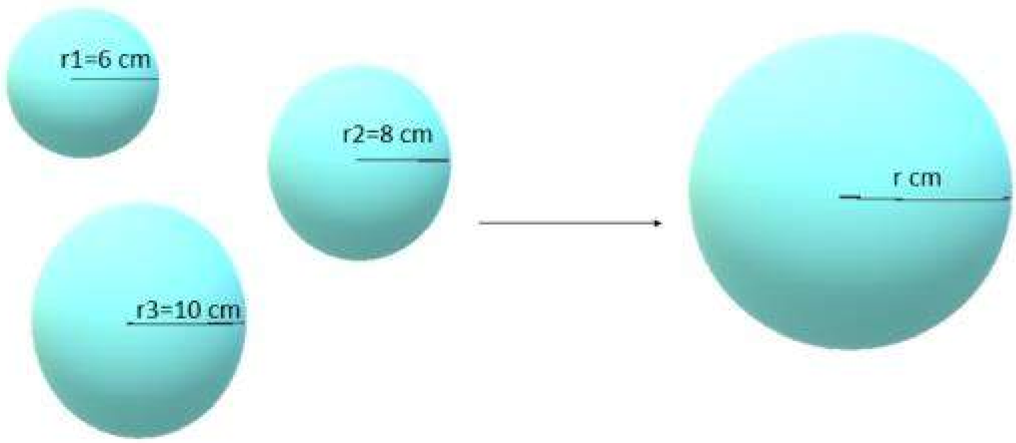
$h = \frac{4\times4.2\times4.2\times4.2}{3\times6\times6}$

h = 2.74 cm

Hence, the height of the cylinder = 2.74 cm

Q. 2 Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Answer:



Radius (r_1) of 1st sphere = 6 cm

Radius (r_2) of 2nd sphere = 8 cm

Radius (r_3) of 3rd sphere = 10 cm

Let the radius of the resulting sphere be r.

The object formed by recasting these spheres will be the same in volume as the sum of the volumes of these spheres.

The volume of 3 spheres = Volume of resulting sphere

$\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi r^3$

$\frac{4}{3}\pi\ [r_1^3 + r_2^3 + r_3^3] = \frac{4}{3}\pi r^3$

$\frac{4}{3}\pi\ [6^3 + 83 + 103] = \frac{4}{3}\pi\ r^3$

$r^3 = 216 + 512 + 1000$

$r^3 = 1728$

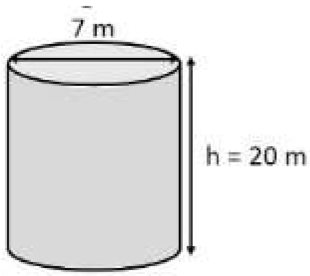
r = 12 cm

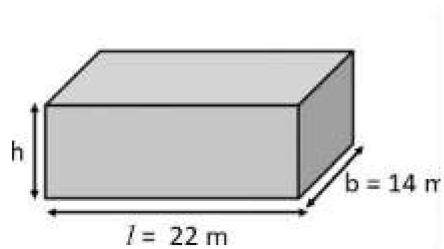
Hence, the radius of the resulting sphere, r = 12 cm.

Q. 3 A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Answer:

The diagram for the situation is as follows:





As per the question,

The shape of the well will be cylindrical

Depth (h) of well = 20 m

Radius (r) of circular end of well = $\frac{7}{2}$

Area of platform = Length × Breadth = 22 × 14 m^2

Let height of the platform = H

Volume of soil dug from the well would be equal to the volume of soil that is scattered on the platform.

Volume of soil from well = Volume of soil used to make such platform

$\pi r^2 h$ = Area × Height

$\pi \times \left(\frac{7}{2}\right)^2 \times 20 = 22 \times 14 \times H$

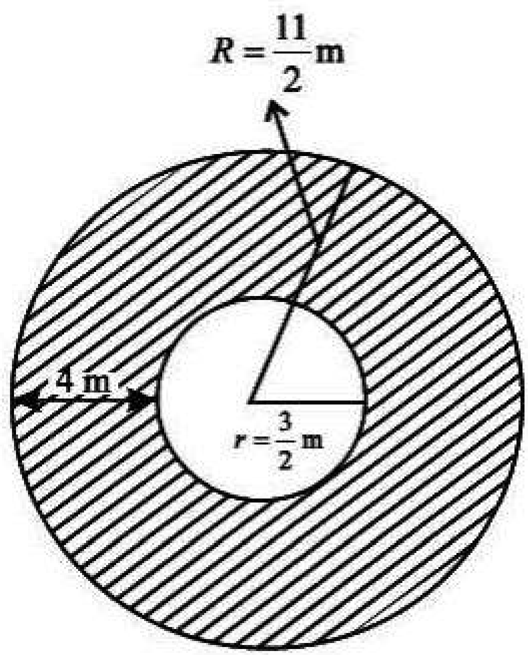
$H = \frac{22}{7} \times \frac{49}{4} \times \frac{20}{22*14}$

= 2.5 m

Hence, the height of the platform = 2.5 m

Q. 4 A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Answer:



The shape of the well will be cylindrical.

