2 Arithmetic Progressions

TOPICS COVERED

1. Sequence/Progression
2. Arithmetic Progressions and its nth term
3. Sum of First n Terms of an AP

1. SEQUENCE/PROGRESSION

- **Sequence/Progression:** A sequence/progression is a succession of numbers or terms formed according to some pattern or rule. Various numbers occurring in a sequence are called terms or elements.

  Consider the following arrangements of numbers:

  (*i*) 1, 8, 27, 64, 125, ... (*ii*) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...$ (*iii*) 2, 4, 6, 8, 10, ...

  In each of the above arrangements, numbers are arranged in a definite order according to some rule. So, they are sequences.

  A sequence is generally written as $< a_n > : a_1, a_2, a_3, ..., a_n$ where $a_1, a_2, a_3, ...$ are the first, second and third terms of the sequence.
- A sequence with finite number of terms or numbers is called a **finite sequence.**
- A sequence with infinite number of terms or numbers is called an **infinite sequence.**

Example 1. Write first four terms of each of the following sequence, whose general terms are:

(*i*) $a_n = 3n - 7$ (*ii*) $a_n = (-1)^{n+1} \times 3^n$

Solution. (*i*) $a_n = 3n - 7$

$\therefore$ $a_1 = 3 \times 1 - 7 = 3 - 7 = -4, a_2 = 3 \times 2 - 7 = 6 - 7 = -1,$

$a_3 = 3 \times 3 - 7 = 9 - 7 = 2$ and $a_4 = 3 \times 4 - 7 = 12 - 7 = 5$

(*ii*) $a_n = (-1)^{n+1} \times 3^n$

$\Rightarrow$ $a_1 = (-1)^{1+1} \times 3^1 = 3,$

$\therefore$ $a_2 = (-1)^{2+1} \times 3^2 = (-1)^3 \times 3^2 = -9,$

$a_3 = (-1)^4 \times 3^3 = 27$ and $a_4 = (-1)^5 \times 3^4 = -81$

Example 2. What is 18^{th} term of the sequence defined by $a_n = \frac{n(n-3)}{n+4}$?

Solution. We have, $a_n = \frac{n(n-3)}{n+4}$

Putting $n = 18$, we get $a_{18} = \frac{18 \times (18-3)}{18+4}$

$= \frac{18 \times 15}{22} = \frac{135}{11}$

Exercise 2.1

I. Very Short Answer Type Questions [1 Mark]

1. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

(1) If $a_n = 5n - 4$ is a sequence, then a_{12} is

(*a*) 48 (*b*) 52 (*c*) 56 (*d*) 62

(2) If $a_n = 3n - 2$, then the value of $a_7 + a_8$ is

(*a*) 39 (*b*) 41 (*c*) 47 (*d*) 53

(3) The second term of the sequence defined by $a_n = 3n + 2$ is

(*a*) 2 (*b*) 4 (*c*) 6 (*d*) 8

2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(*c*) Assertion (A) is true but reason (R) is false.
(*d*) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** The arrangement of numbers, *i.e.*, – 4, 16, – 64, 256, – 1024, 4096, ... form a sequence.
Reason (R): An arrangement of numbers which are arranged in a definite order according to some rule, is called a sequence.

(2) **Assertion (A):** Sequence 1, 5, 9, 13, 17, 21, ... is a finite sequence.
Reason (R): A sequence with finite number of terms or numbers is called a finite sequence.

3. Answer the following:

(1) Write down the first six terms of each of the following sequences, whose general terms are:

(*a*) $a_n = 5n - 3$ (*b*) $a_n = (-1)^n \cdot 2^{2n}$ (*c*) $a_n = \dfrac{2n+1}{n+2}$ (*d*) $a_n = (-1)^{n-1} \cdot n^2$

(2) Find the 10^{th} term of the sequence defined by $a_n = (-1)^{2n-1} \cdot 5^n$.

(3) Find the difference between the 12^{th} term and 10^{th} term of the sequence whose general term is given by $a_n = 5n - 1$.

Answers

1. (1) (*c*) 56 (1) (2) (*b*) 41 (1)
(3) (*d*) 8 (1)

2. (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(2) (*d*) Assertion (A) is false but reason (R) is true. (1)

3. (1) (*a*) 2, 7, 12, 17, 22, 27
(*b*) –4, 16, –64, 256, –1024, 4096
(*c*) $1, \frac{5}{4}, \frac{7}{5}, \frac{3}{2}, \frac{11}{7}, \frac{13}{8}$
(*d*) 1, –4, 9, –16, 25, –36 (1)
(2) –9765625 (1)
(3) 10 (1)

2. Arithmetic Progression and its *n*th Term

- An arithmetic progression is a sequence of numbers in which each term is obtained by adding a fixed number '*d*' to the preceding term, except the first term '*a*'. This fixed number is known as **common difference** of the AP. Common difference of an AP can be negative, positive or zero.
  The general form of an AP is $a, a + d, a + 2d, a + 3d, ...$
  Examples:
  (*i*) The sequence 1, 4, 7, 10, 13, ... is an AP whose first term is 1 and the common difference is equal to 3.
  (*ii*) The sequence 11, 7, 3, –1, ... is an AP whose first term is 11 and the common difference is equal to –4.
- In the list of numbers $a_1, a_2, a_3, ...$ if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, ...$ give the same value, *i.e.*, if $a_{k+1} - a_k$ is the same for different values of k, then the given list of numbers is an AP.
- The n^{th} term a_n (or the general term) of an AP is $a_n = a + (n - 1) d$, where a is the first term, d is the common difference and n is the number of terms. Also, $d = a_{n+1} - a_n$.
- If three terms a, b and c are in AP, then $b - a = c - b$ or $2b = a + c$.
- If l is the last term of an AP, then n^{th} term from the end of the AP $= l + (n - 1)(-d) = l - (n - 1)d$.

Example 1. In an AP, if $d = -4$, $n = 7$, $a_n = 4$, then find the value of a. [CBSE Standard SP 2020-21, Delhi 2018]

Solution. We have $a_n = 4$ for $n = 7$

$\therefore$ $a_n = a + (n - 1)\, d \Rightarrow 4 = a + 6(-4) \Rightarrow a = 28$

Example 2. Is 0 a term of the AP: 31, 28, 25, ...? Justify your answer. [NCERT Exemplar]

Solution. Given AP is 31, 28, 25, ...

Here, $a = 31, d = 28 - 31 = -3 = 25 - 28$

For 0 be a term of this AP, $0 = a_n$ for some 'n' $\Rightarrow 0 = a + (n - 1)d$

$\Rightarrow$ $0 = 31 + (n - 1)(-3) \Rightarrow 31 - 3n + 3 = 0$

$\Rightarrow$ $-3n = -34 \Rightarrow n = \dfrac{34}{3} = 11\dfrac{1}{3}$

which is not possible as n cannot be a fraction.

Therefore, 0 cannot be a term of this AP.

Example 3. Find the 12th term from the end of the AP: –2, – 4, – 6, ..., – 100. **[Imp]**

Solution. Let a be the first term, d the common difference and l the last term of AP.

Here, $a = -2$, $d = (-4 + 2) = -2$, $l = -100$ and $n = 12$

$\therefore$ n^{th} term from end $= l - (n - 1)d$

$\Rightarrow$ 12^{th} term from end $= -100 - (12 - 1)(-2) = -100 + 24 - 2 = -78$

Example 4. For what value of x: $2x$, $x + 10$ and $3x + 2$ are in AP?

Solution. Since, given numbers are in AP.

So, $(x + 10) - 2x = (3x + 2) - (x + 10)$

$\Rightarrow$ $-x + 10 = 2x - 8$ or $3x = 18$ or $x = 6$

Example 5. Find the 25th term of the AP: $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$

Solution. We have, $a = -5,\ d = \frac{-5}{2} - (-5) = \frac{5}{2}$

$\because$ $a_n = a + (n - 1)d$

$\therefore$ $a_{25} = (-5) + (25 - 1)\frac{5}{2} = (-5) + 24\left(\frac{5}{2}\right) = -5 + 60 = 55$

Example 6. Find the 20th term from the last term of the AP: 3, 8, 13,..., 253. [NCERT] **[Imp.]**

Solution. Given, last term $= l = 253$

And, common difference $= d = 8 - 3 = 5 = 13 - 8$

$\therefore$ 20^{th} term from end $= l - (n - 1) \times d = 253 - 19 \times 5 = 253 - 95 = 158$

Example 7. Which term of the AP: 3, 8, 13, 18, ..., is 78? [NCERT] **[Imp.]**

Solution. Let a_n be the required term of the AP: 3, 8, 13, 18,...

Here, $a = 3$, $d = 8 - 3 = 5$ and $a_n = 78$

Now, $a_n = a + (n - 1)d$

$\Rightarrow$ $78 = 3 + (n - 1) \times 5 \quad \Rightarrow \quad 78 - 3 = (n - 1) \times 5$

$\Rightarrow$ $75 = (n - 1) \times 5 \quad \Rightarrow \quad \frac{75}{5} = n - 1$

$\Rightarrow$ $15 = n - 1 \quad \Rightarrow \quad n = 15 + 1 = 16$

Hence, 16th term of given AP is 78.

Example 8. The sum of the 5th and 7th terms of an AP is 52 and the 10th term is 46. Find the AP.

[NCERT Exemplar] **[Imp.]**

Solution. Let the first term and the common difference of an AP be 'a' and 'd'.

$\therefore$ $a_5 = a + 4d$ and $a_7 = a + 6d$

So, $a_5 + a_7 = 2a + 10d = 52 \quad \Rightarrow \quad 2a + 10d = 52$ ...(i)

Also, $a_{10} = a + 9d = 46 \quad \Rightarrow \quad a + 9d = 46$ ...(ii)

From (i) and (ii), $d = 5$ and $a = 1$

So, the AP is as follows 1, 6, 11, 16, 21, ...

Example 9. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

[CBSE SP 2018-19] [NCERT] **[Imp.]**

Solution. Let a be the first term and d be the common difference.

