INVERSE TRIGONOMETRIC FUNCTIONS [JEE ADVANCED PREVIOUS YEAR SOLVED PAPER]

JEE ADVANCED

Single Correct Answer Type

1. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$ is

 a. $\frac{6}{17}$ **b.** $\frac{7}{16}$ **c.** $\frac{16}{7}$ **d.** none of these

 (IIT-JEE 1983)

2. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

 a. $-\frac{2\pi}{3}$ **b.** $\frac{2\pi}{3}$ **c.** $\frac{4\pi}{3}$ **d.** $\frac{5\pi}{3}$

 e. none of these **(IIT-JEE 1986)**

3. If we consider only the principal values of the inverse trigonometric functions, then the value of

 $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right)$ is

 a. $\frac{\sqrt{29}}{3}$ **b.** $\frac{29}{3}$ **c.** $\frac{\sqrt{3}}{29}$ **d.** $\frac{3}{29}$

 (IIT-JEE 1994)

4. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\pi/2$ is

 a. zero **b.** one **c.** two **d.** infinite

 (IIT-JEE 1999)

5. If $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}-\cdots\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}-\cdots\right)=\frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$, then x equals

a. 1/2 **b.** 1 **c.** –1/2 **d.** –1

(IIT-JEE 2001)

6. Domain of the definition of the function $f(x) = \sqrt{\sin^{-1}(2x)+\pi/6}$ is

a. [–1/4, 1/2] **b.** [–1/2, 1/9]
c. [–1/2, 1/2] **d.** [–1/4, 1/4]

(IIT-JEE 2003)

7. The value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1} x)$ is

a. 1/2 **b.** 1 **c.** 0 **d.** –1/2

(IIT-JEE 2004)

8. If $0 < x < 1$, then $\sqrt{1+x^2}\ [\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2}$ is equal to

a. $\frac{x}{\sqrt{1+x^2}}$ **b.** x **c.** $x\sqrt{1+x^2}$ **d.** $\sqrt{1+x^2}$

(IIT-JEE 2008)

9. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is

a. $\frac{23}{25}$ **b.** $\frac{25}{23}$ **c.** $\frac{23}{24}$ **d.** $\frac{24}{23}$

(JEE Advanced 2013)

Multiple Correct Answers Type

1. If $= 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are)

a. $\cos\beta > 0$ **b.** $\sin\beta < 0$
c. $\cos(\alpha+\beta) > 0$ **d.** $\cos\alpha < 0$

(JEE Advanced 2015)

Matching Column Type

1. Match the statements/expressions given in Column I with the values given in Column II.

Column I	Column II
(i) $\sum_{i=1}^{\infty}\tan^{-1}\left(\frac{1}{2i^2}\right) = t$, then $\tan t =$	(a) 0
(ii) Sides a, b, c of a triangle ABC are in A.P. and $\cos\theta_1 = \frac{a}{b+c}$, $\cos\theta_2 = \frac{b}{a+c}$, $\cos\theta_3 = \frac{c}{a+b}$, then $\tan^2\left(\frac{\theta_1}{2}\right)+\tan^2\left(\frac{\theta_3}{2}\right) =$	(b) 1
(iii) A line is perpendicular to $x + 2y + 2z = 0$ and passes through (0, 1, 0). The perpendicular distance of this line from the origin is	(c) $\frac{\sqrt{5}}{3}$
	(d) 2/3

(IIT-JEE 2006)

2. Let (x, y) be such that $\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \pi/2$. Match the statements in column I with statements in column II.