Depth (h_1) of well = 14 m

Radius (r) of the circular end of well = 3/2 m

Width of embankment = 4 m

As per the question,

Our embankment will be in a cylindrical shape having outer radius (R)as 11/2 m and inner radius (r) = 3/2 m

Let the height of embankment be h_2

Volume of soil dug from well = Volume of earth used to form embankment

$\pi r^2 \times h_1 = \pi \times (R^2 - r^2) \times h_2$

$= \pi \times \left(\frac{3}{2}\right)^2 \times 14 = \pi \times \left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] \times \text{h}$

$= \pi \times \left(\frac{3}{2}\right)^2 \times 14 = \pi \times \left[\frac{121}{4} - \frac{9}{4}\right] \times \text{h}$

$= \pi \times \frac{9}{4} \times 14 = \pi \times \left[\frac{112}{4}\right] \times$

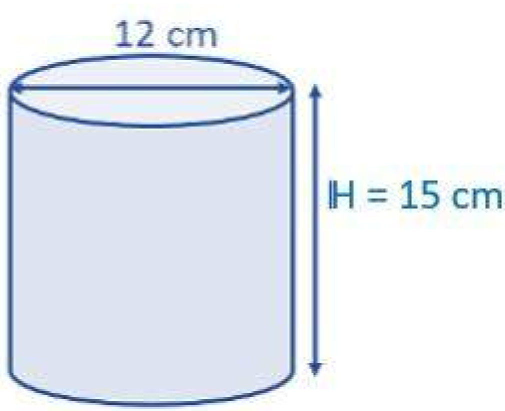
$\Rightarrow 31.5 = 28\ \text{h}$

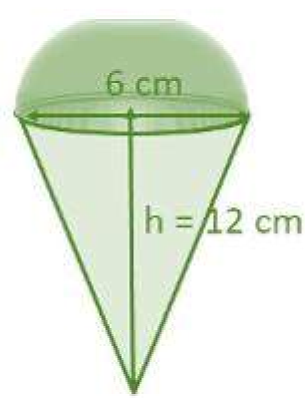
$= \frac{31.5}{28} = \text{h}$

$\Rightarrow \text{h} = 1.125\ \text{m}$

Q. 5 A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Answer:





Height (H) of cylindrical container = 15 cm

Radius (r_1) of circular end of container = $\frac{12}{2}$ = 6 cm

Radius (r_2) of circular end of ice-cream cone = $\frac{6}{2}$ = 3 cm

Height (h) of conical part of ice-cream cone = 12 cm

Let n cones be filled with ice-cream of the container

Volume of ice-cream in cylinder = n × (Volume of 1 ice-cream cone + Volume of hemispherical shape on the top)

$$\pi \times (r_1)^2 \times H = n\left[\left(\frac{1}{3}\pi \times (r_2)^2 \times h\right) + \left(\frac{2}{3}\pi \times (r_2)^2\right)\right]$$

n = $\frac{\pi\times(r_1)^2\times H}{\pi\left(\frac{1}{3}\times(r_2)^2\times h\right)+\left(\frac{2}{3}\times(r_2)^2\right)}$

n = $\frac{(r_1)^2\times H}{\left(\frac{1}{3}\times(r_2)^2\times h\right)+\left(\frac{2}{3}\times(r_2)^2\right)}$

= n = $\frac{6\times6\times15}{\left(\frac{1}{3}\times9\times12\right)+\left(\frac{2}{3}\times3\times3\times3\right)}$

= n = $\frac{540}{36+18} = \frac{540}{54}$

= n = 10

Q. 6 How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Answer:

Coins are cylindrical in shape.

Height (h_1) of cylindrical coins = 2 mm = 0.2 cm

Radius (r) of circular end of coins = $\frac{1.75}{2}$ = 0.875 cm

Let n coins be melted to form the required cuboids

Volume of n coins = Volume of cuboids

$n \times \pi \times r^2 \times h_1 = l \times b \times h$

$n \times \pi \times (0.875)^2 \times 0.2 = 5.5 \times 10 \times 3.5$

$n = \frac{5.5*1.*3.5*7}{0.875*0.875*0.2*22}$

= 400

Hence,

The number of coins melted to form the given cuboid is 400.

