Chapter 9

DIFFERENTIAL EQUATIONS

9.1 Overview

(i) An equation involving derivative (derivatives) of the dependent variable with respect to independent variable (variables) is called a differential equation.

(ii) A differential equation involving derivatives of the dependent variable with respect to only one independent variable is called an ordinary differential equation and a differential equation involving derivatives with respect to more than one independent variables is called a partial differential equation.

(iii) Order of a differential equation is the order of the highest order derivative occurring in the differential equation.

(iv) Degree of a differential equation is defined if it is a polynomial equation in its derivatives.

(v) Degree (when defined) of a differential equation is the highest power (positive integer only) of the highest order derivative in it.

(vi) A relation between involved variables, which satisfy the given differential equation is called its solution. The solution which contains as many arbitrary constants as the order of the differential equation is called the general solution and the solution free from arbitrary constants is called particular solution.

(vii) To form a differential equation from a given function, we differentiate the function successively as many times as the number of arbitrary constants in the given function and then eliminate the arbitrary constants.

(viii) The order of a differential equation representing a family of curves is same as the number of arbitrary constants present in the equation corresponding to the family of curves.

(ix) 'Variable separable method' is used to solve such an equation in which variables can be separated completely, i.e., terms containing x should remain with dx and terms containing y should remain with dy.

(x) A function F (x, y) is said to be a homogeneous function of degree n if F $(\lambda x, \lambda y) = \lambda^n$ F (x, y) for some non-zero constant λ.

(xi) A differential equation which can be expressed in the form $\frac{dy}{dx} = F(x, y)$ or $\frac{dx}{dy} = G(x, y)$, where F (x, y) and G (x, y) are homogeneous functions of degree zero, is called a homogeneous differential equation.

(xii) To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y)$, we make substitution $y = vx$ and to solve a homogeneous differential equation of the type $\frac{dx}{dy} = G(x, y)$, we make substitution $x = vy$.

(xiii) A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x only is known as a first order linear differential equation. Solution of such a differential equation is given by $y\,(\text{I.F.}) = \int (Q \times \text{I.F.})\, dx + C$, where I.F. (Integrating Factor) = $e^{\int P dx}$.

(xiv) Another form of first order linear differential equation is $\frac{dx}{dy} + P_1 x = Q_1$, where P_1 and Q_1 are constants or functions of y only. Solution of such a differential equation is given by $x\,(\text{I.F.}) = \int (Q_1 \times \text{I.F.})\, dy + C$, where I.F. = $e^{\int P_1 dy}$.

9.2 Solved Examples

Short Answer (S.A.)

Example 1 Find the differential equation of the family of curves $y = Ae^{2x} + B.e^{-2x}$.

Solution $y = Ae^{2x} + B.e^{-2x}$

$$\frac{dy}{dx} = 2Ae^{2x} - 2\,B.e^{-2x} \text{ and } \frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x}$$

Thus $\frac{d^2y}{dx^2} = 4y$ i.e., $\frac{d^2y}{dx^2} - 4y = 0.$

Example 2 Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

Solution $\frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dy}{y} = \frac{dx}{x} \Rightarrow \int\frac{dy}{y} = \int\frac{dx}{x}$

$$\Rightarrow \log y = \log x + \log c \Rightarrow y = cx$$

Example 3 Given that $\frac{dy}{dx} = ye^x$ and $x = 0$, $y = e$. Find the value of y when $x = 1$.

Solution $\frac{dy}{dx} = ye^x \Rightarrow \int\frac{dy}{y} = \int e^x dx \Rightarrow \log y = e^x + c$

Substituting $x = 0$ and $y = e$, we get $\log e = e^0 + c$, i.e., $c = 0$ ($\because \log e = 1$)

Therefore, $\log y = e^x$.

Now, substituting $x = 1$ in the above, we get $\log y = e \Rightarrow y = e^e$.

Example 4 Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$.

Solution The equation is of the type $\frac{dy}{dx} + Py = Q$, which is a linear differential equation.

Now I.F. $= \int \frac{1}{x} dx = e^{\log x} = x.$

Therefore, solution of the given differential equation is

$$y.x = \quad x\,x^2\,dx \text{, i.e. } yx = \frac{x^4}{4} \quad c$$

Hence $y = \dfrac{x^3}{4} \quad \dfrac{c}{x}$.

Example 5 Find the differential equation of the family of lines through the origin.

Solution Let $y = mx$ be the family of lines through origin. Therefore, $\dfrac{dy}{dx} = m$

Eliminating m, we get $y = \dfrac{dy}{dx} . x$ or $x\dfrac{dy}{dx} - y = 0$.

Example 6 Find the differential equation of all non-horizontal lines in a plane.

Solution The general equation of all non-horizontal lines in a plane is $ax + by = c$, where $a \neq 0$.

Therefore, $a\dfrac{dx}{dy} \quad b = 0$.

Again, differentiating both sides w.r.t. y, we get

$$a\frac{d^2x}{dy^2} = 0 \Rightarrow \frac{d^2x}{dy^2} = 0.$$

Example 7 Find the equation of a curve whose tangent at any point on it, different from origin, has slope $y \quad \dfrac{y}{x}$.