Column I	Column II
(a) If $a = 1$ and $b = 0$, then (x, y)	(p) lies on the circle $x^2 + y^2 = 1$
(b) If $a = 1$ and $b = 1$, then (x, y)	(q) lies on $(x^2 - 1)(y^2 - 1) = 0$
(c) If $a = 1$ and $b = 2$, then (x, y)	(r) lies on $y = x$
(d) If $a = 2$ and $b = 2$, then (x, y)	(s) lies on $(4x^2 - 1)(y^2 - 1) = 0$

(IIT-JEE 2007)

3. Match the statements in Column I with those in Column II.

Column I	Column II
(a) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length $PQ = d$, then d^2 is	(p) –4
(b) The value of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(c) Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b} = 0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c}) = 0$ and $2\|\vec{b}+\vec{c}\| = \|\vec{b}-\vec{a}\|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are	(r) 4
(d) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(s) 5
	(t) 6

(IIT-JEE 2010)

4. Match List I with List II and select the correct answer using the codes given below the lists:

List I	List II
(p) $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2+y^4\right)^{1/2}$ takes value	(1) $\frac{1}{2}\sqrt{\frac{5}{3}}$
(q) If $\cos x+\cos y+\cos z=0=\sin x+\sin y+\sin z$ then possible value of $\cos\frac{x-y}{2}$ is	(2) $\sqrt{2}$
(r) If $\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x=\cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$ then possible value of $\sec x$ is	(3) 1/2
(s) If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}(x\sqrt{6})\right)$, $x\neq 0$, then possible value of x is	(4) 1

Codes:

	(p)	(q)	(r)	(s)
a.	(4)	(3)	(1)	(2)
b.	(4)	(3)	(2)	(1)
c.	(3)	(4)	(2)	(1)
d.	(3)	(4)	(1)	(2)

(JEE Advanced 2013)

5. Match List I with List II and select the correct answer using the codes given below the lists:

List I	List II
(p) Let $y(x)=\cos(3\cos^{-1}x)$, $x\in[-1,1]$, $x\neq\pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}\right\}$ equals	(1) 1
(q) Let $A_1, A_2, \ldots, A_n$ $(n>2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point A_k, $k=1,2,\ldots n$. If $\left\lvert\sum_{k=1}^{n-1}(\vec{a}_k\times\vec{a}_{k+1})\right\rvert=\left\lvert\sum_{k=1}^{n-1}(\vec{a}_k\cdot\vec{a}_{k+1})\right\rvert$, then the minimum value of n is	(2) 2
(r) If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line $x+y=8$, then the value of h is	(3) 8
(s) Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is	(4) 9

Codes:

	(p)	(q)	(r)	(s)
a.	(4)	(3)	(2)	(1)
b.	(2)	(4)	(3)	(1)
c.	(4)	(3)	(1)	(2)
d.	(2)	(4)	(1)	(3)

(JEE Advanced 2014)

Integer Answer Type

1. Let $f: [0, 4\pi]\to[0,\pi]$ be defined by $f(x)=\cos^{-1}(\cos x)$. The number of points $x\in[0, 4\pi]$ satisfying the equation $f(x)=\frac{10-x}{10}$ is ______ **(JEE Advanced 2014)**

Fill in the Blanks Type

1. Let a, b, and c be positive real numbers.
Let $\theta=\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}}+\tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$. Then $\tan\theta=$ ________.
(IIT-JEE 1981)

2. The numerical value of $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)$ is equal to ________.
(IIT-JEE 1984)

3. The greater of the two angles $A=2\tan^{-1}(2\sqrt{2}-1)$ and $B=3\sin^{-1}(1/3)+\sin^{-1}(3/5)$ is ________.
(IIT-JEE 1989)

Subjective Type

1. Find the value of $\cos(2\cos^{-1}x+\sin^{-1}x)$ at $x=1/5$, where $0\le\cos^{-1}x\le\pi$ and $-\pi/2\le\sin^{-1}x\le\pi/2$.
(IIT-JEE 1981)

2. Prove that $\cos\tan^{-1}\sin\cot^{-1}x=\sqrt{\frac{x^2+1}{x^2+2}}$.
(IIT-JEE 2002)

Answer Key

JEE Advanced

Single Correct Answer Type

1. d.	**2.** e.	**3.** d.	**4.** c.	**5.** b.
6. a.	**7.** d.	**8.** c.	**9.** b.	

