CHAPTER 19

Magnetic Effects of Current & Magnetism

Force on a Charged Particle in a Uniform Magnetic Field

- **Magnetic force on a moving charge**

$$\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B})$$

or $$F_m = Bqv\sin\theta$$

- Path of charged particle in a uniform magnetic field
  - (i) If $\theta = 0°$ or $180°$, path is straight line.
  - (ii) If $\theta = 90°$, path is circle.
  - (iii) For any other angle path is helix.

Note *In the above formulae, θ is the angle between* **v** *and* **B**.

- **List of formulae in circular path**
  - (i) $r = \dfrac{mv}{Bq} = \dfrac{p}{Bq} = \dfrac{\sqrt{2Km}}{Bq} = \dfrac{\sqrt{2qVm}}{Bq}$
  - (ii) $T = \dfrac{2\pi m}{Bq}$; $\omega = \dfrac{Bq}{m}$; $f = \dfrac{Bq}{2\pi m}$
- **List of formulae in helical path**
  - (i) $r = \dfrac{mv\sin\theta}{Bq}$
  - (ii) $T = \dfrac{2\pi m}{Bq}$; $f = \dfrac{Bq}{2\pi m}$
  - (iii) Pitch of helical path, $p = (v\cos\theta)\,T = \dfrac{2\pi mv\cos\theta}{Bq}$
- Path of a charged particle in uniform electric and magnetic field will remain unchanged, if

$$\mathbf{F}_{\text{net}} = 0$$

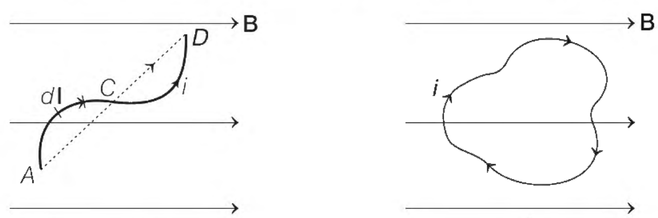
or $$\mathbf{F}_e + \mathbf{F}_m = 0$$

or $$q\mathbf{E} + q\,(\mathbf{v} \times \mathbf{B}) = 0$$

or $$\mathbf{E} = -(\mathbf{v} \times \mathbf{B}) = (\mathbf{B} \times \mathbf{v})$$

Force on Current Carrying Conductor

- Magnetic force on a straight current carrying conductor
$$\mathbf{F}_m = i\,(\mathbf{l} \times \mathbf{B}) \quad \text{or} \quad F_m = ilB\sin\theta$$
Here, θ is the angle between $\mathbf{l}$ and $\mathbf{B}$ or between direction of current and magnetic field.
- For the magnetic force on an arbitrarily shaped wire segment, let us consider the magnetic force exerted on a small segment of vector length $d\mathbf{l}$.



(a) (b)

$$\mathbf{F}_m = i\int_A^D (d\mathbf{l} \times \mathbf{B})$$

$\therefore$ $$\mathbf{F}_m = i\left(\int_A^D d\mathbf{l}\right) \times \mathbf{B}$$

But the quantity $\int_A^D d\mathbf{l}$ represents the vector sum of all length elements from A to D. From the polygon law of vector addition, the sum equals the vector $\mathbf{l}$ directed from A to D. Thus,
we can write $\mathbf{F}_{ACD} = \mathbf{F}_{AD} = i\,(\mathbf{AD} \times \mathbf{B})$ in uniform field.
Using the above result ,we can see that net megnetic force in current carrying loop in uniform magnetic field is zero.

- Force per unit length between two parallel current carrying wires,
$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$$
There will be attraction between the wires, if currents are in the same direction and repulsion, if currents are in opposite directions.

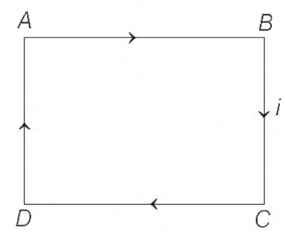
Magnetic Dipole

- Every current carrying loop is a magnetic dipole.
- Magnetic dipole moment of magnetic dipole is given by
$$M = NiA$$
- Direction of $\mathbf{M}$ is perpendicular to the plane of the loop and given by right hand screw law.

In addition to the method discussed above for finding $\mathbf{M}$, here are two more methods for calculating $\mathbf{M}$.

Method 1 This method is useful for calculating **M** for a rectangular or square loop.