Solution Given $\dfrac{dy}{dx} \quad y \quad \dfrac{y}{x} \qquad = y \quad 1 \quad \dfrac{1}{x}$

$$\Rightarrow \frac{dy}{y} \quad 1 \quad \frac{1}{x} \quad dx$$

Integrating both sides, we get

$$\log y = x + \log x + c \Rightarrow \quad \log \quad \frac{y}{x} \quad = x + c$$

$$\Rightarrow \frac{y}{x} = e^{x+c} = e^{x}.e^{c} \quad \Rightarrow \quad \frac{y}{x} = k \,.\, e^{x}$$

$$\Rightarrow y = kx \,.\, e^{x}.$$

Long Answer (L.A.)

Example 8 Find the equation of a curve passing through the point (1, 1) if the perpendicular distance of the origin from the normal at any point P(x, y) of the curve is equal to the distance of P from the x – axis.

Solution Let the equation of normal at P(x, y) be Y – y = $\frac{-dx}{dy}(\mathrm{X} - x)$,i.e.,

$$\mathrm{Y} + \mathrm{X}\frac{dx}{dy} - \quad y \quad x\frac{dx}{dy} \quad = 0 \qquad \text{...(1)}$$

Therefore, the length of perpendicular from origin to (1) is

$$\frac{y \quad x\frac{dx}{dy}}{\sqrt{1 \quad \frac{dx}{dy}^{2}}} \qquad \text{...(2)}$$

Also distance between P and x-axis is $|y|$. Thus, we get

$$\frac{y \quad x\frac{dx}{dy}}{\sqrt{1 \quad \frac{dx}{dy}^{2}}} = |y|$$

$$\Rightarrow \left(y + x\frac{dx}{dy}\right)^{2} = y^{2} \quad 1 \quad \frac{dx}{dy}^{2} \quad \Rightarrow \frac{dx}{dy} \quad \frac{dx}{dy} \quad x^{2} - y^{2} \quad 2xy \quad 0 \Rightarrow \frac{dx}{dy} \quad 0$$

or $$\frac{dx}{dy} = \frac{2xy}{y^{2} - x^{2}}$$

Case I: $\frac{dx}{dy} = 0 \Rightarrow dx = 0$

Integrating both sides, we get $x = k$, Substituting $x = 1$, we get $k = 1$.

Therefore, $x = 1$ is the equation of curve (not possible, so rejected).

Case II: $\frac{dx}{dy} = \frac{2xy}{y^2 \quad x^2} \quad \frac{dy}{dx} \quad \frac{y^2 \quad x^2}{2xy}$. Substituting $y = vx$, we get

$$v \quad x\frac{dv}{dx} \quad \frac{v^2x^2 \quad x^2}{2vx^2} \quad \Rightarrow \quad x.\frac{dv}{dx} \quad \frac{v^2 \quad 1}{2v} \quad v$$

$$= \frac{-(1+v^2)}{2v} \qquad \Rightarrow \quad \frac{2v}{1 \quad v^2}dv \quad \frac{dx}{x}$$

Integrating both sides, we get

$$\log(1 + v^2) = -\log x + \log c \quad \Rightarrow \log(1 + v^2)(x) = \log c \Rightarrow (1 + v^2)\, x = c$$

$\Rightarrow \quad x^2 + y^2 = cx$. Substituting $x = 1$, $y = 1$, we get $c = 2$.

Therefore, $x^2 + y^2 - 2x = 0$ is the required equation.

Example 9 Find the equation of a curve passing through $1, \frac{}{4}$ if the slope of the tangent to the curve at any point P (x, y) is $\frac{y}{x} - \cos^2\frac{y}{x}$.

Solution According to the given condition

$$\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x} \qquad \text{... (i)}$$

This is a homogeneous differential equation. Substituting $y = vx$, we get

$$v + x\frac{dv}{dx} = v - \cos^2 v \qquad \Rightarrow \qquad x\frac{dv}{dx} = -\cos^2 v$$

$$\Rightarrow \quad \sec^2 v \, dv = -\frac{dx}{x} \qquad \Rightarrow \quad \tan v = -\log x + c$$

$$\Rightarrow \quad \tan \frac{y}{x} + \log x = c \qquad \text{...(ii)}$$

Substituting $x = 1$, $y = \frac{\;}{4}$, we get. $c = 1$. Thus, we get

$$\tan \frac{y}{x} \quad + \log x = 1, \text{ which is the required equation.}$$

Example 10 Solve $x^2 \frac{dy}{dx} \quad xy = 1 + \cos\left(\frac{y}{x}\right)$, $x \neq 0$ and $x = 1$, $y = \frac{\pi}{2}$

Solution Given equation can be written as

$$x^2 \frac{dy}{dx} \quad xy = 2\cos^2\left(\frac{y}{2x}\right), x \neq 0.$$

$$\Rightarrow \quad \frac{x^2 \frac{dy}{dx} \quad xy}{2\cos^2 \quad \frac{y}{2x}} \quad 1 \quad \Rightarrow \quad \frac{\sec^2 \quad \frac{y}{2x}}{2} \quad x^2 \frac{dy}{dx} \quad xy \quad 1$$