Multiple Correct Answers Type

1. b., c., d.

Matching Column Type

1. (i) – (b).
2. (a) – (p); (b) – (q); (c) – (p); (d) – (s)
3. (b) – (p), (r).
4. (b)
5. (a)

Integer Answer Type

1. (3)

Fill in the Blanks Type

1. 0 **2.** $\frac{-7}{17}$ **3.** A

Subjective Type

1. $\frac{-2\sqrt{6}}{5}$

Hints and Solutions

2. e. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ = Principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$

3. d. $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right)$

$$= \tan[\tan^{-1}7-\tan^{-1}4] = \tan\left(\tan^{-1}\left(\frac{3}{29}\right)\right)=\frac{3}{29}$$

4. c. We have $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

$\Rightarrow \quad \tan^{-1}\sqrt{x(x+1)}=\frac{\pi}{2}-\sin^{-1}\sqrt{x^2+x+1}$

$\Rightarrow \quad \tan^{-1}\sqrt{x(x+1)}=\cos^{-1}\sqrt{x^2+x+1}$

$\Rightarrow \quad \tan^{-1}\sqrt{x(x+1)}=\tan^{-1}\frac{\sqrt{-x^2-x}}{\sqrt{x^2+x+1}}$

$\Rightarrow \quad \sqrt{x^2+x}=\frac{\sqrt{-x^2-x}}{\sqrt{x^2+x+1}}$

$\Rightarrow \quad x^2+x=0$

$\Rightarrow \quad x=0,-1$

Alternative method:

$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

$\Rightarrow \quad \cos^{-1}\frac{1}{\sqrt{x^2+x+1}}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

$\Rightarrow \quad \frac{1}{\sqrt{x^2+x+1}}=\sqrt{x^2+x+1}$

$\Rightarrow \quad x^2+x+1=1$

$\Rightarrow \quad x=0,-1$

5. b. $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}+\cdots\right)=\frac{\pi}{2}$

or $\quad \cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}\cdots\right)$

$$=\frac{\pi}{2}-\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}\cdots\right)$$

$$=\cos^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}\cdots\right)$$

or $\quad x^2-\frac{x^4}{2}+\frac{x^6}{4}\cdots=x-\frac{x^2}{2}+\frac{x^3}{4}\cdots$

We have G.P. of infinite terms on both sides. Therefore,

$$\frac{x}{1-\left(-\frac{x}{2}\right)}=\frac{x^2}{1-\left(\frac{-x^2}{2}\right)}$$

or $\quad \frac{2x^2}{2+x^2}=\frac{2x}{2+x}$ $\quad$ or $\quad 2x^2+x^3=2x+x^3$

or $\quad x(x-1)=0$ $\quad$ or $\quad x=0,1$

but $\quad 0<|x|<\sqrt{2}$ $\quad \Rightarrow \quad x=1$

JEE Advanced

Single Correct Answer Type

1. d. $\tan\left(\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right)$

$$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{(3/4)+(2/3)}{1-(3/4)\times(2/3)}\right)\right)$$

$$=\frac{17}{12}\times\frac{12}{6}=\frac{17}{6}$$

6. a. For $f(x)=\sqrt{\sin^{-1}(2x)+\frac{\pi}{6}}$ to be defined and real

$$\sin^{-1}2x + (\pi/6) \geq 0$$

$$\Rightarrow \quad \sin^{-1}2x \geq -\frac{\pi}{6} \qquad \text{(i)}$$

But we know that $-\pi/2 \leq \sin^{-1}2x \leq \pi/2$ (ii)

