Parabola Exercise 1 :
Single Option Correct Type Questions

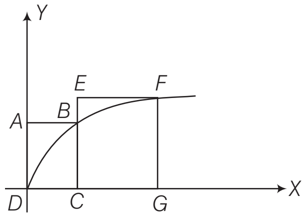
- This section contains **30 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct

1. A common tangent is drawn to the circle $x^2 + y^2 = a^2$ and the parabola $y^2 = 4bx$. If the angle which this tangent makes with the axis of x is $\frac{\pi}{4}$, then the relationship between a and b is $(a, b > 0)$
(a) $b = \sqrt{2}a$ (b) $a = b\sqrt{2}$ (c) $c = 2a$ (d) $a = 2c$

2. The equation of parabola whose vertex and focus lie on the axis of x at distances a and a_1 from the origin respectively, is
(a) $y^2 = 4(a_1 - a)x$ (b) $y^2 = 4(a_1 - a)(x - a)$
(c) $y^2 = 4(a_1 - a)(x - a_1)$ (d) $y^2 = 4aa_1 x$

3. If parabolas $y^2 = ax$ and $25[(x-3)^2 + (y+2)^2] = (3x - 4y - 2)^2$ are equal, then the value of a is
(a) 3 (b) 6 (c) 7 (d) 9

4. $ABCD$ and $EFGC$ are squares and the curve $y = \lambda\sqrt{x}$ passes through the origin D and the points B and F. The ratio $\frac{FG}{BC}$ is



(a) $\frac{\sqrt{3}+1}{4}$ (b) $\frac{\sqrt{3}+1}{2}$ (c) $\frac{\sqrt{5}+1}{4}$ (d) $\frac{\sqrt{5}+1}{2}$

5. Let A and B be two points on a parabola $y^2 = x$ with vertex V such that VA is perpendicular to VB and θ is the angle between the chord VA and the axis of the parabola. The value of $\frac{|VA|}{|VB|}$ is
(a) $\tan\theta$ (b) $\cot^2\theta$ (c) $\tan^3\theta$ (d) $\cot^3\theta$

6. The vertex of the parabola whose parametric equation is $x = t^2 - t + 1, y = t^2 + t + 1, t \in R$, is
(a) (1, 1) (b) (2, 2) (c) (3, 3) (d) $\left(\frac{1}{2}, \frac{1}{2}\right)$

7. The circle $x^2 + y^2 + 2px = 0, p \in R$, touches the parabola $y^2 = 4x$ externally, then
(a) $p > 0$ (b) $p < 0$ (c) $p > 1$ (d) $p > 2$

8. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then
(a) $d^2 + (2b + 3c)^2 = 0$ (b) $d^2 + (3b + 2c)^2 = a^2$
(c) $d^2 + (2b - 3c)^2 = 0$ (d) $d^2 + (2b + 3c)^2 = a^2$

9. A parabola $y = ax^2 + bx + c$ crosses the X-axis at $(\alpha, 0)$ and $(\beta, 0)$ both to the right of the origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is
(a) $\sqrt{\frac{bc}{a}}$ (b) ac^2 (c) $\frac{b}{a}$ (d) $\sqrt{\frac{c}{a}}$

10. Two mutually perpendicular tangent of the parabola $y^2 = 4ax$ meet the axis at P_1 and P_2. It S is the focus of the parabola, then $\frac{1}{SP_1} + \frac{1}{SP_2}$ is equal to
(a) $\frac{1}{4a}$ (b) $\frac{1}{a}$ (c) $\frac{2}{a}$ (d) $\frac{4}{a}$

11. If the normal to the parabola $y^2 = 4ax$ at P meets the curve again at Q and if PQ and the normal at Q make angles α and β respectively with the X-axis, then $\tan\alpha(\tan\alpha + \tan\beta)$ has the value equal to
(a) -2 (b) -1 (c) $-\frac{1}{2}$ (d) 0

12. If the normals to the parabola $y^2 = 4ax$ at three points P, Q and R meet at A and S is the focus, then $SP \cdot SQ \cdot SR$ is equal to
(a) $(SA)^2$ (b) $(SA)^3$ (c) $a(SA)^2$ (d) $a(SA)^3$

13. The length of the shortest normal chord of the parabola $y^2 = 4ax$ is
(a) $2a\sqrt{27}$ (b) $9a$ (c) $a\sqrt{54}$ (d) $18a$

14. The largest value of a for which the circle $x^2 + y^2 = a^2$ falls totally in the interior of the parabola $y^2 = 4(x + 4)$ is
(a) $4\sqrt{3}$ (b) 4 (c) $\frac{4\sqrt{6}}{7}$ (d) $2\sqrt{3}$

15. From a point $(\sin\theta, \cos\theta)$, if three normals can be drawn to the parabola $y^2 = 4ax$, then the value of a is
(a) $\left(\frac{1}{2}, 1\right)$ (b) $\left[-\frac{1}{2}, 0\right)$
(c) $\left[\frac{1}{2}, 1\right]$ (d) $\left(\frac{-1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

16. If two different tangents of $y^2 = 4x$ are the normals to $x^2 = 4$ by, then

(a) $|b| < \frac{1}{2\sqrt{2}}$ (b) $|b| < \frac{1}{\sqrt{2}}$

(c) $|b| > \frac{1}{2\sqrt{2}}$ (d) $|b| > \frac{1}{\sqrt{2}}$

17. The shortest distance between the parabolas $2y^2 = 2x - 1$ and $2x^2 = 2y - 1$ is

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $2\sqrt{2}$ (d) 4

18. Normals at two points (x_1, y_1) and (x_2, y_2) of the parabola $y^2 = 4x$ meet again on the parabola, where $x_1 + x_2 = 4$, then $|y_1 + y_2|$ is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) $8\sqrt{2}$

19. A line is drawn from $A(-2, 0)$ to intersect the curve $y^2 = 4x$ at P and Q in the first quadrant such that $\frac{1}{AP} + \frac{1}{AQ} < \frac{1}{4}$. Then the slope of the line is always

(a) $< \frac{1}{\sqrt{3}}$ (b) $> \frac{1}{\sqrt{3}}$ (c) $> \sqrt{2}$ (d) $> \sqrt{3}$

20. An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having its focus at S. If chord AB lies towards the left of S, then the side length of this triangle is

(a) $a(2 - \sqrt{3})$ (b) $2a(2 - \sqrt{3})$

(c) $4a(2 - \sqrt{3})$ (d) $8a(2 - \sqrt{3})$

21. Let C be a circle with centre (0, 1) and radius unity. P is the parabola $y = ax^2$. The set of values of a for which they meet at a point other than origin is

(a) $(0, \infty)$ (b) $\left(0, \frac{1}{2}\right)$ (c) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \infty\right)$

22. Let S be the focus of $y^2 = 4x$ and a point P be moving on the curve such that its abscissa is increasing at the rate of 4 units/s. Then the rate of increase of the projection of SP on $x + y = 1$ when p is at (4, 4) is

(a) $-\sqrt{2}$ (b) $-\frac{3}{\sqrt{2}}$ (c) -1 (d) $\sqrt{2}$

23. If P is a point on the parabola $y^2 = 3(2x - 3)$ and M is the foot of perpendicular drawn from P on the directrix of the parabola, then the length of each side of the equilateral triangle SMP, where S is the focus of the parabola, is

(a) 2 (b) 4 (c) 6 (d) 8

24. Consider the parabola $y^2 = 4x$. Let $A \equiv (4, -4)$ and $B \equiv (9, 6)$ be two fixed points on the parabola. Let C be a moving point on the parabola between A and B such that the area of the triangle ABC is maximum. Then the coordinates of C are

(a) $\left(\frac{1}{4}, 1\right)$ (b) $(3, -2\sqrt{3})$

(c) $(3, 2\sqrt{3})$ (d) $(4, 4)$

25. Through the vertex O of the parabola $y^2 = 4ax$, two chords OP and OQ are drawn and the circles on OP and OQ as diameters intersect at R. If θ_1, θ_2 and ϕ are the angles made with the axis by the tangents at P and Q on the parabola and by OR, then the value of $\cot\theta_1 + \cot\theta_2$ is

(a) $-2\tan\phi$ (b) $2\tan\phi$

(c) 0 (d) $2\cot\phi$

26. AB is a double ordinate of the parabola $y^2 = 4ax$. Tangents drawn to the parabola at A and B meet the Y-axis at A_1 and B_1 respectively. If the area of trapezium AA_1B_1B is equal to $24a^2$, then the angle subtended by A_1B_1 at the focus of the parabola is equal to

(a) $\tan^{-1} 2$ (b) $\tan^{-1} 3$ (c) $2\tan^{-1} 2$ (d) $2\tan^{-1} 3$

27. If the 4th term in the expansion of $\left(px + \frac{1}{x}\right)^n$, $n \in N$ is $\frac{5}{2}$ and three normals to the parabola $y^2 = x$ are drawn through a point $(q, 0)$, then

(a) $q = p$ (b) $q > p$ (c) $q < p$ (d) $pq = 1$

28. The set of points on the axis of the parabola $y^2 - 4x - 2y + 5 = 0$ from which all the three normals to the parabola are real is

(a) $(k, 0)$; $k > 1$ (b) $(k, 1)$; $k > 3$

(c) $(k, 2)$; $k > 6$ (d) $(k, 3)$; $k > 8$

29. The triangle formed by the tangent to the parabola $y = x^2$ at the point whose abscissa is x_0 ($x_0 \in [1, 2]$), the Y-axis and the straight line $y = x_0^2$ has the greatest area if x_0 is equal to

(a) 0 (b) 1 (c) 2 (d) 3

30. The set of points (x, y) whose distance from the line $y = 2x + 2$ is the same as the distance from (2, 0) is a parabola. This parabola is congruent to the parabola in standard from $y = kx^2$ for some k which is equal to

(a) $\frac{4}{\sqrt{5}}$ (b) $\frac{12}{\sqrt{5}}$ (c) $\frac{\sqrt{5}}{4}$ (d) $\frac{\sqrt{5}}{12}$

Parabola Exercise 2 :
More than One Correct Option Type Questions

- This section contains **15 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **MORE THAN ONE** may be correct.

