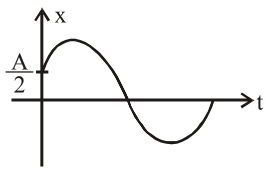
GUIDED REVISION

SECTION-I

Single Correct Answer Type **12 Q. [3 M (–1)]**

1. Following graph shows a particle performing S.H.M. about mean position x = 0. The equation of particle if $t = \frac{T}{4}$ is taken as starting time is (Notations have usual meanings)



(A) $A\sin\left(\omega t + \frac{2\pi}{3}\right)$ (B) $A\sin\left(\omega t + \frac{\pi}{3}\right)$ (C) $A\sin\left(\omega t + \frac{\pi}{6}\right)$ (D) $A\cos\left(\omega t + \frac{2\pi}{3}\right)$

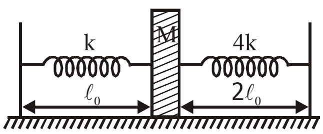
2. A bob is attached to a long, light string. The string is deflected by 3° initially with respect to vertical. The length of the string is 1 m . The value of θ at any time t after the bob released can be approximately written as (Use : $g = \pi^2$)

(A) $3° \cos \pi t$ (B) $3° \sin \pi t$ (C) $3° \sin (\pi t + \frac{\pi}{6})$ (D) $3° \cos (\pi t + \frac{\pi}{6})$

3. Potential energy of a particle is given as $U(x) = 2x^3 - 9x^2 + 12x$ where U is in joule and x is in metre. If the motion of a particle is S.H.M., then find the approx potential energy of the particle :-

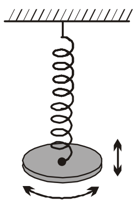
(A) – 36 J (B) 4 J (C) 5 J (D) None of these

4. A block of mass M is kept in gravity free space and touches the two springs as shown in the figure. Initially springs are in their natural lengths. Now, the block is shifted ($\ell_0/2$) from the given position in such a way that it compresses a spring and is released. The time-period of oscillation of mass will be:-



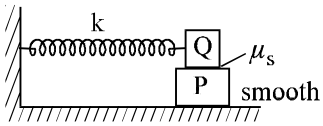
(A) $\frac{\pi}{2}\sqrt{\frac{M}{K}}$ (B) $2\pi\sqrt{\frac{m}{5K}}$ (C) $\frac{3\pi}{2}\sqrt{\frac{M}{K}}$ (D) $\pi\sqrt{\frac{M}{2K}}$

5. A solid disk of radius R is suspended from a spring of linear spring constant k and torsional constant c, as shown in figure. In terms of k and c, what value of R will give the same period for the vertical and torsional oscillations of this system?



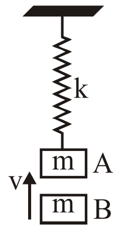
(A) $\sqrt{\frac{2c}{k}}$ (B) $\sqrt{\frac{c}{2k}}$ (C) $2\sqrt{\frac{c}{k}}$ (D) $\frac{1}{2}\sqrt{\frac{c}{k}}$

6. A block P of mass m is placed on a frictionless horizontal surface. Another block Q of same mass is kept on P and connected to the wall with the help of a spring of spring constant k as shown in the figure. μ_s is the coefficient of friction between P and Q. The blocks move together performing SHM of amplitude A. The maximum value of the friction force between P and Q is :- **[IIT JEE 2004]**



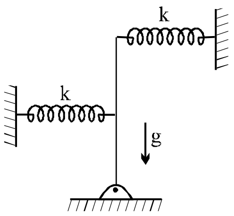
(A) kA (B) $\frac{kA}{2}$ (C) zero (D) $\mu_s mg$

7. Block A is hanging from a vertical spring and is at rest. Block B strikes the block A with velocity v and sticks to it. Then the value of v for which the spring just attains natural length is



(A) $\sqrt{\frac{60mg^2}{k}}$ (B) $\sqrt{\frac{6mg^2}{k}}$ (C) $\sqrt{\frac{10mg^2}{k}}$ (D) $\sqrt{\frac{8mg^2}{k}}$

8. In the figure shown, the spring are connected to the rod at one end and at the midpoint. The rod is hinged at its lower end. Rotational SHM of the rod (Mass m, length L) will occur only if



(A) k > mg/3L (B) k > 2mg/3L (C) k > 2mg/5L (D) k > 0

9. Time period of a particle executing SHM is 8 sec. At t = 0 it is at the mean position. The ratio of the distance covered by the particle in the 1st second to the 2nd second is :

(A) $\frac{1}{\sqrt{2}+1}$ (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}+1$

10. A particle is executing SHM of amplitude A, about the mean position x = 0. Which of the following cannot be a possible phase difference between the positions of the particle at x = + $A/2$ and x = − $A/\sqrt{2}$.