From Eqs. (i) and (ii), we get

$$-\frac{\pi}{6} \leq \sin^{-1}2x \leq \frac{\pi}{2}$$

or $\sin\left(-\frac{\pi}{6}\right) \leq 2x \leq \sin\left(\frac{\pi}{2}\right)$

$$\Rightarrow \quad -\frac{1}{2} \leq 2x \leq 1$$

or $-\frac{1}{4} \leq x \leq \frac{1}{2}$

$$\therefore \quad D_f = \left[-\frac{1}{4}, \frac{1}{2}\right]$$

7. d. $\sin\left[\cot^{-1}(x+1)\right] = \sin\left(\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right)$

$$= \frac{1}{\sqrt{x^2+2x+2}}$$

$$\cos\left(\tan^{-1}x\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$$= \frac{1}{\sqrt{1+x^2}}$$

Thus, $\frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}}$

or $x^2+2x+2 = 1+x^2$ or $x = -\frac{1}{2}$

8. c. $\sqrt{1+x^2}[x\cos\cot^{-1}x + \sin\cot^{-1}x)^2 - 1]^{1/2}$

$$= \sqrt{1+x^2}\left[\left(x\cos\cos^{-1}\frac{x}{\sqrt{1+x^2}} + \sin\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}\left[\left(\frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}(x^2+1-1)^{1/2} = x\sqrt{1+x^2}$$

9. b. $\cot\left(\sum_{n=1}^{23}\cot^{-1}(n^2+n+1)\right)$

$$= \cot\left(\sum_{n=1}^{23}\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)\right)$$

$$= \cot\left(\sum_{n=1}^{23}(\tan^{-1}(n+1) - \tan^{-1}n)\right)$$

$$= \cot(\tan^{-1}24 - \tan^{-1}1) \Rightarrow \cot\left(\tan^{-1}\left(\frac{23}{25}\right)\right)$$

$$= \frac{25}{23}$$

Multiple Correct Answers Type

1. b., c., d.

$$\alpha = 3\sin^{-1}\frac{6}{11} > 3\sin^{-1}\frac{6}{12} = 3\sin^{-1}\frac{1}{2} = \frac{\pi}{2}$$

and $\beta = 3\cos^{-1}\frac{4}{9} > 3\cos^{-1}\frac{4}{8} = \pi$

So, $\alpha > \frac{\pi}{2}$ and $\beta > \pi$

$$\therefore \quad \alpha + \beta > \frac{3\pi}{2}$$

Matching Column Type

1. (i) – (b)

$$S_n = \sum_{i=1}^{n}\tan^{-1}\left[\frac{1}{2i^2}\right]$$

$$= \sum_{i=1}^{n}\tan^{-1}\left[\frac{2}{4i^2}\right]$$

$$= \sum_{i=1}^{n}\tan^{-1}\left[\frac{2}{1+4i^2-1}\right]$$

$$= \sum_{i=1}^{n}\tan^{-1}\left[\frac{(2i+1)-(2i-1)}{1+(2i+1)(2i-1)}\right]$$

$$= \sum_{i=1}^{n}[\tan^{-1}(2i+1) - \tan^{-1}(2i-1)]$$

$$= [(\tan^{-1}3 - \tan^{-1}1) + (\tan^{-1}5 - \tan^{-1}3) + \ldots + (\tan^{-1}(2n+1) - \tan^{-1}(2n-1))]$$

$$= \tan^{-1}(2n+1) - \tan^{-1}1$$

$$\therefore \quad S_\infty = \tan^{-1}(2\times\infty+1) - \tan^{-1}1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Note: Solutions of the remaining parts are given in their respective chapters.

2. (a) – (p); (b) – (q); (c) – (p); (d) – (s)

$$\sin^{-1}(ax) + \cos^{-1}y + \cos^{-1}(bxy) = \frac{\pi}{2}$$

or $\cos^{-1}y + \cos^{-1}(bxy) = \frac{\pi}{2} - \sin^{-1}(ax)$

$$= \cos^{-1}(ax)$$

Let $\cos^{-1}y = \alpha$, $\cos^{-1}(bxy) = \beta$, $\cos^{-1}(ax) = \gamma$.