Since, given AP has 50 terms, so $n = 50$

$\because$ $a_3 = 12 \quad \Rightarrow \quad a + (3 - 1)d = 12$

$\Rightarrow$ $a + 2d = 12$ ...(i)

Also, $a_{50} = 106 \quad \Rightarrow \quad a + (50 - 1)d = 106$

$\Rightarrow$ $a + 49d = 106$ ...(ii)

Subtracting (i) from (ii), we get

$$47d = 94 \quad \Rightarrow \quad d = \frac{94}{47} = 2$$

Putting the value of d in equation (i), we get

$$a + 2 \times 2 = 12 \quad \Rightarrow \quad a = 12 - 4 = 8$$

Here, $a = 8, d = 2$

So, 29th term of the AP is given by

$$a_{29} = a + (29 - 1)d = 8 + 28 \times 2 \quad \Rightarrow \quad a_{29} = 8 + 56 \quad \Rightarrow \quad a_{29} = 64$$

Example 10. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73. [NCERT]

Solution. Let the first term be a and common difference be d.

Now, given $a_{11} = 38 \quad \Rightarrow \quad a + (11 - 1)d = 38$

$\Rightarrow \quad a + 10d = 38$ ...(*i*)

Also, $a_{16} = 73 \quad \Rightarrow \quad a + (16 - 1)d = 73$

$\Rightarrow \quad a + 15d = 73$ ...(*ii*)

Now, subtracting (*ii*) from (*i*), we get

$$\begin{array}{r} a + 10d = 38 \\ a + 15d = 73 \\ - \quad - \quad\quad - \\ \hline -5d = -35 \end{array} \quad \Rightarrow \quad 5d = 35$$

$$\Rightarrow \quad d = \frac{35}{5} = 7$$

Putting the value of d in equation (*i*), we get

$$a + 10 \times 7 = 38 \quad \Rightarrow \quad a + 70 = 38$$

$$\Rightarrow \quad a = 38 - 70 \quad \Rightarrow \quad a = -32$$

We have $a = -32$ and $d = 7$

Therefore, $a_{31} = a + (31 - 1)d = a + 30d$

$$\Rightarrow \quad a_{31} = (-32) + 30 \times 7 = -32 + 210 \quad \Rightarrow \quad a_{31} = 178$$

Example 11. The first term of an AP is x and its common difference is y. Find its 12th term.

Solution. $a_{12} = a + 11d = x + 11y$.

Exercise 2.2

I. Very Short Answer Type Questions [1 Mark]

1. **Multiple Choice Questions (MCQs)**

Choose the correct answer from the given options:

(1) In an AP, if $d = -4$, $n = 7$, $a_n = 4$, then a is

(*a*) 6 (*b*) 7 (*c*) 20 (*d*) 28

(2) The n^{th} term of the AP: $a, 3a, 5a, \ldots$ is

(*a*) na (*b*) $(2n - 1)a$ (*c*) $(2n + 1)a$ (*d*) $2na$

(3) The first term of an AP is p and the common difference is q, then its 10th term is

(*a*) $q + 9p$ (*b*) $p - 9q$ (*c*) $p + 9q$ (*d*) $2p + 9p$

(4) If $\frac{4}{5}$, a, 2 are three consecutive terms of an AP, then the value of a is

(*a*) $\frac{5}{2}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{5}{7}$ (*d*) $\frac{7}{5}$

2. **Assertion-Reason Type Questions**

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** Common difference of the AP: –5, –1, 3, 7, ... is 4.
Reason (R): Common difference of the AP : $a, a + d, a + 2d, \ldots$ is given by $d = 2^{nd}$ term – 1st term.

(2) **Assertion (A):** If n^{th} term of an AP is $7 - 4n$, then its common difference is – 4.
Reason (R): Common difference of an AP is given by $d = a_{n+1} - a_n$.

(3) **Assertion (A):** Common difference of an AP in which $a_{21} - a_7 = 84$ is 14.
Reason (R): n^{th} term of an AP is given by $a_n = a + (n - 1)d$.

3. **Answer the following:**

(1) Write first four terms of the AP, whose first term and the common difference are given as follows: $a = 10, d = 10$

(2) Find the 10^{th} term of the AP: 2, 7, 12, ... [NCERT] **[Imp.]**

(3) In the given AP, find the missing terms: ...., 13, ..., 3. [NCERT]

(4) Find the 6^{th} term from the end of the AP: 17, 14, 11, ..., –40. **[Imp.]**

(5) Which term of the AP: 21, 18, 15, ... is zero? [Delhi 2008 (C)] **[Imp.]**

(6) Write the next term of the AP: $\sqrt{8}, \sqrt{18}, \sqrt{32}$, ... [AI 2008]

(7) Find a, b, and c such that the numbers a, 7, b, 23, c are in AP. [NCERT Exemplar]

(8) Find the 9^{th} term from the end (towards the first term) of the AP: 5, 9, 13, ..., 185. [Delhi 2016]

(9) For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an AP? [Delhi 2016]

(10) For what value of k will the consecutive terms $2k + 1$, $3k + 3$ and $5k - 1$ form an AP? [Foreign 2016]

(11) Find the eleventh term from the last term of the AP: 27, 23, 19, ..., –65. [CBSE Sample Paper 2018]

(12) If the first three terms of an AP are b, c and $2b$, then find the ratio of b and c. [CBSE Standard SP 2019-20]

(13) Find the value of x so that –6, x, 8 are in AP.

(14) Find the 11^{th} term of the AP: –27, –22, –17, –12, ...

(15) The n^{th} term of an AP is $(7 - 4n)$, then what is its common difference?

(16) Find the common difference of the AP whose first term is 12 and fifth term is 0.

II. Short Answer Type Questions - I [2 Marks]

4. Find how many integers between 200 and 500 are divisible by 8. [AI 2017]

5. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$ is the first negative term? [AI 2017]

6. Is –150 a term of the AP: 17, 12, 7, 2, ...? [Delhi 2011]

7. Find the number of two-digit numbers which are divisible by 6. [AI 2011]

8. Which term of the AP: 3, 14, 25, 36, ... will be 99 more than its 25^{th} term? [AI 2011]

9. Which term of the AP: 3, 15, 27, 39, ... will be 120 more than its 21^{st} term? [Delhi 2019]

10. How many natural numbers are there between 200 and 500, which are divisible by 7? [AI 2011]

11. How many two-digit numbers are divisible by 7? [Foreign 2011]

12. How many two digits numbers are divisible by 3? [Delhi 2019]

13. If $\frac{1}{x+2}, \frac{1}{x+3}$ and $\frac{1}{x+5}$ are in AP, find the value of x. [Foreign 2011]

14. How many three digit numbers are divisible by 11? [AI 2012]

15. In an AP, the first term is 12 and the common difference is 6. If the last term of the AP is 252, find its middle term. [Foreign 2017]

16. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5. [AI 2014]

17. The 4^{th} term of an AP is zero. Prove that the 25^{th} term of the AP is three times its 11^{th} term. [AI 2016]

18. Find the middle term of the AP: 6, 13, 20, ..., 216. [Delhi 2015]

19. The n^{th} term of an AP is $6n + 2$. Find its common difference. [Delhi 2008]

20. Find the 10^{th} term from end of the AP: 4, 9, 14, ..., 254. **[Imp.]**

21. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP. [NCERT Exemplar]

22. Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both. [CBSE Standard SP 2019-20]

III. Short Answer Type Questions - II [3 Marks]

23. Which term of the AP: 115, 110, 105, ... is its first negative term?

24. If the 9^{th} term of an AP is zero, prove that its 29^{th} term is double of its 19^{th} term. [NCERT Exemplar]

25. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle. [NCERT Exemplar]

26. For what value of n, the n^{th} term of two APs: 63, 65, 67, ... and 3, 10, 17, ... are equal. [NCERT]

27. The 8^{th} term of an AP is 37 and its 12^{th} term is 57. Find the AP. **[Imp.]**

28. The p^{th}, q^{th} and r^{th} terms of an AP are a, b and c respectively. Show that $a(q - r) + b(r - p) + c(p - q) = 0$. [Foreign 2016]

29. If the n^{th} terms of two APs: 23, 25, 27, ... and 5, 8, 11, 14, ... are equal, then find the value of n.

IV. Long Answer Type Questions **[5 Marks]**

30. If m times the m^{th} term of an Arithmetic Progression is equal to n times its n^{th} term and $m \neq n$, show that the $(m + n)^{th}$ term of the AP is zero. [AI2019]

31. The 19^{th} term of an AP is equal to three times its sixth term. If its 9^{th} term is 19, find the AP. [AI 2013]

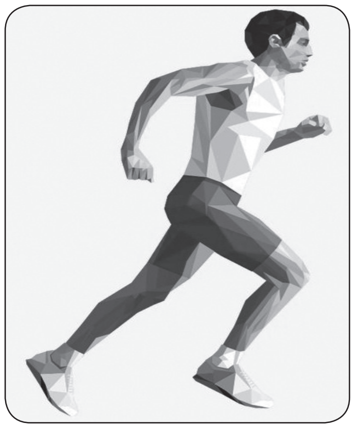
32. The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the AP. **[Imp.]**

33. The eighth term of an AP is half its second term and the eleventh term exceeds one-third of its fourth term by 1. Find the 15^{th} term. **[Imp.]**

34. If 4 times the 4^{th} term of an AP is equal to 18 times the 18^{th} term, then find the 22^{nd} term.

Case Study Based Questions

I. Your friend Veer wants to participate in a 200 m race. Presently, he can run 200 m in 51 seconds and during each day practice it takes him 2 seconds less. He wants to do in 31 seconds.