(A) 75° (B) 165° (C) 135° (D) 195°

11. Two particles are performing SHM with same angular frequency and amplitudes A and 2A respectively along same straight line with same mean position. They cross each other at position A/2 distance from mean position in opposite direction. The phase between them is :

(A) $\frac{5\pi}{6}-\sin^{-1}\left(\frac{1}{4}\right)$ (B) $\frac{\pi}{6}-\sin^{-1}\left(\frac{1}{4}\right)$ (C) $\frac{5\pi}{6}-\cos^{-1}\left(\frac{1}{4}\right)$ (D) $\frac{\pi}{6}-\cos^{-1}\left(\frac{1}{4}\right)$

12. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'. Then, 'T' is equal to :- **[AIEEE-2011]**

(A) $2\pi\sqrt{\frac{\ell\rho}{(\rho-d)g}}$ (B) $2\pi\sqrt{\frac{\ell d}{\rho g}}$ (C) $2\pi\sqrt{\frac{\ell\rho}{dg}}$ (D) $2\pi\sqrt{\frac{\ell d}{(\rho-d)g}}$

Multiple Correct Answer Type **4 Q. [4 M (–1)]**

13. The position vector of a particle that is moving in space is given by

$\vec{r}=(1+2\cos 2\omega t)\hat{i}+(3\sin^2 \omega t)\hat{j}+(3)\hat{k}$

in the ground frame. All units are in SI. Choose the correct statement (s) :

(A) The particle executes SHM in the ground frame about the mean position $\left(1,\frac{3}{2},3\right)$

(B) The particle executes SHM in a frame moving along the z-axis with a velocity of 3 m/s.

(C) The amplitude of the SHM of the particle is $\frac{5}{2}$ m.

(D) The direction of the SHM of the particle is given by the vector $\left(\frac{4}{5}\hat{i}-\frac{3}{5}\hat{j}\right)$

14. A particle is executing SHM between points $-X_m$ and X_m, as shown in figure-I. The velocity V(t) of the particle is partially graphed and shown in figure-II. Two points A and B corresponding to time t_1 and time t_2 respectively are marked on the V(t) curve.

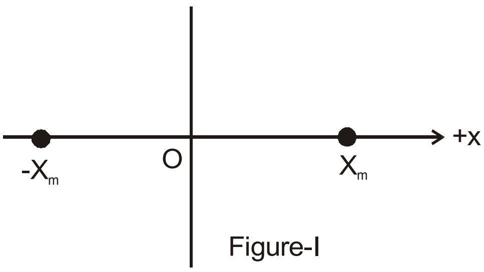


Figure-I

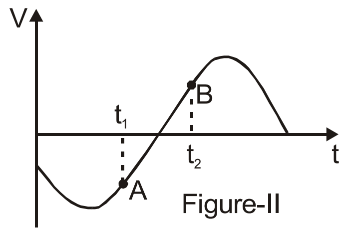
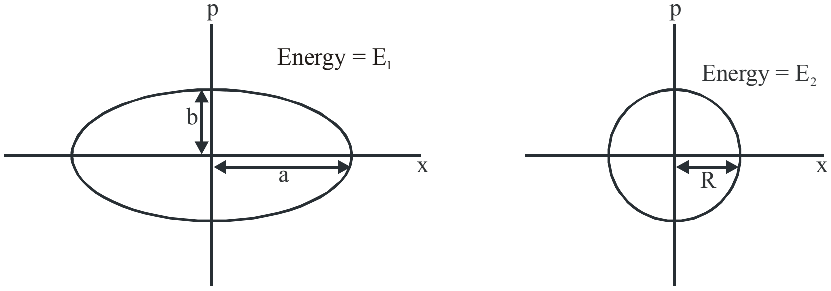


Figure-II

(A) At time t_1 , it is going towards X_m.
(B) At time t_1, its speed is decreasing.
(C) At time t_2, its position lies in between $-X_m$ and O.
(D) The phase difference $\Delta\phi$ between points A and B must be expressed as $90° < \Delta\phi < 180°$.

15. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b}=n^2$ and $\frac{a}{R}=n$, then the correct equation (s) is (are)

[JEE-Advanced-2015]



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1}=n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1}=\frac{E_2}{\omega_2}$

16. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M? **[JEE-Advanced-2016]**

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

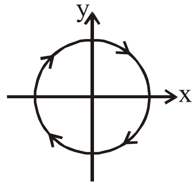
(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

Linked Comprehension Type (1 Para × 2Q.) [3 M (-1)]
(Single Correct Answer Type)

Paragraph for Questions no. 17 and 18

Lissajous figures are produced by superposition of 2 SHM's in mutually perpendicular directions. If both SHM have same frequency, the lissajous figure is simple. It may be a circle, ellipse or a straight line depending on the phase difference between two SHMs. Interesting case occurs when one of the SHM is at double frequency of another. Suppose a body executes SHM vertically with frequency f_0, but horizontally with a frequency $2f_0$ and is initially at A, mean position of vertical as well as horizontal SHM. It can be seen to trace out a figure of 8 in space as shown. Since horizontal SHM has half the time period, it executes two horizontal oscillations by the time it completes a vertical oscillation.