31. Equation of the common tangent to the circle $x^2 + y^2 = 50$ and the parabola $y^2 = 40x$ can be

(a) $x + y - 10 = 0$ (b) $x - y + 10 = 0$
(c) $x + y + 10 = 0$ (d) $x - y - 10 = 0$

32. Let PQ be a chord of the parabola $y^2 = 4x$. A circle drawn with PQ as a diameter passes through the vertex V of the parabola. If area of $\Delta PVQ = 20$ unit^2, then the coordinates of P are

(a) $(16, 8)$ (b) $(16, -8)$
(c) $(-16, 8)$ (d) $(-16, -8)$

33. Let $y^2 = 4ax$ be a parabola and $x^2 + y^2 + 2bx = 0$ be a circle. If parabola and circle touch each other externally, then

(a) $a > 0, b < 0$ (b) $a > 0, b > 0$
(c) $a < 0, b > 0$ (d) $a < 0, b < 0$

34. Tangent is drawn at any point (x_1, y_1) other than the vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of chords of contact pass through a fixed point (x_2, y_2), then

(a) x_1, a, x_2 are in GP (b) $\frac{y_1}{2}, a, y_2$ are in GP
(c) $-4, \frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in GP (d) $x_1x_2 + y_1y_2 = a^2$

35. Let P, Q and R are three co-normal points on the parabola $y^2 = 4ax$. Then the correct statement(s) is/at

(a) algebraic sum of the slopes of the normals at P, Q and R vanishes
(b) algebraic sum of the ordinates of the points P, Q and R vanishes
(c) centroid of the triangle PQR lies on the axis of the parabola
(d) Circle circumscribing the triangle PQR passes through the vertex of the parabola

36. Let P be a point whose coordinates differ by unity and the point does not lie on any of the axes of reference. If the parabola $y^2 = 4x + 1$ passes through P, then the ordinate of P may be

(a) 3 (b) -1
(c) 5 (d) 1

37. If a point P on $y^2 = 4x$, the foot of the perpendicular from P on the directrix and the focus form an equilateral triangle, then the coordinates of P may be

(a) $(3, -2\sqrt{3})$ (b) $(-3, 2\sqrt{3})$
(c) $(3, 2\sqrt{3})$ (d) $(-3, -2\sqrt{3})$

38. The locus of the foot of the perpendicular from the focus on a tangent to the parabola $y^2 = 4ax$ is

(a) the directrix (b) the tangent at the vertex
(c) $x = a$ (d) $x = 0$

39. The extremities of latusrectum of a parabola are (1, 1) and (1, −1). Then the equation of the parabola can be

(a) $y^2 = 2x - 1$ (b) $y^2 = 1 - 2x$
(c) $y^2 = 2x - 3$ (d) $y^2 = 2x - 4$

40. If from the vertex of a parabola $y^2 = 4ax$ a pair of chords be drawn at right angles to one another and with these chords as adjacent sides a rectangle be made, then the locus of the further angle of the rectangle is

(a) an equal parabola
(b) a parabola with focus at $(8a, 0)$
(c) a parabola with directrix as $x - 7a = 0$
(d) not a parabola

41. If two chords drawn from the point (4, 4) to the parabola $x^2 = 4y$ are divided by the line $y = mx$ in the ratio 1 : 2, then

(a) $m \in (-\infty, -\sqrt{3})$ (b) $m \in (-\infty, -\sqrt{3} - 1)$
(c) $m \in (\sqrt{3}, \infty)$ (d) $m \in (\sqrt{3} - 1, \infty)$

42. Through a point $P(-2, 0)$, tangents PQ and PR are drawn to the parabola $y^2 = 8x$. Two circles each passing through the focus of the parabola and one touching at Q and the other at R are drawn. Which of the following point (s) with respect to the triangle PQR lie (s) on the common chord of the two circles?

(a) centroid (b) orthocentre
(c) incentre (d) circumcentre

43. The set of points on the axis of the parabola $(y - 2)^2 = 4\left(x - \frac{1}{2}\right)$ from which three distinct normals can be drawn to the parabola are

(a) (3, 2) (b) (1, 2)
(c) (4, 2) (d) (5, 2)

44. Three normals are drawn from the point (14, 7) to the curve $y^2 - 16x - 8y = 0$. Then the coordinates of the feet of the normals are

(a) (3, – 4) (b) (8, 16)
(c) (0, 0) (d) (2, 2)

45. A quadrilateral is inscribed in a parabola, then

(a) the quadrilateral may be cyclic
(b) diagonals of the quadrilateral may be equal
(c) all possible pairs of adjacent sides may be perpendicular
(d) None of the above

Parabola Exercise 3 :
Paragraph Based Questions

- This section contains **8 paragraphs** based upon each of the paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph I
(Q. Nos. 46 to 48)

Consider a parabola P touches coordinate axes at $(4, 0)$ and $(0, 3)$.

46. If focus of parabola P is (a, b), then the value of $b - a$ is

(a) $\frac{1}{25}$ (b) $\frac{3}{25}$ (c) $\frac{4}{25}$ (d) $\frac{12}{25}$

47. Length of latus rectum of parabola P is

(a) $\frac{72}{125}$ (b) $\frac{144}{125}$
(c) $\frac{288}{125}$ (d) $\frac{576}{125}$

48. Equation of directrix of parabola P is

(a) $4x + 3y = 0$ (b) $3x + 4y = 12$
(c) $3x + 4y = 0$ (d) $4x + 3y = 12$

Paragraph II
(Q. Nos. 49 to 51)

Let C be the locus of the circumcentre of a variable triangle having sides Y-axis, $y = 2$ and $ax + by = 1$, where (a, b) lies on the parabola $y^2 = 4\lambda x$.

49. For $\lambda = 2$, the product of coordinates of the vertex of the curve C is

(a) – 8 (b) – 6 (c) 6 (d) 8

50. For $\lambda = \frac{1}{32}$, the length of smallest focal chord of the curve C is

(a) $\frac{8}{3}$ (b) 2 (c) 4 (d) 8

51. The curve C is symmetrical about the line

(a) $x = -\frac{3}{2}$ (b) $y = -\frac{3}{2}$
(c) $x = \frac{3}{2}$ (d) $y = \frac{3}{2}$

Paragraph III
(Q. Nos. 52 to 54)

Consider a parabola (P) $x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0$.

52. The focus of the parabola (P) is

(a) (2, 1) (b) (– 2, 1) (c) (– 2, – 1) (d) (2, – 1)

53. Length of latusrectum of the parabola (P) is

(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{6}{\sqrt{5}}$ (c) $\frac{12}{\sqrt{5}}$ (d) $\frac{24}{\sqrt{5}}$

54. Equation of directrix of parabola (P) is

(a) $x - 2y - 4 = 0$ (b) $2x + y - 3 = 0$
(c) $x - 2y + 4 = 0$ (d) $2x + y + 3 = 0$

Paragraph IV
(Q. Nos. 55 to 57)

If l and m are variable real numbers such that $5l^2 - 4lm + 6m^2 + 3l = 0$, then the variable line $lx + my = 1$ always touches a fixed parabola, whose axis is parallel to the X-axis.

55. If (a, b) is the vertex of the parabola, then the value of $|a - b|$ is

(a) 2 (b) 3
(c) 4 (d) 5

56. If (c, d) is the focus of the parabola, then the value of $2^{|d - c|}$ is

(a) 1 (b) 2 (c) 4 (d) 8

57. If $ex + f = 0$ is directrix of the parabola and e, f are prime numbers, then the value of $|e - f|$ is

(a) 2 (b) 4
(c) 6 (d) 8

Paragraph V
(Q. Nos. 58 to 60)

C_1 is a curve $y^2 = 4x$, C_2 is curve obtained by rotating C_1, 120° in anti-clockwise direction C_3 is reflection of C_2 with respect to $y = x$ and S_1, S_2, S_3 are focii of C_1, C_2 and C_3, respectively, where O is origin.

58. If $(t^2, 2t)$ are parametric form of curve C_1, then the parametric form of curve C_2 is

(a) $\left(\frac{1}{2}(t^2 + 2\sqrt{3}t), \frac{1}{2}(\sqrt{3}t^2 + 2t)\right)$

(b) $\left(\frac{1}{2}(-t^2 + 2\sqrt{3}t), \frac{1}{2}(\sqrt{3}t^2 + 2t)\right)$

(c) $\left(\frac{1}{2}(-t^2 + 2\sqrt{3}t), \frac{1}{2}(-\sqrt{3}t^2 + 2t)\right)$

(d) $\left(\frac{1}{2}(-t^2 + 2\sqrt{3}t), \frac{1}{2}(-\sqrt{3}t^2 - 2t)\right)$

59. Area of ΔOS_2S_3 is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) 1

60. If $S_1(x_1, y_1)$, $S_2(x_2, y_2)$ and $S_3(x_3, y_3)$, then the value of $\Sigma x_1^2 + \Sigma y_1^2$ is

(a) 2 (b) 3
(c) 4 (d) 5

Paragraph VI

(Q. Nos. 61 to 63)

Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the Y-axis also touches the circle $x^2 + y^2 = c^2$. It is known that no point of the parabola is below X-axis.