1. Which of the following terms are in AP for the given situation?
(*a*) 51, 53, 55, ... (*b*) 51, 49, 47, ... (*c*) –51, –53, –55, ... (*d*) 51, 55, 59, …

2. What is the minimum number of days he needs to practice till his goal is achieved?
(*a*) 10 (*b*) 12 (*c*) 11 (*d*) 9

3. Which of the following term is not in the AP of the above given situation?
(*a*) 41 (*b*) 30 (*c*) 37 (*d*) 39

4. If n^{th} term of an AP is given by $a_n = 2n + 3$ then common difference of an AP is
(*a*) 2 (*b*) 3 (*c*) 5 (*d*) 1

5. The value of x, for which $2x$, $x + 10$, $3x + 2$ are three consecutive terms of an AP is
(*a*) 6 (*b*) – 6 (*c*) 18 (*d*) –18

II. India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6^{th} year and 22600 in 9^{th} year.

1. The production during first year is
(*a*) 3000 TV sets (*b*) 5000 TV sets (*c*) 7000 TV sets (*d*) 10000 TV sets

2. The production during 8^{th} year is
(*a*) 10500 (*b*) 11900 (*c*) 12500 (*d*) 20400

3. The production during first 3 years is
(*a*) 12800 (*b*) 19300 (*c*) 21600 (*d*) 25200

4. In which year, the production is 29,200?
(*a*) 10^{th} year (*b*) 12^{th} year (*c*) 15^{th} year (*d*) 18^{th} year

5. The difference of the production during 7^{th} year and 4^{th} year is
(*a*) 6600 (*b*) 6800 (*c*) 5400 (*d*) 7200

Answers and Hints

1. (1) (*d*) 28 (1) (2) (*b*) $(2n-1)a$ (1)
(3) (*c*) $p+9q$ (1) (4) (*a*) $=\frac{7}{5}$ (1)

2. (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(2) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(3) (*d*) Assertion (A) is false but reason (R) is true. (1)

3. (1) 10, 20, 30, 40 (1) (2) 47 (1)
(3) 18, 8 (1) (4) –25 (1)
(5) 8 (1) (6) $\sqrt{50}$ or $5\sqrt{2}$ (1)
(7) $a=-1, b=15, c=31$ (1)
(8) Reversing the given AP, we get
185, 181, 174, ..., 9, 5

Ninth term $a_9 = a+(9-1)d$
$= 185 + 8\times(-4)$
$= 185 - 32$
$= 153$ (1)

(9) Given that $k+9$, $2k-1$ and $2k+7$ are in AP
Then,
$(2k-1)-(k+9) = (2k+7)-(2k-1)$
$\Rightarrow k-10 = 8$
$\Rightarrow k = 18$ (1)

(10) Given that $2k+1$, $3k+3$ and $5k-1$ are in AP.
So, $(3k+3)-(2k+1) = (5k-1)-(3k+3)$
$\Rightarrow k+2 = 2k-4$
$\Rightarrow 2k-k = 2+4$
$\Rightarrow k = 6$ (1)

(11) $a_{11} = -25$ (1)

(12) b, c and $2b$ are in AP
$\Rightarrow c = \frac{3b}{2}$
$\therefore b : c = 2 : 3$ (1)

(13) 1 (1)

(14) 23 (1)

(15) $a_n = 7-4n$
$\Rightarrow a_1 = 7-4\times1 = 3$
$\Rightarrow a_2 = 7-4\times2 = 7-8 = -1$
$a_3 = 7-4\times3 = 7-12 = -5$
Now, $a_2 - a_1 = -1-3 = -4$
$a_3 - a_2 = -5-(-1)$
$= -5+1 = -4$
So, the common difference of AP is –4. (1)

(16) $A_5 = a_1 + 4d = 0$
$12 + 4d = 0$
$d = -3$ (1)

4. AP formed is 208, 216, 224, ..., 496
$a_n = 496$ (1)
$\Rightarrow 208 + (n-1)\times 8 = 496$
$\Rightarrow n = 37$ (1)

5. Here $d = \frac{-3}{4}$ (½)
Let the n^{th} term be first negative term.
$\therefore 20 + (n-1)\left(\frac{-3}{4}\right) < 0 \Rightarrow 3n > 83$ (1)
$\Rightarrow n > 27\frac{2}{3}$
Hence, 28^{th} term is first negative term. (½)

6. Let $a_n = -150$
$a + (n-1)d = -150$
$\Rightarrow 17 + (n-1)(-5) = -150$ (1)
$\Rightarrow (n-1)(-5) = -167$
$\Rightarrow n = \frac{167+5}{5} = \frac{172}{5} = 34\frac{2}{5}$
Here, n is not a natural number.
$\therefore$ –150 is not a term of the given AP. (1)

7. Two-digit numbers which are divisible by 6 are 12, 18, 24, ..., 96
$\because$ Last term,
$a_n = 96$
$\Rightarrow 12 + (n-1)6 = 96$ (1)
$\Rightarrow (n-1)6 = 96-12 = 84$
$\Rightarrow n = 15$
$\therefore$ There are 15 two-digit numbers divisible by 6. (1)

8. Let a_n be the term which is 99 more than 25^{th} term of given AP.
ATQ, $a_n = a_{25} + 99$
$\Rightarrow a + (n-1)d = a + 24d + 99$ (1)
$\Rightarrow 11(n-1) = 24\times 11 + 99$
$\Rightarrow n = 34$
Hence, 34^{th} is the required term. (1)

9. AP: 3, 15, 27, 39, ...
$a = 3, d = 15-3 = 12$
$a_{21} = a + 20d = 3 + 20\times 12$
$= 3 + 240 = 243$
120 more than $a_{21} = 243 + 120 = 363$ (1)
Let 363 be n^{th} term.
So, $363 = 3 + (n-1)\,12$
$\Rightarrow 360 = 12(n-1)$
$30 = n-1 \Rightarrow n = 31$
Thus, 31^{st} term of the given AP is 120 more than its 21^{st} term. (1)

10. Natural numbers between 200 and 500 which are divisible by 7 are as 203, 210, 217, ..., 497

Let above are n numbers and $a_n = 497$

$a + (n - 1)d = 497$ (1)

$\Rightarrow 203 + 7(n - 1) = 497$

$\Rightarrow n = 43$

$\therefore$ There are 43 natural numbers between 200 and 500 divisible by 7. (1)

11. Two-digit numbers which are divisible by 7 are 14, 21, 28, ..., 98.

Let $a_n = 98$

$\Rightarrow a + (n - 1)d = 98$ (1)

$\Rightarrow 14 + 7(n - 1) = 98$

$n = 13$

Hence, there are 13 two-digit numbers which are divisible by 7. (1)

12. 2-digit numbers divisible by 3 are 12, 15, 18, ..., 99 which is in AP.

So, $a_n = 99, d = 15 - 12 = 3$

Now, $a_n = a + (n - 1)\, d$ (1)

$\Rightarrow 99 = 12 + (n - 1)\, 3$

$\Rightarrow 87 = 3\,(n - 1)$

$\Rightarrow 29 = n - 1$

$\Rightarrow n = 30$

Thus, 30, 2-digit numbers are divisible by 3. (1)

13. Given term are in AP

So, $\frac{2}{x+3} = \frac{1}{x+2} + \frac{1}{x+5}$

$\Rightarrow \frac{2}{x+3} = \frac{(x+5)+(x+2)}{(x+2)(x+5)}$ (1)

$\Rightarrow 2x^2 + 14x + 20 = 2x^2 + 13x + 21$

$\therefore x = 1$ (1)

14. Three-digit numbers which are divisible by 11 are 110, 121, 132, ..., 990

Let $a_n = 990$ (1)

$\Rightarrow a + (n - 1)d = 990$

$\Rightarrow 110 + 11(n - 1) = 990$

$\therefore n = 81$

Hence, there are 81 three-digit numbers which are divisible by 11. (1)

15. Let $a_n = 252 = \text{last term}$

$\Rightarrow a + (n - 1)d = 252$

$\Rightarrow 12 + (n - 1)6 = 252$

$\Rightarrow n = 41$ (1)

$\therefore$ Since number of terms is odd, so only one middle term.

Now,middle term $= \left(\frac{41+1}{2}\right)$

$= 21^{st}$ term

$\therefore 21^{st}$ term, $a_{21} = a + 20d$

$= 12 + 20 \times 6$

$= 132$

$=$ middle term value. (1)

16. Numbers between 101 and 999 which are divisible by both 2 and 5 (*i.e.*, by 10) are 110, 120, 130, ... 990.

Now, $a_n = a + (n - 1)d$ (1)

$\Rightarrow 990 = 110 + (n - 1)10$

$\Rightarrow n = 89$

$\therefore$ Natural numbers which are divisible by 2 and 5 both are 89. (1)

17. $a_4 = a + (4 - 1)d$

$0 = a + 3d$

$\Rightarrow a = -3d$ [$\because$ Given, $a_4 = 0$] (1)

Now $a_{25} = a + (25 - 1)d = a + 24d$

$= -3d + 24d = 21d = 3 \times 7d$

Hence, $a_{25} = 3 \times a_{11}$

[$\because$ Since $a_{11} = a + (11 - 1)d = -3d + 10d = 7d$] (1)

18. Given AP is 6, 13, 20, ..., 216

n^{th} term, $a_n = 216$

$\Rightarrow a + (n - 1)d = 216$

$\Rightarrow 6 + 7(n - 1) = 216$

$\Rightarrow 7n = 217$

$\Rightarrow n = 31$ (1)

Since, the number of terms in AP are 31, so, the middle most term is 16^{th} term.

$\left[\because \text{middle term} = \frac{(31+1)}{2} = 16^{th} \text{ term}\right]$

$\therefore 16^{th}$ term, $a_{16} = a + 15d = 6 + 15 \times 7 = 111.$ (1)

19. 6 (2) **20.** 209 (2) **21.** $k = 0$ (2)

22. 110, 120, 130, … , 990

$a_n = 990$

$\Rightarrow 110 + (n - 1) \times 10 = 990$

$\therefore n = 89$ (2)

23. 25^{th} term (3) **25.** 40°, 60°, 80° (3)

26. 13 (3) **27.** 2, 7, 12, 17, 22, ... (3)

28. Let A and d be the first term and common difference of the given AP, then

$a_p = A + (p - 1)d = a$ ...(*i*)

$a_q = A + (q - 1)d = b$ ...(*ii*)

$a_r = A + (r - 1)d = c$ ...(*iii*)

Now, subtracting (*i*) and (*ii*), we get

$(p - q)d = a - b$

$p - q = \frac{a}{d} - \frac{b}{d}$ (1)

Multiplying by 'c' on both sides,

$c(p - q) = \frac{ca}{d} - \frac{cb}{d}$ ...(*iv*)

Now, (*ii*) – (*iii*), we get

$(q - r)d = b - c$

$q - r = \frac{b}{d} - \frac{c}{d}$

Multiplying by 'a' on both sides,

$a(q - r) = \frac{ab}{d} - \frac{ac}{d}$ ...(*v*)(1)

Now, (*iii*) – (*i*), we get

$$(r - p)d = c - a$$

$$(r - p) = \frac{c}{d} - \frac{a}{d}$$

Multiplying by '*b*' on both sides,

$$(r - p)b = \frac{bc}{d} - \frac{ba}{d} \quad ...(vi)$$

Adding (*iv*), (*v*) and (*vi*), we get

$a(q - r) + b(r - p) + c(p - q)$

$$= \frac{ab}{d} - \frac{ac}{d} + \frac{bc}{d} - \frac{ba}{d} + \frac{ca}{d} - \frac{cb}{d} = 0 \quad (1)$$

