# Chapter 3

# Trigonometric Functions-I

## Solutions (Set-1)

**Very Short Answer Type Questions :**

1. A circular wire of radius 3 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 48 cm. Find the angle, in degrees, which is subtended at the centre of the hoop.

**Sol.** Given that the circular wire is of radius 3 cm, so when it is cut then its length = $2\pi \times 3 = 6\pi$ cm. Again, it is being placed along a circular hoop of radius 48 cm. Here, *s i.e.* the length of arc = $6\pi$ cm and $r$ = 48 cm is the Madius of the circular hoop. Therefore, the angle $\theta$, in radian, subtended by the arc at the centre of the circle is given by $\theta = \dfrac{\text{Arc}}{\text{Radius}} = \dfrac{6\pi}{48} = \dfrac{\pi}{8} = 22.5°$.

2. Find the value of $\cos2\theta\cos2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$.

**Sol.** $\cos2\theta\cos2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$= \cos2\theta \cos2\phi + \sin(\theta - \phi + \theta + \phi) \sin(\theta - \phi - \theta - \phi)$

$= \cos2\theta \cos2\phi - \sin2\theta \sin2\phi$

$= \cos(2\theta + 2\phi)$

$= \cos[2(\theta + \phi)]$

3. Find the angle, in degrees, subtended at the centre of a circle by an arc whose length is 2.2 times the radius.

**Sol.** Let the radius be $r$ and length be $l$.

$\therefore \quad l = 2.2\, r$

now, $l = r\theta$

$\theta = \dfrac{l}{r} = \dfrac{2.2r}{r} = 2.2$ radians

$\theta = \left(2.2 \times \dfrac{180}{\pi}\right)^\circ = \left(2.2 \times \dfrac{180}{22} \times 7\right)$

$\theta = 126°$.

4. A wheel makes 270 revolutions in 1 minute, through how many radians does it turn in 1 second?

**Sol.** Angle subtendend in 1 revolution = $2\pi$

$\therefore$ Amount of rotation in 270 revolutions = $270 \times 2\pi$

$\Rightarrow$ Amount of rotation in minute = $270 \times 2\pi$

$\therefore$ Amount of rotation in 1 sec = $\dfrac{270 \times 2\pi}{60} = 9\pi$

5. Prove that $\cos^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)=\frac{1}{\sqrt{2}}\cos A.$

**Sol.** LHS = $\cos^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)$

$= \cos\left(\frac{\pi}{8}+\frac{A}{2}+\frac{\pi}{8}-\frac{A}{2}\right)\cos\left(\frac{\pi}{8}+\frac{A}{2}-\frac{\pi}{8}+\frac{A}{2}\right)$

$= \cos\left(\frac{\pi}{4}\right)\cos A$

$= \frac{1}{\sqrt{2}}\cos A = \text{RHS}$

Hence proved.

6. Find the value of $\cos 210^\circ + \sin\frac{5\pi}{3}$.

**Sol.** $\cos 210^\circ + \sin\frac{5\pi}{3}$

$= \cos(180^\circ + 30^\circ) + \sin\left(2\pi - \frac{\pi}{3}\right)$

$= -\cos 30^\circ + \left(-\sin\frac{\pi}{3}\right)$

$= \frac{-\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$

$= -\sqrt{3}$

7. Prove that $\sin 75^\circ \cos 15^\circ = \frac{2+\sqrt{3}}{4}$.

**Sol.** LHS = $\sin 75^\circ \cos 15^\circ$

$7 = \frac{1}{2}(2\sin 75^\circ \cos 15^\circ)$

$= \frac{1}{2}[\sin(75^\circ + 15^\circ) + \sin(75^\circ - 15^\circ)]$

$= \frac{1}{2}(\sin 90^\circ + \sin 60^\circ)$

$= \frac{1}{2}\left[1 + \frac{\sqrt{3}}{2}\right]$

$= \frac{2+\sqrt{3}}{4}$ = RHS

Hence proved.

8. Find the value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + ... + \sin^2 90°$.

**Sol.** This question is based on complementary function $\sin(90° - \theta) = \cos\theta$

Also, $\sin^2\theta + \cos^2\theta = 1$

$\sin 85° = \sin(90° - 5°) = \cos 5°$

Similarly, on converting the other functions also, we get

$$(\sin^2 5° + \cos^2 85°) + (\sin^2 10° + \cos^2 80°) + ...... + (\sin^2 45° + \sin^2 90°)$$

$= 1 + 1 + .....$ 8 terms $+\frac{1}{2} + 1 = 9\frac{1}{2}$

9. Find the angle between the minute hand of a clock and the hour hand when the time is 7.20 AM.

**Sol.** We know that the hour hand completes one rotation in 12 hours while the minute hand completes one rotation in 60 minutes.

$\therefore$ Angle traced by hour hand in 12 hours = 360°.

Angle traced by hour hand in 7 hours and 20 minutes *i.e.,* $\frac{22}{3}$ hours $= \left(\frac{360}{12} \times \frac{22}{3}\right)^{\circ} = 220^{\circ}$.

Also, angle traced by minute hand in 60 minutes = 360°.

$\Rightarrow$ Angle traced by minute hand in 20 minutes $= \left(\frac{360}{60} \times 20\right)^{\circ} = 120°$

$\therefore$ The required angle = 220° – 120° = 100°

10. If the arcs of same length in 2 circles subtend angles 65° and 110° at the centre, then find the ratio of their radii.