Dividing both sides by x^3, we get

$$\frac{\sec^2\left(\frac{y}{2x}\right)}{2}\left[\frac{x\frac{dy}{dx} - y}{x^2}\right] = \frac{1}{x^3} \qquad \Rightarrow \qquad \frac{d}{dx} \quad \tan \quad \frac{y}{2x} \qquad \frac{1}{x^3}$$

Integrating both sides, we get

$$\tan \quad \frac{y}{2x} \qquad \frac{1}{2x^2} \qquad k.$$

Substituting $x = 1$, $y = \frac{}{2}$, we get

$k = \frac{3}{2}$, therefore, $\tan \frac{y}{2x} \quad \frac{1}{2x^2} \quad \frac{3}{2}$ is the required solution.

Example 11 State the type of the differential equation for the equation. $xdy - ydx = \sqrt{x^2 \quad y^2}\, dx$ and solve it.

Solution Given equation can be written as $xdy = \sqrt{x^2 \quad y^2} \quad y \; dx$, i.e.,

$$\frac{dy}{dx} \quad \frac{\sqrt{x^2 \quad y^2} \quad y}{x} \qquad \text{... (1)}$$

Clearly RHS of (1) is a homogeneous function of degree zero. Therefore, the given equation is a homogeneous differential equation. Substituting $y = vx$, we get from (1)

$$v \quad x\frac{dv}{dx} \quad \frac{\sqrt{x^2 \quad v^2x^2} \quad vx}{x} \qquad \text{i.e. } v \quad x\frac{dv}{dx} \quad \sqrt{1 \quad v^2} \quad v$$

$$x\frac{dv}{dx} \quad \sqrt{1 \quad v^2} \quad \Rightarrow \quad \frac{dv}{\sqrt{1 \quad v^2}} \quad \frac{dx}{x} \qquad \text{... (2)}$$

Integrating both sides of (2), we get

$$\log (v + \sqrt{1 \quad v^2}) = \log x + \log c \Rightarrow v + \sqrt{1 \quad v^2} = cx$$

$$\Rightarrow \frac{y}{x} + \sqrt{1 \quad \frac{y^2}{x^2}} = cx \qquad \Rightarrow y + \sqrt{x^2 \quad y^2} = cx^2$$

Objective Type Questions

Choose the correct answer from the given four options in each of the Examples 12 to 21.

Example 12 The degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3=\left(\frac{d^2y}{dx^2}\right)^2$ is

(A) 1 (B) 2 (C) 3 (D) 4

Solution The correct answer is (B).

Example 13 The degree of the differential equation

$$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2\log\left(\frac{d^2y}{dx^2}\right) \text{ is}$$

(A) 1 (B) 2 (C) 3 (D) not defined

Solution Correct answer is (D). The given differential equation is not a polynomial equation in terms of its derivatives, so its degree is not defined.

Example 14 The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2=\frac{d^2y}{dx^2}$ respectively, are

(A) 1, 2 (B) 2, 2 (C) 2, 1 (D) 4, 2

Solution Correct answer is (C).

Example 15 The order of the differential equation of all circles of given radius a is:

(A) 1 (B) 2 (C) 3 (D) 4

Solution Correct answer is (B). Let the equation of given family be $(x-h)^2+(y-k)^2=a^2$. It has two orbitrary constants h and k. Threrefore, the order of the given differential equation will be 2.

Example 16 The solution of the differential equation $2x.\frac{dy}{dx}-y=3$ represents a family of

(A) straight lines (B) circles (C) parabolas (D) ellipses

Solution Correct answer is (C). Given equation can be written as

$$\frac{2dy}{y \quad 3} \quad \frac{dx}{x} \Rightarrow 2\log(y+3) = \log x + \log c$$

$\Rightarrow$ $(y+3)^2 = cx$ which represents the family of parabolas

Example 17 The integrating factor of the differential equation

$$\frac{dy}{dx}(x \log x) + y = 2\log x \text{ is}$$

(A) e^x (B) $\log x$ (C) $\log(\log x)$ (D) x

Solution Correct answer is (B). Given equation can be written as $\frac{dy}{dx} \quad \frac{y}{x \log x} \quad \frac{2}{x}$.

Therefore, I.F. $= e^{\int \frac{1}{x \log x} dx} = e^{\log (\log x)} = \log x$.

Example 18 A solution of the differential equation $\frac{dy}{dx}^{\;2} \quad x\frac{dy}{dx} \quad y \quad 0$ is

(A) $y = 2$ (B) $y = 2x$ (C) $y = 2x - 4$ (D) $y = 2x^2 - 4$

Solution Correct answer is (C).

Example 19 Which of the following is not a homogeneous function of x and y.

(A) $x^2 + 2xy$ (B) $2x \quad - y$ (C) $\cos^2 \frac{y}{x} \quad \frac{y}{x}$ (D) $\sin x - \cos y$

Solution Correct answer is (D).