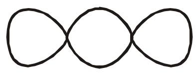
17. A lissajous figure as shown here can be produced by



(A) $x = A \sin \omega t$; $y = A \cos \omega t$ (B) $x = A \cos \omega t$; $y = A \sin \omega t$

(C) $x = A \sin \omega t$; $y = A \sin\left(\omega t + \frac{\pi}{4}\right)$ (D) $x = A \sin \omega t$; $y = A \sin\left(\omega t + \frac{3\pi}{4}\right)$

18. For the Lissajous figure shown here, the frequency in vertical direction is _______ times the frequency in horizontal direction :-

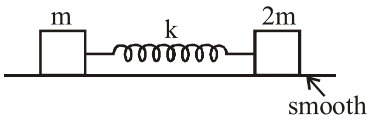


(A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$

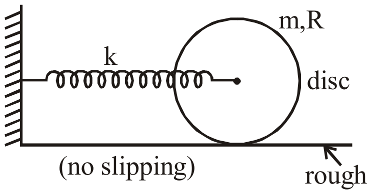
Matching List Type (4 × 4) 1 Q. [3 M (–1)]

19. In list-I, the systems are performing SHM and in list-II, the time period of SHM is shown then match list-I with list-II.

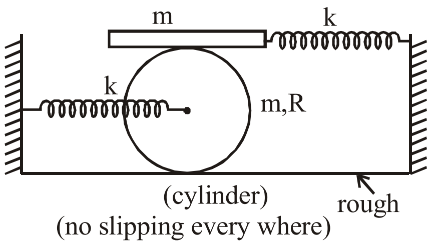
List-I | **List-II**

(P) 

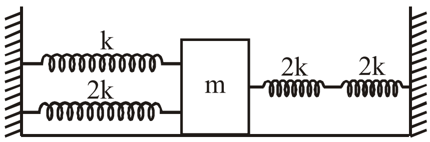
(1) $\left[2\pi\sqrt{\frac{11m}{10k}}\right]$

(Q) 

(2) $2\pi\sqrt{\frac{m}{4k}}$

(R) 

(3) $2\pi\sqrt{\frac{3m}{2k}}$

(S) 

(4) $2\pi\sqrt{\frac{2m}{3k}}$

Codes :

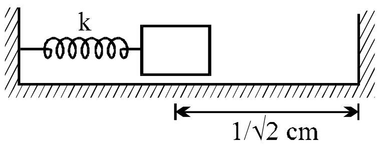
	P	Q	R	S
(A)	3	2	1	4
(B)	4	3	2	1
(C)	3	4	2	1
(D)	4	3	1	2

SECTION-II

Numerical Answer Type Question 2 Q. [3(0)]
(upto second decimal place)

1. An object of mass 0.2 kg executes SHM along the x-axis with frequency of $(25/\pi)$ Hz. At the point x = 0.04m the object has KE 0.5 J and PE 0.4 J. The amplitude of oscillation is ________.

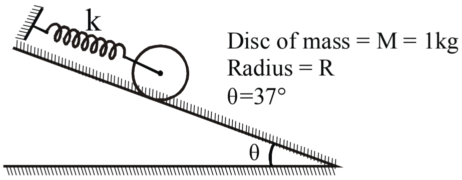
2. A block of mass 0.9 kg attached to a spring of force constant k is lying on a frictionless floor. The spring is compressed to $\sqrt{2}$ cm and the block is at a distance $1/\sqrt{2}$ cm from the wall as shown in the figure. When the block is released, it makes elastic collision with the wall and its period of motion is 0.2 sec. Find the approximate value of k.



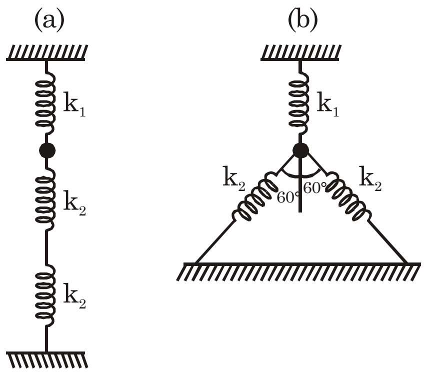
SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 2 Q. [4 M (0)]

1. A disc of mass m is connected to an ideal spring of force constant 'k'. If disc is released from rest, then what is maximum friction force on disc (in N). Assuming friction is sufficient for pure rolling

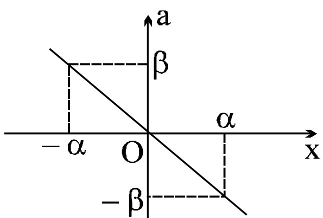


2. Mass m is suspended by ideal massless springs in two different ways, indicated by (a) and (b) in the figure. The mass is displaced upwards by a small amount from equilibrium position and is then released resulting in a SHM of the mass in the vertical direction. We denote the oscillation frequencies associated with the two cases (a) and (b) by f_a and f_b respectively. Find $\frac{f_a}{f_b}$. Given $k_2 = 2k_1$.