29. $AP_1 = 23, 25, 27, ...$

Here, $a_1 = 23$

$d_1 = 25 - 23 = 27 - 25 = 2$

$\therefore$ n^{th} term $= a_1 + (n - 1)d_1$

$= 23 + (n - 1)2$

$AP_2 = 5, 8, 11, 14, ...$ (1)

Here, $a_2 = 5$

$d_2 = 8 - 5 = 11 - 8 = 3$

$\therefore$ n^{th} term $= a_2 + (n - 1)d_2$

$= 5 + (n - 1)3$ (1)

Now, $23 + (n - 1)2 = 5 + (n - 1)3$

$\Rightarrow 23 + 2n - 2 = 5 + 3n - 3$

$\Rightarrow 3n - 2n = 23 - 2 - 5 + 3$

$\Rightarrow n = 26 - 7 = 19$ (1)

30. We know that $a_n = a + (n - 1)d$

From the given conditions,

$m[a + (m - 1) d] = n[a + (n - 1)d]$ (1)

$\Rightarrow m[a + (md - d)] = n[a + nd - d]$

$\Rightarrow am + m^2d - md = an + n^2d - nd$ (1)

$\Rightarrow am - an + m^2d - n^2d - md + nd = 0$

$\Rightarrow a(m - n) + d(m^2 - n^2) - d(m - n) = 0$

$\Rightarrow a(m - n) + (m + n)(m - n)d - (m - n)d = 0$ (1)

$\Rightarrow (m - n)[a + (m + n) d - d] = 0$

$\Rightarrow a + md + nd - d = 0$ (1)

$\Rightarrow a + (m + n - 1)d = 0$

Since, $m \neq n$, it is clear that $(m + n)^{th}$ term of the AP is zero. (1)

31. 3, 5, 7, 9,... (5) **32.** –13, –8, –3 (5)

33. 3 (5)

34. Let $a_1, a_2, a_3, ... a_n, ...$ be the AP with its first term *a* and common difference *d*.

It is given that

$4a_4 = 18a_{18}$ (1)

$\Rightarrow 4(a + 3d) = 18(a + 17d)$ (1)

$\Rightarrow 4a + 12d = 18a + 306d$ (1)

$\Rightarrow 14a + 294d = 0 \Rightarrow 14(a + 21d) = 0$ (1)

$\Rightarrow a + 21d = 0 \Rightarrow a + (22 - 1)d = 0$

$\Rightarrow a_{22} = 0$

Thus, 22^{nd} term is 0. (1)

Case Study Based Questions

I. 1. (*b*) 51, 49, 47…. **2.** (*c*) 11

3. (*b*) 30 **4.** (*a*) 2

5. (*a*) 6

II.1. (*b*) 5000 TV sets **2.** (*d*) 20400

3. (*c*) 21600 **4.** (*b*) 12^{th} year

5. (*a*) 6600

3. Sum of First *n* Terms of an AP

- If first term of an AP be *a* and its common difference is *d*, then the sum S_n of the first *n* terms of an AP is given by

  $S_n = \frac{n}{2}[2a + (n - 1) d]$ or, $S_n = \frac{n}{2}(a + a_n)$ where

  a_n = *n*th term of the AP.
- If *l* is the last term of an AP of *n* terms, then the sum of all '*n*' terms can also be given by

  $S_n = \frac{n}{2}(a + l)$. Sometimes S_n is also denoted by *S*.
- The sum of first *n* positive integers is given by

  $S_n = \frac{n(n+1)}{2}$.
- If S_n is the sum of the first *n* terms of an AP, then its n^{th} term is given by $a_n = S_n - S_{n-1}$, *i.e.*, the n^{th} term of an AP is the difference of the sum to first *n* terms and the sum to first $(n - 1)$ terms of it.

Example 1. Find the sum of the given AP: – 5 + (– 8) + (– 11) + ... + (– 230). [NCERT][CBSE Standard 2020]

Solution. We have, $a = -5$ and $d = -8 + 5 = -3$

So, $a_n = a + (n - 1)d$

$\Rightarrow -230 = -5 + (n - 1)(-3) \Rightarrow -230 = -5 - 3n + 3$

$\Rightarrow -230 + 2 = -3n \Rightarrow -228 = -3n \Rightarrow n = \frac{228}{3} = 76$

$\therefore S_n = \frac{n}{2}(a + a_n) \Rightarrow S_{76} = \frac{76}{2}[-5 - 230]$

$= 38[-235] = -8930$

Example 2. Find the sum of the AP: $7 + 10\frac{1}{2} + 14 + ... + 84$ [NCERT]

Solution. Let a be the first term, d the common difference and a_n the last term of given AP.

We have, $a = 7, d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2}$ and $a_n = 84$

Now, $a_n = a + (n-1)d \Rightarrow 84 = 7 + (n-1) \times \frac{7}{2}$

$\Rightarrow$ $77 = (n-1) \times \frac{7}{2} \Rightarrow 11 \times 2 = (n-1) \Rightarrow 22 = n - 1$

$\therefore$ $n = 22 + 1 = 23$

Now, $S_n = \frac{n}{2}[2a + (n-1)\,d]$

$\Rightarrow$ $S_{23} = \frac{23}{2}\left[2 \times 7 + (23-1) \times \frac{7}{2}\right]$

$\Rightarrow$ $S_{23} = \frac{23}{2}\left[14 + 22 \times \frac{7}{2}\right] = \frac{23}{2}[14 + 77] = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}$

Example 3. How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636? [NCERT] **[Imp.]**

Solution. Let sum of n terms be 636.

Then, $S_n = 636, a = 9, d = 17 - 9 = 8$

$\Rightarrow$ $\frac{n}{2}[2a + (n-1)d] = 636 \Rightarrow \frac{n}{2}[2 \times 9 + (n-1) \times 8] = 636$

$\Rightarrow$ $\frac{n}{2} \times 2[9 + (n-1) \times 4] = 636 \Rightarrow n(9 + 4n - 4) = 636$

$\Rightarrow$ $n[5 + 4n] = 636 \Rightarrow 5n + 4n^2 = 636 \Rightarrow 4n^2 + 5n - 636 = 0$

$\therefore$ $n = \frac{-5 \pm \sqrt{(5)^2 - 4 \times 4 \times (-636)}}{2 \times 4} = \frac{-5 \pm \sqrt{25 + 10176}}{8}$

$= \frac{-5 \pm \sqrt{10201}}{8} = \frac{-5 \pm 101}{8} = \frac{96}{8}, \frac{-106}{8} = 12, -\frac{53}{4}$

But $n \neq \frac{-53}{4}$, So, $n = 12$

Thus, the sum of 12 terms of the given AP is 636.

Example 4. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms. [Delhi 2008] **[Imp.]**

Solution. We have, $S_7 = 49$

$\Rightarrow$ $49 = \frac{7}{2}[2a + (7-1) \times d] \Rightarrow 7 \times 2 = [2a + 6d]$

$\Rightarrow$ $14 = 2a + 6d \Rightarrow a + 3d = 7$ ...(i)

and $S_{17} = 289$

$\Rightarrow$ $289 = \frac{17}{2}[2a + (17-1)\,d] \Rightarrow 2a + 16d = \frac{289 \times 2}{17} = 34$

$\Rightarrow$ $a + 8d = 17$ ...(ii)

Now subtracting equation (i) from (ii), we get

$5d = 10 \Rightarrow d = 2$

Putting the value of d in equation (i), we get

$a + 3 \times 2 = 7 \Rightarrow a = 7 - 6 = 1$

Here $a = 1$ and $d = 2$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1) \times 2] = \frac{n}{2}[2 + 2n - 2] = \frac{n}{2} \times 2n = n^2$

Example 5. The sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161. Find the 28th term of this AP. [Foreign 2014]

Solution. We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_7 = \frac{7}{2}[2a + (7-1)d] \Rightarrow S_7 = \frac{7}{2}[2a + 6d]$

$\Rightarrow$ $63 = 7a + 21d$ $[S_7 = 63 \text{ (given)}]$

$\Rightarrow$ $a = \frac{63 - 21d}{7}$ ...(*i*)

Also, $S_{14} = \frac{14}{2}[2a + 13d]$

$\Rightarrow$ $S_{14} = 14a + 91d$

But according to question, $S_{1-7} + S_{8-14} = S_{14}$

$\Rightarrow$ $63 + 161 = 14a + 91d \Rightarrow 224 = 14a + 91d$

$\Rightarrow$ $2a + 13d = 32 \Rightarrow 2\left(\frac{63 - 21d}{7}\right) + 13d = 32$ ...(*ii*)

$\Rightarrow$ $126 - 42d + 91d = 224$

$\Rightarrow$ $49d = 98 \Rightarrow d = 2$

$\therefore$ $a = \frac{63 - 21 \times 2}{7} = \frac{63 - 42}{7} = \frac{21}{7} = 3$

Thus, $a_{28} = a + 27d = 3 + 27 \times 2$

$\Rightarrow$ $a_{28} = 3 + 54 = 57$

Example 6. Find the sum of the integers between 100 and 200 that are:

(*i*) divisible by 9 (*ii*) not divisible by 9 [NCERT Exemplar]

Solution. (*i*) Numbers divisible by 9 between 100 and 200 are 108, 117, 126, ..., 198.

Here, $a = 108, d = 9, a_n = 198$

$\therefore$ $a_n = a + (n-1)d \Rightarrow 198 = 108 + (n-1)9$

$\Rightarrow$ $198 = 108 + 9n - 9 \Rightarrow 198 = 99 + 9n$

$\Rightarrow$ $198 - 99 = 9n \Rightarrow \frac{99}{9} = n$

$\Rightarrow$ $n = 11$

Thus, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{11} = \frac{11}{2}[2 \times 108 + 10 \times 9] = \frac{11}{2}[216 + 90] = 1683$

(*ii*) Numbers between 100 and 200 are 101, 102,..., 199.

