Cube and Cube Root

Solution 1:

Sr. No.	Number	Digit at the units place of the given number	Digit at the units place obtained by cubing the number
(1)	711	1	1
(2)	408	8	2
(3)	544	4	4
(4)	57	7	3
(5)	26	6	6
(6)	70	0	0

Solution 2:

Sr. No.	Number	Digit at the units place of the given number	Digit at the units place of the cube root
(1)	216	6	6
(2)	2197	7	3
(3)	2744	4	4
(4)	6859	9	9
(5)	42875	5	5
(6)	125000	0	0

Solution 3:

(1) 400

$$\begin{array}{r|r} 2 & 400 \\ \hline 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$400 = \underline{2 \times 2 \times 2} \times \underline{2} \times \underline{5 \times 5}$

Here, in the first group, 2 appears three times but in second group, 2 appears only once and in third group, 5 appears only two times.

∴ 400 is not a perfect cube.

(2) 9000

2	9000
2	4500
2	2250
3	1125
3	375
5	125
5	25
5	5
	1

$9000 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5 \times 5}$

Here, in the first group, 2 appears three times, in the third group, 5 appears three times, but in the second group, 3 appears only two times.

∴ 9000 is not a perfect cube.

(3) 343

$343 = 7 \times 7 \times 7 = 7^3$

∴ 343 is a perfect cube.

(4) 17576

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

$17576 = \underline{2 \times 2 \times 2} \times \underline{13 \times 13 \times 13}$

$= 2^3 \times 13^3$

$= (2 \times 13)^3$

$= 26^3$

∴ 17576 is a perfect cube.

Solution 4(1):

675

3	675
3	225
3	75
5	25
5	5
	1

$675 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5}$

$= 3^3 \times 5 \times 5$

Here, in the first group, the prime factor 3 forms a group of three, but in the second group, the prime factor 5 appears only twice. Thus,

$675 \times 5 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$= 3^3 \times 5^3$

$= (3 \times 5)^3$

$= 15^3$

Hence, 675 should be multiplied by the smallest number 5 to obtain a perfect cube.

Solution 4(2):

392

2	392
2	196
2	98
7	49
7	7
	1

$392 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7}$

$= 2^3 \times 7 \times 7$

Here, in the first group, the prime factor 2 forms a group of three, but in the second group, the prime factor 7 appears only twice. Thus,

$392 \times 7 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

$= 2^3 \times 7^3$

$= (2 \times 7)^3$

$= 14^3$

Hence, 392 should be multiplied by the smallest number 7 to obtain a perfect cube.

Solution 4(3):

968

2	968
2	484
2	242
11	121
11	11
	1

$968 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11}$

$= 2^3 \times 11 \times 11$

Here, in the first group, the prime factor 2 forms a group of three, but in the second group, the prime factor 11 appears only twice. Thus,

$968 \times 11 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

$= 2^3 \times 11^3$

$= (2 \times 11)^3$

$= 22^3$

Hence, 968 should be multiplied by the smallest number 11 to obtain a perfect cube.

Solution 4(4):

875

5	875
5	175
5	35
7	7
	1

$875 = \underline{5 \times 5 \times 5} \times \underline{7}$

$= 5^3 \times 7$

Here, in the first group, the prime factor 5 forms a group of three, but in the second group, the prime factor 7 appears only once. Thus,

$875 \times 7 \times 7 = \underline{5 \times 5 \times 5} \times \underline{7 \times 7 \times 7}$

$= 5^3 \times 7^3$

$= (5 \times 7)^3$

$= 35^3$

Hence, 875 should be multiplied by the smallest number 7 × 7 = 49 to obtain a perfect cube.

Solution 5(1):

1536

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

$1536 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3}$

Here, in the first three groups, the prime factor 2 forms a group of three, but in the fourth group, the prime factor 3 appears only once.

Hence, 1536 should be divided by the smallest number 3 to obtain a perfect cube.

Solution 5(2):

8019

3	8019
3	2673
3	891
3	297
3	99
3	33
11	11
	1

$8019 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{11}$

Here, in the first two groups, the prime factor 3 forms a group of three, but in the third group, the prime factor 11 appears only once.

Thus, 8019 should be divided by the smallest number 11 to obtain a perfect cube.

Solution 5(3):

7000

2	7000
2	3500
2	1750
5	875
5	175
5	35
7	7
	1

$7000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} \times \underline{7}$

Here, in the first group, the prime factor 2 and in the second group, the prime factor 5, form a group of three, but in the third group, the prime factor 7 appears only once.