Q. 7 A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap

Answer:

Given:

Height (h_1) of cylindrical bucket = 32 cm

Radius (r_1) of circular end of bucket = 18 cm

Height (h_2) of conical heap = 24 cm

Let the radius of the circular end of conical heap be r_2

The volume of sand in the cylindrical bucket will be equal to the volume of sand in the conical heap

Volume of sand in the cylindrical bucket = Volume of sand in conical heap

$\pi\, r_1^2 \times h_1 = \frac{1}{3}\pi \times r_2^2 \times h_2$

$\pi \times (18)^2 \times 32 = \frac{1}{3} \times \frac{22}{7} \times r_2^2 \times 24$

$r_2^2 = \frac{18 \times 18 \times 32 \times 3}{24}$

$r_2 = \sqrt{18 \times 18 \times 4}$

= 36 cm

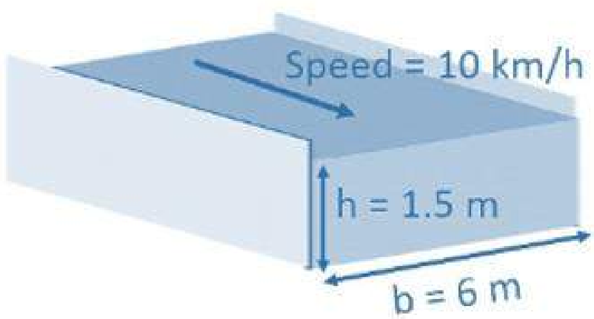
If h is the height and r is the radius of cone, then slant height is given by

$l = \sqrt{h^2 + r^2}$

$l = \sqrt{36^2 + 24^2}$

$l = 12\sqrt{13}$ cm

Q. 8 Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?



Area of cross-section = 6 × 1.5 = 9 m^2

Speed of water = 10 km/h

Water flows through canal in 60 min = 10 km

Water flows through canal in 1 min = $\frac{1}{60} \times 10$ km

Water flows through canal in 30 min = $\frac{30}{60} \times 10$

= 5 km = 5000 m

Hence length of the canal is 5000 m.

The volume of canal = L × B × H= (5000 × 6 × 1.5) = 45000 m^3

Let the irrigated area be A.

Now, we know that,

The volume of water irrigating the required area will be equal to the volume of water that flowed in 30 minutes from the canal.

Vol. of water flowing in 30 minutes from canal = Vol. of water irrigating the reqd. area ⇒ 45000 = Area × height

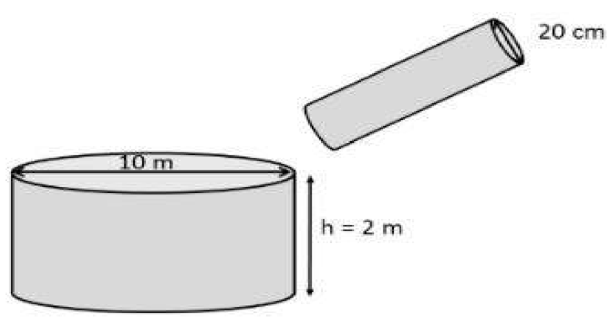
Given height = 8 cm = 0.08 m⇒ 45000 = Area × 0.08

= area = $\frac{45000}{0.08}$

Area = 562500 m^2

Q. 9 A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Answer:



As we know that 1m = 100cmTherefore, 1cm = $\frac{1}{100}$ m

20 cm = $\frac{20}{200}$ m

Radius (r1) of circular end of pipe = $\frac{20}{200}$ = 0.1 m

Area of cross-section =$\pi \times r_1^2$

$= \pi \times (0.1)^2$

$= 0.01\ \pi\ m^2$

Speed of water = 3 km/h

$= \frac{3000}{60}$

$= \frac{300}{6}$

= 50 meter/min

The volume of water that flows in 1 minute from pipe = $50 \times 0.01\ \pi$

$= 0.5\pi\ m^3$

The volume of water that flows in t minutes from pipe = $t \times 0.5\pi\ m^3$

Radius (r_2) of circular end of cylindrical tank = $\frac{10}{2}$ = 5 m

Depth (h_2) of cylindrical tank = 2 m

Let the tank be filled completely in t minutes

The volume of water filled in the tank in t minutes is equal to the volume of water flowed in t minutes from the pipe

The volume of water that flows in t minutes from pipe = Volume of water in the tank

$t\times 0.5\pi = \pi \times (r_2)^2 \times h_2$

$t\times 0.5 = (5)^2 \times 2$

t = 100

Hence, the cylindrical tank will be filled in 100 minutes.