Then, $y = \cos\alpha$, $bxy = \cos\beta$, $ax = \cos\gamma$

Therefore, we get $\alpha + \beta = \gamma$

$$\Rightarrow \quad \cos(\gamma - \alpha) = bxy$$

$$\Rightarrow \quad \cos\gamma\cos\alpha + \sin\gamma\sin\alpha = bxy$$

$$\Rightarrow \quad (a-b)xy = -\sin\alpha\sin\gamma$$

$$\Rightarrow \quad (a-b)^2x^2y^2 = \sin^2\alpha\sin^2\gamma$$

$$= (1-\cos^2\alpha)(1-\cos^2\gamma)$$

$$\Rightarrow \quad (a-b)^2x^2y^2 = (1-a^2x^2)(1-y^2) \qquad \text{(i)}$$

a. For $a = 1$, $b = 0$, Eq. (i) reduces to

$$x^2y^2 = (1-x^2)(1-y^2) \text{ or } x^2+y^2 = 1$$

b. For $a = 1$, $b = 1$, Eq. (i) becomes $(1 - x^2)(1 - y^2) = 0$

or $\quad (x^2 - 1)(y^2 - 1) = 0$

c. For $a = 1$, $b = 2$, Eq. (i) reduces to

$$x^2y^2 = (1 - x^2)(1 - y^2)$$

or $\quad x^2 + y^2 = 1$

d. For $a = 2$, $b = 2$, Eq. (i) reduces to

$$0 = (1 - 4x^2)(1 - y^2)$$

or $\quad (4x^2 - 1)(y^2 - 1) = 0$

3. (b) – (p), (r)

$$\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}(3/5)$$

$$\Rightarrow \quad \tan^{-1}\frac{(x+3)-(x-3)}{1+(x^2-9)} = \tan^{-1}\frac{3}{4}$$

$$\Rightarrow \quad \frac{6}{x^2-8} = \frac{3}{4}$$

$$\therefore \quad x^2 - 8 = 8$$

or $\quad x = \pm 4$

Clearly, both values satisfy the equation.

Note: Solutions of the remaining parts are given in their respective chapters.

4. (b)

(p) $\dfrac{\cos(\tan^{-1} y) + y\sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)}$

$$= \frac{\dfrac{1}{\sqrt{1+y^2}} + \dfrac{y^2}{\sqrt{1+y^2}}}{\dfrac{\sqrt{1-y^2}}{y} + \dfrac{y}{\sqrt{1-y^2}}} = \frac{\sqrt{1+y^2}}{\dfrac{1}{y\sqrt{1-y^2}}}$$

$$= y\sqrt{1-y^4}$$

$$\Rightarrow \quad \frac{1}{y^2}\left(\frac{\cos(\tan^{-1} y) + y\sin(\tan^{-1} y)}{\cos(\sin^{-1} y) + \tan(\sin^{-1} y)}\right)^2 + y^4$$

$$= \frac{1}{y^2}(y^2(1 - y^4)) + y^4$$

$$= 1 - y^4 + y^4 = 1$$

(s) $\cot(\sin^{-1}\sqrt{1-x^2}) = \sin(\tan^{-1}(x\sqrt{6}))$

$$\Rightarrow \quad \cot\left(\cot^{-1}\frac{x}{\sqrt{1-x^2}}\right) = \sin\left(\sin^{-1}\frac{x\sqrt{6}}{\sqrt{1+6x^2}}\right)$$

$$\Rightarrow \quad \frac{x}{\sqrt{1-x^2}} = \frac{x\sqrt{6}}{\sqrt{6x^2+1}}$$

$$\Rightarrow \quad 6x^2 + 1 = 6 - 6x^2$$

$$\Rightarrow \quad 12x^2 = 5$$

$$\Rightarrow \quad x = \sqrt{\frac{5}{12}} = \frac{1}{2}\sqrt{\frac{5}{3}}$$

Note: Solutions of the remaining parts are given in their respective chapters.