**Sol.** Let the length of the arc be '$l$' and 65° angle be subtended at the centre of circle with radius $r_1$, and 110° at centre of circle with radius $r_2$.

$\therefore$ $l = r_1\theta_1 = r_2\theta_2$

$\Rightarrow$ $65° r_1 = 110° r_2$

$\Rightarrow$ $\frac{r_1}{r_2} = \frac{110°}{65°} = \frac{22}{13}$

$\Rightarrow$ $r_1 : r_2 = 22 : 13$

**Short Answer Type Questions :**

11. Find the value of $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$.

**Sol.** We have,

$$\sqrt{3}\,\text{cosec}\,20° - \sec 20° = \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$$

$$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°} = 4\left(\frac{\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°}{2\sin 20° \cos 20°}\right)$$

$$= 4\left(\frac{\sin 60° \cos 20° - \cos 60° \sin 20°}{\sin 40°}\right) \quad [\sin 2A = 2\sin A \cos A]$$

$$= 4\left(\frac{\sin(60° - 20°)}{\sin 40°}\right) = 4 \quad [\sin(A - B) = \sin A \cos B - \cos A \sin B]$$

12. If $A = \cos^2\theta + \sin^4\theta$ for all values of $\theta$, then prove that $\frac{3}{4} \le A \le 1$.

**Sol.** We have, $A = \cos^2\theta + \sin^4\theta = \cos^2\theta + \sin^2\theta\sin^2\theta \le \cos^2\theta + \sin^2\theta$

Therefore, $A \le 1$

Also, $A = \cos^2\theta + \sin^4\theta = (1 - \sin^2\theta) + \sin^4\theta$

$$= \left(\sin^2\theta - \frac{1}{2}\right)^2 + \left(1 - \frac{1}{4}\right) = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \ge \frac{3}{4}$$

Hence, $\frac{3}{4} \le A \le 1$.

13. Prove that $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$.

**Sol.** LHS $= \sin 4A = 2\sin 2A\cos 2A$

$= 2 \times 2\sin A\cos A\,(\cos^2 A - \sin^2 A)$

$= 4\sin A\cos A(\cos^2 A - \sin^2 A)$

$= 4\sin A\cos^3 A - 4\sin^3 A\cos A$

$=$ RHS

Hence proved.

14. If $\cot\theta = \sin 2\theta$, ($\theta = n\pi$, $n$ is integer), then find the value of $\theta$.

**Sol.** $\cot\theta = \sin 2\theta$

$\Rightarrow \frac{\cos\theta}{\sin\theta} = 2\sin\theta\cos\theta$

$\Rightarrow \cos\theta(1 - 2\sin^2\theta) = 0$

$\Rightarrow \cos\theta.\cos 2\theta = 0$

$\Rightarrow \cos\theta = 0$ and $\cos 2\theta = 0$

$\Rightarrow \cos\theta = 0, \quad \cos 2\theta = 0$

$\Rightarrow \theta = (2n+1)\frac{\pi}{2}, \quad 2\theta = (2n+1)\frac{\pi}{2}$

$, \quad \theta = (2n+1)\frac{\pi}{4}$

15. The difference between two acute angles of a right angled triangle is $\frac{3\pi}{10}$ radians. Express the angles in degrees.

**Sol.** $A + B + C = 180°$

Let $A = 90°$

$\Rightarrow B + C = 90°$

$\therefore \quad B + C = \frac{\pi}{2}$

$B - C = \frac{3\pi}{10}$

$2B = \frac{\pi}{2} + \frac{3\pi}{10}$

$2B = \frac{8\pi}{10}$

$B = \frac{2\pi}{5}$

$\Rightarrow \quad C = \frac{\pi}{2} - \frac{2\pi}{5} = \frac{\pi}{10}$

$B = \frac{2}{5} \times 180° = 72°$ and $C = \frac{180}{10} = 18°$

16. Prove that $\cos^2 A + \cos^2(A + 120°) + \cos^2(A - 120°) = \frac{3}{2}$

**Sol.** LHS = $\cos^2 A + \cos^2(A + 120°) + \cos^2(A - 120°)$

$= \cos^2 A + \cos^2(A + 120°) + 1 - \sin^2(A - 120°)$

$= 1 + \cos^2 A + \cos(A + 120° + A - 120°) x \cos(A + 120° - A + 120°)$

$= 1 + \cos^2 A + \cos 2A \cos 240°$

$= 1 + \cos^2 A + \cos 2A\left(-\frac{1}{2}\right)$

$= 1 + \cos^2 A - \frac{1}{2}(2\cos^2 A - 1)$

$= 1 + \frac{1}{2} = \frac{3}{2} = \text{RHS}$

Hence proved.

17. Find the general solution of $\tan x + \tan 2x + \tan x \tan 2x = 1$.

**Sol.** The given equation is $\tan x + \tan 2x + \tan x \tan 2x = 1$

$\tan x + \tan 2x = 1 - \tan x \tan 2x$

$\frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = 1$

$\tan(x + 2x) = 1$

$\tan 3x = \tan \frac{\pi}{4}$

$3x = n\pi + \frac{\pi}{4}$

$x = \frac{n\pi}{3} + \frac{\pi}{12}, n \in Z$

18. In $\Delta ABC$ if $\sin A : \sin B : \sin C = 1 : 2 : 2$, then find the value of $\cos A$.

**Sol.** Given $\sin A : \sin B : \sin C = 1 : 2 : 2$

By using Sine rule, we have

$a : b : c = \sin A : \sin B : \sin C$

Let $a = k$, $b = 2k$ and $c = 2k$

$\therefore \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4k^2 + 4k^2 - k^2}{2(2k)(2k)} = \frac{7}{8}$

19. Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \begin{cases} \text{cosec}\theta+\cot\theta, & 0<\theta<\pi \\ -\text{cosec}\theta-\cot\theta, & \pi<\theta<2\pi \end{cases}$

**Sol.** $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \frac{1+\cos\theta}{\sqrt{\sin^2\theta}} = \frac{1+\cos\theta}{|\sin\theta|}$

$$\Rightarrow \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \begin{cases} \frac{1+\cos\theta}{\sin\theta}, 0<\theta<\pi \\ \frac{1+\cos\theta}{-\sin\theta}, \pi<\theta<2\pi \end{cases} = \begin{cases} \text{cosec}\theta+\cot\theta, 0<\theta<\pi \\ -\text{cosec}\theta-\cot\theta, \pi<\theta<2\pi \end{cases}$$

Hence proved.

20 Prove that $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$.

**Sol.** LHS = $\cot 4x(\sin 5x + \sin 3x)$

$= \cot 4x \times 2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)$

$= \frac{\cos 4x}{\sin 4x} \times 2\sin 4x \cos x$

$= 2\cos 4x \cos x$

RHS = $\cot x(\sin 5x - \sin 3x)$

$= \cot x 2\sin\left(\frac{5x-3x}{2}\right)\cos\left(\frac{5x+3x}{2}\right)$

$= \frac{\cos x}{\sin x} \times 2\sin x \cos 4x$

$= 2\cos 4x \cos x$

LHS = RHS

Hence proved.

**Long Answer Type Questions :**

21. If $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $\alpha$ lie between 0 and $\frac{\pi}{4}$, then find the value of $\tan 2\alpha$.

[**Hint**: Express $\tan 2\alpha$ as $\tan(\alpha + \beta + \alpha - \beta)$]

**Sol.** $\cos(\alpha+\beta) = \frac{4}{5} \Rightarrow \tan(\alpha+\beta) = \frac{3}{4}$

$\sin(\alpha-\beta) = \frac{5}{13} \Rightarrow \tan(\alpha-\beta) = \frac{5}{12}$

Now, $\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)]$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{\frac{9+5}{12}}{\frac{48-15}{48}} = \frac{56}{33}$$

22. Prove that $\dfrac{\sin(A+B)+\cos(B-A)}{\sin(B-A)+\cos(B+A)} = \dfrac{\cos A+\sin A}{\cos A-\sin A}$.

**Sol.** $\dfrac{\sin(A+B)+\cos(B-A)}{\sin(B-A)+\cos(B+A)}$

$$= \frac{\sin(B+A)+\sin\left(\frac{\pi}{2}-(B-A)\right)}{\sin(B-A)+\sin\left(\frac{\pi}{2}-(B+A)\right)} = \frac{2\sin\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)}{2\sin\left(\frac{\pi}{4}-A\right)\cos\left(\frac{\pi}{4}-B\right)}$$

$$= \frac{\sin\left(\frac{\pi}{4}+A\right)}{\sin\left(\frac{\pi}{4}-A\right)} = \frac{(\cos A+\sin A)}{(\cos A-\sin A)} = \text{RHS}$$

Hence proved.

23. If sinθ + cosθ = $m$ and secθ + cosecθ = $n$, then prove that $n(m + 1)(m - 1) = 2m$.

**Sol.** $\sin\theta + \cos\theta = m$ ...(i)

$\sec\theta + \operatorname{cosec}\theta = n$

$$\Rightarrow \frac{1}{\cos\theta}+\frac{1}{\sin\theta} = n$$

$$\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta} = n$$

$$\sin\theta\cos\theta = \frac{m}{n} \quad \text{[From (i)]}$$

Now, $(m + 1)(m - 1) = (\sin\theta + \cos\theta + 1)(\sin\theta + \cos\theta - 1)$

$= (\sin\theta + \cos\theta)^2 - 1^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1$

$= 1 + 2\sin\theta\cos\theta - 1$

$= \dfrac{2m}{n}$

$$\therefore \quad (m+1)(m-1) = \frac{2m}{n}$$

$n(m + 1)(m - 1) = 2m$

Hence proved.

24. Prove that $\tan 189° = \dfrac{\cos 36° - \sin 36°}{\cos 36° + \sin 36°}$

**Sol.** LHS = tan189° = tan(180° + 9°) = tan9° [$\because$ tan(180° + θ) = tanθ]

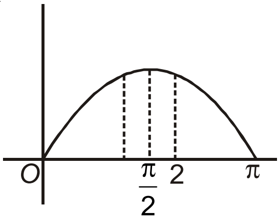
$$= \tan(45°-36°) = \frac{\tan 45° - \tan 36°}{1+\tan 45°\tan 36°}$$

$$= \frac{1-\tan 36°}{1+\tan 36°} = \frac{1-\frac{\sin 36°}{\cos 36°}}{1+\frac{\sin 36°}{\cos 36°}} = \frac{\cos 36° - \sin 36°}{\cos 36° + \sin 36°} = \text{RHS}$$

Hence proved.