The magnetic moment (**M**) of the rectangular loop shown in figure is

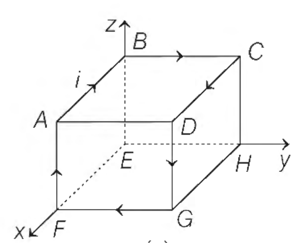


$$\mathbf{M} = i\,(\mathbf{AB} \times \mathbf{BC}) = i\,(\mathbf{BC} \times \mathbf{CD}) = i\,(\mathbf{CD} \times \mathbf{DA}) = i\,(\mathbf{DA} \times \mathbf{AB})$$

Here, the cross product of any two consecutive sides (taken in order) gives the area as well as the correct direction of **M**.

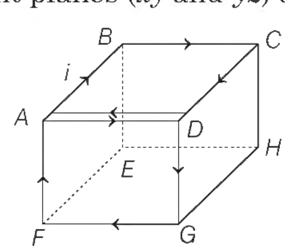
Method 2 Sometimes, a current carrying loop does not lie in a single plane. By assuming two equal and opposite currents in one branch (which obviously makes no change in the given circuit) two (or more) closed loops are completed in different planes.

Now, the net magnetic moment of the given loop is the vector sum of individual loops.



(a)

For example, in Fig. (a), six sides of a cube of length l carry a current i in the directions shown. By assuming two equal and opposite currents in wire AD, two loops in two different planes (xy and yz) are completed.



(b)

$$\mathbf{M}_{ABCDA} = -il^2\,\hat{\mathbf{k}}$$

$$\mathbf{M}_{ADGFA} = -il^2\,\hat{\mathbf{i}}$$

$$\therefore \quad \mathbf{M}_{\text{net}} = -il^2\,(\hat{\mathbf{i}} + \hat{\mathbf{k}})$$

- $\dfrac{M}{L} = \dfrac{q}{2m}$ for any system of charges in rotation.

Magnetic Dipole in Uniform Magnetic Field

- $\mathbf{F} = 0$
- $\tau = \mathbf{M} \times \mathbf{B}$ or $\tau = MB\sin\theta$
- $U = -\mathbf{M} \cdot \mathbf{B} = -MB\cos\theta$
- $W_{\theta_1 \to \theta_2} = U_{\theta_2} - U_{\theta_1} = MB(\cos\theta_1 - \cos\theta_2)$
- $\theta = 0°$ is stable equilibrium position of the loop. In this condition,
  $F = 0, \tau = 0$ and $U = -MB =$ minimum
- $\theta = 180°$ is unstable equilibrium position of the loop. Under this condition,
  $F = 0, \tau = 0$ and $U = +MB =$ maximum
- Every current carrying loop is like a bar magnet (or a magnetic dipole).
- Magnetic field due to a bar magnet at a point lying on its axis is
  $$B = \frac{\mu_0}{4\pi}\frac{2M}{r^3} \quad \text{(Along } \mathbf{M}\text{)}$$
  where, $r >>$ size of bar magnet
- Magnetic field due to a bar magnet at a point on equatorial line is
  $$B = \frac{\mu_0}{4\pi}\frac{M}{r^3} \quad \text{(Opposite to } \mathbf{M}\text{)}$$
  Again, $r >>$ size of bar magnet

Biot-Savart Law

- $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i(d\mathbf{l} \times \mathbf{r})}{r^3}$ or $dB = \frac{\mu_0}{4\pi}\frac{i(dl\sin\theta)}{r^2}$
- SI unit of magnetic field is tesla (T) or Wb/m^2.
  $1\text{T} = 1\text{ Wb/m}^2 = 10^4$ gauss.