Example 20 Solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is

(A) $\frac{1}{x} + \frac{1}{y} = c$ (B) $\log x \,.\, \log y = c$ (C) $xy = c$ (D) $x + y = c$

Solution Correct answer is (C). From the given equation, we get $\log x + \log y = \log c$ giving $xy = c$.

Example 21 The solution of the differential equation $x\frac{dy}{dx} \quad 2y \quad x^2$ is

(A) $y = \frac{x^2 + c}{4x^2}$ (B) $y = \frac{x^2}{4} + c$ (C) $y = \frac{x^4 + c}{x^2}$ (D) $y = \frac{x^4 + c}{4x^2}$

Solution Correct answer is (D). I.F. = $e^{\frac{2}{x}dx} \quad e^{2\log x} \quad e^{\log x^2} \quad x^2$. Therefore, the solution is $y \cdot x^2 = \int x^2 . x dx = \frac{x^4}{4} + k$, i.e., $y = \frac{x^4 \quad c}{4x^2}$.

Example 22 Fill in the blanks of the following:

(i) Order of the differential equation representing the family of parabolas $y^2 = 4ax$ is ____________ .

(ii) The degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^2y}{dx^2}\right)^2 = 0$ is __________ .

(iii) The number of arbitrary constants in a particular solution of the differential equation $\tan x\, dx + \tan y\, dy = 0$ is ____________ .

(iv) $F(x, y) = \frac{\sqrt{x^2 + y^2} + y}{x}$ is a homogeneous function of degree____________ .

(v) An appropriate substitution to solve the differential equation

$$\frac{dx}{dy} = \frac{x^2 \log \frac{x}{y} \quad x^2}{xy \log \frac{x}{y}}$$ is____________ .

(vi) Integrating factor of the differential equation $x\frac{dy}{dx} - y = \sin x$ is ____________ .

(vii) The general solution of the differential equation $\frac{dy}{dx} = e^{x-y}$ is ____________ .

(viii) The general solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=1$ is __________ .

(ix) The differential equation representing the family of curves y = A sin$x$ + B cos$x$ is __________ .

(x) $\frac{e^{2\sqrt{x}}}{\sqrt{x}} \quad \frac{y}{\sqrt{x}} \quad \frac{dx}{dy} \quad 1 (x \quad 0)$ when written in the form $\frac{dy}{dx}+\text{P}y=\text{Q}$, then P = __________ .

Solution

(i) One; a is the only arbitrary constant.

(ii) Two; since the degree of the highest order derivative is two.

(iii) Zero; any particular solution of a differential equation has no arbitrary constant.

(iv) Zero.

(v) $x = vy$.

(vi) $\frac{1}{x}$; given differential equation can be written as $\frac{dy}{dx}-\frac{y}{x}=\frac{\sin x}{x}$ and therefore

I.F. = $e^{\quad \frac{1}{x}dx} = e^{-\log x}=\frac{1}{x}$.

(vii) $e^y = e^x + c$ from given equation, we have $e^y dy = e^x dx$.

(viii) $xy=\frac{x^2}{2} \quad c$; I.F. = $e^{\quad \frac{1}{x}dx} = e^{\log x} = x$ and the solution is $y \,.\, x = \quad x.1dx = \frac{x^2}{2}+\text{C}$.

(ix) $\frac{d^2y}{dx^2}+y=0$; Differentiating the given function w.r.t. x successively, we get

$$\frac{dy}{dx} = \text{A}\cos x - \text{B}\sin x \quad \text{and} \quad \frac{d^2y}{dx^2} = -\text{A}\sin x - \text{B}\cos x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + y = 0 \text{ is the differential equation.}$$

(x) $\frac{1}{\sqrt{x}}$; the given equation can be written as

$$\frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \quad \frac{y}{\sqrt{x}} \qquad \text{i.e.} \qquad \frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

This is a differential equation of the type $\frac{dy}{dx} + \text{P}y = \text{Q}$.

Example 23 State whether the following statements are **True** or **False**.

(i) Order of the differential equation representing the family of ellipses having centre at origin and foci on x-axis is two.

(ii) Degree of the differential equation $\sqrt{1+\frac{d^2y}{dx^2}} = x + \frac{dy}{dx}$ is not defined.

(iii) $\frac{dy}{dx} \quad y \quad 5$ is a differential equation of the type $\frac{dy}{dx} + \text{P}y = \text{Q}$ but it can be solved using variable separable method also.

(iv) $\text{F}(x, y) = \dfrac{y\cos\left(\frac{y}{x}\right) + x}{x\cos\left(\frac{y}{x}\right)}$ is not a homogeneous function.

(v) $\text{F}(x, y) = \dfrac{x^2 \quad y^2}{x \quad y}$ is a homogeneous function of degree 1.

(vi) Integrating factor of the differential equation $\frac{dy}{dx} \quad y \quad \cos x$ is e^x.

(vii) The general solution of the differential equation $x(1 + y^2)dx + y\,(1 + x^2)dy = 0$ is $(1 + x^2)\,(1 + y^2) = k$.

(viii) The general solution of the differential equation $\frac{dy}{dx} + y \sec x = \tan x$ is $y\,(\sec x - \tan x) = \sec x - \tan x + x + k$.