25. Compare sin 1 and sin 2, cos 1 and cos 2, tan 1 and tan 2.

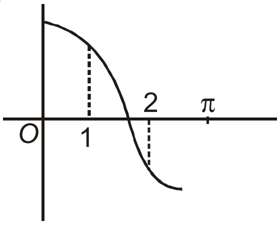
**Sol.** From the graph of $y = \sin x$




$\because \quad \sin\frac{\pi}{2}$ is maximum and $2-\frac{\pi}{2}<\frac{\pi}{2}-1$

So, sin 2 > sin 1

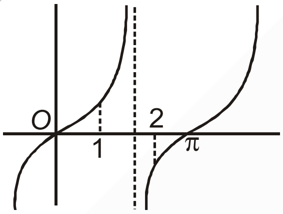
From the graph of $y = \cos x$




cos 1 is positive and cos 2 is negative.

$\Rightarrow$ cos 1 > cos 2

From the graph of $y = \tan x$




tan 1 is positive and tan 2 is negative.

So, tan 1 > tan 2

26. Prove that $\sin\alpha+\sin\left(\alpha+\frac{2\pi}{3}\right)+\sin\left(\alpha+\frac{4\pi}{3}\right)=0$

**Sol.** LHS $= \sin\alpha+\sin\left(\alpha+\frac{2\pi}{3}\right)+\sin\left(\alpha+\frac{4\pi}{3}\right)$

$$= \sin\alpha+\left[2\sin\left(\frac{\alpha+\frac{2\pi}{3}+\alpha+\frac{4\pi}{3}}{2}\right)\cos\left(\frac{\alpha+\frac{2\pi}{3}-\alpha-\frac{4\pi}{3}}{2}\right)\right]$$

$$= \sin\alpha+2\sin(\alpha+\pi)\cos\frac{\pi}{3}$$

$$= \sin\alpha+2(-\sin\alpha)\frac{1}{2}$$

$= \sin\alpha - \sin\alpha = 0 =$ RHS

Hence proved.

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# Chapter 3

# Trigonometric Functions-I

## Solutions (Set-2)

1. If tan $\theta = 3$ and $\theta$ lies in the III quadrant, then the value of $\sin\theta$ is

(1) $\frac{1}{\sqrt{10}}$ (2) $-\frac{1}{\sqrt{10}}$ (3) $\frac{-3}{\sqrt{10}}$ (4) $\frac{3}{\sqrt{10}}$

**Sol.** Answer (3)

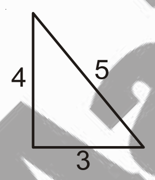
2. If *A* lies in the second quadrant and $3\tan A + 4 = 0$, then the value of $2\cot A - 5\cos A + \sin A$ is equal to

(1) $\frac{-53}{10}$ (2) $\frac{23}{10}$ (3) $\frac{37}{10}$ (4) $\frac{7}{10}$

**Sol.** Answer (2)

$3\tan A = -4$

$\tan A = -\frac{4}{3}$




$\therefore\ 2\cot A - 5\cos A + \sin A$

$= 2\left(\frac{-3}{4}\right) - 5\left(\frac{-3}{5}\right) + \frac{4}{5}$

$= \frac{-6}{4} + 3 + \frac{4}{5} = \frac{-30+60+16}{20} = \frac{46}{20} = \frac{23}{10}$

3. The value of $(\sec A + \tan A - 1)(\sec A - \tan A + 1) - 2\tan A$ is equal to

(1) $\sec A$ (2) $2\sec A$ (3) 0 (4) 1

**Sol.** Answer (3)

$= (\sec A + \tan A - 1)(\sec A - \tan A + 1) - 2\tan A$

$= [\sec A + (\tan A - 1)][\sec A - (\tan A - 1)] - 2\tan A$

$= [\sec^2 A - (\tan A - 1)^2] - 2\tan A$

$= \sec^2 A - \tan^2 A - 1 + 2\tan A - 2\tan A$

$= 1 - 1 = 0$

4. The circular wire of diameter 10 cm is cut and placed along the circumference of a circle of diameter 1 m. The angle subtended by the wire at the centre of the circle is equal to

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{5}$ (4) $\frac{\pi}{10}$

**Sol.** Answer (3)

Diameter of the wire = 10 cm

Length of wire = $10\pi$ cm

Another circle is of diameter = 1 m = 100 cm

$\therefore$ Radius = 50 cm

Required angle = $\frac{\text{Length of arc}}{\text{Radius}} = \frac{10\pi}{50} = \frac{\pi}{5}$

5. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$

(1) 3 (2) 2 (3) 1 (4) 0

**Sol.** Answer (4)

$\sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1.$

$\therefore \theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2}$

6. The value of $\cos 10° - \sin 10°$ is

(1) Positive (2) Negative (3) 0 (4) 1

**Sol.** Answer (1)

7. The value of cos1°.cos2°.....cos179° is

(1) $\frac{1}{\sqrt{2}}$ (2) 0 (3) 1 (4) –1

**Sol.** Answer (2)

cos1°.cos2°……cos179° = 0

(as there will be a term cos 90° in this series whose value is zero)

8. The value of $(\text{cosec}A.\text{cosec}B + \cot A.\cot B)^2 - (\text{cosec}A.\cot B + \text{cosec}B.\cot A)^2$ is

(1) 1 (2) 2 (3) 0 (4) –1

**Sol.** Answer (1)

$(\text{cosec}A.\text{cosec}B + \cot A.\cot B)^2 - (\text{cosec}A.\cot B + \text{cosec}B.\cot A)^2$

$= \text{cosec}^2 A(\text{cosec}^2 B - \cot^2 B) - \cot^2 A(\text{cosec}^2 B - \cot^2 B)$