Here, $a = 101, d = 1, a_n = 199$

Now, $a_n = a + (n-1)d$

$\Rightarrow$ $199 = 101 + (n-1)1 \Rightarrow 199 = 100 + n \Rightarrow n = 99$

So, $S_n = \frac{n}{2}(a + l)$, where l is the last term

$= \frac{99}{2}(101 + 199) = \frac{99}{2} \times 300 = 14850$

Sum of the numbers which are not divisible by 9

= Sum of total numbers – sum of numbers which are divisible by 9

$= S_{99} - S_{11} = 14850 - 1683 = 13167$

Example 7. Find the sum: $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \ldots$ to 11 terms. [NCERT Exemplar]

Solution. The first term, $a_1 = \frac{a-b}{a+b}$

Common difference $\qquad d = \dfrac{3a-2b}{a+b} - \dfrac{(a-b)}{a+b} = \dfrac{2a-b}{a+b}$

$$\therefore \qquad S_{11} = \frac{11}{2}\left[\frac{2(a-b)}{a+b} + 10\left(\frac{2a-b}{a+b}\right)\right]$$

$$= \frac{11}{2(a+b)}\ [2a - 2b + 20a - 10b] = \frac{11}{a+b}\ [11a - 6b]$$

Example 8. The sum of the first n terms of an AP is $3n^2 + 6n$. Find the n^{th} term of this AP. [Foreign 2014]

Solution. We have, $\qquad S_n = 3n^2 + 6n$

$$S_{n-1} = 3(n-1)^2 + 6(n-1)$$
$$= 3(n^2 + 1 - 2n) + 6n - 6$$
$$= 3n^2 + 3 - 6n + 6n - 6 = 3n^2 - 3$$

The nth term will be a_n

$$S_n = S_{n-1} + a_n$$
$$\Rightarrow \qquad a_n = S_n - S_{n-1}$$
$$= 3n^2 + 6n - 3n^2 + 3 = 6n + 3$$

Example 9. In an AP, the sum of first ten terms is – 150 and the sum of next ten terms is – 550. Find the AP. [Delhi 2010]

Solution. Let a be the first term and d the common difference of the AP.

We have, $\qquad S_{10} = -150$

$$\Rightarrow \qquad \frac{10}{2}[2a + 9d] = -150 \quad \Rightarrow \quad 2a + 9d = -30 \qquad \text{...}(i)$$

and $\qquad S_{20} - S_{10} = -550$

$$\Rightarrow \qquad S_{20} = -550 - 150 = -700$$

$$\Rightarrow \qquad \frac{20}{2}\ [2a + 19d] = -700 \qquad \left[\because\ S_{20} = \frac{20}{2}[2a + 19d]\right]$$

$$\Rightarrow \qquad 2a + 19d = -70 \qquad \text{...}(ii)$$

From (i) and (ii),

$$d = -4 \text{ and } a = 3$$

So, the AP is: 3, –1, –5, ...

Example 10. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, \ldots$ form an AP. Also find S_{20}. [NCERT Exemplar]

Solution. We have, $\qquad a_n = 3 - 4n$

$$\therefore \qquad a_1 = -1,\ a_2 = -5,\ a_3 = -9, \ldots$$

Since $\qquad a_2 - a_1 = -4 = a_3 - a_2$

So, –1, –5, –9, ... form an AP.

$$S_{20} = \frac{20}{2}\ [-2 + 19 \times (-4)] = 10\ [-2 - 76] = 10 \times (-78) = -780$$

Example 11. If the sum of the first p terms of an AP is $ap^2 + bp$, find its common difference. [CBSE 2010]

Solution.

$$a_p = S_p - S_{p-1} = (ap^2 + bp) - [a(p-1)^2 + b(p-1)]$$
$$= ap^2 + bp - (ap^2 + a - 2ap + bp - b)$$
$$= ap^2 + bp - ap^2 - a + 2ap - bp + b = 2ap + b - a$$
$$\therefore \qquad a_1 = 2a + b - a = a + b$$
$$a_2 = 4a + b - a = 3a + b$$
$$\Rightarrow \qquad d = a_2 - a_1 = (3a + b) - (a + b) = 2a$$

Example 12. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference 'd'. [Delhi 2014]

Solution. We have, $\qquad a = 5,\ T_n = 45,\ S_n = 400$

$$\therefore \qquad T_n = a + (n-1)d$$

$$\Rightarrow \qquad 45 = 5 + (n-1)d \quad \Rightarrow \quad (n-1)d = 40 \qquad \text{...}(i)$$

$$S_n = \frac{n}{2}(a + T_n)$$

$$\Rightarrow \quad 400 = \frac{n}{2}(5 + 45) \quad \Rightarrow \quad n = 2 \times 8 = 16$$

Substituting the value of n in (i), we get

$$(16 - 1)d = 40 \quad \Rightarrow \quad 15d = 40$$

$$\therefore \quad d = \frac{40}{15} = \frac{8}{3}$$

Example 13. The sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Determine the AP and the 12th term. [Delhi 2019] **[Imp.]**

Solution. We have, $S_n = 3n^2 - 4n$ ...(i)

Replacing n by $n - 1$, we get

$$S_{n-1} = 3(n-1)^2 - 4(n-1) \quad ...(ii)$$

Since,

$$a_n = S_n - S_{n-1} = \{3n^2 - 4n\} - \{3(n-1)^2 - 4(n-1)\}$$
$$= \{3n^2 - 4n\} - \{3n^2 + 3 - 6n - 4n + 4\}$$
$$= 3n^2 - 4n - 3n^2 - 3 + 6n + 4n - 4 = 6n - 7$$

So, nth term, $a_n = 6n - 7$ ...(iii)

Substituting $n = 1, 2, 3, ...$ respectively in (iii), we get

$$a_1 = 6 \times 1 - 7 = -1, \; a_2 = 6 \times 2 - 7 = 5$$

and $a_3 = 6 \times 3 - 7 = 11$

Hence, AP is – 1, 5, 11, ...

12th term, $a_{12} = 6 \times 12 - 7 = 72 - 7 = 65$ [From (iii)]

Example 14. If the m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then show that its $(mn)^{th}$ term is 1. [Delhi 2017]

Solution. Let a and d be the first term and the common difference of the AP respectively.

Then, $a_m = \frac{1}{n}$ and $a_n = \frac{1}{m}$

$$\Rightarrow \quad a + (m-1)d = \frac{1}{n} \quad ...(i)$$

$$\text{and} \quad a + (n-1)d = \frac{1}{m} \quad ...(ii)$$

Subtracting (ii) from (i), we get

$$a + (m-1)d - [a + (n-1)d] = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow \quad (m-n)d = \frac{m-n}{mn} \quad \Rightarrow \quad d = \frac{1}{mn}$$

Putting the value of d in (i), we get

$$a + (m-1)\frac{1}{mn} = \frac{1}{n} \quad \Rightarrow \quad a - \frac{1}{mn} = 0$$

$$\Rightarrow \quad a = \frac{1}{mn}$$

$$\therefore \quad a_{mn} = a + (mn-1)d = \frac{1}{mn} + (mn-1)\frac{1}{mn} = 1$$

Example 15. If S_n denotes the sum of the first n terms of an AP, prove that $S_{30} = 3(S_{20} - S_{10})$. [Foreign 2014]

Solution. We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \quad S_{30} = \frac{30}{2}[2a + (30-1)d]$$

$$\Rightarrow \quad S_{30} = 15(2a + 29d) = 30a + 435d \quad ...(i)$$

$$\text{and} \quad S_{20} = \frac{20}{2}[2a + (20-1)d]$$

$$S_{20} = 10(2a + 19d) = 20a + 190d$$

$$S_{10} = \frac{10}{2}[2a + (10-1)d]$$

$$\Rightarrow \quad S_{10} = 5(2a + 9d) = 10a + 45d$$

$$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d]$$

$$= 3[10a + 145d] = 30a + 435d = S_{30} \quad \text{[From } (i)]$$

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

Example 16. The sum of n, $2n$, $3n$ terms of an AP are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$. **[HOTS]**

Solution. Let a be the first term and d the common difference of the AP

$$\therefore \quad S_1 = \frac{n}{2}[2a + (n-1)\,d] \quad \text{...}(i)$$

$$S_2 = \frac{2n}{2}[2a + (2n-1)\,d] \quad \text{...}(ii)$$

$$S_3 = \frac{3n}{2}[2a + (3n-1)d] \quad \text{...}(iii)$$

Now,

$$S_2 - S_1 = \frac{2n}{2}[2a + (2n-1)\,d] - \frac{n}{2}[2a + (n-1)\,d]$$

$$= \frac{n}{2}[2\{2a + (2n-1)\,d\} - \{2a + (n-1)\,d\}]$$

$$= \frac{n}{2}[4a + 4nd - 2d - 2a - nd + d]$$

$$= \frac{n}{2}[2a + 3nd - d] = \frac{n}{2}[2a + (3n-1)\,d]$$

$$\therefore \quad 3(S_2 - S_1) = \frac{3n}{2}[2a + (3n-1)\,d] = S_3 \quad \text{[From } (iii)]$$

$$\Rightarrow \quad 3(S_2 - S_1) = S_3$$

Example 17. If the sum of m terms of an AP is the same as the sum of its n terms, show that the sum of its $(m + n)$ terms is zero. **[HOTS]** [CBSE Standard SP 2019-20]

Solution. Let a and d be the first term and the common difference of the given AP respectively.

Then, $S_m = S_n$

$$\Rightarrow \quad \frac{m}{2}\{2a + (m-1)\,d\} = \frac{n}{2}\{2a + (n-1)\,d\}$$

$$\Rightarrow \quad 2a(m - n) + \{m(m-1) - n\,(n-1)]\,d = 0$$

$$\Rightarrow \quad 2a(m - n) + \{(m^2 - n^2) - (m - n)\}d = 0$$

$$\Rightarrow \quad (m - n)\{2a + (m + n - 1)d\} = 0$$

$$\Rightarrow \quad 2a + (m + n - 1)\,d = 0 \quad \text{...}(i) \quad [\because m - n \neq 0]$$

Now,

$$S_{m+n} = \frac{m+n}{2}\{2a + (m + n - 1)\,d\}$$

$$\Rightarrow \quad S_{m+n} = \frac{m+n}{2} \times 0 = 0 \quad \text{[From } (i)]$$

Example 18. Find the common difference of an AP whose first term is 5 and the sum of its first four terms is half the sum of the next four terms. [CBSE 2012]

Solution. Given the first term of the AP, $a = 5$. Let d be the common difference.