(ix) $x + y = \tan^{-1}y$ is a solution of the differential equation $y^2\frac{dy}{dx} \quad y^2 \quad 1 \quad 0$

(x) $y = x$ is a particular solution of the differential equation $\frac{d^2y}{dx^2} \quad x^2\frac{dy}{dx} \quad xy \quad x$.

Solution

(i) True, since the equation representing the given family is $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$, which has two arbitrary constants.

(ii) True, because it is not a polynomial equation in its derivatives.

(iii) True

(iv) True, because $f(\lambda x, \lambda y) = \lambda^{\circ} f(x, y)$.

(v) True, because $f(\lambda x, \lambda y) = \lambda^1 f(x, y)$.

(vi) False, because I.F = $e^{\quad 1dx} \quad e^{-x}$.

(vii) True, because given equation can be written as

$$\frac{2x}{1 \quad x^2}dx \quad \frac{2y}{1 \quad y^2}dy$$

$\Rightarrow$ $\log(1 + x^2) = -\log(1 + y^2) + \log k$

$\Rightarrow$ $(1 + x^2)(1 + y^2) = k$

(viii) False, since I.F. = $e^{\quad \sec x dx} \quad e^{\log(\sec x \quad \tan x)} =$ $\sec x + \tan x$, the solution is,

$y(\sec x + \tan x) = \quad (\sec x \quad \tan x)\tan x dx = \int(\sec x \tan x + \sec^2 x - 1)dx =$

$\sec x + \tan x - x + k$

(ix) True, $x + y = \tan^{-1}y \Rightarrow 1 \quad \frac{dy}{dx} \quad \frac{1}{1 \quad y^2}\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{1+y^2} - 1\right) = 1$, i.e., $\frac{dy}{dx} \quad \frac{(1 \quad y^2)}{y^2}$ which satisfies the given equation.

(x) False, because $y = x$ does not satisfy the given differential equation.

9.3 EXERCISE

Short Answer (S.A.)

1. Find the solution of $\frac{dy}{dx}\ \ 2^{y\ \ x}$.

2. Find the differential equation of all non vertical lines in a plane.

3. Given that $\frac{dy}{dx}\ \ e^{\ 2y}$ and $y = 0$ when $x = 5$.

Find the value of x when $y = 3$.

4. Solve the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2\ \ 1}$.

5. Solve the differential equation $\frac{dy}{dx}\ \ 2xy\ \ y$

6. Find the general solution of $\frac{dy}{dx}\ \ ay\ \ e^{mx}$

7. Solve the differential equation $\frac{dy}{dx}\ \ 1\ \ e^{x\ \ y}$

8. Solve: $ydx - xdy = x^2ydx$.

9. Solve the differential equation $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0$, $x = 0$.

10. Find the general solution of $(x + 2y^3)\frac{dy}{dx} = y$.

11. If $y(x)$ is a solution of $\frac{2\ \ \sin x}{1\ \ y}\ \frac{dy}{dx} = -\cos x$ and y (0) = 1, then find the value

of $y\ \ \frac{\ }{2}$.

12. If $y(t)$ is a solution of $(1 + t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then show that

$y(1) = -\frac{1}{2}$.

13. Form the differential equation having $y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$, where A and B are arbitrary constants, as its general solution.

14. Form the differential equation of all circles which pass through origin and whose centres lie on y-axis.

15. Find the equation of a curve passing through origin and satisfying the differential equation $(1 \quad x^2)\frac{dy}{dx} \quad 2xy \quad 4x^2$.

16. Solve : $x^2 \frac{dy}{dx} = x^2 + xy + y^2$.

17. Find the general solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$.

18. Find the general solution of $y^2 dx + (x^2 - xy + y^2)\, dy = 0$.

19. Solve : $(x + y)(dx - dy) = dx + dy$.[**Hint:** Substitute $x + y = z$ after seperating dx and dy]

20. Solve : $2(y + 3) - xy\frac{dy}{dx} = 0$, given that $y(1) = -2$.

21. Solve the differential equation $dy = \cos x\,(2 - y\,\mathrm{cosec}\,x)\,dx$ given that $y = 2$ when $x = \frac{\pi}{2}$.

22. Form the differential equation by eliminating A and B in $Ax^2 + By^2 = 1$.

23. Solve the differential equation $(1 + y^2)\tan^{-1}x\, dx + 2y(1 + x^2)\, dy = 0$.

24. Find the differential equation of system of concentric circles with centre (1, 2).

Long Answer (L.A.)

25. Solve : $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

26. Find the general solution of $(1 + \tan y)(dx - dy) + 2x\,dy = 0$.

27. Solve : $\frac{dy}{dx} = \cos(x + y) + \sin(x + y)$.[**Hint:** Substitute $x + y = z$]

28. Find the general solution of $\frac{dy}{dx} \quad 3y \quad \sin 2x$.

29. Find the equation of a curve passing through (2, 1) if the slope of the tangent to the curve at any point (x, y) is $\frac{x^2 \quad y^2}{2xy}$.