5. (a)

(s) $\tan^{-1}\left(\dfrac{1}{2x+1}\right) + \tan^{-1}\left(\dfrac{1}{4x+1}\right) = \tan^{-1}\dfrac{2}{x^2}$

$$\Rightarrow \quad \tan^{-1}\left(\frac{3x+1}{4x^2+3x}\right) = \tan^{-1}\frac{2}{x^2}$$

$$\Rightarrow \quad 3x^2 - 7x - 6 = 0$$

$$\Rightarrow \quad x = -\frac{2}{3}, 3$$

But for $x = -\dfrac{2}{3}$, L.H.S. is negative and R.H.S. is positive.

Hence, the only solution is $x = 3$.

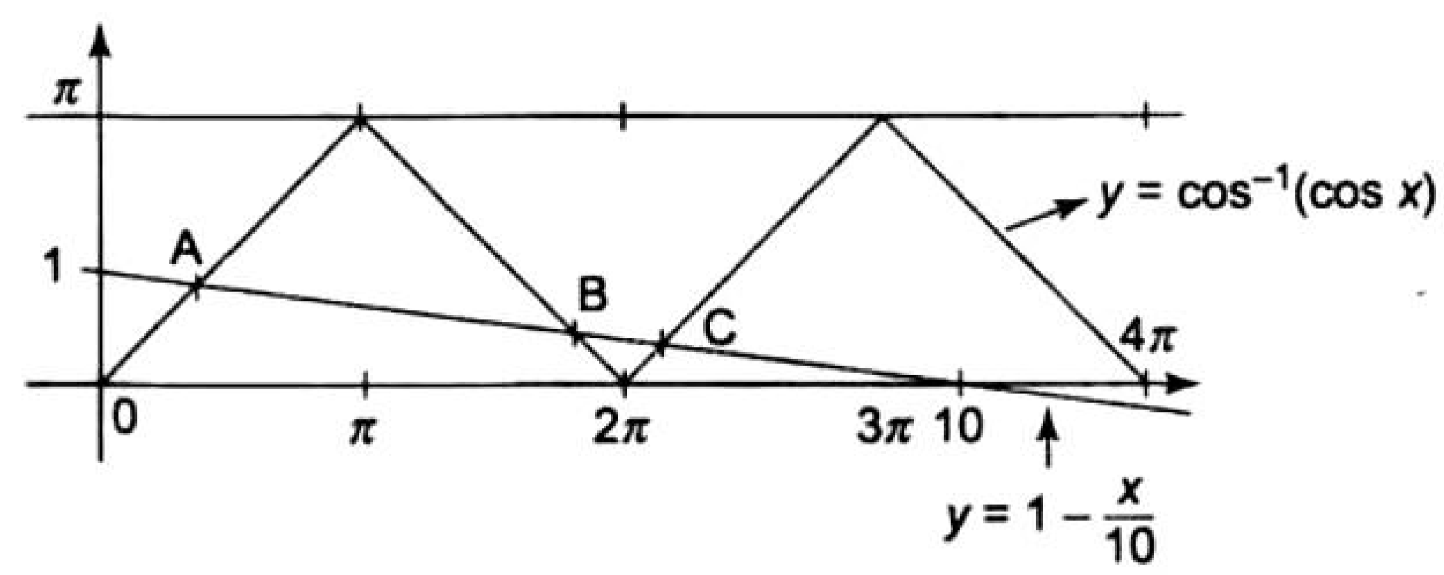
Note: Solutions of the remaining parts are given in their respective chapters.

