7. Ratio, Mixture & Partnership

7.1 Ratio & Proportion

A **Ratio** is the relation between two quantities of the same kind. This relation indicates how many times one quantity is equal to the other. In other words, ratio is a number, which expresses one quantity as a fraction of the other.
Example: Ratio of 12 to 13 is 12/13 or 12 : 13.

The numbers forming the ratio are called terms. The numerator, i.e. '12', is known as

the antecedent and the denominator, i.e. ‘13’, in this case, is known as the consequent.

The ratio between two quantities a and b if expressed as a/b, is called fractional form and, a : b is called linear form.

If two different ratios, a : b and c : d are expressed in different units, then the two are compounded to obtain a combined ratio. Compounding of a : b and c : d yields a × c / b × d .

- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then each of these ratios is equal to $\frac{a+c+e}{b+d+f}$
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$ (Invertendo)

- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$ (Alterendo)
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo dividendo)

Continued Proportion: Four numbers a, b, c and d are said to be in proportion if a : b = c : d. If on the other hand, a : b = b : c = c : d, then the four numbers are said to be in continued proportion.

Let us consider the ratios, a : b = b : c. Here b is called the mean proportional and is equal to the square root of the product of a and c i.e. $b^2 = a \times c \Rightarrow b = \sqrt{ac}$

If a/b = b/c = c/d etc., then a, b, c, d are in geometric progression.
Let a/b = b/c = c/d = k, then, c = dk; b = ck and a = bk

Since c = dk, b = dk $\times$ k = dk^2 and a = bk = dk^2

$\times k = dk^3$, implying that they are in a geometric progression.

If the three ratios, a : b, b : c, c : d are known, we can find a : d by multiplying these three ratios a/d = a/b × b/c × c/d

If a, b, c and d are four terms and the ratios a : b, b : c, c : d are known, then one can find the ratio a : b : c : d.

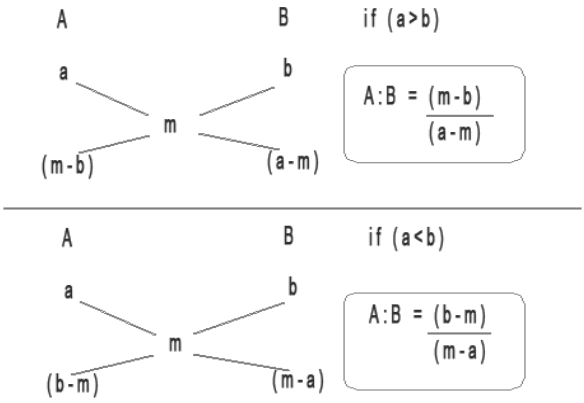
Variation - Direct, Inverse

(i) If a is directly proportional to b, then the ratio of a and b is constant.
(ii) If a is inversely proportional to b, then the product of a and b is constant.
(iii) If a α b and b α c, then a α c.
(iv) If a α b and a α c, then a α (b $\pm$ c).
(v) If a α b and c α d, then ac α bd.

7.2 Alligation & Mixture

In Mixture problems, different substances are combined and the characteristics of the resulting mixture have to be determined.

In solving mixture problems, we use the Alligation Rule (Alligation means 'linking').The rule states, that "When different quantities of different ingredients are mixed together to produce a mixture of a mean value, the ratio of their quantities is inversely proportional to the differences in their cost from the mean value."



One case in mixtures is the repeated dilution of a mixture with one of the ingredients, by removing, say n litres of the mixture and replacing it with n litres of one of the ingredients. Say there are m litres of water initially. We now remove n litres of the water and replace it by n litres of wine. This operation is done t times. Then,

Quantity of water left in the vessel = $m(1 - n/m)^t$

Where

m = Total quantity

n = Quantity drawn every time

t = No. of times

7.3 Partnership

Partnership is an association of two or more persons who invest money together in order to carry out a certain business. Partnerships are of two types:

(i) Simple Partnership: When all partners invest in the business at the same time i.e. their capital remains in the business for the same duration it is called simple partnership. In this kind of partnership, the profit is simply distributed amongst the

partners, in the ratio of their respective invested capital.

(ii) Compound Partnership: When capital of the partners is invested in the business for different time periods, the partnership is known as compound partnership. In this, the profit sharing ratio is calculated by multiplying the capital invested with the unit of time (mostly months).

The sharing of profit and loss can be better understood with the help of the following illustrations:

Rule 1: In a simple partnership, the loss or profit is distributed amongst the partners in the ratio of their respective investments.

Example: Say, P and Q invested Rs. a and b for one year in a business. Then, the share of profit or loss will be,
P's profit/ loss : Q's profit/ loss = a : b

Rule 2: In a compound partnership, the profit or loss ratio is calculated as capital multiplied by the duration of investment.

Example: P's profit/loss : Q's profit/loss = $a \times t_1 : b \times t_2$
Where, t_1 = P's duration of investment and, t_2 = Q's duration of investment