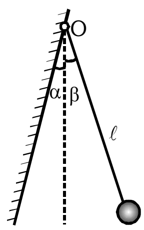
Subjective Type 7 Q. [4 M (0)]

1. The acceleration-displacement $(a - x)$ graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.

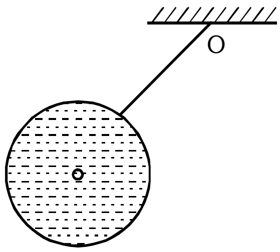


2. A small body of mass m is fixed to the middle of a stretched string of length 2ℓ. In the equilibrium position the string tension is equal to T_0. Find the angular frequency of small oscillations of the body in the transverse direction. The mass of the string is negligible, the gravitational field is absent.

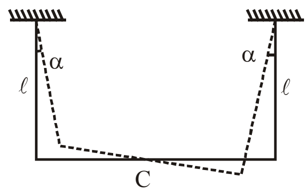
3. A ball is suspended by a thread of length ℓ at the point O on the wall, forming a small angle α with the vertical as shown in figure. Then the thread with the ball was deviated through a small angle β $(\beta > \alpha)$ and set free. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.



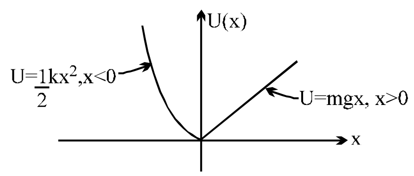
4. A pendulum is constructed as a light thin-walled sphere of radius R filled up with water and suspended at the point O from a light rigid rod. The distance between the point O and the centre of the sphere is equal to ℓ. How many times will the small oscillations of such a pendulum change after the water freezes? The viscosity of water and the change of its volume on freezing are to be neglected.



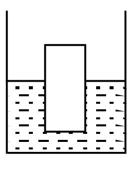
5. A uniform rod of mass m = 1.5 kg suspended by two identical threads ℓ = 90 cm in length was turned through a small angle about the vertical axis passing through its middle point C. The threads deviated in the process through an angle $\alpha = 5.0°$. Then the rod was released to start performing small oscillations. Find : (a) the oscillation period; (b) the rod's oscillation energy.



6. A particle of mass *m* moves in the potential energy U shown above. Find the period of the motion when the particle has total energy E.



7. A liquid of density 2ρ is filled in a cylindrical vessel, whose cross-sectional area is 2A. A wooden cylinder of height H, cross-sectional area A and density ρ is floating in the liquid at equilibrium with its axis vertical. The cylinder is pushed down by a small distance x from its equilibrium position and released. Find its initial acceleration.



SECTION-I

Single Correct Answer Type **12 Q. [3 M (–1)]**

1. Ans. (A)	**2. Ans. (A)**	**3. Ans. (B)**	**4. Ans. (C)**
5. Ans. (A)	**6. Ans. (B)**	**7. Ans. (B)**	**8. Ans. (C)**
9. Ans. (D)	**10. Ans. (C)**	**11. Ans. (A)**	**12. Ans. (B)**

Multiple Correct Answer Type **4 Q. [4 M (–1)]**

13. Ans. (A,C,D)	**14. Ans. (B, C)**	**15. Ans. (B,D)**	**16. Ans. (A,B,D)**

Linked Comprehension Type **(1 Para × 2Q.) [3 M (-1)]**
(Single Correct Answer Type)

17. Ans. (A) **18. Ans. (A)**

Matching List Type (4 × 4) **1 Q. [3 M (–1)]**

19. Ans. (D)

SECTION-II

Numerical Answer Type Question **2 Q. [3(0)]**
(upto second decimal place)

1. Ans. 0.06m **2. Ans.** 100 Nm^{-1}

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **2 Q. [4 M (0)]**

1. Ans. 2 **2. Ans. 1**

Subjective Type **7 Q. [4 M (0)]**

1. Ans. $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$ **2. Ans.** $\omega = \sqrt{\frac{2T_0}{m\ell}}$ **3. Ans.** $T = 2\sqrt{\ell/g}\,[\pi/2 + \sin^{-1}(\alpha/\beta)]$

4. Ans. Will increase $\sqrt{1+\frac{2}{5}\left(\frac{R}{\ell}\right)^2}$ times. It is taken into account here that the water (when in liquid phase) moves translationwise, and the system behaves as a mathematical pendulum.