30. Find the equation of the curve through the point (1, 0) if the slope of the tangent to the curve at any point (x, y) is $\frac{y \quad 1}{x^2 \quad x}$.

31. Find the equation of a curve passing through origin if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abcissa and ordinate of the point.

32. Find the equation of a curve passing through the point (1, 1). If the tangent drawn at any point P (x, y) on the curve meets the co-ordinate axes at A and B such that P is the mid-point of AB.

33. Solve : $x\frac{dy}{dx} \quad y\ (\log y - \log x + 1)$

Objective Type

Choose the correct answer from the given four options in each of the Exercises from 34 to 75 (M.C.Q)

34. The degree of the differential equation $\frac{d^2y}{dx^2}\ ^2 \quad \frac{dy}{dx}\ ^2 \quad x\sin \frac{dy}{dx}$ is:

(A) 1 (B) 2 (C) 3 (D) not defined

35. The degree of the differential equation $1 \quad \frac{dy}{dx}\ ^2\ ^{\frac{3}{2}} \quad \frac{d^2y}{dx^2}$ is

(A) 4 (B) $\frac{3}{2}$ (C) not defined (D) 2

36. The order and degree of the differential equation $\frac{d^2y}{dx^2} \quad \frac{dy}{dx}\ ^{\frac{1}{4}} + x^{\frac{1}{5}} \quad 0$,

respectively, are

(A) 2 and not defined (B) 2 and 2 (C) 2 and 3 (D) 3 and 3

37. If $y = e^{-x}(A\cos x + B\sin x)$, then y is a solution of

(A) $\frac{d^2y}{dx^2} \quad 2\frac{dy}{dx} \quad 0$ (B) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(C) $\frac{d^2y}{dx^2} \quad 2\frac{dy}{dx} \quad 2y \quad 0$ (D) $\frac{d^2y}{dx^2} + 2y = 0$

38. The differential equation for $y = A\cos \alpha x + B\sin \alpha x$, where A and B are arbitrary constants is

(A) $\frac{d^2y}{dx^2} \quad {}^2 y \quad 0$ (B) $\frac{d^2y}{dx^2} \quad {}^2 y \quad 0$

(C) $\frac{d^2y}{dx^2} \quad y \quad 0$ (D) $\frac{d^2y}{dx^2} \quad y \quad 0$

39. Solution of differential equation $xdy - ydx = 0$ represents :

(A) a rectangular hyperbola
(B) parabola whose vertex is at origin
(C) straight line passing through origin
(D) a circle whose centre is at origin

40. Integrating factor of the differential equation $\cos x \frac{dy}{dx} + y\sin x = 1$ is :

(A) $\cos x$ (B) $\tan x$ (C) $\sec x$ (D) $\sin x$

41. Solution of the differential equation $\tan y \sec^2 x\, dx + \tan x \sec^2 y\, dy = 0$ is :

(A) $\tan x + \tan y = k$ (B) $\tan x - \tan y = k$

(C) $\frac{\tan x}{\tan y} \quad k$ (D) $\tan x \,.\, \tan y = k$

42. Family $y = Ax + A^3$ of curves is represented by the differential equation of degree :

(A) 1 (B) 2 (C) 3 (D) 4

43. Integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$ is :

(A) x (B) $\log x$ (C) $\frac{1}{x}$ (D) $-x$

44. Solution of $\frac{dy}{dx} \quad y \quad 1$, $y(0) = 1$ is given by

(A) $xy \quad = -e^x$ (B) $xy \quad = -e^{-x}$ (C) $xy \quad = -1$ (D) $y \;= 2e^x - 1$

45. The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$ when $y(1) = 2$ is :

(A) none (B) one (C) two (D) infinite

46. Which of the following is a second order differential equation?

(A) $(y')^2 + x = y^2$ (B) $y'y'' + y = \sin x$

(C) $y''' + (y'')^2 + y = 0$ (D) $y' = \ y^2$

47. Integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} - xy = 1$ is

(A) $-x$ (B) $\frac{x}{1 \quad x^2}$ (C) $\sqrt{1 \quad x^2}$ (D) $\frac{1}{2}\log(1 - x^2)$

48. $\tan^{-1}x + \tan^{-1}y = c$ is the general solution of the differential equation:

(A) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ (B) $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$

(C) $(1 + x^2)\, dy + (1 + y^2)\, dx = 0$ (D) $(1 + x^2)\, dx + (1 + y^2)\, dy = 0$