Thus, 7000 should be divided by the smallest number 7 to obtain a perfect cube.

Solution 5(4):

5400

2	5400
2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

5400 = $\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5}$

Here, in the first group, the prime factor 2 and in the second group, the prime factor 3, form a group three, but in the third group, the prime factor 5 appears only twice.

Thus, $\frac{5400}{5 \times 5} = \frac{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5}{5 \times 5}$

$\therefore \frac{5400}{5 \times 5} = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 3^3$

$= (2 \times 3)^3$

$= 6^3$

Thus, 5400 should be divided by the smallest number 5 × 5 = 25, to obtain a perfect cube.

Solution 6(1):

512

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$512 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3 \times 2^3$

$= (2 \times 2 \times 2)^3$

$= 8^3$

$\therefore \sqrt[3]{512} = 8$

Solution 6(2):

3375

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$
$= 3^3 \times 5^3$
$= (3 \times 5)^3$
$= 15^3$

$\therefore \sqrt[3]{3375} = 15$

Solution 6(3):

17576

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

$17576 = \underline{2 \times 2 \times 2} \times \underline{13 \times 13 \times 13}$
$= 2^3 \times 13^3$
$= (2 \times 13)^3$
$= 26^3$

$\therefore \sqrt[3]{17576} = 26$

Solution 6(4):

35937

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

$35937 = \underline{3 \times 3 \times 3} \times \underline{11 \times 11 \times 11}$
$= 3^3 \times 11^3$
$= (3 \times 11)^3$
$= 33^3$

$\therefore \sqrt[3]{35937} = 33$

Solution 6(5):

32768

2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$32768 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$

$= (2 \times 2 \times 2 \times 2 \times 2)^3$

$= 32^3$

$\therefore \sqrt[3]{32768} = 32$

Solution 6(6):

29791

31	29791
31	961
31	31
	1

$29{,}791 = \underline{31 \times 31 \times 31}$

$= 31^3$

$\therefore \sqrt[3]{29791} = 31$

Solution 7(1):

4096

$\sqrt[3]{4096}$

$\downarrow \; \downarrow$

$1 \quad 6$

Step 1: Divide the given number into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number.

Here, the first part is 096 and the second part is 4.

Step 2: Here, the first part is 096 whose units place digit is 6. Hence, the digit at the units place of the cube root is 6.

Step 3: The number 4 in the second part lies between 1 and 8, i.e. $1^3 < 4 < 2^3$. Considering the smaller number out of 1 and 3, the digit at the tens place of the cube root is 1.

$\therefore \sqrt[3]{4096} = 16$

Solution 7(2):

42,875

$$\sqrt[3]{42875}$$
$\downarrow \quad \downarrow$
3 5

Step 1: Divide the given number into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number.
Here, the first part is 875 and the second part is 42.

Step 2: Here, the first part is 875 whose units place digit is 5. Hence, the digit at the units place of the cube root is 5.
Step 3: The number 42 in the second part lies between 27 and 64, i.e. $3^3 < 42 < 4^3$.
Considering the smaller number out of 3 and 4, the digit at the tens place of the cube root is 3.

$\therefore \sqrt[3]{42875} = 35$

Solution 7(3):
85184

$$\sqrt[3]{85184}$$
$\downarrow \quad \downarrow$
4 4

Step 1: Divide the given number into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number.
Here, the first part is 184 and the second part is 85.

Step 2: Here, the first part is 184 whose units place digit is 4. Hence, the digit at the units place of the cube root is 4.
Step 3: The number 85 in the second part lies between 64 and 125, i.e. $4^3 < 85 < 5^3$.
Considering the smaller number out of 4 and 5, the digit at the tens place of the cube root is 4.

$\therefore \sqrt[3]{85184} = 44$

Solution 7(4):
54, 872

$$\sqrt[3]{54872}$$
$\downarrow \quad \downarrow$
3 8

Step 1: Divide the given number into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number.
Here, the first part is 872 and the second part is 54.

Step 2: Here, the first part is 872 whose units place digit is 2. Hence, the digit at the units place of the cube root is 8.

Step 3: The number 54 in the second part lies between 27 and 64, i.e. $3^3 < 54 < 4^3$.
Considering the smaller number out of 3 and 4, the digit at the tens place of the cube root is 3.