Then, as per the question

$$\sum_{n=1}^{4} a_n = \frac{1}{2}\sum_{n=5}^{8} a_n$$

$$\Rightarrow \quad a_1 + a_2 + a_3 + a_4 = \frac{1}{2}[a_5 + a_6 + a_7 + a_8]$$

$$\Rightarrow \quad [a + (a + d) + (a + 2d) + (a + 3d)] = \frac{1}{2}[(a + 4d) + (a + 5d) + (a + 6d) + (a + 7d)]$$

$$\Rightarrow \quad 4a + 6d = \frac{1}{2}(4a + 22d) \Rightarrow 2(4a + 6d) = 4a + 22d$$

$$\Rightarrow \quad 2(20 + 6d) = 20 + 22d \qquad [\because a = 5 \text{ (given)}]$$

$$\Rightarrow \quad 40 + 12d = 20 + 22d \quad \Rightarrow \quad 20 = 22d - 12d$$

$$\Rightarrow \quad 20 = 10d \quad \Rightarrow \quad d = 2$$

Exercise 2.3

I. Very Short Answer Type Questions [1 Mark]

1. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

(1) The sum of first five terms of the AP: 3, 7, 11, 15, ... is:

(*a*) 44 (*b*) 55 (*c*) 22 (*d*) 11

(2) If the first term of an AP is 1 and the common difference is 2, then the sum of first 26 terms is

(*a*) 484 (*b*) 576 (*c*) 676 (*d*) 625

(3) If the sum to n terms of an AP is $3n^2 + 4n$, then the common difference of the AP is

(*a*) 7 (*b*) 5 (*c*) 8 (*d*) 6

(4) If a, b, c are in AP then $ab + bc =$

(*a*) b (*b*) b^2 (*c*) $2b^2$ (*d*) $\frac{1}{b}$

(5) The sum of all natural numbers which are less than 100 and divisible by 6 is

(*a*) 412 (*b*) 510 (*c*) 672 (*d*) 816

2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation ofassertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** Sum of the first 10 terms of the arithmetic progression –0.5, –1.0, –1.5,... is 27.5.

Reason (R): Sum of first n terms of an AP is given as $S_n = \frac{n}{2}[2a + (n-1)d]$ where a = first term, d = common difference.

(2) **Assertion (A):** The sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Then its n^{th} term, $a_n = 6n - 7$.

Reason (R): n^{th} term of an AP, whose sum of n terms is S_n, is given by $a_n = S_n - S_{n-1}$.

(3) **Assertion (A):** Sum of first hundred even natural numbers divisible by 5 is 500.

Reason (R): Sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[a + l]$ where l = last term.

3. Answer the following:

(1) Find the sum of first 10 terms of the AP: 2, 7, 12, ... [NCERT] **[Imp.]**

(2) If the sum of first m terms of an AP is $2m^2 + 3m$, then what is its second term? [Foreign 2010]

(3) Find the sum of first 10 multiples of 6. [AI 2019]

(4) What is the sum of five positive integers divisible by 6? [CBSE Sample Paper 2012]

(5) If the sum of the first q terms of an AP is $2q + 3q^2$, what is its common difference? [AI 2010]

(6) If n^{th} term of an AP is $(2n + 1)$, what is the sum of its first three terms? [CBSE SP 2018-19]

(7) Find the sum of first 100 natural numbers. [CBSE Standard 2020]

II. Short Answer Type Questions - I [2 Marks]

4. Find the sum of first 8 multiples of 3. [CBSE 2018]

5. Find the number of terms of the AP: 54, 51, 48, ... so that their sum is 513. **[Imp.]**

6. In an AP, the first term is –4, the last term is 29 and the sum of all its terms is 150. Find its common difference. [Foreign 2016]

7. Find the sum of all three digit natural numbers, which are multiples of 11. [Delhi 2012]

8. The first and the last terms of an AP are 8 and 65 respectively. If sum of all its terms is 730, find its common difference. [Delhi 2014]

9. The sum of the first n terms of an AP is $4n^2 + 2n$. Find the n^{th} term of this AP. [Foreign 2013]

10. How many terms of the AP: 18, 16, 14, ... be taken so that their sum is zero? [Delhi 2016]

11. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first n terms. [AI 2015]

12. The sum of first n terms of an AP is given by $S_n = 2n^2 + 3n$. Find the sixteenth term of the AP.

III. Short Answer Type Questions - II [3 Marks]

13. How many multiples of 4 lie between 10 and 250? Also find their sum. [AI 2011]

14. Find the sum of first n terms of an AP whose n^{th} term is $5n - 1$. Hence find the sum of first 20 terms. [AI 2011]

15. The sum of first six terms of an AP is 42. The ratio of its 10^{th} term to its 30^{th} term is 1 : 3. Calculate the first and the thirteenth terms of the AP. [AI 2009]

16. Find the sum of all multiples of 7 lying between 500 and 900. [AI 2010]

17. If M, N and T are in AP, prove that $(M + 2N - T)(2N + T - M)(T + M - N) = 4MNT$.

18. In an AP, if the 6th and 13th terms are 35 and 70 respectively, find the sum of its first 20 terms. [Foreign 2011]

19. The sum of the 2^{nd} and the 7^{th} terms of an AP is 30. If its 15^{th} term is 1 less than twice its 8^{th} term, find the AP. [AI 2014]

20. If the ratio of the sum of first n terms of two AP's is $(7n + 1) : (4n + 27)$, find the ratio of their m^{th} terms. [AI 2016]

21. The digits of a positive number of three digits are in AP and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number. [AI 2016]

22. The sums of first n terms of three A.Ps' are S_1, S_2 and S_3. The first term of each AP is 5 and their common differences are 2, 4 and 6 respectively. Prove that $S_1 + S_3 = 2S_2$. **[Imp]**

23. Find the sum of n terms of the series $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ...$ [Delhi 2017]

24. Solve the equation: $1 + 4 + 7 + 10 + ... + x = 287$.

IV. Long Answer Type Questions [5 Marks]

25. The sum of the first three numbers in an arithmetic progression is 18. If the product of the first and the third terms is 5 times the common difference, find the three numbers. [Al 2019]

26. If m times the m^{th} term of an arithmetic progression is equal to n times its n^{th} term and $m \neq n$, show that the $(m + n)^{th}$ term of the AP is zero. [Al 2019]

27. The first and the last term of an AP are 8 and 350 respectively. If its common difference is 9, how many terms are there and what is their sum? [AI 2011]

28. Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to $\frac{(a + c)(b + c - 2a)}{2(b - a)}$. [NCERT Exemplar][CBSE Standard 2020]

29. If the p^{th} term of an AP is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of the pq terms is $\frac{1}{2}(pq + 1)$. [CBSE 2012]

30. The ratio of the 11^{th} term to the 18^{th} term of an AP is 2 : 3. Find the ratio of the 5^{th} term to the 21^{st} term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms. [NCERT Exemplar]

31. The sum of the first five terms of an AP is 55 and sum of the first ten terms of this AP is 235, find the sum of its first 20 terms. **[Imp.]**

32. The sums of n terms of two APs are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 25^{th} terms. **[Imp.]**

33. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also, find the sum of all numbers on both sides of the middle terms separately. [Foreign 2015]

34. If the ratio of the sum of the first n terms of two APs is $(7n + 1) : (4n + 27)$, then find the ratio of their 9^{th} terms. [AI 2017]

35. If the sum of first 14 terms of an AP is 1050 and its first term is 10, find the 20^{th} term.

36. The first term of an AP is 5, the last term is 45 and sum is 400. Find the number of terms and the common difference.

37. How many terms of the AP: 24, 21, 18, ... must be taken so that their sum is 78?

Case Study Based Questions

I. **Pollution—A Major Problem:** One of the major serious problems that the world is facing today is the environmental pollution. Common types of pollution include light, noise, water and air pollution.

In a school, students thoughts of planting trees in and around the school to reduce noise pollution and air pollution.

Condition I: It was decided that the number of trees that each section of each class will plant be the same as the class in which they are studying, *e.g.* a section of class I will plant 1 tree a section of class II will plant 2 trees and so on a section of class XII will plant 12 trees.

Condition II: It was decided that the number of trees that each section of each class will plant be the double of the class in which they are studying, *e.g.* a section of class I will plant 2 trees, a section of class II will plant 4 trees and so on a section of class XII will plant 24 trees.

Refer to Condition I

1. The AP formed by sequence *i.e.* number of plants by students is
   (*a*) 0, 1, 2, 3, ..., 12 (*b*) 1, 2, 3, 4, ..., 12 (*c*) 0, 1, 2, 3, ..., 15 (*d*) 1, 2, 3, 4, ..., 15
2. If there are two sections of each class, how many trees will be planted by the students?
   (*a*) 126 (*b*) 152 (*c*) 156 (*d*) 184
3. If there are three sections of each class, how many trees will be planted by the students?
   (*a*) 234 (*b*) 260 (*c*) 310 (*d*) 326

Refer to Condition II

4. If there are two sections of each class, how many trees will be planted by the students?
   (*a*) 422 (*b*) 312 (*c*) 360 (*d*) 540
5. If there are three sections of each class, how many trees will be planted by the students?
   (*a*) 468 (*b*) 590 (*c*) 710 (*d*) 620

II. Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of ₹ 1,18,000 by paying every month starting with the first instalment of ₹ 1000. If he increases the instalment by ₹ 100 every month , answer the following:

1. The amount paid by him in 30th installment is
   (*a*) ₹ 3900 (*b*) ₹ 3500 (*c*) ₹ 3700 (*d*) ₹ 3600
2. The total amount paid by him upto 30 installments is
   (*a*) ₹ 37000 (*b*) ₹ 73500 (*c*) ₹ 75300 (*d*) ₹ 75000
3. What amount does he still have to pay after 30th installment?
   (*a*) ₹ 45500 (*b*) ₹ 49000 (*c*) ₹ 44500 (*d*) ₹ 54000
4. If total installments are 40, then amount paid in the last installment is
   (*a*) ₹ 4900 (*b*) ₹ 3900 (*c*) ₹ 5900 (*d*) ₹ 9400
5. The ratio of the 1st installment to the last installment is
   (*a*) 1 : 49 (*b*) 10 : 49 (*c*) 10 : 39 (*d*) 39 : 10