5. Ans. (a) $T = 2\pi\sqrt{\ell/3g} = 1.1$ s; (b) $E = \frac{1}{2}mg\ell\alpha^2 = 0.05$ J

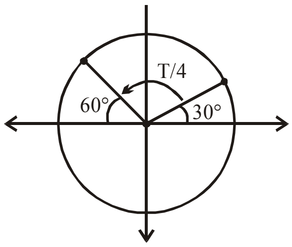
6. Ans. $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$ **7. Ans.** $\left(\frac{4g}{H}\right)x$

SOLUTIONS
SECTION-I

Single Correct Answer Type **12 Q. [3 M (–1)]**

1. Ans. (A)

Sol. $x = A\sin\left(\omega t + \frac{2\pi}{3}\right)$



2. Ans. (A)

Sol. $\omega = \sqrt{\frac{g}{\ell}} = \pi$

$\theta = \theta_A \sin(\omega t + \phi)$

$\Rightarrow \theta = 3° \cos \pi t$

3. Ans. (B)

Sol. $F = 6x^2 - 18x + 12$

$F' = 12x - 18$

$= 6(x-1)(x-2)$

As $F_{(mean)} = 0$

$\therefore$ x may be 1 or 2

U_{min} must occur at mean

$F' > 0$ $x = 2$

$\therefore U \approx 4$ J $\rightarrow$ As for SHM amplitude is very small and hence is same for all positions.

4. Ans. (C)

Sol. $T = \pi\left[\sqrt{\frac{M}{K}} + \frac{1}{2}\sqrt{\frac{M}{K}}\right] = \frac{3\pi}{2}\sqrt{\frac{M}{K}}$

5. Ans. (A)

Sol. $\frac{2c}{mR^2} = \frac{k}{m}$

$R = \sqrt{\frac{2c}{k}}$

6. Ans. (B)

Sol. $a_{max} = \frac{kA}{m+m} = \frac{f}{m}$

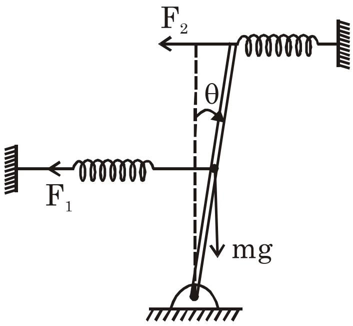
$f_{max} = \frac{kA}{2} \le \mu_s mg$

7. Ans. (B)

Sol. $mv = 2mv' \Rightarrow v' = \frac{v}{2}$; $\frac{1}{2}2m\left(\frac{v}{2}\right)^2 = 2mgx - \frac{1}{2}kx^2 \Rightarrow kx = mg \Rightarrow x = \frac{mg}{k} \Rightarrow v = \sqrt{\frac{6mg^2}{k}}$

8. Ans. (C)

Sol. $F_1 = k\left(\frac{\ell}{2}\theta\right)$



$F_2 = k(\ell\theta)$

$\because \theta << 1$

$\Rightarrow \sin\theta \approx \theta$

$\tau_{net} = -F_1 \times \frac{\ell}{2} - F_2 \times \ell + mg\sin\theta \times \frac{\ell}{2}$

$= -\frac{k\ell^2}{4}\theta - k\ell^2\theta + \frac{mg\ell}{2}\theta$

$\Rightarrow \tau_{net} = -\left(\frac{5k\ell^2}{4} - \frac{mg\ell}{2}\right)\theta$

For S.H.M.

$\frac{5k\ell^2}{4} - \frac{mg\ell}{2} > \theta$

$\Rightarrow k > \frac{2mg}{5\ell}$

9. Ans. (D)

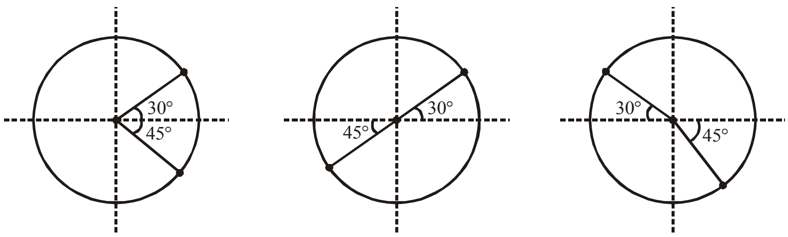
Sol.

t	0	1	2
ϕ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
pos.	0	$\frac{A}{\sqrt{2}}$	A

$d = \frac{\frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$

10. Ans. (C)

Sol.

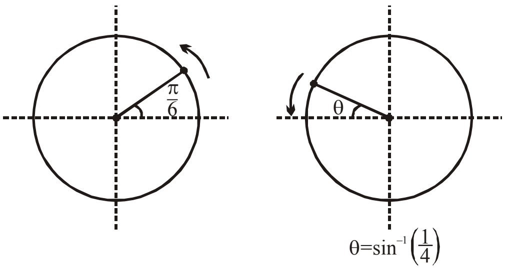


$\Delta\phi = 75°$ (A) $\Delta\phi = 165°$ (B) $\Delta\phi = 195°$ (D)

11. Ans. (A)

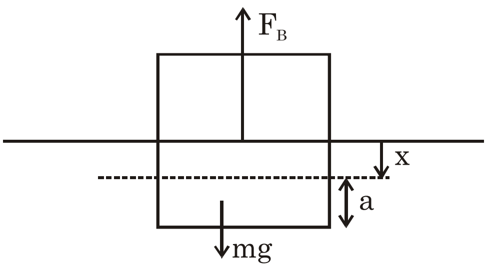
Sol. $\therefore \Delta\phi = \pi - \sin^{-1}\left(\frac{1}{4}\right) - \frac{\pi}{6}$

$= \frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (A)



12. Ans. (B)

Sol.