$\therefore \sqrt[3]{54872} = 38$

Solution 7(5):

74,088

$$\sqrt[3]{74088}$$
$$\downarrow \quad \downarrow$$
$$4 \quad 2$$

Step 1: Divide the given number into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number.

Here, the first part is 088 and the second part is 74.

Step 2: Here, the first part is 088 whose units place digit is 8. Hence, the digit at the units place of the cube root is 2.

Step 3: The number 74 in the second part lies between 64 and 125, i.e. $4^3 < 74 < 5^3$.

Considering the smaller number out of 4 and 5, the digit at the tens place of the cube root is 4.

$\therefore \sqrt[3]{74088} = 42$

Solution 7(6):

140608

$$\sqrt[3]{140608}$$
$$\downarrow \quad \downarrow$$
$$5 \quad 2$$

Step 1: Divide the given number into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number.

Here, the first part is 608 and the second part is 140.

Step 2: Here, the first part is 608 whose units place digit is 8. Hence, the digit at the units place of the cube root is 2.

Step 3: The number 140 in the second part lies between 125 and 216, i.e. $5^3 < 140 < 6^3$.

Considering the smaller number out of 5 and 6, the digit at the tens place of the cube root is 5.

$\therefore \sqrt[3]{140608} = 52$

Practice 1

Solution 1:

Sr. No.	Number	Digit at the units place of the number	Digit at the units place obtained by cubing the number
(1)	401	1	1
(2)	258	8	2
(3)	344	4	4
(4)	47	7	3
(5)	66	6	6
(6)	25	5	5
(7)	79	9	9
(8)	10	0	0

Solution 2:

Sr. No.	Number	Digit at the units place of the number	Digit at the units place of the cube root
(1)	729	9	9
(2)	4096	6	6
(3)	15625	5	5
(4)	13824	4	4
(5)	12167	7	3
(6)	8000	0	0
(7)	5832	2	8
(8)	1331	1	1

Practice 2

Solution 1:

729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$729 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$= 3^3 \times 3^3$

$= (3 \times 3)^3$

$= 9^3$

∴ 729 is a perfect cube.

Solution 2:

100

2	100
2	50
5	25
5	5
	1

$100 = \underline{2 \times 2} \times \underline{5 \times 5}$

Here, in the first as well as second group, the prime factor 2 and the prime factor 5 appear only two times. None of them form a group of three.

∴ 100 is not a perfect cube.

Solution 3:

243

3	243
3	81
3	27
3	9
3	3
	1

$243 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3}$

Here, in the first group, the prime factor 3 forms a group of three, but in the second group, the prime factor 3 appears only two times.

∴ 243 is not a perfect cube.

Solution 4:

400

2	400
2	200
2	100
2	50
5	25
5	5
	1

$400 = \underline{2 \times 2 \times 2} \times \underline{2} \times \underline{5 \times 5}$

Here, in the first group, the prime factor 2 forms a group of three, but in the second group, the prime factor 2 appears once and in the third group, the prime factor 5 appears only two times.

∴ 400 is not a perfect cube.

Solution 5:

3375

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$= 3^3 \times 5^3$

$= (3 \times 5)^3$

$= 15^3$

∴ 3375 is a perfect cube.

Solution 6:

127000

2	127000
2	63500
2	31750
5	15875
5	3175
5	635
127	127
	1

$127000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} \times \underline{127}$

Here, in the first and the second group, the prime factor 2 and prime factor 5 form a group of 3, but in the third group, the prime factor 127 appears only once.

∴ 127000 is not a perfect cube.

*Remark: The answer given in the textbook is calculated for the cube root of 27000.

Solution 7:

4913

17	4913
17	289
17	17
	1

$4913 = \underline{17 \times 17 \times 17}$

$= 17^3$

∴ 4913 is a perfect cube.

Solution 8:

4096

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$4096 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3 \times 2^3 \times 2^3$

$= (2 \times 2 \times 2 \times 2)^3$

$= 16^3$

∴ 4096 is a perfect cube.

Practice 3

Solution 1(1):

256

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$256 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2}$

$= 2^3 \times 2^3 \times 2 \times 2$

Here, in the first two groups, the prime factor 2 forms a group of three, but in the third group, the prime factor 2 appears only twice. Thus, we have

$256 \times 2 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3 \times 2^3$

$= (2 \times 2 \times 2)^3$

$= 8^3$

Hence, 256 should be multiplied by the smallest number 2 to obtain a perfect cube.