Integer Answer Type

1. (3) $f: [0, 4\pi] \to [0, \pi]$, $f(x) = \cos^{-1}(\cos x)$

For number of roots of equation $\cos^{-1}(\cos x) = \dfrac{10-x}{10}$, draw the graph of $y = \cos^{-1}(\cos x)$

and $\quad y = \dfrac{10-x}{10}$ and find the points of intersection.



From the graph number of solutions is 3.

Fill in the Blanks Type

1. $\theta = \tan^{-1}\sqrt{\dfrac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\dfrac{b(a+b+c)}{ca}}$

$$+ \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$$

Now, $\sqrt{\dfrac{a(a+b+c)}{bc}}\sqrt{\dfrac{b(a+b+c)}{ca}}$

$$= \frac{a+b+c}{c} = 1 + \frac{b}{c} + \frac{a}{c} > 1$$

$$\Rightarrow \quad \theta = \pi + \tan^{-1}\frac{\sqrt{\dfrac{a(a+b+c)}{bc}} + \sqrt{\dfrac{b(a+b+c)}{ca}}}{1 - \sqrt{\dfrac{a(a+b+c)}{bc}} \times \sqrt{\dfrac{b(a+b+c)}{ca}}}$$

$$+ \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$$

$$= \pi + \tan^{-1}\frac{\sqrt{\dfrac{a+b+c}{c}}\left(\sqrt{\dfrac{a}{b}} + \sqrt{\dfrac{b}{a}}\right)}{1 - \dfrac{a+b+c}{c}}$$

$$+ \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$$

$$= \pi + \tan^{-1}\left(-\sqrt{\frac{c(a+b+c)}{ab}}\right)$$

$$+ \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}} = \pi \quad \text{or} \quad \tan\theta = 0$$

2. $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$

$$= \tan\left(\tan^{-1}\left(\frac{2/5}{1-(1/5)^2}\right) - \tan^{-1}(1)\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}(1)\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{(5/12)-1}{1+(5/12)}\right)\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{-7}{17}\right)\right) = \frac{-7}{17}$$

3. We have

$$A = 2\tan^{-1}(2\sqrt{2}-1)$$

$$= 2\tan^{-1}(2\times 1.414 - 1)$$

$$= 2\tan^{-1}(1.828) > 2\tan^{-1}\sqrt{3} = \frac{2\pi}{3}$$

$$\Rightarrow \quad A > \left(\frac{2\pi}{3}\right) \qquad \text{(i)}$$

Also, $B = 3\sin^{-1}(1/3) + \sin^{-1}(3/5)$

$$= \sin^{-1}\left[3\times\frac{1}{3} - 4\times\frac{1}{27}\right] + \sin^{-1}(3/5)$$

$$= \sin^{-1}\left(\frac{23}{27}\right) + \sin^{-1}(0.6)$$

$$= \sin^{-1}(0.852) + \sin^{-1}(0.6)$$

$$< \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \quad B < (2\pi/3) \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we conclude $A > B$.

Subjective Type

1. $\cos(2\cos^{-1}x + \sin^{-1}x)$

$$= \cos\left(\cos^{-1}x + \frac{\pi}{2}\right)$$

$$= -\sin(\cos^{-1}x) = -\sin\left(\sin^{-1}\sqrt{1-x^2}\right)$$

$$= -\sqrt{1-x^2}$$

At $x = \frac{1}{5}$, value $= -\sqrt{1-\frac{1}{25}} = -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$

2. L.H.S. = $\cos(\tan^{-1}(\sin(\cot^{-1}x)))$

$$= \cos\left(\tan^{-1}\left(\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right)\right) \text{ if } x > 0$$

and $$\cos\left(\tan^{-1}\left(\sin\left(\pi - \sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right)\right) \text{ if } x < 0$$

In each case,

$$\text{L.H.S.} = \cos\left(\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \cos\left(\cos^{-1}\sqrt{\frac{1+x^2}{2+x^2}}\right)$$

$$= \sqrt{\frac{x^2+1}{x^2+2}}$$