61. The value of $5c^2$ when a attains its maximum value is

(a) 1 (b) 3
(c) 5 (d) 7

62. The slope of the tangent when C is maximum, is

(a) – 1 (b) 0 (c) 1 (d) 2

63. Let Δ be the minimum area bounded by the tangent and the coordinate axes, then the value of 8Δ is

(a) 1 (b) 2 (c) 4 (d) 8

Paragraph VII

(Q. Nos. 64 to 66)

A parabola (P) touches the conic

$$x^2 + xy + y^2 - 2x - 2y + 1 = 0$$

at the points when it is cut by the line $x + y + 1 = 0$.

64. If equation of parabola (P) is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, then the value of $|a + b + c + f + g + h|$ is

(a) 8 (b) 10 (c) 12 (d) 14

65. The length of latusrectum of parabola (P) is

(a) $\sqrt{2}$ (b) $3\sqrt{2}$ (c) $5\sqrt{2}$ (d) $7\sqrt{2}$

66. If (a, b) is the vertex of the parabola (P), then the value of $|a - b|$ is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{2}$

Paragraph VIII

(Q. Nos. 67 to 69)

$y = 3x$ is tangent to the parabola $2y = ax^2 + b$.

67. The minimum value of $a + b$ is

(a) 2 (b) 4 (c) 6 (d) 8

68. If (2, 6) is the point of contact, then the value of $2a$ is

(a) 2 (b) 3 (c) 4 (d) 5

69. If $b = 18$, then the point of contact is

(a) (1, 3) (b) (2, 6)
(c) (3, 9) (d) (6, 18)

Parabola Exercise 4 :
Single Integer Answer Type Questions

- This section contains **10 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

70. Two tangents are drawn from the point $(-2, -1)$ to the parabola $y^2 = 4x$. If θ is the angle between these tangents, then the value of $\tan\theta$ is

71. If the distances of two points P and Q from the focus of a parabola $y^2 = 4x$ are 4 and 9 respectively, the distance of the point of intersection of tangents at P and Q from the focus is

72. The tangents and normals are drawn at the extremities of the latusrectum of the parabola $y^2 = 4x$. The area of quadrilateral so formed is λ sq units, the value of λ is

73. Three normals are drawn from the point $(a, 0)$ to the parabola $y^2 = x$. One normal is the X-axis. If other two normals are perpendicular to each other, then the value of $4a$ is

74. AB is the chord of the parabola $y^2 = 6x$ with the vertex at A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis is λ units, then the value of λ is

75. The parabolas $y = x^2 - 9$ and $y = \lambda x^2$ intersect at points A and B. If length of AB is equal to $2a$ and if $\lambda a^2 + \mu = a^2$, then the value of μ is

76. Let n be the number of integral points lying inside the parabola $y^2 = 8x$ and circle $x^2 + y^2 = 16$, then the sum of the digits of number n is

77. Radius of the largest circle which passes through the focus of the parabola $y^2 = 4x$ and contained in it, is

78. If the circle $(x-6)^2 + y^2 = r^2$ and the parabola $y^2 = 4x$ have maximum number of common chords, then the least integral value of r is

79. The slope of the line which belongs to the family of lines $(1+a)x + (a-1)y + 2(1-a) = 0$ and makes shortest intercept on $x^2 - 4y + 4 = 0$ is

Parabola Exercise 5 :
Matching Type Questions

- This section contains **3 questions.** Each question has four statements (A, B, C and D) given in **Column** I and four statements (p, q, r and s) in **Column** II. Any given statement in **Column** I can have correct matching with one or more statement(s) given in **Column** II.

80. Match the following.

	Column I		Column II
(A)	The number of common chords of the parabola $x = y^2 - 6y + 11$ and $y = x^2 - 6x + 11$ is	(p)	Prime number
(B)	AB is a chord of the parabola $y^2 = 4x$ with vertex A, BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is	(q)	Composite number
(C)	The maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ is	(r)	Perfect number
(D)	If the locus of the middle of point of contact of tangents drawn to the parabola $y^2 = 8x$ and the foot of perpendicular drawn from its focus to the tangents is a conic, then the length of latusrectum of this conic is	(s)	Even number

81. Match the following.

	Column I		Column II
(A)	If the parabola $x^2 = ay$ makes an intercept of length $\sqrt{40}$ on the line $y - 2x = 1$, then the values of a are	(p)	-2
(B)	If the tangents drawn from the point $(0, 2)$ to the parabola $y^2 = 4ax$ are inclined at an angle $\frac{3\pi}{4}$, then the values of a are	(q)	1
(C)	If two distinct chords of a parabola $y^2 = 4ax$ passing through $(a, 2a)$ are bisected on the line $x + y = 1$, then the length of latusrectum can be	(r)	2
(D)	If the focus of the parabola $x^2 - ay + 3 = 0$ is $(0, 2)$ and if two values of a are a_1, a_2 such that $a_1 > a_2$, then the value of $\frac{a_1}{a_2}$ is	(s)	3

82. Match the following.

	Column I		Column II
(A)	The common chord of the circle $x^2 + y^2 = 5$ and the parabola $6y = 5x^2 + 7x$ will passes through the point (s)	(p)	$(1, 2)$
(B)	Tangents are drawn from point $(2, 3)$ to the parabola $y^2 = 4x$. Then, the points of contact are	(q)	$(4, 4)$
(C)	From a point P on the circle $x^2 + y^2 = 5$, the equation of chord of contact to the parabola $y^2 = 4x$ is $y = 2(x-2)$. Then, the coordinates of point P will be	(r)	$(-2, 1)$
(D)	$P(4, -4)$ and Q are points on the parabola $y^2 = 4x$ such that the area of ΔPOQ is 6 sq units, where O is the vertex. Then, the coordinates of Q may be	(s)	$(9, -6)$

Parabola Exercise 6 :
Statement I and II Type Questions

- **Directions** (Q. Nos. 83 to 90) are Assertion-Reason type questions. Each of these questions contains two statements:
Statement I (Assertion) and
Statement II (Reason)
Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below :
(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

83. **Statement I** The equation of the common tangent to the parabolas $y^2 = 4x$ and $x^2 = 4y$ is $x + y + 1 = 0$.

Statement II Both the parabolas are reflected to each other about the line $y = x$.

84. **Statement I** Two perpendicular normals can be drawn from the point $\left(\frac{5}{2}, -2\right)$ to the parabola $(y+2)^2 = 2(x-1)$.

Statement II Two perpendicular normals can be drawn from the point $(3a, 0)$ to the parabola $y^2 = 4ax$.

85. **Statement I** The line $y = mx + \frac{a}{m}$ is tangent to the parabola $y^2 = 4ax$ for all values of m.

Statement II A straight line $y = mx + c$ that intersects the parabola $y^2 = 4ax$ one point is a tangent line.

86. **Statement I** The conic $\sqrt{ax} + \sqrt{by} = 1$ represents a parabola.

Statement II Conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a parabola, if $h^2 = ab$.

87. **Statement I** The lines from the vertex to the two extremities of a focal chord of the parabola $y^2 = 4ax$ are perpendicular to each other.

Statement II If extremities of focal chord of a parabola are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$, then $t_1t_2 = -1$.

88. **Statement I** Length of focal chord of a parabola $y^2 = 8x$ making an angle of $60°$ with X-axis is 32/3.

Statement II Length of focal chord of a parabola $y^2 = 4ax$ making an angle α with X-axis is $4a\sec^2(\alpha/2)$.

89. **Statement I** Straight line $x + y = \lambda$ touch the parabola $y = x - x^2$, if $k = 1$.

Statement II Discriminant of $(x-1)^2 = x - x^2$ is zero.

90. **Statement I** Length of latusrectum of parabola $(3x + 4y + 5)^2 = 4(4x + 3y + 2)$ is 4.

Statement II Length of latusrectum of parabola $y^2 = 4ax$ is $4a$.

Parabola Exercise 7 :
Subjective Type Questions

- In this section, there are **15 subjective questions**.

91. If the tangent to the parabola $y^2 = 4ax$ meets the axis in T and tangent at the vertex A in Y and the rectangle $TAYG$ is completed, show that the locus of G is $y^2 + ax = 0$.

92. If incident ray from point $(-1, 2)$ parallel to the axis of the parabola $y^2 = 4x$ strikes the parabola, find the equation of the reflected ray.

93. Prove that the normal chord to a parabola at the point whose ordinate is equal to the abscissa subtends a right angle at the focus.

94. Find the shortest distance between the parabola $y^2 = 4x$ and circle $x^2 + y^2 - 24y + 128 = 0$.

95. Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1 : 2 is a parabola. Find the vertex of this parabola.

96. Show that the locus of the points of intersection of tangents to $y^2 = 4ax$, which intercept a constant length d on the directrix is $(y^2 - 4ax)(x + a)^2 = d^2x^2$.

97. Through the vertex O of the parabola $y^2 = 4ax$ two chords OP and OQ are drawn and the circle on OP and OQ as diameters intersect in R. If θ_1 and θ_2 are the angles made with the axes by the tangents at P and Q to the parabola and ϕ is the angle made by OR with the axis of the parabola, then prove that $\cot\theta_1 + \cot\theta_2 + 2\tan\phi = 0$.

98. Three normals with slopes m_1, m_2 and m_3 are drawn from a point P not on the axis of the parabola $y^2 = 4x$. If $m_1m_2 = \alpha$, results in the locus of P being a part of the parabola, find the value of α.