Solution 1(2):

100

2	100
2	50
5	25
5	5
	1

$100 = \underline{2 \times 2} \times \underline{5 \times 5}$

Here, in the first group, the prime factor 2 and in the second group, the prime factor 5 appear only twice. Thus, we have

$100 \times 2 \times 5 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

$= 2^3 \times 5^3$

$= (2 \times 5)^3$

$= 10^3$

Hence, 100 should be multiplied by the smallest number 2 × 5 = 10 to obtain a perfect cube.

Solution 1(3):

576

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

$576 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3}$

Here, in the first two groups, the prime factor 2 forms a group of three each, but in the third group, the prime factor 3 appears only twice. Thus, we have

$576 \times 3 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 3^3$

$= (2 \times 2 \times 3)^3$

$= 12^3$

Hence, 576 should be multiplied by the smallest number 3 to obtain a perfect cube.

Solution 1(4):

81

3	81
3	27
3	9
3	3
	1

$81 = \underline{3 \times 3 \times 3} \times \underline{3}$

Here, in the first group, the prime factor 3 forms a group of three, but in the second group, the prime factor 3 appears only once. Thus, we have

$81 \times 3 \times 3 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$= 3^3 \times 3^3$

$= (3 \times 3)^3$

$= 9^3$

Hence, 81 should be multiplied by the smallest number 3 × 3 = 9, to obtain a perfect cube.

Solution 1(5):

1715

5	1715
7	343
7	49
7	7
	1

$1715 = \underline{5} \times \underline{7 \times 7 \times 7}$

Here, in the second group, the prime factor 7 forms a group of three, but in the first group, the prime factor 5 appears only once. Thus, we have

$1715 \times 5 \times 5 = \underline{5 \times 5 \times 5} \times \underline{7 \times 7 \times 7}$

$= 5^3 \times 7^3$

$= (5 \times 7)^3$

$= 35^3$

Hence, 1715 should be multiplied by the smallest number 5 × 5 = 25, to obtain a perfect cube.

Solution 2(1):

88

2	88
2	44
2	22
11	11
	1

$88 = \underline{2 \times 2 \times 2} \times \underline{11}$

Here, in the first group, the prime factor 2 forms a group of three, but in the second group, the prime factor 11 appears only once.

Hence, 88 should be divided by the smallest number 11 to obtain a perfect cube.

Solution 2(2):

875

5	875
5	175
5	35
7	7
	1

$875 = \underline{5 \times 5 \times 5} \times \underline{7}$

Here, in the first group, the prime factor 5 forms a group of three, but in the second group, the prime factor 7 appears only once.

Hence, 875 should be divided by the smallest number 7 to obtain a perfect cube.

Solution 2(3):

1512

2	1512
2	756
2	378
3	189
3	63
3	21
7	7
	1

$1512 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7}$

Here, in the first group and the second group, the prime factor 2 and the prime factor 3 form a group of three, but in the third group, the prime factor 7 appears only once.

$$\text{Thus, } \frac{1512}{7} = \frac{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7}{7}$$

$$\therefore \frac{1512}{7} = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$$

$$= 2^3 \times 3^3 = 6^3$$

Hence, 1512 should be divided by the smallest number 7 to obtain a perfect cube.

Solution 2(4):

625

5	625
5	125
5	25
5	5
	1

$625 = \underline{5 \times 5 \times 5} \times \underline{5}$

Here, in the first group, the prime factor 5 forms a group of three, but in the second group, the prime factor 5 appears only once.

$$\text{Thus, } \frac{625}{5} = \frac{5 \times 5 \times 5 \times 5}{5}$$

$$\therefore \frac{625}{5} = \underline{5 \times 5 \times 5}$$

$$= 5^3$$

Hence, 625 should be divided by the smallest number 5 to obtain a perfect cube.

Solution 2(5):

13500

2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$13500 = \underline{2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

Here, in the second group, the prime factor 3 and in the third group, the prime factor 5 form a group of three, but in the first group, the prime factor 2 appears only twice.

Thus, $\frac{13500}{2\times2}=\frac{2\times2\times3\times3\times3\times5\times5\times5}{2\times2}$

$\therefore \frac{13500}{2\times2}=\underline{3\times3\times3}\times\underline{5\times5\times5}$

$=3^3\times5^3$

$=15^3$

Hence, 13500 should be divided by the smallest number 2 × 2 = 4, to obtain a perfect cube.