Answers and Hints

1. (1) (*b*) 55 (1) (2) (*c*) 676 (1)
(3) (*d*) 6 (1) (4) (*c*) $2b^2$ (1)
(5) (*d*) 816 (1)

2. (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(2) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (1)
(3) (*d*) Assertion (A) is false but reason (R) is true. (1)

3. (1) 245 (1) (2) 9 (1)
(3) First 10 multiples of 6 are 6, 12, 18, ..., 60.
This is an AP in which $a = 6$, $n = 10$ and $d = 6$.
$\therefore$ Sum of first 10 multiples of 6 = S_{10} (½)

$$\Rightarrow S_{10} = \frac{n}{2}[2a+(n-1)d]$$
$$= \frac{10}{2}[2\times6+(10-1)6]$$
$$= 5\,(12 + 54)$$
$$= 5 \times 66 = 330 \quad (½)$$

(4) 90 (1)
(5) Given that,
$$S_q = 2q + 3q^2$$
$$S_1 = 2 + 3 = 5 = T_1 = \text{First term} \quad [\text{put } q = 1]$$
$$S_2 = 4 + 3(4) = 16 \quad [\text{put } q = 2]$$
$$S_3 = 6 + 3(9) = 33 \quad [\text{put } q = 3](½)$$
$\therefore$ 2^{nd} term,
$$T_2 = S_2 - S_1 = 16 - 5 = 11$$
$\therefore$ 3^{rd} term,
$$T_3 = S_3 - S_2 = 33 - 16 = 17$$
Common difference
$$= T_3 - T_2 = 17 - 11 = 6 \quad (½)$$

(6) $a_1 = 3, a_3 = 7, S_3 = \frac{3}{2}(3+7) = 15$ [½+½]

(7) Natural numbers are 1, 2, 3, 4, ...
The sum of first 100 natural numbers is given by
$$S_n = \frac{n(n+1)}{2} = \frac{100\times(100+1)}{2} \quad (½)$$
$$= \frac{100\times101}{2}$$
$$= 50 \times 101 = 5050 \quad (½)$$

4.
$$S_8 = 3 + 6 + 9 + 12 + ... + 24$$
$$= 3(1 + 2 + 3 + ... + 8) \quad (1)$$
$$= 3 \times \frac{8\times9}{2} = 108 \quad (1)$$

5. 18 or 19 (2)

6.
$$150 = \frac{n}{2}(-4+29) \qquad \left|\because S_n = \frac{n}{2}(a+l)\right|$$
$\Rightarrow$ $300 = 25n \Rightarrow n = 12$ (1)
$\therefore$ Then, $l = a_{12} = 29 = -4 + 11d$
$\Rightarrow$ $11d = 33 \Rightarrow d = 3$ (1)

7. 3-digit natural numbers which are multiples of 11 are 110, 121, 132, ..., 990
n^{th} term, $990 = 110 + (n-1)11$
$\Rightarrow$ $n = 81$ (1)
$\therefore$ Sum of 'n' terms,
$$S_n = \frac{n}{2}[a+l]$$
$$= \frac{81}{2}[110+990] = 44550$$
$\therefore$ Sum of all three-digit natural numbers, which are multiples of 11 is 44550. (1)

8.
$$S_n = \frac{n}{2}(a + a_n)$$
$$730 = \frac{n}{2}(8+65) \Rightarrow \frac{73n}{2} = 730$$
$\Rightarrow$ $n = 20$ (1)
$\therefore$ Given $a_{20} = 65$, where $a_n = a + (n-1)d$
$\Rightarrow$ $a + 19d = 65 \Rightarrow 8 + 19d = 65$
$\Rightarrow$ $19d = 57$
Hence, common differences = $d = 3$. (1)

9. Given, $S_n = 4n^2 + 2n$
So,
$$S_{n-1} = 4(n-1)^2 + 2(n-1)$$
$$= 4(n^2 - 2n + 1) + 2n - 2$$
$$= 4n^2 - 8n + 4 + 2n - 2$$
$$= 4n^2 - 6n + 2 \quad (1)$$
$$a_n = S_n - S_{n-1} = n^{th} \text{ term}$$
$$= (4n^2 + 2n) - (4n^2 - 6n + 2)$$
$$= 4n^2 + 2n - 4n^2 + 6n - 2$$
$$= 8n - 2 \quad (1)$$

10. Let the number of terms taken for sum to be zero be n.
Then, sum of n terms
$$(S_n) = 0 \quad \text{(Given)}$$
$$\Rightarrow S_n = \frac{n}{2}[2a + (n-1)d] \quad (1)$$
$$\Rightarrow 0 = \frac{n}{2}[2\times18+(n-1)(-2)]$$
$$\Rightarrow n = 19$$
Hence, sum of 19 terms is 0. (1)

11.
$$S_5 + S_7 = 167$$
$$\Rightarrow \frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d) = 167$$
$$\left\{\because S_n = \frac{n}{2}[2a+(n-1)d]\right\}$$
$$\Rightarrow 5a + 10d + 7a + 21d = 167$$
$$\Rightarrow 12a + 31d = 167 \quad ...(i)$$
Also,
$$S_{10} = 235$$
$$\Rightarrow \frac{10}{2}(2a+9d) = 235$$
$$\Rightarrow 2a + 9d = 47 \quad ...(ii)(1)$$
Multiplying eq. (*ii*) by 6, we get
$$6(2a + 9d) = 6 \times 47$$

$\Rightarrow \quad 12a + 54d = 282$ ...(iii)

$\therefore$ Subtracting eq. (i) from (iii), we get

$$\begin{array}{r} 12a + 54d = 282 \\ 12a + 31d = 167 \\ - \quad - \quad\quad - \\ \hline 23d = 115 \end{array}$$

$\therefore \quad d = 5$

Putting 'd' in (ii) equation, $a = 1$

$\therefore$ Required AP is 1, 6, 11, ... (1)

12. $S_n = 2n^2 + 3n$

$S_1 = 5 = a_1$

$S_2 = a_1 + a_2 = 14 \Rightarrow a_2 = 9$ (1)

$d = a_2 - a_1 = 4$

$a_{16} = a_1 + 15d = 5 + 15(4) = 65$ (1)

13. 60, 7800 (3)

14. Given: $a_n = 5n - 1$

$a_1 = 4$

$\therefore \quad a_2 = 5(2) - 1 = 9$

$d = a_2 - a_1 = 9 - 4 = 5$ (1)

Now, sum of first 'n' terms,

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}[2\times 4+5(n-1)]$$

$$= \frac{n}{2}(8+5n-5) = \frac{n(5n+3)}{2}$$ (1)

Now, sum of first 20 terms,

$$S_{20} = \frac{20(5\times 20+3)}{2}$$

$= 10 \times 103 = 1030$ (1)

15. 2 and 26 (3) **16.** 39, 900 (3)

18. Given that, $a_6 = 35 \Rightarrow a + 5d = 35$ ...(i)

and also $a_{13} = 70 \Rightarrow a + 12d = 70$ ...(ii)(1)

On solving the above equations, we get

$a = 10; \quad d = 5$ (1)

Now, sum of first 20 terms,

$$S_{20} = \frac{20}{2}[2\times 10+19\times 5]$$

$$\left|\because S_n = \frac{n}{2}[2a+(n-1)d]\right|$$

$= 1150$ (1)

19. Given, $a_2 + a_7 = 30$

$\Rightarrow \quad a + d + a + 6d = 30$

$\Rightarrow \quad 2a + 7d = 30$ ...(i)(1)

$[\because a_n = a + (n-1)d]$

Also, given $a_{15} = 2a_8 - 1$

$\Rightarrow \quad a + 14d = 2(a + 7d) - 1 \quad \Rightarrow a = 1$ (1)

Putting the value of a in (i), we get

$2 + 7d = 30 \quad \Rightarrow \quad d = 4$

$\therefore \quad a = 1, d = 4$

Hence, AP is 1, 5, 9, 13, 17, ... (1)

20. Let S_n and S'_n be the sum of n terms of two APs. Let a, a' and d, d' be first terms and common differences of two APs. Then

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']}$$

$$= \frac{a+\left(\frac{n-1}{2}\right)d}{a'+\left(\frac{n-1}{2}\right)d'} = \frac{7n+1}{4n+27}$$ ...(i)(1)

Since $\dfrac{t_m}{t'_m} = \dfrac{a+(m-1)d}{a'+(m-1)d'}$

[$\because$ Let t_m, t'_m be m^{th} terms of two APs]

So, replacing $\dfrac{n-1}{2}$ by $m - 1$, i.e., $n = 2m - 1$ in (i)

$$\frac{t_m}{t'_m} = \frac{a+(m-1)d}{a'+(m-1)d'}$$ (1)

$$= \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Thus, the ratio of their m^{th} terms is

$14m - 6 : 8m + 23.$ (1)

21. Let the required numbers in AP are $a - d$, a, $a + d$ respectively.

Now, $a - d + a + a + d = 15$ [$\because$ Sum of digits = 15]

$\Rightarrow \quad 3a = 15 \Rightarrow a = 5$ (1)

According to question, number is

$100(a - d) + 10a + a + d$, i.e. $111a - 99d$

Number on reversing the digits is

$100(a + d) + 10a + a - d$, i.e. $111a + 99d$

Now, as per given condition in question,

$(111a - 99d) - (111a + 99d) = 594$ (1)

$\Rightarrow \quad d = -3$

$\therefore$ Digits of number are $[5 - (-3), 5, (5 + (-3))]$

$= 8, 5, 2.$

$\therefore$ Required number is $111 \times (5) - 99(-3)$

$= 555 + 297 = 852.$ (1)

23. $S_n = \left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+ \ldots$ upto n terms

$$= (4+4+\ldots+4)-\frac{1}{n}(1+2+3+\ldots+n)$$ (1)

$$= 4n-\frac{1}{n}\times\frac{n(n+1)}{2}$$ (1)

$$= \frac{7n-1}{2}$$ (1)

24. Given equation: $1 + 4 + 7 + 10 + \ldots + x = 287$

Here, $a = 1$, $d = 4 - 1 = 7 - 4 = 3$

$S_n = 287$

Bur, $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$287 = \dfrac{n}{2}[2 \times 1 + (n-1)3]$

$\Rightarrow \quad 287 \times 2 = n(2 + 3n - 3)$

$\Rightarrow \quad 574 = n(3n - 1) = 3n^2 - n$

$\Rightarrow \quad 3n^2 - n - 574 = 0$ (1)