$m = d\ell^3$

$mg = F_B \Rightarrow d \times \ell^3 \times g = \rho\ell^2 \times ag$

$\Rightarrow a = \frac{d\ell}{\rho}$

Now cube is pushed slightly down and released,

$F_{net} = mg - F_B$

$= d\ell^3 g - \rho\ell^2(a + x)g$

$F_{net} = -\rho\ell^2 gx = -k_{eq}x$

& $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{d\ell^3}{\rho\ell^2 g}}$

$= 2\pi\sqrt{\frac{d\ell}{\rho g}}$

Multiple Correct Answer Type 4 Q. [4 M (–1)]

13. Ans. (A,C,D)

Sol. (A) $\vec{r} = (1 + 2\cos 2\omega t)\hat{i} + \left(\frac{3}{2} - \frac{3}{2}\cos 2\omega t\right)\hat{j} + (3)\hat{k}$

$\therefore \; m_P = \left(1, \frac{3}{2}, 3\right)$

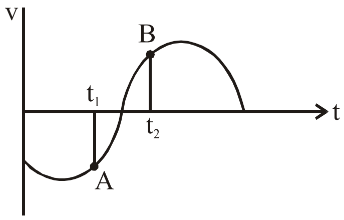
(B) It does not excute SHM about x-axis

(C) $A = \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \frac{5}{2}$

(D)Direction of SHM is $\frac{2}{\frac{5}{2}}\hat{i} - \frac{\frac{3}{2}}{\frac{5}{2}}\hat{j} \Rightarrow \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$

14. Ans. (B, C)

Sol. (A) If t_1 it is moving towards x_m



(B) At t_1 speed is decreasing
(C) At t_2 it lies between $-x_m$ and O
(D) $\Delta\phi$ lies between 0° and 180°

15. Ans. (B,D)

Sol. $P_{1\,max} = ma\omega_1 = b$
$P_{2\,max} = mR\omega_2 = R$

$\frac{\omega_1}{\omega_2} = \frac{1}{n^2}$

$\frac{\omega_2}{\omega_1} = n^2$

$E_1 = \frac{1}{2}m\omega_1^2a^2$

$E_2 = \frac{1}{2}m\omega_2^2R^2$

$\frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2}\frac{a^2}{R^2} = \frac{\omega_1^2}{\omega_2^2}n^2 = \frac{\omega_1^2}{\omega_2^2}\frac{\omega_2}{\omega_1}$

$\frac{E_1}{E_2} = \frac{\omega_1}{\omega_2}$

$\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

16. Ans. (A,B,D)

Sol. $T_i = 2\pi\sqrt{\frac{M}{K}}, T_f = 2\pi\sqrt{\frac{M+m}{K}}$

case (i) :
$M(A\omega) = (M+m)V$
$\therefore$ Velocity decreases at equilibrium position.

By energy conservation

$$A_f = A_i\sqrt{\frac{M}{M+m}}$$

case (ii) :
No energy loss, amplitude remains same
At equilibrium (x_0) velocity = $A\omega$.
In both cases ω decreases so velocity decreases in both cases

Linked Comprehension Type (1 Para × 2Q.) [3 M (-1)]
(Single Correct Answer Type)

17. Ans. (A)

Sol. $x^2 + y^2 = A^2$ which represents a circle of radius A
And starting point is x = 0 so answer will be (A)

18. Ans. (A)

Sol. Time period for vertical S.H.M. is one third of horizontal S.H.M.

Matching List Type (4 × 4) 1 Q. [3 M (–1)]

19. Ans. (D)

Sol. (A) $T = 2\pi\sqrt{\frac{2m}{3k}}$

(B) $\frac{1}{2}kx^2 + \frac{1}{2}\left(\frac{3}{2}mR^2\right)\frac{v^2}{R^2} = \text{constant}$

$$kx + \frac{3}{2}mv\frac{dv}{dx} = 0$$

$$a = \left(\frac{2k}{3m}\right)dx$$

$$T = 2\pi\sqrt{\frac{3m}{2k}}$$

(C) $\frac{1}{2}kx^2 + \frac{1}{2}k(2x)^2 + \frac{1}{2}\left(\frac{3}{2}mR^2\right)\frac{v^2}{R^2} + \frac{1}{2}m(2V)^2 = \text{constant}$

$$kx + 4kx + \frac{3}{2}ma + 4ma = 0$$

$$5kx + \frac{11}{2}ma = 0$$

$$a = -\left(\frac{10k}{11m}\right)x$$

$$\left[T = 2\pi\sqrt{\frac{11m}{10k}}\right]$$

(D) $T = 2\pi\sqrt{\frac{m}{4k}}$

SECTION-II

Numerical Answer Type Question **2 Q. [3(0)]**
(upto second decimal place)