Practice 4

Solution 1(1):

$\frac{8}{27}$

Numerator : 8

$$\begin{array}{c|c} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Denominator:27

$$\begin{array}{c|c} 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$\frac{8}{27}=\frac{2\times2\times2}{3\times3\times3}=\frac{2^3}{3^3}$

$\therefore \sqrt[3]{\frac{8}{27}}=\sqrt[3]{\frac{2^3}{3^3}}=\frac{2}{3}$

Solution 1(2):

$\frac{27}{125}$

Numerator : 27

$$\begin{array}{c|c} 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Denominator : 125

$$\begin{array}{c|c} 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\frac{27}{125}=\frac{3\times3\times3}{5\times5\times5}=\frac{3^3}{5^3}$

$\therefore \sqrt[3]{\frac{27}{125}}=\sqrt[3]{\frac{3^3}{5^3}}=\frac{3}{5}$

Solution 1(3):

$\frac{125}{729}$

Numerator : 125

5	125
5	25
5	5
	1

Denominator : 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$$\frac{125}{729} = \frac{5 \times 5 \times 5}{3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{5^3}{3^3 \times 3^3}$$

$$\therefore \sqrt[3]{\frac{27}{125}} = \sqrt[3]{\frac{5^3}{3^3 \times 3^3}} = \frac{5}{3 \times 3} = \frac{5}{9}$$

Solution 1(4):

$\frac{2744}{2197}$

Numerator : 2744

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

Denominator : 2197

13	2197
13	169
13	13
	1

$$\frac{2744}{2197} = \frac{2 \times 2 \times 2 \times 7 \times 7 \times 7}{13 \times 13 \times 13} = \frac{2^3 \times 7^3}{13^3}$$

$$\therefore \sqrt[3]{\frac{2744}{2197}} = \sqrt[3]{\frac{2^3 \times 7^3}{13^3}} = \frac{2 \times 7}{13} = \frac{14}{13}$$

Solution 1(5):

$\frac{3375}{4096}$

Numerator : 3375

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

Denominator : 4096

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$\frac{3375}{4096} = \frac{3 \times 3 \times 3 \times 5 \times 5 \times 5}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$

$$= \frac{3^3 \times 5^3}{2^3 \times 2^3 \times 2^3 \times 2^3}$$

$$\therefore \sqrt[3]{\frac{3375}{4096}} = \sqrt[3]{\frac{3^3 \times 5^3}{2^3 \times 2^3 \times 2^3 \times 2^3}} = \frac{3 \times 5}{2 \times 2 \times 2 \times 2} = \frac{15}{16}$$

Solution 1(6):

$0.8 = \frac{8}{10}$

Numerator : 8

2	8
2	4
2	2
	1

Denominator : 10

2	10
5	5
	1

Thus, 10 is not a perfect cube.

*Remark: The answer given in the textbook is 0.2 for which the question should be 0.008. Hence, the solution.

0.008

$0.008 = \frac{8}{1000}$

Numerator : 8

2	8
2	4
2	2
	1

Denominator : 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{8}{1000}=\frac{2\times2\times2}{2\times2\times2\times5\times5\times5}=\frac{2^3}{2^3\times5^3}$$

$$\therefore \sqrt[3]{\frac{8}{1000}}=\sqrt[3]{\frac{2^3}{2^3\times5^3}}=\frac{2}{2\times5}=\frac{2}{10}=0.2$$

Thus, $\sqrt[3]{0.008}=0.2$

Solution 1(7):

0.125

$$0.125=\frac{125}{1000}$$

Numerator : 125

5	125
5	25
5	5
	1

Denominator : 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{125}{1000}=\frac{5\times5\times5}{2\times2\times2\times5\times5\times5}=\frac{5^3}{2^3\times5^3}$$

$$\therefore \sqrt[3]{\frac{125}{1000}}=\sqrt[3]{\frac{5^3}{2^3\times5^3}}=\frac{5}{2\times5}=\frac{5}{10}=0.5$$

Thus, $\sqrt[3]{0.125}=0.5$

Solution 1(8):

0.216

$$0.216 = \frac{216}{1000}$$

Numerator : 216

2	216
2	108
2	54
3	27
3	9
3	3
	1

Denominator : 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{216}{1000} = \frac{2 \times 2 \times 2 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5} = \frac{2^3 \times 3^3}{2^3 \times 5^3}$$

$$\therefore \sqrt[3]{\frac{216}{1000}} = \sqrt[3]{\frac{2^3 \times 3^3}{2^3 \times 5^3}} = \frac{2 \times 3}{2 \times 5} = \frac{6}{10} = 0.6$$