We know that,

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 3 \times (-574)}}{2 \times 3}$$

$$= \frac{1 \pm \sqrt{1 + 6888}}{6} = \frac{1 \pm \sqrt{6889}}{6} = \frac{1 \pm 83}{6} \quad (1)$$

Either $n = \frac{1 \pm 83}{6}$ or $n = \frac{1 - 83}{6}$

$\Rightarrow \quad n = \frac{84}{6}$ or $n = \frac{-82}{6}$

$\Rightarrow \quad n = 14$ or $n = \frac{-41}{3}$

$\therefore \quad n = 14$

Now, $S_n = \frac{n}{2}(a + l) \quad \Rightarrow \quad 287 = \frac{14}{2}(1 + x)$

$\Rightarrow \quad 287 = 7(1 + x) \quad \Rightarrow \quad 287 = 7 + 7x$

$\Rightarrow \quad 7x = 280 \quad \Rightarrow \quad x = \frac{280}{7} = 40$ (1)

25. Let the three numbers in AP are $a - d, a, a + d$

Then $a - b + a + a + d = 18$

$\Rightarrow \quad 3a = 18 \quad \Rightarrow \quad a = 6$ (1)

Given: $(a - d)(a + d) = 5d$

$\Rightarrow \quad a^2 - d^2 = 5d \quad \Rightarrow \quad a^2 = 5d + d^2$

$\Rightarrow \quad 36 = 5d + d^2 \quad [\because \ a = 6]$(1)

$\Rightarrow \quad d^2 + 5d - 36 = 0$

$\Rightarrow \quad d^2 + 9d - 4d - 36 = 0$ (1)

$\Rightarrow \quad d(d + 9) - 4(d + 9) = 0$

$\Rightarrow \quad (d - 4)(d + 9) = 0$

$\Rightarrow \quad d - 4 = 0$ or $d + 9 = 0$

$\Rightarrow \quad d = 4$ or $d = -9$ [Reject]

$\Rightarrow \quad d = 4$ (1)

Thus, three numbers are $a - d, a, a + d$

$= 6 - 4, 6, 6 + 4$

$= 2, 6, 10$ (1)

26. We know that $a_n = a + (n - 1)d$

From the given conditions,

$m[a + (m - 1)d] = n[a + (n - 1)d]$

$\Rightarrow \quad m[a + (md - d)] = n[a + nd - d]$ (1)

$\Rightarrow \quad am + m^2d - md = an + n^2d - nd$

$\Rightarrow \quad am - an + m^2d - n^2d - md + nd = 0$ (1)

$\Rightarrow \quad a(m - n) + d(m^2 - n^2) - d(m - n) = 0$

$\Rightarrow \quad a(m - n) + (m + n)(m - n)d - (m - n)d = 0$ (1)

$\Rightarrow \quad (m - n)[a + (m + n)d - d = 0$

$\Rightarrow \quad a + md + nd - d = 0$ (1)

$\Rightarrow \quad a + (m + n - 1)d = 0$

Since, $m \neq n$, it is clear that $(m + n)^{th}$ term of the AP is zero. (1)

27. 39; 6981 (5) **30.** 1 : 3, 5 : 49 (5)

31. 970 (5) **32.** 249 : 447 (5)

33. List of 3-digit number leaving remainder 3 when divided by 4, are 103, 107, 111, ..., 999.

Now, $a_n = 999 \quad \Rightarrow \quad a + (n - 1)d = 999$

$103 + (n - 1)4 = 999 \quad \Rightarrow \quad n = 225$ (1)

Since, number of terms is odd, so there will be only one middle term

Middle term $= \frac{225 + 1}{2} = 113$ (1)

$\therefore \quad a_{113} = a + 112d$

$= 103 + 112 \times 4 = 551$ (1)

There are 112 numbers before 113^{th} term.

$\therefore$ Sum of all terms before middle term

$S_{112} = \frac{112}{2}[2 \times 103 + 111 \times 4]$

$= 36400$ (1)

$\therefore$ Sum of all terms $= S_{225} = 123975$

$\therefore$ Sum of terms after middle term

$= S_{225} - (S_{112} + 551)$

$= 87024$ (1)

34. Let the first terms be a and a' and d and d' be their respective common differences.

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2a' + (n-1)d']}$$

$$= \frac{7n + 1}{4n + 27} \quad (1)$$

$$\Rightarrow \quad \frac{a + \left(\frac{n-1}{2}\right)d}{a' + \left(\frac{n-1}{2}\right)d'} = \frac{7n+1}{4n+27} \quad (1)$$

To get ratio of 9^{th} terms, replacing $\frac{n - 1}{2} = 8$ (1)

$\Rightarrow \quad n = 17$ (1)

Hence, $\frac{t_9}{t'_9} = \frac{a + 8d}{a' + 8d'} = \frac{120}{95}$ or $\frac{24}{19}$ (1)

35. Let common difference be d.

$\Rightarrow \quad \frac{14}{2}[2(10) + (n - 1)d] = 1050$ (2)

$\Rightarrow \quad d = 10$ (1)

$a_{20} = a + 19d$

$= 10 + 19(10) = 200$ (2)

36.

$$a = 5$$
$$a_n = 45$$
$$S_n = 400$$

$$\Rightarrow \quad \frac{n}{2}(5 + 45) = 400 \qquad (2)$$

$$50n = 800 \qquad (1)$$
$$n = 16$$

also $\quad a_n = 45$

$$5 + 15d = 45$$
$$15d = 40$$
$$d = \frac{8}{3} \qquad (2)$$

37. AP is 24, 21, 18, ...

Here, $a = 24$ and $d = 21 - 24 = 18 - 21 = -3$ (1)

Let the sum of n terms of the AP be 78.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 78 = \frac{n}{2}[2 \times 24 + (n-1)(-3)]$$

$$\Rightarrow \quad 78 \times 2 = n[48 - 3n + 1]$$

$$\Rightarrow \quad 156 = n(49 - 3n)$$

$$\Rightarrow \quad 156 = 49n - 3n^2$$

$$\Rightarrow \quad 3n^2 - 49n + 156 = 0 \qquad (1)$$

We know that

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{49 \pm \sqrt{(49)^2 - 4 \times 3 \times 156}}{2 \times 3}$$

$$= \frac{49 \pm \sqrt{2401 - 1872}}{6}$$

$$= \frac{49 \pm \sqrt{529}}{6}$$

$$= \frac{49 \pm 23}{6} \qquad (1)$$

Either $n = \dfrac{49 + 23}{6}$ or $n = \dfrac{49 - 23}{6}$

$$n = \frac{72}{6} \text{ or } n = \frac{26}{6} = \frac{13}{3}$$

$$n = 12 \text{ or } n = 4\frac{1}{3}$$

Thus, $\quad n = 12.$ (2)

Case Study Based Questions

I. **1.** (*b*) 1, 2, 3, 4, ..., 12
2. (*c*) 156 **3.** (*a*) 234
4. (*b*) 312 **5.** (*a*) 468

II. **1.** (*a*) ₹ 3900 **2.** (*b*) ₹ 73500
3. (*c*) ₹ 44500 **4.** (*a*) ₹ 4900
5. (*b*) 10 : 49

Experts' Opinion

Questions based on following types are very important for Exams. So, students are advised to revise them thoroughly.

1. Finding n^{th} term of given AP.
2. Finding n^{th} term of given AP from the end.
3. Finding n when n^{th} term of an AP is given.
4. Finding AP or n^{th} term or both when its two terms are given.
5. Finding sum of first n terms of an AP.
6. Finding number of terms when sum of first n terms and AP are given.

IMPORTANT FORMULAE

- The n^{th} term of an AP, $a_n = a + (n-1)d$
- The n^{th} term of an AP from end, $a_n = l - (n-1)d$
- Sum of finite terms of an AP

$$S_n = \frac{n}{2}[2a + (n-1)d] \text{ or } S_n = \frac{n}{2}(a + a_n)$$

- If there are only n terms in an AP, then

$$a_n = l, \text{ the last term}$$

$$S_n = \frac{n}{2}(a + l)$$

Note: $\quad a_n = S_n - S_{n-1}$

where, a = first term, n = number of terms, d = common difference, and $a_n = n^{th}$ term, l = last term.

QUICK REVISION NOTES

- A succession of numbers or terms formed and arranged according to some rule is called a sequence/progression.
- An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. This fixed number is called the common difference of the AP, which can be positive, negative or zero.
- The sequence $a_1, a_2, a_3, a_4, ..., a_n$ is an AP of n terms with common difference 'd' iff

$$a_n - a_{n-1} = a_{n-1} - a_{n-2} = ... = a_2 - a_1 = d$$

- General term (n^{th} term) of an AP (from the beginning) is given by

$$a_n = a + (n - 1)d.$$

- Three numbers a, b, c are in AP if and only if $b - a = c - b$ or $2b = c + a$.

$$\Rightarrow \quad b = \frac{c + a}{2}$$

Note that b is known as arithmetic mean of a and c.

- If 'l' is the last term of an AP, then n^{th} term from the end of an AP

$$= l + (n - 1)(-d) = l - (n - 1)d.$$

- Let a be the 1st term, 'd' the common difference and 'n' the number of terms of an AP, then S_n, the sum of 'n' terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Also $$S_n = \frac{n}{2}(a + a_n)$$

where, a_n is the last term.

COMMON ERRORS

Errors	Corrections
(*i*) Finding incorrectly the common difference (d) when the numbers in AP is in descending order or if the succeeding term is smaller.	(*i*) For finding the common difference, we should subtract the preceding term from the succeeding term, even if the numbers in AP is in descending order or the succeeding term is smaller.
(*ii*) When 'd' is negative in questions to find 'n', multiplying $(n - 1)$ incorrectly by positive value of 'd'.	(*ii*) Put 'd' with negative sign in bracket so that multiplication will be taken up in the next step.
(*iii*) Incorrectly differentiating a_n and S_n.	(*iii*) S_n represents the sum of n terms whereas a_n represents n^{th} term.
(*iv*) Trying incorrectly to find n^{th} term when sum to first n terms and the sum to first $(n - 1)$ terms are given.	(*iv*) The n^{th} term of an AP is the difference of the sum to first n terms and the sum to first $(n - 1)$ terms of it. *i.e.*, $a_n = S_n - S_{n-1}$