49. The differential equation $y\frac{dy}{dx} + x = c$ represents :

(A) Family of hyperbolas (B) Family of parabolas

(C) Family of ellipses (D) Family of circles

50. The general solution of $e^x \cos y\, dx - e^x \sin y\, dy = 0$ is :

(A) $e^x \cos y = k$ (B) $e^x \sin y = k$

(C) $e^x \ = k \cos y$ (D) $e^x \ = k \sin y$

51. The degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y^5 = 0$ is :

(A) 1 (B) 2 (C) 3 (D) 5

52. The solution of $\frac{dy}{dx} + y = e^{-x}, y(0) = 0$ is :

(A) $y = e^x (x - 1)$ (B) $y = xe^{-x}$

(C) $y = xe^{-x} + 1$ (D) $y = (x + 1)e^{-x}$

53. Integrating factor of the differential equation $\frac{dy}{dx} \quad y\tan x - \sec x \quad 0$ is:

(A) $\cos x$ (B) $\sec x$

(C) $e^{\cos x}$ (D) $e^{\sec x}$

54. The solution of the differential equation $\frac{dy}{dx} \quad \frac{1 \quad y^2}{1 \quad x^2}$ is:

(A) $y = \tan^{-1}x$ (B) $y - x = k(1 + xy)$

(C) $x = \tan^{-1}y$ (D) $\tan(xy) = k$

55. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is:

(A) $\frac{x}{e^x}$ (B) $\frac{e^x}{x}$

(C) xe^x (D) e^x

56. $y = ae^{mx} + be^{-mx}$ satisfies which of the following differential equation?

(A) $\frac{dy}{dx} \quad my \quad 0$ (B) $\frac{dy}{dx} \quad my \quad 0$

(C) $\frac{d^2y}{dx^2} \quad m^2y \quad 0$ (D) $\frac{d^2y}{dx^2} \quad m^2y \quad 0$

57. The solution of the differential equation $\cos x \sin y \, dx + \sin x \cos y \, dy = 0$ is :

(A) $\frac{\sin x}{\sin y} \quad c$ (B) $\sin x \sin y = c$

(C) $\sin x + \sin y = c$ (D) $\cos x \cos y = c$

58. The solution of $x\frac{dy}{dx} + y = e^x$ is:

(A) $y = \frac{e^x}{x} \quad \frac{k}{x}$ (B) $y = xe^x + cx$

(C) $y = xe^x + k$ (D) $x = \frac{e^y}{y} \quad \frac{k}{y}$

59. The differential equation of the family of curves $x^2 + y^2 - 2ay = 0$, where a is arbitrary constant, is:

(A) $(x^2 - y^2)\dfrac{dy}{dx} = 2xy$ (B) $2\,(x^2 + y^2)\dfrac{dy}{dx} = xy$

(C) $2\,(x^2 - y^2)\dfrac{dy}{dx} = xy$ (D) $(x^2 + y^2)\dfrac{dy}{dx} = 2xy$

60. Family $y = Ax + A^3$ of curves will correspond to a differential equation of order

(A) 3 (B) 2 (C) 1 (D) not defined

61. The general solution of $\dfrac{dy}{dx} = 2x\, e^{x^2 - y}$ is :

(A) $e^{x^2 - y} = c$ (B) $e^{-y} + e^{x^2} = c$

(C) $e^{y} = e^{x^2} + c$ (D) $e^{x^2 + y} = c$

62. The curve for which the slope of the tangent at any point is equal to the ratio of the abcissa to the ordinate of the point is :

(A) an ellipse (B) parabola

(C) circle (D) rectangular hyperbola

63. The general solution of the differential equation $\dfrac{dy}{dx}\quad e^{\frac{x^2}{2}} + xy$ is :

(A) $y \quad ce^{\frac{x^2}{2}}$ (B) $y \quad ce^{\frac{x^2}{2}}$

(C) $y = (x+c)e^{\frac{x^2}{2}}$ (D) $y \quad (c \quad x)e^{\frac{x^2}{2}}$

64. The solution of the equation $(2y - 1)\, dx - (2x + 3)dy = 0$ is :

(A) $\dfrac{2x \quad 1}{2y \quad 3} \quad k$ (B) $\dfrac{2y+1}{2x-3} = k$

(C) $\dfrac{2x \quad 3}{2y \quad 1} \quad k$ (D) $\dfrac{2x \quad 1}{2y \quad 1} \quad k$

65. The differential equation for which $y = a\cos x + b\sin x$ is a solution, is :

(A) $\frac{d^2y}{dx^2} + y = 0$ (B) $\frac{d^2y}{dx^2} - y = 0$

(C) $\frac{d^2y}{dx^2} + (a + b)\, y = 0$ (D) $\frac{d^2y}{dx^2} + (a - b)\, y = 0$

66. The solution of $\frac{dy}{dx} + y = e^{-x}$, $y(0) = 0$ is :

(A) $y = e^{-x}(x - 1)$ (B) $y = xe^{x}$

(C) $y = xe^{-x} + 1$ (D) $y = xe^{-x}$

67. The order and degree of the differential equation

$$\frac{d^3y}{dx^3}^{2} \quad 3\frac{d^2y}{dx^2} \quad 2 \quad \frac{dy}{dx}^{4} \quad y^4 \text{ are :}$$

(A) 1, 4 (B) 3, 4 (C) 2, 4 (D) 3, 2

68. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right] = \frac{d^2y}{dx^2}$ are :

(A) 2, $\frac{3}{2}$ (B) 2, 3 (C) 2, 1 (D) 3, 4

69. The differential equation of the family of curves $y^2 = 4a(x + a)$ is :

(A) $y^2 = 4\frac{dy}{dx}\left(x+\frac{dy}{dx}\right)$ (B) $2y\frac{dy}{dx} \quad 4a$