99. Find the locus of centres of a family of circles passing through the vertex of the parabola $y^2 = 4ax$ and cutting the parabola orthogonally at the other point of intersection.

100. TP and TQ are tangents to the parabola $y^2 = 4ax$. The normals at P and Q intersect at R on the curve. Prove that the circle circumscribing the ΔTPQ lies on the parabola $2y^2 = a(x - a)$.

101. A family of chords of the parabola $y^2 = 4ax$ is drawn so that their projections on a straight line inclined equally to both the axes are all of a constant length c; prove that the locus of their middle points is the curve $(y^2 - 4ax)(y + 2a)^2 + 2a^2c^2 = 0$.

102. The normals at P, Q, R are concurrent and PQ meets the diameter through R on the directrix $x = -a$. Prove that PQ touches [or PQ envelopes] the parabola $y^2 + 16a(x + a) = 0$.

103. If the normals to the parabola $y^2 = 4ax$ at three points P, Q and R meet at A and S be the focus, prove that $SP \cdot SQ \cdot SR = a(SA)^2$.

104. From a point A common tangents are drawn to the circle $x^2 + y^2 = (a^2/2)$ and the parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chords of contact of the point A, with respect to the circle and the parabola.

105. Prove that the any three tangents to a parabola whose slopes are in harmonic progression enclose a triangle of constant area.

Parabola Exercise 8 :
Questions Asked in Previous 13 Year's Exams

- This section contains questions asked in **IIT-JEE, AIEEE, JEE Main & JEE Advanced** from year **2005** to **2017**.

106. Tangent to the curve $y = x^2 + 6$ at a point (1, 7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then the coordinates of Q are **[IIT-JEE 2005, 3M]**

(a) $(-6, -11)$ (b) $(-9, -13)$
(c) $(-10, -15)$ (d) $(-6, -7)$

107. Let P be a point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid-point of PQ is **[AIEEE 2005, 3M]**

(a) $x^2 - 4y + 2 = 0$ (b) $x^2 + 4y + 2 = 0$
(c) $y^2 + 4x + 2 = 0$ (d) $y^2 - 4x + 2 = 0$

108. The axis of a parabola is along the line $y = x$ and the distance of its vertex from origin is $\sqrt{2}$ and that from its focus is $2\sqrt{2}$. If vertex and focus both lie in the first quadrant, the equation of the parabola is **[IIT-JEE 2006, 3M]**

(a) $(x + y)^2 = (x - y - 2)$ (b) $(x - y)^2 = (x + y + 2)$
(c) $(x - y)^2 = 4(x + y - 2)$ (d) $(x - y)^2 = 8(x + y - 2)$

109. The equations of the common tangents to the parabolas $y = x^2$ and $y = -(x - 2)^2$ is/are **[IIT-JEE 2006, 5M]**

(a) $y = 4(x - 1)$ (b) $y = 0$
(c) $y = -4(x - 1)$ (d) $y = -30x - 50$

110. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is **[AIEEE 2006, 4.5 *M*]**

(a) $xy = \frac{105}{64}$ (b) $xy = \frac{3}{4}$
(c) $xy = \frac{35}{16}$ (d) $xy = \frac{64}{105}$

111. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is **[AIEEE 2006, 4.5 *M*]**

(a) $\pi/3$ (b) $\pi/2$
(c) $\pi/6$ (d) $\pi/4$

112. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and fourth quadrants, respectively. Tangents to the circle at P and Q intersect the X-axis at R and tangents to the parabola at P and Q intersect the X-axis at S.

(i) The ratio of the areas of the ΔPQS and ΔPQR is

(a) $1:\sqrt{2}$ (b) 1 : 2
(c) 1 : 4 (d) 1 : 8

(ii) The radius of the circumcircle of the ΔPRS is

(a) 5 (b) $3\sqrt{3}$
(c) $3\sqrt{2}$ (d) $2\sqrt{3}$

(iii) The radius of the incircle of the ΔPQR is **[IIT-JEE 2007, (4 + 4 + 4) M]**

(a) 4 (b) 3
(c) 8/3 (d) 2

113. Statement I The curve $y = -\frac{x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$ because

Statement II A parabola is symmetric about its axis. **[IIT-JEE 2007, 3M]**

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

114. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is **[AIEEE 2007, 3M]**

(a) $(-1, 1)$ (b) $(0, 2)$
(c) $(2, 4)$ (d) $(-2, 0)$

115. Consider the two curves $C_1 : y^2 = 4x$, $C_2 : x^2 + y^2 - 6x + 1 = 0$, then **[IIT-JEE 2008, 3M]**

(a) C_1 and C_2 touch each other only at one point
(b) C_1 and C_2 touch each other exactly at two points
(c) C_1 and C_2 intersect (but do not touch) at exactly two points
(d) C_1 and C_2 neither intersect nor touch each other

116. A parabola has the origin as its focus and the line $x = 2$ as the directrix. The vertex of the parabola is at **[AIEEE 2008, 3M]**

(a) (0, 2) (b) (1, 0)
(c) (0, 1) (d) (2, 0)

117. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the ΔPTN is a parabola whose **[IIT-JEE 2009, 4M]**

(a) vertex is $\left(\frac{2a}{3}, 0\right)$ (b) directrix is at $x = 0$
(c) latusrectum is $\frac{2a}{3}$ (d) focus is $(a, 0)$

118. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, The slope of the line joining A and B can be **[IIT-JEE 2010, 3M]**

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$ (c) $\frac{2}{r}$ (d) $-\frac{2}{r}$

119. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, the locus of P is **[AIEEE 2010, 4M]**

(a) $2x + 1 = 0$ (b) $x = -1$
(c) $2x - 1 = 0$ (d) $x = 1$

120. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latusrectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola and Δ_2 be the area of the triangle formed by drawing tangent at P and at the end points of the latusrectum. Then, $\frac{\Delta_1}{\Delta_2}$ is **[IIT-JEE 2011, 4M]**

121. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then, the locus of P is **[IIT-JEE 2011, 3M]**

(a) $x^2 = y$ (b) $y^2 = 2x$
(c) $y^2 = x$ (d) $x^2 = 2y$

122. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by **[IIT-JEE 2011, 4M]**

(a) $y - x + 3 = 0$ (b) $y + 3x - 33 = 0$
(c) $y + x - 15 = 0$ (d) $y - 2x + 12 = 0$

123. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is **[AIEEE 2011, 4M]**

(a) $\frac{3\sqrt{2}}{8}$ (b) $\frac{8}{3\sqrt{2}}$
(c) $\frac{4}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{4}$

124. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the ΔPQS is **[IIT-JEE 2012, 4M]**

Paragraph

(Q. Nos. 125 and 126)

Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangent to the parabola at P and Q meet at a point lying on the line $y = 2x + a, a > 0$. **[JEE Advanced 2013, 3+3 M]**

125. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, them $\tan\theta$ is equal to

(a) $\frac{2}{3}\sqrt{7}$ (b) $-\frac{2}{3}\sqrt{7}$ (c) $\frac{2}{3}\sqrt{5}$ (d) $-\frac{2}{3}\sqrt{5}$

126. Length of chord PQ is

(a) $7a$ (b) $5a$ (c) $2a$ (d) $3a$

127. The slope of the line touching the parabolas $y^2 = 4x$ and $x^2 = -32y$ is **[JEE Main 2014, 4M]**

(a) 1/8 (b) 2/3
(c) 1/2 (d) 3/2

128. The common tangent to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then, the area of the quadrilateral $PQRS$ is **[JEE Advanced 2014, 3 M]**

(a) 3 (b) 6 (c) 9 (d) 15

Paragraph

(Q. Nos. 129 and 130)

Let a, r, s and t be non-zero real numbers. Let $P(at^2\ 2at)$, $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point $(2a, 0)$. **[JEE Advanced 2014, (3 + 3) M]**

129. The value of r is

(a) $-\frac{1}{t}$ (b) $\frac{t^2+1}{t}$ (c) $\frac{1}{t}$ (d) $\frac{t^2-1}{t}$

130. If $st = 1$, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(a) $\frac{(t^2+1)^2}{2t^3}$ (b) $\frac{a(t^2+1)^2}{2t^3}$

(c) $\frac{a(t^2+1)^2}{t^3}$ (d) $\frac{a(t^2+2)^2}{t^3}$

131. Let O be the vertex and Q be any point on the parabola $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is **[JEE Main 2015, 4M]**

(a) $x^2 = y$ (b) $y^2 = x$
(c) $y^2 = 2x$ (d) $x^2 = 2y$

132. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latusrectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is **[JEE Advanced 2015, 4M]**

133. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, the distance between A and B is **[JEE Advanced 2015, 4M]**

134. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P? **[JEE Advanced 2015, 4M]**

(a) $(4, 2\sqrt{2})$ (b) $(9, 3\sqrt{2})$

(c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (d) $(1, \sqrt{2})$

135. Let P be the point on the parabola $y^2 = 8x$, which is at a minimum distance from the centre C of the circle $x^2 + (y+6)^2 = 1$, the equation of the circle passing through C and having its centre at P, is **[JEE Main 2016, 4M]**

(a) $x^2 + y^2 - 4x + 8y + 12 = 0$
(b) $x^2 + y^2 - x + 4y - 12 = 0$
(c) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$
(d) $x^2 + y^2 - 4x + 9y + 18 = 0$

136. The circle $C_1 : x^2 + y^2 = 3$ with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the Y-axis, then **[JEE Advanced 2016, 4M]**

(a) $Q_2Q_3 = 12$
(b) $R_2R_3 = 4\sqrt{6}$
(c) area of ΔOR_2R_3 is $6\sqrt{2}$
(d) area of ΔPQ_2Q_3 is $4\sqrt{2}$

137. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the centre S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then,

(a) $SP = 2\sqrt{5}$ **[JEE Advanced 2016, 4M]**

(b) $SQ : QP = (\sqrt{5} + 1) : 2$

(c) the x-intercept of the normal to the parabola at P is 6

(d) the slope of the tangent to the circle at Q is $\frac{1}{2}$

138. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is **[JEE Main 2017, 4M]**

(a) $4(\sqrt{2} + 1)$

(b) $2(\sqrt{2} + 1)$

(c) $2(\sqrt{2} - 1)$

(d) $4(\sqrt{2} - 1)$

139. If a chord, which is not a tangent of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and mid-point (h, k), then which of the following is (are) possible value(s) of p, h and k ? **[JEE Advanced 2017, 4M]**

(a) $p = 2, h = 3, k = -4$

(b) $p = -1, h = 1, k = -3$

(c) $p = -2, h = 2, k = -4$

(d) $p = 5, h = 4, k = -3$

Answers

Chapter Exercise

1. (a) **2.** (b) **3.** (b) **4.** (d) **5.** (d) **6.** (a)
7. (a) **8.** (a) **9.** (d) **10.** (b) **11.** (a) **12.** (c)
13. (a) **14.** (d) **15.** (d) **16.** (a) **17.** (a) **18.** (c)
19. (d) **20.** (c) **21.** (d) **22.** (a) **23.** (c) **24.** (a)

25. (a) **26.** (c) **27.** (b) **28.** (b) **29.** (c) **30.** (d)
31. (b,c) **32.** (a,b) **33.** (b,d) **34.** (b,c,d) **35.** (a,b,c, d) **36.** (a, c)
37. (a,c) **38.** (b,d) **39.** (a,c) **40.** (a,c) **41.** (b,c,d) **42.** (a,b,c,d)
43. (a,c,d) **44.**(a,b,c) **45.** (a,b) **46.** (d) **47.** (d) **48.** (c)
49. (b) **50.** (c) **51.** (d) **52.** (d) **53.** (c) **54.** (d)
55. (b) **56.** (b) **57.** (d) **58.** (d) **59.** (b) **60.** (b)
61. (a) **62.** (b) **63.** (b) **64.** (c) **65.** (d) **66.** (a)
67. (c) **68.** (b) **69.** (d) **70.** (3) **71.** (6) **72.** (8)
73. (3) **74.** (6) **75.** (9) **76.** (8) **77.** (4) **78.** (5)
79. (0)
80. (A) $\rightarrow$ (q,r,s); (B) $\rightarrow$ (q,s); (C) $\rightarrow$ (p); (D) $\rightarrow$ (q)
81. (A) $\rightarrow$ (p,q); (B) $\rightarrow$ (p,r); (C) $\rightarrow$ (q,r,s); (D) $\rightarrow$ (s)
82. (A) $\rightarrow$ (p,r); (B) $\rightarrow$ (p,q); (C) $\rightarrow$ (r); (D) $\rightarrow$ (p,s)
83. (a) **84.** (a) **85.** (a) **88.** (c) **87.** (d) **88.** (c)
89. (c) **90.** (d) **92.** $x = 1$
94. $4(\sqrt{5} - 1)$ **95.** $\left(\frac{8}{9}, \frac{2}{9}\right)$ **98.** (2)
99. $2y^2(2y^2 + x^2 - 12ax) = ax(3x - 4a)^2$
104. $\frac{15a^2}{4}$ sq units **106.** (d) **107.** (d) **108.** (d) **109.** (a, b)
110.(a) **111.** (b) **112.** [i] (c) [ii] (b) [iii] (d) **113.** (a)
114.(d) **115.** (b) **116.** (b) **117.** (a,d) **118.** (c,d) **119.** (b)
120. (2) **121.** (c) **122.** (a,b,d) **123.** (a) **124.** 4 sq units
125. (d) **126.** (b) **127.** (c) **128.** (d) **129.** (d) **130.** (b)
131. (d) **132.** (2) **133.** (4) **134.** (a,d)
135. (a) **136.** (a,b,c) **137.** (a,c,d) **138.** (d) **139.** (a)

Solutions

1. Equation of tangent of $y^2 = 4bx$ is

$$y = mx + \frac{b}{m} \quad \text{...(i)}$$

Here, $m = \tan\frac{\pi}{4} = 1$

From Eq. (i), $y = x + b$

For common tangent $y = x + b$ is also tangent of circle $x^2 + y^2 = a^2$, then

$$\frac{|0 - 0 + b|}{\sqrt{1+1}} = a$$

$\Rightarrow$ $b = a\sqrt{2}$ $[\because a > 0, b > 0]$

2. The coordinates of vertex and focus of required parabola are $(a, 0)$ and $(a_1, 0)$ respectively. Therefore, the distance between the vertex and the focus is $AS = a_1 - a$. So, the length of latusrectum is $4(a_1 - a)$. Thus, the equation of the required parabola is

$$(y - 0)^2 = 4(a_1 - a)(x - a)$$

or $y^2 = 4(a_1 - a)(x - a)$.

3. The parabolas are equal if the lengths of their latusrectum are equal.

The length of latusrectum of $y^2 = ax$ is a

The equation of second parabola can be written as

$$\sqrt{(x-3)^2 + (y+2)^2} = \left(\frac{3x - 4y - 2}{5}\right)$$

Here, focus is $(3, -2)$ and the equation of directrix is $3x - 4y - 2 = 0$.

$\therefore$ Length of latusrectum $= 2 \times$ Distance between focus and directrix

$$= 2\left|\frac{9 - 4 \times -2 - 2}{\sqrt{(9+16)}}\right| = 6$$

Thus, the two parabolas are equal if $a = 6$.

4. Let $DC = CB = BA = AD = k$

$\therefore$ Coordinates of B are (k, k),

which lie on $y = \lambda\sqrt{x}$

$\therefore$ $k = \lambda\sqrt{k}$

$\therefore$ $k = \lambda^2$

$\therefore$ $BC = k = \lambda^2$ ...(i)

Also, let $CG = GF = FE = EC = k_1$

$\therefore$ Coordinates of F are $(\lambda^2 + k_1, k_1)$,

which lie on $y = \lambda\sqrt{x}$

Then $k_1 = \lambda\sqrt{(\lambda^2 + k_1)}$

$\Rightarrow$ $k_1^2 = \lambda^4 + \lambda^2 k_1$

or $k_1^2 - \lambda^2 k_1 - \lambda^4 = 0$

$\therefore$ $k_1 = \dfrac{\lambda^2 \pm \sqrt{(\lambda^4 + 4\lambda^4)}}{2}$ $[\because k_1 > 0]$

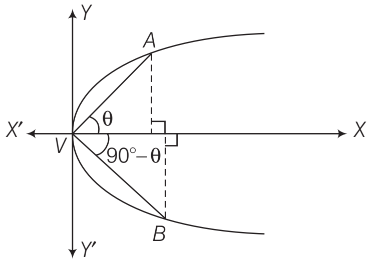
or $\dfrac{k_1}{\lambda^2} = \dfrac{1 + \sqrt{5}}{2}$

or $\dfrac{FG}{BC} = \dfrac{\sqrt{5} + 1}{2}$

5. Coordinates of A and B are

$(VA\cos\theta, VA\sin\theta)$ and $(VB\sin\theta, -VB\cos\theta)$ respectively

$\because$ A and B lie on $y^2 = x$, then $(VA\sin\theta)^2 = VA\cos\theta$



$\Rightarrow$ $VA = \dfrac{\cos\theta}{\sin^2\theta}$

Similarly, $VB = \dfrac{\sin\theta}{\cos^2\theta}$

$\therefore$ $\dfrac{|VA|}{|VB|} = \cot^3\theta$

6. $\because$ $x = t^2 - t + 1, y = t^2 + t + 1$

$\therefore$ $x + y = 2t^2 + 2$ and $y - x = 2t$

or $2(x + y - 2) = 4t^2 = (2t)^2 = (y - x)^2$

or $(x - y)^2 = 2(x + y - 2)$

or $\left(\dfrac{x - y}{\sqrt{2}}\right)^2 = \sqrt{2}\left(\dfrac{x + y - 2}{\sqrt{2}}\right)$

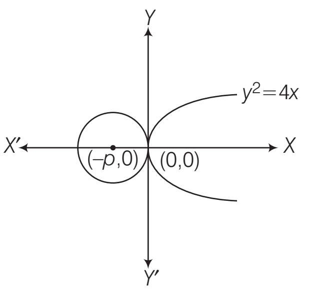
For vertex, $\dfrac{x - y}{\sqrt{2}} = 0$ and $\dfrac{x + y - 2}{\sqrt{2}} = 0$

or $x = y$ and $x + y = 2$

$\therefore$ $x = y = 1$

Hence, vertex is (1, 1).

7. It is clear from figure



$-p < 0$ or $p > 0$

8. The point of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ are $(0, 0)$ and $(4a, 4a)$ but $a \neq 0$.

Now, $2bx + 3cy + 4d = 0$ passes through $(0, 0)$ and $(4a, 4a)$.

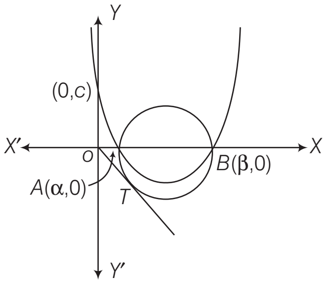
Therefore, $d = 0$ and $2b(4a) + 3c(4a) = 0$ i.e.,

$$2b + 3c = 0 \qquad [\because a \neq 0]$$

or $$d^2 + (2b + 3c)^2 = 0$$

9. $\because (OT)^2 = OA \cdot OB$

$$= \alpha \cdot \beta$$



or $$OT = \sqrt{\alpha\beta} \qquad \text{...(i)}$$

Parabola cuts X-axis at α and β.

