Baye's Theorem

Q.1. A class consists of 50 students out of which there are 10 girls. In the class 2 girls and 5 boys are rank holders in an examination. If a student is selected at random from the class and is found to be a rank holder, what is the probability that the student selected is a girl ?

Solution : 1

Probability of girl rank holder = P(G/R)

= [P(R/G).P(G)]/[P(R/G).P(G)+P(R/B).P(B)]

= [(1/5)×(1/5)]/[(1/5)×(1/5)+(4/5)×(5/40)]

= [1/25]/[(1/25)+(1/10)]

= [1/25]/[(2+5)/50]

=[1/25]/[7/50]

= (1/25)×(50/7) = 2/7 .

Q.2. An insurance company insured 1500 scooter drivers, 2500 car drivers and 4500 truck drivers. The probability of a scooter, a car and a truck meeting with an accident is 0.01, 0.02 and 0.04 respectively. If one of the insured persons meets with an accident, find the probability that he is a scooter driver.

Solution : 2

Probability of accident by scooter

= P(Accident/Scooter) = P(A/S)

= [P(S).P(S/A)]/[P(S).P(S/A) + P(C).P(C/A) + P(T).P(T/A)]

= [(1500/8500)×0.01]/[(1500/8500)×0.01 + (2500/8500)×0.02 + (4500/8500)×0.04]

= 15/245 = 3/49 .

Q.3. A company has two plants which manufacture scooters. Plant I manufactures 80% of the scooters while Plant II manufactures 20% of the scooters. At Plant I, 85 out of 100 scooters are rated as being of standard quality, while at Plant II only 65 out of 100 scooters are rated as being of standard quality. If a scooter is of standard quality , what is the probability that it come from Plant I.

Solution : 3

Let A and B denotes the scooters manufactured in Plant I and Plant II respectively , then

Probability P(A) = 80/100 = 0.8 & P(B) = 20/100 = 0.2

Let X represent the event that scooter manufactured is of standard quality , then P(X/A) = 85/100 = 0.85 & P(X/B) = 65/100 = 0.65 .

Applying Baye's Theorem , the probability of selected scooter is of standard quality produced by Plant I ,

$P(A/X) = [P(A)\ P(X/A)]/[P(A)P(X/A) + P(B)P(X/B)]$

$= [0.8\times0.85]/[(0.8\times0.85) + (0.2\times0.65)]$

$= 0.68/(0.68 + 0.13) = 0.68/0.81 = 68/81 = 0.84$.

Q.4. A manufacturing firm produces steel pipes in three plants A, B and C with daily production of 500, 1000 and 2000 units respectively. The fractions of defective steel pipes output produced by the plant A, B and C are respectively 0.005, 0.008 and 0.010. If a pipe is selected from a day's total production and found to be defective, find out the probability that it came from the first plant.

Solution : 4

We have probability of production,

P(A) = 500/3500 = 1/7,

P(B) = 1000/3500 = 2/7,

P(C) = 2000/3500 = 4/7.

And probability of defective pipes,

By plant A = P(A/E) = 0.005,

By plant B = P(B/E) = 0.008,

By plant C = P(C/E) = 0.010.

Therefore by Baye's Theorem, probability of defective pipe from first plant

= P(E/A) = [P(A)×P(A/E)]/[P(A).P(A/E) + P(B).P(B/E) + P(C).P(C/E)]

= [1/7×0.005]/[1/7×0.005 + 2/7×0.008 + 4/7×0.010]

= 0.005/[0.005 + 0.016 + 0.040]

= 0.005/0.061 = 5/61

Q.5. An insurance company insured 6000 scooter drivers, 3000 car drivers and 9000 truck drivers. The probability of an accident involving a scooter, a car and a truck is 0.02, 0.06 and 0.30 respectively. One of the insured persons meets with an accident. Find the probability that he is a car driver.

Solution : 5

Let A, B and C be the event that the insured person is scooter driver, car driver and truck driver and E be the event when an insured person meets with an accident.

Therefore, the probability P(A) = 6000/(6000 + 3000 + 9000) = 6000/18000 = 1/3,

P(B) = 3000/(6000 + 3000 + 9000) = 3000/18000 = 1/6,

P(C) = 9000/(6000 + 3000 + 9000) = 9000/18000 = 1/2 .

It is given that P(E/A) = 0.02, P(E/B) = 0.06 and P(E/C) = 0.30.

Hence by Baye's Theorem,

P(B/E) = [P(B)P(E/B)]/[P(A)P(E/A) + P(B)P(E/B) + P(C)P(E/C)

= {1/6(6/100)}/[{1/3(2/100) + 1/6(6/100) + 1/2(30/100)}

= 1/[(2/3) + 1 + 15] = 3/50 = 0.06.