$= \text{cosec}^2 A - \cot^2 A = 1$

9. If $\tan\alpha + \cot\alpha = a$, then the value of $\tan^4\alpha + \cot^4\alpha$ is equal to

(1) $a^4 + 4a^2 + 2$ (2) $a^4 - 4a^2 + 2$ (3) $a^4 - 4a^2 - 2$ (4) $-a^4 + 2a^2 + 4$

**Sol.** Answer (2)

$\tan\alpha + \cot\alpha = a$ , $\tan^4\alpha + \cot^4\alpha = ?$

$\tan^4\alpha + \cot^4\alpha = (\tan\alpha + \cot\alpha)^4 - 4\tan\alpha.\cot\alpha.(\tan^2\alpha + \cot^2\alpha) - 6\tan^2\alpha.\cot^2\alpha$

$= a^4 - 4\{(\tan\alpha + \cot\alpha)^2 - 2\tan\alpha.\cot\alpha\} - 6$

$= a^4 - 4a^2 + 2$

10. If $a\cos\theta + b\sin\theta = 3$ and $a\sin\theta - b\cos\theta = 4$, then $a^2 + b^2$ has the value

(1) 25 (2) 14 (3) 7 (4) 15

**Sol.** Answer (1)

$a\cos\theta + b\sin\theta = 3$

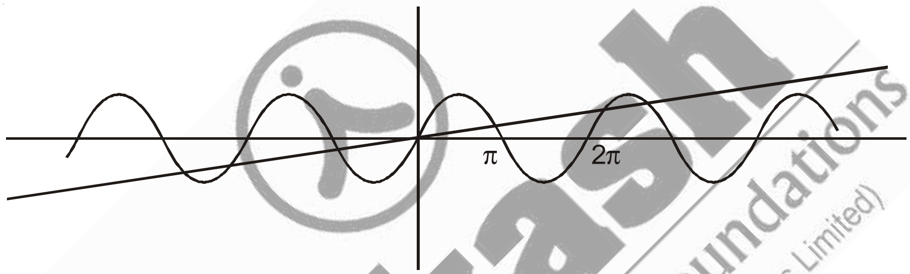
$a\sin\theta - b\cos\theta = 4$

Squaring and adding, $a^2 + b^2 = 25$

11. The number of intersecting points on the graph for $\sin x = \frac{x}{10}$ for $x \in [-\pi, \pi]$ is

(1) 3 (2) 4 (3) 2 (4) 1

**Sol.** Answer (1)




$\sin x = \frac{x}{10}$, $x \in [-\pi, \pi]$

Clearly the total number of points of intersection is 3.

12. $\left|(3 - \sec^2 x)_{max} - (4 + \tan^2 y)_{min}\right|$ equals

(1) 0 (2) 1 (3) 2 (4) 3

**Sol.** Answer (3)

$\left|(3 - \sec^2 x)_{max} - (4 + \tan^2 y)_{min}\right| = |2 - 4| = 2$

13. If $\tan\theta = \frac{p}{q}$, then the value of $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$ is

(1) $\frac{p^2 - q^2}{p^2 + q^2}$ (2) $\frac{p^2 + q^2}{p^2 - q^2}$ (3) 0 (4) 1

**Sol.** Answer (1)

$\tan\theta = \frac{p}{q}, \frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} = ?$

Divide numerator and denominator by cos θ

$\frac{p\tan\theta - q}{p\tan\theta + q} = \frac{p^2 - q^2}{p^2 + q^2}$

14. The value of $\sin 1.\cos 2.\tan 3.\cot 4.\sec 5.\operatorname{cosec} 6$ is

(1) Positive (2) Negative

(3) Zero (4) May be positive and Negative

**Sol.** Answer (2)

$$\begin{array}{cccccc} \sin 1 & \cos 2 & \tan 3 & \cot 4 & \sec 5 & \operatorname{cosec} 6 \\ + & - & - & + & + & - \end{array}$$

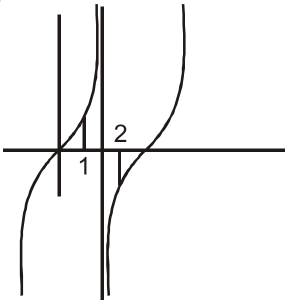
= negative

15. Which of the following is correct?

(1) tan1 > tan2 (2) tan2 > tan1 (3) sin1 < cos1 (4) cos3 > cos2

**Sol.** Answer (1)

By graph we can say




tan 1 > tan 2 (from graph)

16. If $\cos\theta + \sec\theta = -2, \theta \in [0, 2\pi]$, then $\sin^8\theta + \cos^8\theta$ equal to

(1) –2 (2) 1 (3) $2^4$ (4) $2^5$

**Sol.** Answer (2)

$\cos\theta + \sec\theta = -2$, $\theta \in [0,\ 2\pi]$

$\Rightarrow \cos\theta = \sec\theta = -1$

$\theta = \pi$

$\therefore \sin^8\theta + \cos^8\theta = 0 + 1 = 1$

17. If $\dfrac{2\sin A}{1+\sin A+\cos A} = k$, then $\dfrac{1+\sin A-\cos A}{1+\sin A}$ is

(1) $\dfrac{k}{2}$ (2) $k$ (3) $2k$ (4) $\dfrac{1}{k}$

**Sol.** Answer (2)

$$\frac{2\sin A}{1+\sin A+\cos A} = k, \quad \frac{1+\sin A-\cos A}{1+\sin A} = ?$$

$$k = \frac{2\sin A.(1+\sin A-\cos A)}{(1+\sin A)^2 - \cos^2 A}$$

$$\Rightarrow k = \frac{2\sin A.(1+\sin A-\cos A)}{(1+\sin A)^2 - (1-\sin^2 A)} = \frac{2\sin A.(1+\sin A-\cos A)}{(1+\sin A).\{(1+\sin A)-(1-\sin A)\}} = \frac{1+\sin A-\cos A}{1+\sin A}$$

18. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then the value of $\theta + \phi$ is

(1) $\frac{\pi}{6}$ (2) $\pi$ (3) 0 (4) $\frac{\pi}{4}$

**Sol.** Answer (4)

$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$

19. The value of $\cos^2 48° - \sin^2 12°$ is

(1) $\frac{\sqrt{5}+1}{8}$ (2) $\frac{\sqrt{5}-1}{8}$ (3) $\frac{\sqrt{5}+1}{5}$ (4) $\frac{\sqrt{5}+1}{2\sqrt{2}}$

**Sol.** Answer (1)

$\cos^2 A - \sin^2 B = \cos(A + B)\cos(A - B)$

20. The value of $4(\sin 24° + \sin 84°)$ is

(1) $\sqrt{15}-\sqrt{3}$ (2) $\sqrt{15}+\sqrt{3}$ (3) $\frac{\sqrt{15}-\sqrt{3}}{2}$ (4) $\frac{\sqrt{15}+\sqrt{3}}{2}$

**Sol.** Answer (2)

Given expression

$= 4(2\sin 54° \cos 30°)$

$= 8 \cdot \frac{\sqrt{5}+1}{4} \cdot \frac{\sqrt{3}}{2}$

$= \sqrt{15} + \sqrt{3}$

21. The value of sin50° – sin70° + sin10° is equal to

(1) 1 (2) 0 (3) $\frac{1}{2}$ (4) 2

**Sol.** Answer (2)

sin50° + sin10° – sin70°

$= 2\sin\left(\frac{60}{2}\right)^\circ \cos\left(\frac{40}{2}\right)^\circ - \sin 70°$

= (cos20°) – cos20°

= 0

22. The value of $\sin\frac{\pi}{18} + \sin\frac{\pi}{9} + \sin\frac{2\pi}{9} + \sin\frac{5\pi}{18}$ is

(1) $\sin\frac{7\pi}{18} + \sin\frac{4\pi}{9}$ (2) 1 (3) $\cos\frac{\pi}{6} + \cos\frac{3\pi}{7}$ (4) $\cos\frac{\pi}{9} + \sin\frac{\pi}{9}$

**Sol.** Answer (1)

Let

$\therefore$ $\sin\theta + \sin2\theta + \sin4\theta + \sin5\theta$

$= \sin\theta + \sin5\theta + \sin2\theta + \sin4\theta$

$= 2\sin3\theta\cos2\theta + 2\sin3\theta\cos\theta$

$= 2\sin3\theta\,(\cos2\theta + \cos\theta)$

$= 2\sin\frac{3\pi}{18}\left(\cos\frac{2\pi}{18}+\cos\frac{\pi}{18}\right)$

$= \cos\frac{\pi}{9}+\cos\frac{\pi}{18}$

$= \sin\left(\frac{\pi}{2}-\frac{\pi}{9}\right)+\sin\left(\frac{\pi}{2}-\frac{\pi}{18}\right)$

$= \sin\frac{7\pi}{18}+\sin\frac{8\pi}{18}$

$= \sin\frac{7\pi}{18}+\sin\frac{4\pi}{9}$

23. The value of $\sin\frac{\pi}{10}\sin\frac{13\pi}{10}$ is

(1) $\frac{1}{2}$ (2) $\frac{-1}{2}$ (3) $\frac{-1}{4}$ (4) 1

**Sol.** Answer (3)

$\sin18^\circ\sin\left(\pi+\frac{3\pi}{10}\right)$

$= -\sin18^\circ\sin54^\circ$

$= -\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)$

$= -\frac{4}{16} = -\frac{1}{4}$

24. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then tan (2*A* + *B*) is equal to

(1) 1 (2) 2 (3) 3 (4) 4

**Sol.** Answer (3)

$$\tan2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\times\frac{1}{2}}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\tan(2A+B) = \frac{\tan2A+\tan B}{1-\tan2A\tan B} = \frac{\frac{4}{3}+\frac{1}{3}}{1-\frac{4}{3}\times\frac{1}{3}} = 3$$

25. If $\tan^2\alpha = 1-\lambda^2$, then the value of $\sec\alpha + \tan^3\alpha\,\mathrm{cosec}\,\alpha$ is

(1) $\left(\sqrt{2-\lambda^2}\right)^3$ (2) $\sqrt{2-\lambda^2}$ (3) $\sqrt{2+\lambda^2}$ (4) $\left(\sqrt{2+\lambda^2}\right)^3$

**Sol.** Answer (1)

$$\sec\alpha + \tan^3\alpha\,\mathrm{cosec}\,\alpha = \sec\alpha(1+\tan^3\alpha\cot\alpha) = (1+\tan^2\alpha)^{3/2} = (2-\lambda^2)^{3/2}$$

26. If $\sin\theta = \frac{-4}{5}$ and $\theta$ lies in the III quadrant, then the value of $\cos\frac{\theta}{2}$ is

(1) $\frac{1}{\sqrt{5}}$ (2) $\frac{1}{\sqrt{10}}$ (3) $\frac{-1}{\sqrt{5}}$ (4) $-\frac{1}{\sqrt{10}}$

**Sol.** Answer (3)

$180° < \theta < 270°$

$\Rightarrow\ 90° < \frac{\theta}{2} < 135°$

This means $\frac{\theta}{2}$ lies in the second quadrant.