$\therefore \alpha, \beta$ are the roots of $ax^2 + bx + c = 0$

$$\therefore \quad \alpha\beta = \frac{c}{a} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$OT = \sqrt{\frac{c}{a}}$$

10. $\because$ Two perpendicular tangents meet a point on directrix.

Now, equations of tangents at $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are $t_1 y = x + at_1^2$ and $t_2 y = x + at_2^2$, respectively.

$\therefore \quad P_1 \equiv (-at_1^2, 0), P_2 \equiv (-at_2^2, 0)$

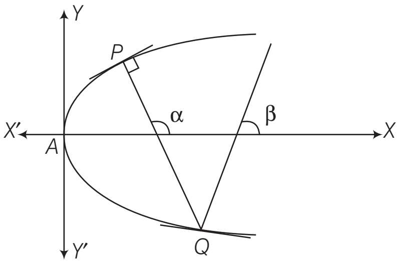
$\Rightarrow SP_1 = a(1 + t_1^2), SP_2 = a(1 + t_2^2)$ and $t_1 t_2 = -1$

$$\therefore \quad \frac{1}{SP_1} + \frac{1}{SP_2} = \frac{1}{a(1 + t_1^2)} + \frac{1}{a(1 + t_2^2)}$$

$$= \frac{1}{a(1 + t_1^2)} + \frac{1}{a\left(1 + \frac{1}{t_1^2}\right)} \qquad \left[\therefore t_2 = -\frac{1}{t_1}\right]$$

$$= \frac{1}{a(1 + t_1^2)} + \frac{t_1^2}{a(1 + t_1^2)} = \frac{1}{a}$$

11. Let $P \equiv (at_1^2, 2at_1)$ and $Q \equiv (at_2^2, 2at_2)$



Equation of normal at P is

$$y + t_1 x = 2at_1 + at_1^3$$

meet the curve again at Q, then

$$t_2 = -t_1 - \frac{2}{t_1} \qquad \text{...(i)}$$

Here $\tan\alpha = -t_1$ and $\tan\beta = -t_2$, from Eq. (i)

$$\therefore \quad -\tan\beta = \tan\alpha + \frac{2}{\tan\alpha}$$

$$\Rightarrow \quad \tan\alpha(\tan\alpha + \tan\beta) = -2$$

12. Let $A \equiv (\alpha, \beta)$

The equation of normal at $(at^2, 2at)$

$$y + tx = 2at + at^3 \qquad \text{...(i)}$$

(α, β) lie on Eq. (i), then

$$at^3 + (2a - \alpha)t - \beta = 0 \qquad \text{...(ii)}$$

Let t_1, t_2 and t_3 be the roots of Eq. (ii), then

$$at^3 + (2a - \alpha)t - \beta = a(t - t_1)(t - t_2)(t - t_3) \qquad \text{...(iii)}$$

Let $P \equiv (at_1^2, 2at_1)$, $Q \equiv (at_2^2, 2at_2)$ and $R \equiv (at_3^2, 2at_3)$

Since, the focus is $S(a, 0)$

$$\therefore \quad SP = a(1 + t_1^2), SQ = a(1 + t_2^2)$$

and $$SR = a(1 + t_3^2)$$

On putting $t = i = \sqrt{-1}$ in Eq. (iii), we get

$$-ai + (2a - \alpha)i - \beta$$
$$= a(i - t_1)(i - t_2)(i - t_3)$$

or $|(a - \alpha)i - \beta| = a|i - t_1||i - t_2|i - t_3|$

$$\Rightarrow \quad \sqrt{(a - \alpha)^2 + \beta^2}$$
$$= a\sqrt{(1 + t_1^2)}\sqrt{(1 + t_2^2)}\sqrt{(1 + t_3^2)}$$

or $a((a - \alpha)^2 + \beta^2) = a(1 + t_1^2) \cdot a(1 + t_2^2) \cdot a(1 + t_3^2)$

$$a(SA)^2 = SP \cdot SQ \cdot SR$$

or $$SP \cdot SQ \cdot SR = a(SA)^2$$

13. Let AB be a normal chord, where $A \equiv (at_1^2, 2at_1)$ and $B \equiv (at_2^2, 2at_2)$, we have $t_2 = -t_1 - \dfrac{2}{t_1}$

Now, $$AB = \sqrt{(at_1^2 - at_2^2)^2 + (2at_1 - 2at_2)^2}$$

$$= a|t_1 - t_2|\sqrt{(t_1 + t_2)^2 + 4}$$

$$= a\left|2\left(t_1 + \frac{1}{t_1}\right)\right|\sqrt{\left(\frac{4}{t_1^2} + 4\right)}$$

$$= 4a\sqrt{\frac{(t_1^2 + 1)^3}{t_1^4}}$$

$$(AB)^2 = 16a^2\frac{(t_1^2 + 1)^3}{t_1^4}$$

$$\frac{d(AB)^2}{dt_1} = 16a^2\left\{\frac{(1 + t_1^2)^2}{t_1^5}(t_1^2 - 2)\right\}$$

For $$\frac{d(AB)^2}{dt_1} = 0 \Rightarrow t_1 = \sqrt{2}$$

For which $(AB)^2$ is minimum, thus

$$AB_{\min} = \sqrt{\frac{16a^2(2 + 1)^3}{4}} = 2a\sqrt{27}$$

14. On solving

$$x^2 + y^2 = a^2 \text{ and } y^2 = 4(x + 4)$$

$\Rightarrow \quad x^2 + 4(x + 4) = a^2$

or $\quad x^2 + 4x + 16 - a^2 = 0$

If the circle and parabola touch each other, then

$$D = 0 \Rightarrow 16 - 4 \cdot 1 \cdot (16 - a^2) = 0$$

$\Rightarrow \quad a^2 = 12 \text{ or } a = 2\sqrt{3}$

15. $\sin\theta > |2a|$ $\quad [\because h > 2a]$

$\Rightarrow \quad 0 < |2a| < 1 \text{ or } 0 < |a| < \frac{1}{2}$

$\therefore \quad a \in \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

16. Tangent to $y^2 = 4x$ in terms of m is

$$y = mx + \frac{1}{m} \quad \text{...(i)}$$

and normal to $x^2 = 4by$ in terms of m is

$$y = mx + 2b + \frac{b}{m^2} \quad \text{...(ii)}$$

$\because$ Eqs. (i) and (ii) are same, then

$$\frac{1}{m} = 2b + \frac{b}{m^2}$$

$\Rightarrow \quad 2bm^2 - m + b = 0$

For two different tangents

$\therefore \quad D > 0 \Rightarrow 1 - 8b^3 > 0$

or $\quad |b| < \frac{1}{2\sqrt{2}}$

17. The given parabolas $2y^2 = 2x - 1$ and $2x^2 = 2y - 1$ are symmetrical about the line $y = x$. The shortest distance occurs along the common normal which is perpendicular to the line $y = x$.

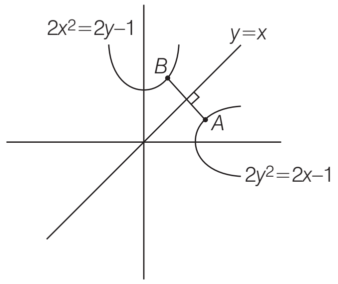
Therefore, the tangent at point A on $2y^2 = 2x - 1$ is parallel to $y = x$. Therefore,

$$4y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{2y} = 1$$

or $\quad y = \frac{1}{2}$ and $x = \frac{3}{4}$

$\therefore \quad A \equiv \left(\frac{3}{4}, \frac{1}{2}\right)$, then coordinates of B on $2x^2 = 2y - 1$ is

$$B \equiv \left(\frac{1}{2}, \frac{3}{4}\right)$$



$\therefore$ Shortest distance $= AB$

$$= \sqrt{\left(\frac{3}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - \frac{3}{4}\right)^2}$$

$$= \sqrt{\left(\frac{1}{16} + \frac{1}{16}\right)} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

18. We know that normals at $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ meet again on the parabola. Then $t_1t_2 = 2$

Here, $\quad a = 1$

and $\quad x_1 = at_1^2 = t_1^2$ and $y_1 = 2at_1 = 2t_1$

$\quad x_2 = at_2^2 = t_2^2$ and $y_2 = 2at_2 = 2t_2$

Given $\quad x_1 + x_2 = 4 \Rightarrow t_1^2 + t_2^2 = 4$

or $\quad (t_1 + t_2)^2 - 2t_1t_2 = 4$

$\Rightarrow \quad (t_1 + t_2)^2 = 8 \quad [\because t_1t_2 = 2]$

or $\quad |t_1 + t_2| = \sqrt{8}$

or $\quad |2t_1 + 2t_2| = 2\sqrt{8}$

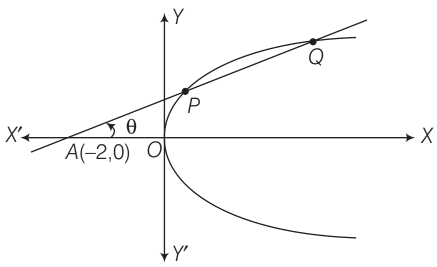
or $\quad |y_1 + y_2| = 2\sqrt{8} = 4\sqrt{2}$