(C) $y\frac{d^2y}{dx^2} \quad \frac{dy}{dx}^{2} \quad 0$ (D) $2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2 - y$

70. Which of the following is the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$?

(A) $y = (Ax + B)e^{x}$ (B) $y = (Ax + B)e^{-x}$

(C) $y = Ae^{x} + Be^{-x}$ (D) $y = A\cos x + B\sin x$

71. General solution of $\frac{dy}{dx} + y \tan x = \sec x$ is :

(A) $y \sec x = \tan x + c$ (B) $y \tan x = \sec x + c$

(C) $\tan x = y \tan x + c$ (D) $x \sec x = \tan y + c$

72. Solution of the differential equation $\frac{dy}{dx} \quad \frac{y}{x} \quad \sin x$ is :

(A) $x (y + \cos x) = \sin x + c$ (B) $x (y - \cos x) = \sin x + c$

(C) $xy \cos x = \sin x + c$ (D) $x (y + \cos x) = \cos x + c$

73. The general solution of the differential equation $(e^x + 1)\, y dy = (y + 1)\, e^x dx$ is:

(A) $(y + 1) = k (e^x + 1)$ (B) $y + 1 = e^x + 1 + k$

(C) $y = \log \{k (y + 1) (e^x + 1)\}$ (D) $y \quad \log \frac{e^x \quad 1}{y \quad 1} \quad k$

74. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is :

(A) $y = e^{x-y} - x^2 e^{-y} + c$ (B) $e^y - e^x = \frac{x^3}{3} + c$

(C) $e^x + e^y = \frac{x^3}{3} + c$ (D) $e^x - e^y = \frac{x^3}{3} + c$

75. The solution of the differential equation $\frac{dy}{dx} \quad \frac{2xy}{1 \quad x^2} \quad \frac{1}{(1 \quad x^2)^2}$ is :

(A) $y (1 + x^2) = c + \tan^{-1}x$ (B) $\frac{y}{1 \quad x^2} = c + \tan^{-1}x$

(C) $y \log (1 + x^2) = c + \tan^{-1}x$ (D) $y (1 + x^2) = c + \sin^{-1}x$

76. Fill in the blanks of the following (i to xi)

(i) The degree of the differential equation $\frac{d^2y}{dx^2} \quad e^{\frac{dy}{dx}} \quad 0$ is __________.

(ii) The degree of the differential equation $\sqrt{1 \quad \frac{dy}{dx}^{2}} \quad x$ is __________.

(iii) The number of arbitrary constants in the general solution of a differential equation of order three is __________.

(iv) $\frac{dy}{dx} \quad \frac{y}{x \log x} \quad \frac{1}{x}$ is an equation of the type __________.

(v) General solution of the differential equation of the type $\frac{dx}{dy} + P_1 x = Q_1$ is given by __________.

(vi) The solution of the differential equation $\frac{xdy}{dx} \quad 2y \quad x^2$ is __________.

(vii) The solution of $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ is __________.

(viii) The solution of the differential equation $ydx + (x + xy)dy = 0$ is ______.

(ix) General solution of $\frac{dy}{dx} \quad y = \sin x$ is __________.

(x) The solution of differential equation $\cot y \, dx = xdy$ is __________.

(xi) The integrating factor of $\frac{dy}{dx} \quad y \quad \frac{1 \quad y}{x}$ is __________.

77. State **True** or **False** for the following:

(i) Integrating factor of the differential of the form $\frac{dx}{dy} + p_1 x = Q_1$ is given by $e^{\int p_1 dy}$.

(ii) Solution of the differential equation of the type $\frac{dx}{dy} + p_1 x = Q_1$ is given by $x.\text{I.F.} = \quad (\text{I.F}) \quad Q_1 dy$.

(iii) Correct substitution for the solution of the differential equation of the type $\frac{dy}{dx} \quad f(x, y)$, where $f(x, y)$ is a homogeneous function of zero degree is $y = vx$.

(iv) Correct substitution for the solution of the differential equation of the type $\frac{dx}{dy} \quad g(x, y)$ where $g(x, y)$ is a homogeneous function of the degree zero is $x = vy$.

(v) Number of arbitrary constants in the particular solution of a differential equation of order two is two.

(vi) The differential equation representing the family of circles $x^2 + (y - a)^2 = a^2$ will be of order two.

(vii) The solution of $\frac{dy}{dx} \quad \frac{y}{x}^{\frac{1}{3}}$ is $y^{\frac{2}{3}} - x^{\frac{2}{3}} = c$.

(viii) Differential equation representing the family of curves $y = e^x (A\cos x + B\sin x)$ is $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} \quad 2y \quad 0$

(ix) The solution of the differential equation $\frac{dy}{dx} = \frac{x+2y}{x}$ is $x + y = kx^2$.

(x) Solution of $\frac{xdy}{dx} \quad y \quad x\tan\frac{y}{x}$ is $\sin \frac{y}{x} \quad cx$

(xi) The differential equation of all non horizontal lines in a plane is $\frac{d^2x}{dy^2} = 0$.