Applications of Biot-Savart Law

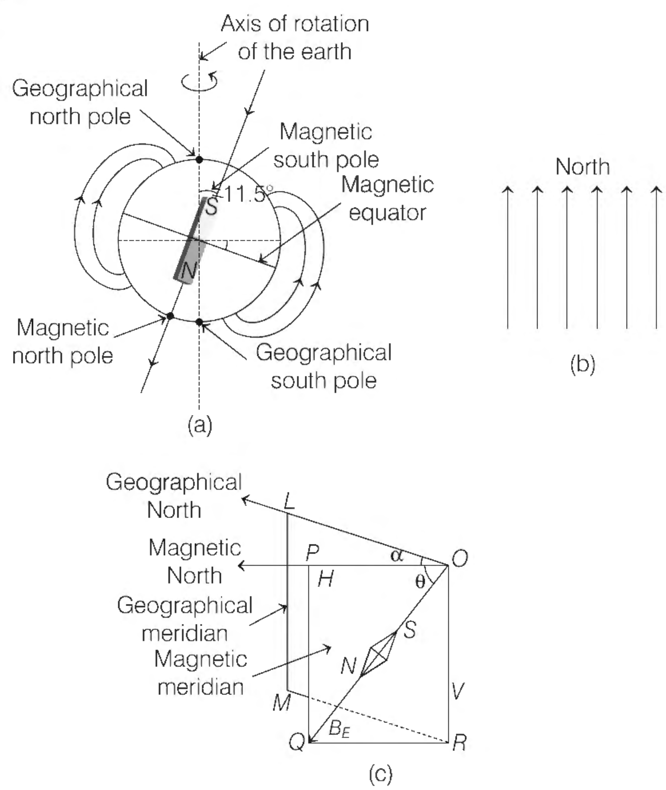
- Magnetic field due to a straight wire of finite length,
  $$B = \frac{\mu_0}{4\pi}\frac{i}{r}(\sin\alpha \pm \sin\beta)$$
- Magnetic field due to a straight wire of infinite length,
  $$B = \frac{\mu_0}{2\pi}\frac{i}{r}$$
- Magnetic field on the axis of a circular loop
  $$B = \frac{\mu_0 NiR^2}{2(R^2 + x^2)^{3/2}}$$
- Magnetic field at the centre of a circular loop
  $$B = \frac{\mu_0 Ni}{2R}$$
- Magnetic field at centre due to arc of circle
  $$B = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 Ni}{2R}\right)$$

- Magnetic field on the axis of a solenoid,
$$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$$
- Magnetic field at centre (on the axis) of a long solenoid,
$$B = \mu_0 ni$$
- Magnetic field at ends of a long solenoid,
$$B = \frac{\mu_0 ni}{2}$$

Ampere's Circuital Law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (i_{\text{net}})$$

Earth's Magnetism



- The value of magnetic field on the surface of earth is of the order of 10^{-5} T.
- The axis of earth makes an angle of approximately $11.5°$ with the earth's rotational axis.
- At any point, the vertical plane passing through the line joining the geographical north and south pole is called the geographical meridian.
- At any point, a vertical plane in the direction of earth's magnetic field is called magnetic meridian.
- At any place, the acute angle between the magnetic meridian and geographical meridian is called angle of declination α.

- The angle of dip(θ) at any place is the angle between the direction of earth's magnetic field and the horizontal.

  Magnetic dip will point downward in northern hemisphere (positive dip) and upward in southern hemisphere (negative dip). The range of dip angle is from $-90°$ (at the south magnetic pole) to $+90°$ (at the north magnetic pole).
- $H = B_e \cos\theta,\ V = B_e \sin\theta,\ B_e = \sqrt{H^2 + V^2}$ and $\theta = \tan^{-1}\left(\frac{V}{H}\right)$

  Here, B_e = total earth's magnetic field at some point.

  H = horizontal component of earth's magnetic field at that point

  and V = vertical component of earth's magnetic field at that point.

Magnetic Substances

- Magnetic field inside a solenoid,

$$B_0 = \mu_0 ni = \mu_0 H$$

  Here, $ni = H$ = magnetic intensity or magnetic field strength (A/m)
- Now if a dia, para or ferromagnetic substance is kept inside the solenoid and let B be the magnetic field inside this substance, then

  B may be $> B_0 \longrightarrow$ in paramagnetic substance

  $>> B_0 \longrightarrow$ in ferromagnetic substance

  $< B_0 \longrightarrow$ in diamagnetic substance

  Thus, $\mathbf{B} = \mathbf{B}_0 + \mathbf{A}$ ($\mathbf{A}$ is a vector quantity)

  Here, A is $\mu_0 I$ or $\mu_0 M$.

  or $\mathbf{B} = \mathbf{B}_0 + \mu_0 \mathbf{M}$

  where, I or M is called intensity of magnetisation.

$$I \text{ or } M = \frac{\text{magnetic moment}}{\text{volume}}$$

  I or M depends upon H and a constant χ (called susceptibility).