19. Let any point at distance r from A on the parabola is

$$(-2 + r\cos\theta, r\sin\theta),$$

then $\quad r^2\sin^2\theta = 4(-2 + r\cos\theta)$

or $\quad r^2\sin^2\theta - 4r\cos\theta + 8 = 0$



Let P and Q are distances r_1 and r_2 from A, then

$$r_1 + r_2 = \frac{4\cos\theta}{\sin^2\theta}$$

and $\quad r_1r_2 = \frac{8}{\sin^2\theta}$

Now, $\quad \frac{1}{AP} + \frac{1}{AQ} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1r_2} = \frac{\cos\theta}{2}$

given that $\quad \frac{1}{AP} + \frac{1}{AQ} < \frac{1}{4}$

$\Rightarrow \quad \frac{\cos\theta}{2} < \frac{1}{4}$

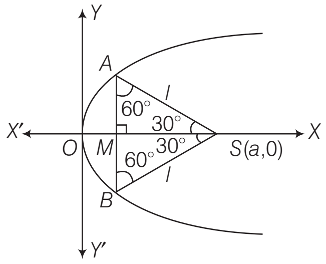
or $\quad \cos\theta < \frac{1}{2}$

or $\quad \tan\theta > \sqrt{3}$

$[\because \cos\theta$ is decreasing and $\tan\theta$ is increasing in $\left(0, \frac{\pi}{2}\right)]$

$\therefore$ Slope $\quad (m) > \sqrt{3}$

20. Coordinates of A are $(a - l\cos 30^\circ, l \sin 30^\circ)$



or $\quad A \equiv \left(a - \dfrac{l\sqrt{3}}{2}, \dfrac{l}{2}\right)$

which lies on $y^2 = 4ax$, then

$$\frac{l^2}{4} = 4a\left(a - \frac{l\sqrt{3}}{2}\right) \Rightarrow l^2 + 8a\sqrt{3}l - 16a^2 = 0$$

$$\therefore \quad l = \frac{-8a\sqrt{3} \pm \sqrt{(192a^2 + 64a^2)}}{2}$$

$$= \frac{-8a\sqrt{3} \pm 16a}{2}$$

$$= 4a(2 - \sqrt{3}) \qquad \text{[taking +ve sign]}$$

Aliter :

Let $\quad A \equiv (at_1^2, 2at_1)$ and $B \equiv (at_1^2 - 2at_1)$.

Slope of $SA = \tan(180^\circ - 30^\circ)$

$$\frac{2at_1}{at_1^2 - a} = -\tan 30^\circ$$

$$\frac{2t_1}{t_1^2 - 1} = -\frac{1}{\sqrt{3}}$$

or $\quad t_1^2 + 2\sqrt{3}t_1 - 1 = 0'$ or $t_1 = -\sqrt{3} \pm 2$

Thus, $\quad t_1 = 2 - \sqrt{3} \qquad [\because\ t_1 = -2 - \sqrt{3}$ rejected]

Here, $\quad AB = 4at_1 = 4a(2 - \sqrt{3})$

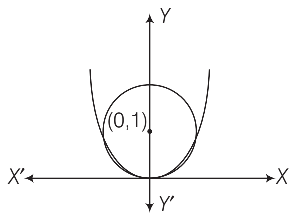
21. $C : x^2 + (y-1)^2 = 1$ and $P : y = ax^2$

Putting $x^2 = \dfrac{y}{a}$ in $x^2 + (y-1)^2 = 1$

or $\quad \dfrac{y}{a} + (y-1)^2 = 1$

or $\quad y^2 - 2y + \dfrac{y}{a} = 0$

$\therefore \quad y = 0$ or $y = 2 - \dfrac{1}{a}$



On substituting $y = 2 - \dfrac{1}{a}$ in $y = ax^2$, then

$$ax^2 = 2 - \frac{1}{a}$$

or $\quad x^2 = \dfrac{2a-1}{a^2} > 0$

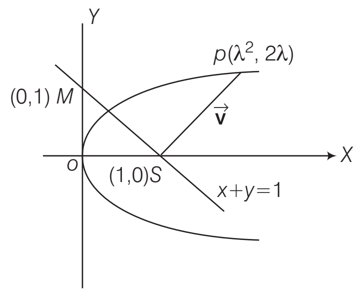
or $\quad 2a - 1 > 0$

$\therefore \quad a > \dfrac{1}{2}$

or $\quad a \in \left(\dfrac{1}{2}, \infty\right)$

22. $\vec{\mathbf{V}} = (\lambda^2 - 1)\,\hat{\mathbf{i}} + (2\lambda - 0)\,\hat{\mathbf{j}}$

$\vec{\mathbf{V}} = (\lambda^2 - 1)\,\hat{\mathbf{i}} + (2\lambda)\,\hat{\mathbf{j}} \qquad$ ...(i)



and $\quad \vec{\mathbf{n}} = (0 - 1)\,\hat{\mathbf{i}} + (1 - 0)\,\hat{\mathbf{j}}$

$\quad = -\hat{\mathbf{i}} + \hat{\mathbf{j}}$

The projection of $\vec{\mathbf{V}}$ on $\vec{\mathbf{n}} = y \qquad$ [given]

$\therefore \quad y = \dfrac{\vec{\mathbf{V}} \cdot \vec{\mathbf{n}}}{|\vec{\mathbf{n}}|} = \dfrac{-(\lambda^2 - 1) + 2\lambda}{\sqrt{2}} \qquad$ ...(ii)

Given, $\quad \dfrac{dx}{dt} = 4$

$\Rightarrow \quad \dfrac{d}{dt}(\lambda^2 - 1) = 4 \quad \Rightarrow \quad 2\lambda\dfrac{d\lambda}{dt} = 4$

When $P \equiv (4, 4)$,

We have $\lambda = 2$, therefore

$\quad \dfrac{d\lambda}{dt} = 1 \qquad$ ...(iii)

From Eq. (ii),

$$\frac{dy}{dt} = \frac{(2 - 2\lambda)\dfrac{d\lambda}{dt}}{\sqrt{2}} \qquad \text{[from Eq. (ii)]}$$

$$= \frac{(2-4) \times 1}{\sqrt{2}} = -\sqrt{2}$$

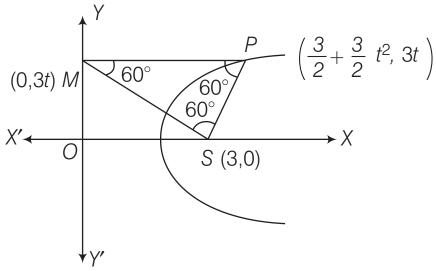
23. Given parabola is

$$y^2 = 6\left(x - \frac{3}{2}\right)$$

The equation of directrix is $x - \dfrac{3}{2} + \dfrac{6}{4} = 0$

i.e. $\quad x = 0$

Let the coordinates of P be $\left(\dfrac{3}{2} + \dfrac{3}{2}t^2, 3t\right)$



Therefore, $MS = MP$

$$\sqrt{(9 + 9t^2)} = \frac{3}{2} + \frac{3}{2}t^2$$

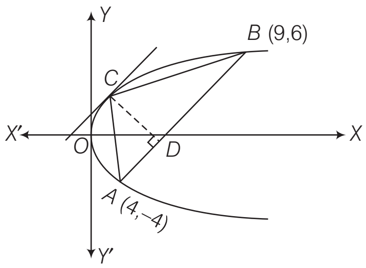
or $$9 + 9t^2 = \frac{9}{4}(1 + t^2)^2$$

or $$1 + t^2 = 4$$

$\therefore$ Length of each side $= MS$

$$= \sqrt{9(1 + t^2)} = \sqrt{36} = 6$$

24. The area of ΔABC is maximum if CD is maximum, because AB is fixed.



It is clear that tangent drawn to the parabola at C should be parallel to AB.

For $y^2 = 4x$

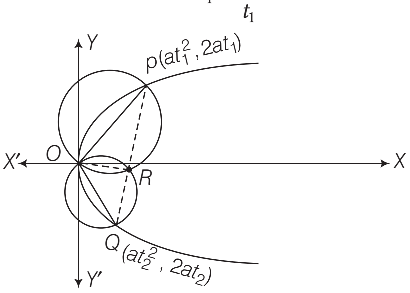
$\therefore$ $$2y\frac{dy}{dx} = 4 \quad \text{or} \quad \frac{dy}{dx} = \frac{2}{y} = \text{slope of } AB$$

$\Rightarrow$ $$\frac{2}{y} = \frac{6+4}{9-4} = 2 \quad \text{or} \quad y = 1, \text{ then } x = \frac{1}{4}$$

Hence, coordinates of C are $\left(\frac{1}{4}, 1\right)$.

25. $\because$ Tangent at $P(at_1^2, 2at_1)$ is $t_1 y = x + at_1^2$.

$\therefore$ $$\tan\theta_1 = \frac{1}{t_1}$$



Similarly, $$\tan\theta_2 = \frac{1}{t_2}$$

$\because$ Angle in a semicircle is 90°.

$\therefore$ $$\angle ORP = \angle ORQ = 90^\circ$$

$\Rightarrow$ P, R, Q are collinear.

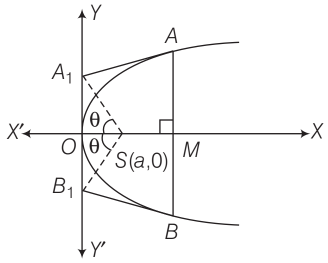
$$\text{Slope of } PQ = \frac{2}{t_1 + t_2} \qquad [\because OR \perp PQ]$$

$\therefore$ $$\text{Slope of } OR = -\frac{(t_1 + t_2)}{2} = \tan\phi$$

or $$t_1 + t_2 = -2\tan\phi$$

$\Rightarrow$ $$\cot\theta_1 + \cot\theta_2 = -2\tan\phi$$

26. Let $A \equiv (at^2, 2at)$ and $B \equiv (at^2, -2at)$



The equations of tangents at A and B are

$$ty = x + at^2 \text{ and } -ty = x + at^2.$$

These tangents meet the Y-axis at $A_1(0, at)$ and $B_1(0, -at)$ respectively.