Thus, $\sqrt[3]{0.216} = 0.6$

Solution 1(9):

4.913

$$4.913 = \frac{4913}{1000}$$

Numerator : 4913

17	4913
17	289
17	17
	1

Denominator : 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{4913}{1000} = \frac{17 \times 17 \times 17}{2 \times 2 \times 2 \times 5 \times 5 \times 5} = \frac{17^3}{2^3 \times 5^3}$$

$$\therefore \sqrt[3]{\frac{4913}{1000}} = \sqrt[3]{\frac{17^3}{2^3 \times 5^3}} = \frac{17}{2 \times 5} = \frac{17}{10} = 1.7$$

Thus, $\sqrt[3]{4.913} = 1.7$

Solution 1(10):

5.832

$5.832=\frac{5832}{1000}$

Numerator : 5832

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Numerator : 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{5832}{1000}=\frac{2\times2\times2\times3\times3\times3\times3\times3\times3}{2\times2\times2\times5\times5\times5}=\frac{2^3\times3^3\times3^3}{2^3\times5^3}$$

$$\therefore\sqrt[3]{\frac{5832}{1000}}=\sqrt[3]{\frac{2^3\times3^3\times3^3}{2^3\times5^3}}=\frac{2\times3\times3}{2\times5}=\frac{18}{10}=1.8$$

Thus, $\sqrt[3]{5.832}=1.8$

Solution 2(1):

8000

$\sqrt[3]{8000}$

$\downarrow\ \downarrow$

2 0

Step 1: Divide the given number into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number.
Here, the first part is 000 and the second part is 8.

Step 2: Here, the first part is 000 whose units place digit is 0. Hence, the digit at the units place of the cube root is 0.

Step 3: The number 8 in the second part itself is a perfect cube whose cube root is 2.
Hence, the digit at the tens place of the cube root is 2.
Thus, $\sqrt[3]{8000}=20$

Solution 2(2):

9261

$\sqrt[3]{9261}$

$\downarrow\ \downarrow$

2 1

Step 1: Divide the given perfect cube into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number. Here, the first part is 261 and the second part is 9.

Step 2: Here, the first part is 261 whose units place digit is 1. Hence, the digit at the units place of the cube root is 1.

Step 3: The number 9 in the second part lies between 8 and 27, i.e. $2^3 < 9 < 3^3$. Considering the smaller number out of 2 and 3, the digit at the tens place of the cube root is 2.

Thus, $\sqrt[3]{9261} = 21$

Solution 2(3):

13824

$$\sqrt[3]{13824}$$
$$\downarrow \quad \downarrow$$
$$2 \quad 4$$

Step 1: Divide the given perfect cube into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number. Here, the first part is 824 and the second part is 13.

Step 2: Here, the first part is 824 whose units place digit is 4. Hence, the digit at the units place of the cube root is 4.

Step 3: The number 13 in the second part lies between 8 and 27, i.e. $2^3 < 13 < 3^3$. Considering the smaller number out of 2 and 3, the digit at the tens place of the cube root is 2.

Thus, $\sqrt[3]{13824} = 24$

Solution 2(4):

15625

$$\sqrt[3]{15625}$$
$$\downarrow \quad \downarrow$$
$$2 \quad 5$$

Step 1: Divide the given perfect cube into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number. Here, the first part is 625 and the second part is 15.

Step 2: Here, the first part is 625 whose units place digit is 5. Hence, the digit at the units place of the cube root is 5.

Step 3: The number 15 in the second part lies between 8 and 27, i.e. $2^3 < 15 < 3^3$. Considering the smaller number out of 2 and 3, the digit at tens place of the cube root is 2.

Thus, $\sqrt[3]{15625} = 25$

Solution 2(5):

19683

$$\begin{array}{c} \sqrt[3]{\overline{19}\,\overline{683}} \\ \downarrow \quad \downarrow \\ 2 \quad 7 \end{array}$$

Step 1: Divide the given perfect cube into two parts such that the first part is made up of the digits at the hundreds, tens and units place of the given number and the second part is made up of the remaining digits of the given number. Here, the first part is 683 and the second part is 19.

Step 2: Here, the first part is 683 whose units place digit is 3. Hence, the digit at the units place of the cube root is 7.

Step 3: The number 19 in the second part lies between 8 and 27, i.e. $2^3 < 19 < 3^3$.

Considering the smaller number out of 2 and 3, the digit at the tens place of the cube root is 2.

Thus, $\sqrt[3]{19683} = 27$