  So, I or $M = \chi H$
- Thus, $B = B_0 + \mu_0 M = \mu_0 H + \mu_0 \chi H$

  $= \mu_0 H (1 + \chi) = \mu_0 H \mu_r$

  or $B = \mu H$
- $\mu_0 \mu_r = \mu$ = permeability of that substance or

  $\mu_r = \frac{\mu}{\mu_0}$ = relative permeability of that substance
- $B = \mu H$ and $B_0 = \mu_0 H$

  Thus, $\frac{B}{B_0} = \frac{\mu}{\mu_0} = \mu_r = (1 + \chi)$

  or $\chi = \mu_r - 1$
- For diamagnetic substance, $B < B_0$

  $\therefore$ $\mu_r < 1$ or χ is slightly negative.

For paramagnetic substance, $B > B_0$

$\therefore \qquad \mu_r > 1$ or χ is slightly positive.

and for ferromagnetic substance, $B >> B_0$

$\mu_r >>$ or χ is highly positive.

- **Curie's law** Magnetic susceptibility of a paramagnetic substance

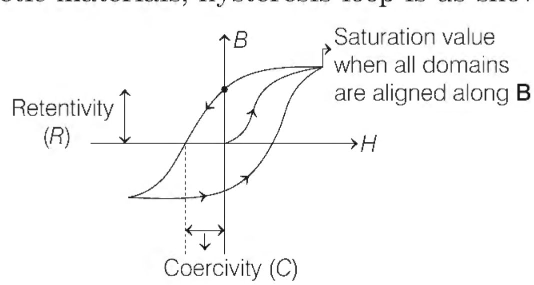
$$\chi_m \propto \frac{1}{T}$$

Difference between Dia, Para and Ferromagnetic Substances

Properties	Ferromagnetic materials	Paramagnetic materials	Diamagnetic materials
State	They are solid.	They can be solid, liquid or gas.	They can be solid, liquid or gas.
Effect of magnet	Strongly attracted by a magnet.	Weakly attracted by a magnet.	Weakly repelled by a magnet.
Behaviour under non-uniform field	tend to move from low to high magnetic field region.	tend to move from low to high magnetic field region.	tend to move from high to low magnetic field region.
Behaviour under external field	They preserve the magnetic properties after the external field is removed.	They do not preserve the magnetic properties once the external field is removed.	They do not preserve the magnetic properties once the external field is removed.
Effect of temperature	Above Curie point, it becomes a paramagnetic.	With the rise of temperature, it becomes a diamagnetic.	No effect.
Permeability	Very high.	Slightly greater than unity.	Slightly less than unity.
Susceptibility	Very high and positive.	Slightly greater than unity and positive.	Slightly less than unity and negative.
Examples	Iron, Nickel, Cobalt	Lithium and Tantalum	Copper, Silver and Gold

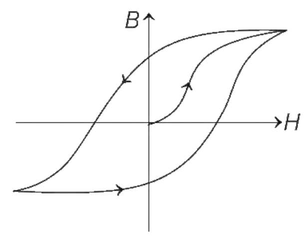
Hysteresis Loop

- For ferromagnetic materials, hysteresis loop is as shown below.

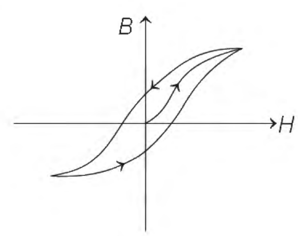


- B is the magnetic field inside the ferromagnetic substance.
- Due to change in direction of H, domains keep on changing their directions. Due to friction, heat is produced.
- Area (A) of the loop is a measure of loss of energy.

- $R_{SI} > R_S \Rightarrow C_{SI} < C_S \Rightarrow A_{SI} < A_S$
  Here, SI is soft iron and S is steel.
- **Permanent magnets** These required high retentivity and coercivity. Hysteresis is immaterial, as this is never put to cyclic changes. So, steel is best suited for it.



- **Electromagnets** These are temporary magnets. They require high initial permeability and low hysteresis loss. These are used in transformer cores. Soft iron is best suited for it.



Cyclotron

- $r = \frac{mv}{Bq} \Rightarrow v = \frac{Bqr}{m}$
- $t = \pi r / v = \frac{\pi m}{Bq}$
- $f = \frac{1}{T} = \frac{1}{2t} = \frac{Bq}{2\pi m}$
- $K = \frac{1}{2} mv^2 = \frac{1}{2} m\,(BqR/m)^2$

  $\Rightarrow \quad K = B^2R^2q^2/2m$

Moving Coil Galvanometer

- $$k\phi = NiAB$$
  $$\Rightarrow \quad i = \left(\frac{k}{NAB}\right)\phi$$
- Galvanometer constant $= \frac{k}{NAB}$
- Sensitivity $= \frac{\phi}{i} = \frac{NAB}{k}$