$\therefore\ \cos\frac{\theta}{2}$ will be negative.

If $\sin\theta = \frac{-4}{5}$, then $\cos\theta = \frac{-3}{5}$

Now, $\cos 2A = 2\cos^2 A - 1$

$\therefore\ \cos\theta = 2\cos^2\frac{\theta}{2} - 1$

$\frac{-3}{5} = 2\cos^2\frac{\theta}{2} - 1$

$2\cos^2\frac{\theta}{2} = \frac{2}{5}$

$\cos^2\frac{\theta}{2} = \frac{1}{5}$

$\cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}}$ [Since $\frac{\theta}{2}$ lies in the II quadrant]

27. If $\tan\theta = \frac{1-\cos\phi}{\sin\phi}$, then tan $3\theta$ is equal to

(1) $\tan 2\phi$ (2) $\tan\frac{3\phi}{2}$ (3) $\tan\phi$ (4) $-\tan 2\phi$

**Sol.** Answer (2)

$$\tan 3\theta = \frac{\tan\theta + \tan 2\theta}{1-\tan\theta\cdot\tan 2\theta} = \frac{\tan\frac{\phi}{2} + \tan\phi}{1-\tan\frac{\phi}{2}\cdot\tan\phi} = \tan\frac{3\phi}{2}$$

28. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha - \cos\beta = b$ then, $\tan\left(\frac{\alpha-\beta}{2}\right)$ is equal to

(1) $-\frac{a}{b}$ (2) $-\frac{b}{a}$ (3) $\sqrt{a^2+b^2}$ (4) $\frac{a}{b}$

**Sol.** Answer (2)

$\sin\alpha + \sin\beta = a$

$\Rightarrow\ 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ ...(i)

$\Rightarrow\ -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = b$ ...(ii)

Divide both

$\Rightarrow\ \cot\left(\frac{\alpha-\beta}{2}\right) = -\frac{a}{b}$

$\Rightarrow\ \tan\left(\frac{\alpha-\beta}{2}\right) = -\frac{b}{a}$

29. If $\tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$, then the expression $\cos 2\alpha + (2+\sqrt{3})\sin 2\alpha$ is

(1) $2+\sqrt{3}$ (2) –1 (3) 1 (4) $-\left(2+\sqrt{3}\right)$

**Sol.** Answer (3)

$\tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \tan 75°$

$\Rightarrow\ \alpha = 75°$

$\cos 2\alpha + \left(2+\sqrt{3}\right)\sin 2\alpha \quad \Rightarrow \quad \cos 150 + \left(2+\sqrt{3}\right)\sin 150$

$-\frac{\sqrt{3}}{2} + \left(2+\sqrt{3}\right)\times\frac{1}{2} = 1$

30. If $0 < \theta_2 < \theta_1 < \frac{\pi}{4}, \cos(\theta_1+\theta_2) = \frac{3}{5}$ and

$\cos(\theta_1 - \theta_2) = \frac{4}{5}$, then $\sin 2\theta_1$ equal to

(1) – 1 (2) 1 (3) 2 (4) – 2

**Sol.** Answer (2)

$\cos\left(\theta_1+\theta_2\right) = \frac{3}{5}, \cos\left(\theta_1-\theta_2\right) = \frac{4}{5}$

$\sin(2\theta_1) = \sin[(\theta_1+\theta_2) + (\theta_1-\theta_2)]$

$= \frac{4}{5}.\frac{4}{5} + \frac{3}{5}.\frac{3}{5} = \frac{25}{25} = 1$

31. In a triangle $ABC$, $\tan A + \tan B + \tan C = 6$ and $\tan A \cdot \tan B = 2$, then value of $\tan A$, $\tan B$ and $\tan C$ are

(1) 3, 1, 2 (2) 1, 2, 4 (3) 1, 2, 3 (4) 2, 2, 2

**Sol.** Answer (3)

$\tan A + \tan B + \tan C = 6$

$\Rightarrow \tan A \tan B \tan C = 6 \quad \Rightarrow \quad \tan C = \frac{6}{2} = 3$

$\tan A = 1, \tan B = 2, \tan C = 3$

32. The minimum value of $8(\cos 2\theta + \cos\theta)$ is equal to

(1) –17 (2) –9 (3) 3 (4) –8

**Sol.** Answer (2)

$8(\cos 2\theta + \cos\theta)$

$= 8(2\cos^2\theta - 1 + \cos\theta)$

$= 8(2\cos^2\theta + \cos\theta - 1)$

$= 16\left(\cos^2\theta + \frac{1}{2}\cos\theta - \frac{1}{2}\right)$

$= 16\left(\left(\cos\theta + \frac{1}{4}\right)^2 - \frac{9}{16}\right)$

Minimum value = $16 \times -\frac{9}{16} = -9$

33. The value of $\tan 7\frac{1}{2}^\circ$ is equal to

(1) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}+1}$ (2) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}-1}$ (3) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}+1}$ (4) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}-1}$