1. **Ans.** 0.06m

Sol. $f = \frac{25}{\pi}$

$\omega = 50$

$K = \frac{0.2}{10} \times 50 \times 50$

$= 500$ N/m

$KE = 0.9 = \frac{1}{2} \times 500 \times A^2$

$\Rightarrow A^2 = \frac{1.8}{500} = \frac{18}{5000} = \frac{9}{2500}$

$\therefore\ A = \frac{3}{50} = 0.06m$

2. **Ans.** 100 Nm^{-1}

Sol. $\frac{2\pi}{3}\sqrt{\frac{0.9}{k}} = 0.2$

$\frac{4 \times 10}{9} \times \frac{9}{10 \times k} = \frac{4}{100}$

$\Rightarrow k \approx 100$ N/m

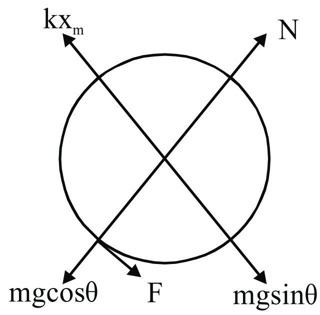
SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **2 Q. [4 M (0)]**

1. **Ans. 2**

Sol. From work energy theorem maximum extension of spring is x_m

$mgx_m \sin\theta = \frac{1}{2}kx_m^2$ or $x_m = \frac{2mg\sin\theta}{k}$



At lower extreme

$kx_m - mg\sin\theta - F = ma$...(i)

$a = R\alpha$...(ii)

$FxR = \frac{mR^2}{2}\alpha$ ...(iii)

Thus $F = \frac{mg\sin\theta}{3}$

2. **Ans. 1**

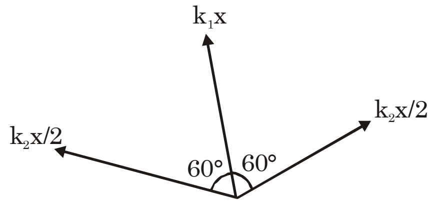
Sol. $f_a = \frac{1}{2\pi}\sqrt{\frac{\left(k_1 + \frac{k_2}{2}\right)}{m}}$, $f_b = \frac{1}{2\pi}\sqrt{\frac{\left(k_1 + \frac{k_2}{2}\right)}{m}}$

(a) In case (a) $k_{eq} = k_1 + \frac{k_2}{2}$

(b) x' = xcos60°

Thus making FBD

$$-\left(k_1 x + \frac{2k_2}{2}\cos 60°\right) = m\frac{d^2x}{dt^2}$$



In this case also $k_{eq} = k_1 + \frac{k_2}{2}$

So $f_b = f_a$

Subjective Type — 7 Q. [4 M (0)]

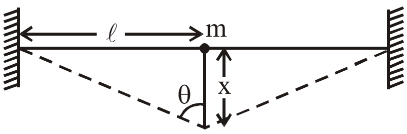
1. Ans. $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

Sol. $\omega^2 = -\frac{\alpha}{x} = \frac{\beta}{\propto}$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$$

2. Ans. $\omega = \sqrt{\frac{2T_0}{m\ell}}$

Sol.



$x << \ell$

Net restoring force = $2T_0 \cos\theta$.

∴ we have,

$$\frac{2T_0 x}{\sqrt{\ell^2 + x^2}} = ma$$

$$\Rightarrow a = \frac{2T_0}{m\ell}\cdot x$$

$$= \omega = \sqrt{\frac{2T_0}{m\ell}}$$

3. Ans. $T = 2\sqrt{\ell/g}\,[\pi/2 + \sin^{-1}(\alpha/\beta)]$

Sol. The system can be assumed as an incomplete SHM.

Thus time period can be calculate as twice the (time taken by the bob to travel to mean position from extreme + time taken to travel from mean to wall)

$$\therefore\ T_{net} = 2\left[\frac{T}{4} + \frac{\sin^{-1}\left(\frac{\alpha}{\beta}\right)T}{2\pi}\right] = 2\left[\frac{2\pi}{4}\cdot\sqrt{\frac{\ell}{g}} + \frac{\sin^{-1}\left(\frac{\alpha}{\beta}\right)}{2\pi}\cdot 2\pi\sqrt{\frac{\ell}{g}}\right]$$

$$= 2\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2} + \sin^{-1}\frac{\alpha}{\beta}\right]$$

4. **Ans.** Will increase $\sqrt{1+\frac{2}{5}\left(\frac{R}{\ell}\right)^2}$ times. It is taken into account here that the water (when in liquid phase) moves translationwise, and the system behaves as a mathematical pendulum.

Sol. If water has frozen,the system consisting of the light rod and the frozen water in the hollow sphere constitute a compound (physical) pendulum to a very good approximation because we can take the whole system to be rigid. For such systems the time period is given by

$T_1 = 2\pi\sqrt{\frac{1}{8}}\sqrt{1+\frac{k^2}{\ell^2}}$ where $k^2 = \frac{2}{5}R^2$ is the radius of gyration of the sphere.