**Sol.** Answer (1)

$\therefore \quad \tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta}$

Put $\theta = \frac{15}{2}$

$$\tan\frac{15^\circ}{2} = \frac{1-\cos 15^\circ}{\sin 15^\circ} = \frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{2\sqrt{6}-3-\sqrt{3}+2\sqrt{2}-\sqrt{3}-1}{3-1}$$

$$= \frac{2\sqrt{6}-4-2\sqrt{3}+2\sqrt{2}}{2} = \sqrt{6}-\sqrt{4}-\sqrt{3}+\sqrt{2}$$

$$\therefore \quad \sqrt{3}-\sqrt{2} \times \sqrt{2}-1 = \left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{2}+1\right) = \sqrt{6}-\sqrt{4}-\sqrt{3}+\sqrt{2} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}+1}$$

34. If $\alpha+\beta=\frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then $\tan\alpha$ equals

(1) $2(\tan\beta + \tan\gamma)$ (2) $\tan\beta + 2\tan\gamma$ (3) $\tan\beta - \tan\gamma$ (4) $\tan\gamma - \tan\beta$

**Sol.** Answer (2)

$$\alpha+\beta=\frac{\pi}{2},\quad \beta+\gamma=\alpha$$

$$\alpha=\frac{\pi}{2}-\beta \qquad \beta=\alpha-\gamma$$

$$\Rightarrow \tan\alpha = \cot\beta \qquad \tan\beta=\frac{\tan\alpha-\tan\gamma}{1+\tan\alpha\tan\gamma}$$

$\Rightarrow$ $\tan\beta + \tan\alpha \tan\beta \tan\gamma = \tan\alpha - \tan\gamma$

$\Rightarrow$ $\tan\beta = \tan\alpha - 2\tan\gamma$

$\Rightarrow$ $\tan\alpha = \tan\beta + 2\tan\gamma$

35. The solution of the equation $\cos^2\theta + \sin\theta + 1 = 0$ lies in the interval

(1) $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$ (2) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (3) $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ (4) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

**Sol.** Answer (4)

$\cos^2\theta + \sin\theta + 1 = 0$

$1 - \sin^2\theta + \sin\theta + 1 = 0$

$\sin^2\theta - \sin\theta - 2 = 0$

$(\sin\theta - 2)(\sin\theta + 1) = 0$

$\therefore$ $\sin\theta = -1$ $\qquad [\because \sin\theta \neq 2 \Rightarrow \sin\theta - 2 \neq 0]$

$$\Rightarrow \theta=\pi+\frac{\pi}{2}=\frac{3\pi}{2}=\frac{6\pi}{4}$$

$$\therefore \frac{6\pi}{4}\in\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$$

36. If $\sqrt{3}\sec\theta+2=0$, then principal value of $\theta$ may be

(1) $\frac{5\pi}{6}$ (2) $-\frac{5\pi}{6}$ (3) $\frac{7\pi}{6}$ (4) $-\frac{7\pi}{6}$

**Sol.** Answer (1)

$$\sec\theta=-\frac{2}{\sqrt{3}}$$

$$\Rightarrow \cos\theta=-\frac{\sqrt{3}}{2}$$

$$\Rightarrow \boxed{\theta=\frac{5\pi}{6}\text{ (Principal)}}$$

37. If $2\sec 2\alpha = \tan\beta + \cot\beta$, then $(\alpha + \beta)$ may be

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) Zero

**Sol.** Answer (1)

$$2\sec 2\alpha = \frac{1}{\sin\beta\cos\beta} = 2\text{cosec}\, 2\beta$$

Now, $2\alpha = 2\beta = \frac{\pi}{4}$ is possible.

$\therefore \quad \alpha + \beta = \frac{\pi}{4}$

38. The number of solution of the equation $\tan 3x - \tan 2x - \tan 3x \cdot \tan 2x = 1$ in $[0, 2\pi]$ is

(1) 1 (2) Zero (3) 3 (4) 2

**Sol.** Answer (2)

$\tan 3x - \tan 2x = 1 + \tan 3x \tan 2x$

$\Rightarrow \frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$

$\Rightarrow \tan x = 1$

$\Rightarrow x = n\pi + \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}, \ldots\ldots..$

But for each of values $\tan 2x$, $\tan 3x$ will not be defined. Therefore no solution exists.

**Trick :** Always check the values by back substitution in case of tan*x*.

39. The general solution of $\sec\theta + \tan\theta = \sqrt{3}$ is

(1) $\theta = 2n\pi + \frac{\pi}{6}, 2n\pi - \frac{\pi}{2}, n \in I$ (2) $\theta = 2n\pi + \frac{\pi}{6}, n \in I$

(3) $2n\pi - \frac{\pi}{2}; n \in I$ (4) $2n\pi - \frac{\pi}{2}; n \neq 2k + 1, n, k \in I$

**Sol.** Answer (2)

$\sec\theta + \tan\theta = \sqrt{3}$

We know that $\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$$\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$$

$$\sec\theta + \tan\theta = \sqrt{3}$$

$$2\sec\theta = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$\sec\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$

40. If $x \neq n\pi,\ n \in I$ and satisfies the equation $\sin 5x \cos 3x = \sin 6x \cos 2x$, then $x$ is

(1) $\frac{\pi}{3} + \frac{n\pi}{6},\ n \in I$ (2) $\frac{\pi}{6} + \frac{n\pi}{3},\ n \in I$ (3) $\frac{\pi}{6} + n\pi,\ n \in I$ (4) $n\pi + \frac{\pi}{3},\ n \in I$

**Sol.** Answer (2)

$2\sin 5x \cos 3x = 2\sin 6x \cos 2x$

$\Rightarrow \sin 2x = \sin 4x$

$\Rightarrow 2\cos 3x \sin x = 0$

$\Rightarrow \cos 3x = 0$

$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{6}$

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