The situation is different when water is unfrozen. When dissipative forces (viscosity) are neglected, we are dealing with ideal fluids. Such fluids instantaneously responds to (unbalanced) internal stresses. Suppose the sphere with liquid water actually executes mall rigid ocillations. Then the portion of the fluids above the centre of the sphere will have a greater acceleration than the portion below the centre because the linear acceleration of any element is in this case, equal to angular acceleration of the element multiplied by the distance of the element from the centre of suspension (Recall that we are considering small oscillations). Then, is obvious in a frame moving with the centre of mass, there will appear an unbalanced couple (not negatedby any pscudoforces) which will cause the fluid to move rotationally so a to destroy differences in acceleration. Thus for this case of ideal fluids the pendulum must move in such a way that the elements of the fluid all undergothe same acceleration. This implies that we have a simple (mathematical) pendulum with the time period :

$$T_0 = 2\pi\sqrt{\frac{\ell}{8}}$$

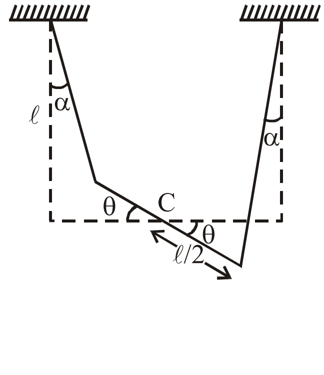
Thus $T_1 = T_0\sqrt{1+\frac{2}{5}\left(\frac{R}{\ell}\right)^2}$

(One expects that a liquid with very small viscosity will have a time period close T_0 while one with high viscosity will have a time period closer to T_1.)

5. **Ans.** (a) $T = 2\pi\sqrt{\ell/3g} = 1.1$ s; (b) $E = \frac{1}{2}mg\ell\alpha^2 = 0.05$ J

Sol. (a) Let us locate the system when the threads are deviated through an angle $\alpha' < \alpha$, during the oscillations of the system (Figure). From the conservation of mechanical energy of the system :

$$\frac{1}{2}\frac{mL^2}{12}0^2 + mg(1-\cos\alpha') = \text{constant} \qquad ...(1)$$



Where L is the length of the rod, 0 is the angular deviation of the rod from its equilibrium position i.e. 0 = 0.

Differentiating Equ. (1) w.r.t. time

$$\frac{1}{2}\frac{mL^2}{12}2\dot{\theta}\ddot{\theta} + mg\ell\sin\alpha'\dot{\alpha}' = 0$$

So, $\frac{L^2}{12}\dot{\theta}\ddot{\theta} + g\ell\alpha'\dot{u}' = 0$ (for small α', sin α' = α') ...(2)

But from the Figure

$$\frac{L}{2}\theta = \ell\alpha' \text{ or } \alpha' = \frac{L}{2\ell}\theta$$

So, $\dot{\alpha}' = \frac{L}{2\ell}\dot{\theta}$

Putting these values of α' and $\frac{d\alpha'}{dt^2}$ in Eqn. (2) we get

$$\frac{d^2\theta}{dt^2} = -\frac{3g}{\ell}\theta$$

Thus the sought time period

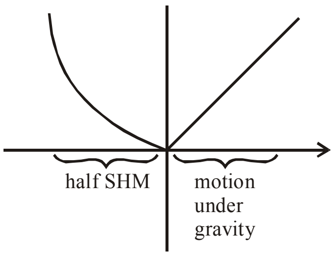
$$T = \frac{2\pi}{\omega_D} = 2\pi\sqrt{\frac{\ell}{3g}}$$

(b) The sought oscillation energy

$$E = U_{extreme} = mg\ell(1-\cos\alpha) = mg\ell 2\sin^2\frac{\alpha}{2}$$

$$= mg\ell 2\frac{\alpha^2}{4} = \frac{mg\ell\alpha^2}{2} \text{ (because for small angle } \sin\theta = \theta)$$

6. Ans. $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$



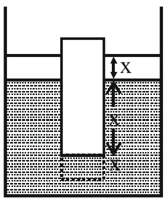
Sol. $t_1 = \frac{T}{2}, \frac{2\pi}{2}\sqrt{\frac{m}{k}}, \pi\sqrt{\frac{m}{k}}$

$$t_2 = \frac{2u}{g} = \frac{2}{g}\sqrt{\frac{2E}{m}}$$

$$\therefore\ t = \pi\sqrt{\frac{m}{k}} + \frac{2}{g}\sqrt{\frac{2E}{m}}$$

7. Ans. $\left(\frac{4g}{H}\right)x$

Sol. ρ(A)(H)g = 2ρ(A)(x')g



$$\left[x' = \frac{H}{2}\right]$$

Δx = Δy

y = x

$f_{net} = 2\rho(A)(2x)g = \rho(AH)a$

$$\left[a = \frac{4g}{H}x\right]$$