Area of trapezium

$$AA_1 B_1 B = \frac{1}{2}(AB + A_1B_1) \times OM$$

$$24a^2 = \frac{1}{2}(4at + 2at) \times at^2$$

$$24a^2 = 3a^2t^3 \Rightarrow t^3 = 8$$

$\therefore$ $$t = 2$$

$\therefore$ Coordinates of A_1 and B_1 are $(0, 2a)$ and $(0, -2a)$ respectively.

If $\angle OSA_1 = \angle OSB_1 = \theta$, then

$\therefore$ $$\tan\theta = \frac{2a}{a} = 2$$

$\Rightarrow$ $$\theta = \tan^{-1} 2$$

Hence, subtended angle $= 2\theta = 2\tan^{-1} 2$

27. Given, $$\frac{5}{2} = {}^nC_3(px)^{n-3}\left(\frac{1}{x}\right)^3 = {}^nC_3 \cdot p^{n-3}x^{n-6} \qquad \text{...(i)}$$

Since, LHS of Eq. (i) is independent of x

$\therefore$ $$n - 6 = 0 \Rightarrow n = 6$$

From Eq. (i),

$$\frac{5}{2} = {}^6C_3p^3 = 20p^3$$

$\Rightarrow$ $$p^3 = \left(\frac{1}{2}\right)^3 \Rightarrow p = \frac{1}{2}$$

Given, parabola is $y^2 = x$

Here, $4a = 1$

$\Rightarrow$ $$a = \frac{1}{4}$$

Since, three normals are drawn from point $(q, 0)$,

$\therefore$ $$q > 2a \text{ or } q > \frac{1}{2} \text{ or } q > p \qquad \left[\because p = \frac{1}{2}\right]$$

28. $$y^2 - 4x - 2y + 5 = 0$$

$\Rightarrow$ $$(y-1)^2 - 4x + 4 = 0$$

$\Rightarrow$ $$(y-1)^2 = 4(x-1)$$

Let $y - 1 = Y$ and $x - 1 = X$

then, $y^2 = 4X$ comparing with $Y^2 = 4ax$

$\therefore$ $$a = 1$$

$\because$ All three normals to the parabola are real and meet on the axis of parabola, then

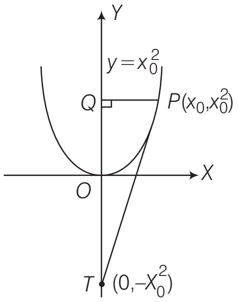
$$X > 2a \text{ and } Y = 0$$

i.e., $$x - 1 > 2 \text{ and } y - 1 = 0$$

$\therefore$ $$x > 3 \text{ and } y = 1$$

or $(k, 1); k > 3$

29. Let $P(x_0, x_0^2)$ be any point on the parabola $y = x^2$



Equation of tangent at $P(x_0, x_0^2)$ is

$$xx_0 = \frac{1}{2}(y + x_0^2)$$

$\Rightarrow$ $$2xx_0 - y - x_0^2 = 0$$

Tangent meets the Y-axis at $T(0, -x_0^2)$.

Hence, the area of the triangle $\Delta PTQ = \frac{1}{2} \times PQ \times QT$

$$= \frac{1}{2} \times x_0 \times 2x_0^2 = x_0^3$$

which increases in the interval $[1, 2]$ and hence is greatest when $x_0 = 2$.

30. $$\left(\frac{2x - y + 2}{\sqrt{5}}\right) = \sqrt{(x-2)^2 + (y-0)^2} \quad \text{[given]}$$

$\Rightarrow$ $$(2x - y + 2)^2 = 5(x^2 + y^2 - 4x + 4)$$

or $$x^2 + 4y^2 + 4xy = 28x - 4y - 16$$

$\Rightarrow$ $$(x + 2y)^2 = 4(7x - y - 4)$$

$\because$ $x + 2y = 0$ and $7x - y - 4 = 0$ and are not perpendicular.

$\therefore$ $(x + 2y + \lambda)^2 = (2\lambda + 28)x + (4\lambda - 4)y + \lambda^2 - 16$ ...(i)

Now, (slope of $x + 2y + \lambda = 0$) $\times$ (slope of $(2\lambda + 28)x + (4\lambda - 4)y + \lambda^2 - 16 = 0$) $= -1$

$\therefore$ $$-\frac{1}{2} \times -\frac{(2\lambda + 28)}{(4\lambda - 4)} = -1$$

$\Rightarrow$ $$2\lambda + 28 = -8\lambda + 8$$

or $$10\lambda = -20$$

$\therefore$ $$\lambda = -2$$

From Eq. (i),

$$(x + 2y - 2)^2 = (24x - 12y - 12)$$

$\Rightarrow$ $$\left(\frac{x + 2y - 2}{\sqrt{5}}\right)^2 = \frac{12}{\sqrt{5}}\left(\frac{2x - y - 1}{\sqrt{5}}\right)$$

Let $$\frac{x + 2y - 2}{\sqrt{5}} = X, \frac{2x - y - 1}{\sqrt{5}} = Y$$

or $$X^2 = \frac{12}{\sqrt{5}} Y \quad \text{or} \quad Y = \frac{\sqrt{5}}{12} X^2$$

$\therefore$ $$k = \frac{\sqrt{5}}{12}$$

31. Equation of tangent of parabola

$$y^2 = 40x \text{ is } y = mx + \frac{10}{m} \quad \text{... (i)}$$

which is also tangent of circle $x^2 + y^2 = 50$, then

$$\frac{\left|\frac{10}{m}\right|}{\sqrt{(m^2 + 1)}} = 5\sqrt{2}$$

$\Rightarrow$ $$m^4 + m^2 - 2 = 0$$

$\Rightarrow$ $$(m^2 + 2)(m^2 - 1) = 0$$

$\therefore$ $$m^2 = 1, m^2 + 2 \neq 0 \quad \text{or} \quad m = \pm 1$$

From Eq. (i), common tangents are

$$y = x + 10 \text{ and } y = -x - 10$$

or $$x - y + 10 = 0 \text{ and } x + y + 10 = 0$$

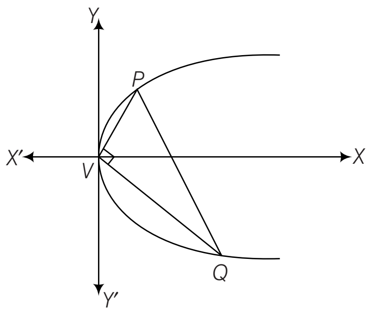
32. Let coordinates of P be $(t^2, 2t)$

$\therefore$ Slope of $PV = \frac{2t - 0}{t^2 - 0} = \frac{2}{t}$

$\Rightarrow$ Slope of QV is $-\frac{t}{2}$

$\therefore$ Equation of QV is $y = \frac{-t}{2}x$

Solving it with $y^2 = 4x$, we get $Q\left(\frac{16}{t^2}, \frac{-8}{t}\right)$



Area of $\Delta PVQ = 20$ (given)

$\Rightarrow$ $$\frac{1}{2}\left|\begin{vmatrix} t^2 & 2t \\ \frac{16}{t^2} & \frac{-8}{t} \end{vmatrix}\right| = 20$$

$\Rightarrow$ $$t + \frac{4}{t} = \pm 5$$

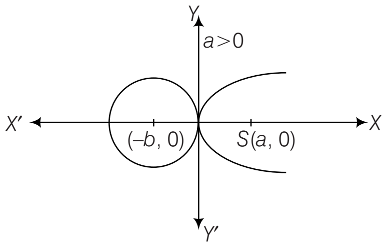
$\Rightarrow$ $t^2 - 5t + 4 = 0$ or $t^2 + 5t + 4 = 0$

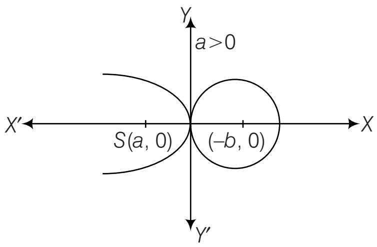
$\therefore$ $$t = 1, 4 \text{ or } t = -1, -4$$

Hence coordinates of P are $(1, 2), (16, 8), (1 - 2), (16, -8)$

33. Parabola is $y^2 = 4ax$ and circle is $(x + b)^2 + (y - 0)^2 = b^2$

If parabola and circle touch each other externally, then





If $\quad a > 0, -b < 0$ and if $a < 0, -b > 0$

or $\quad a > 0, b > 0$ and $a < 0, b < 0$

34. Let $(x_1, y_1) \equiv (at^2, 2at)$

Equation of tangent at $(at^2, 2at)$ is $ty = x + at^2$

Let any point on this tangent is $\left(\lambda, \dfrac{\lambda + at^2}{t}\right)$

The chord of contact of this point w.r.t the circle $x^2 + y^2 = a^2$ is

$$x \cdot \lambda + y \cdot \left(\frac{\lambda + at^2}{t}\right) = a^2$$

$$\Rightarrow \qquad (aty - a^2) + \lambda \cdot \left(x + \frac{y}{t}\right) = 0$$

which is family of straight lines passing through the point of intersection of

$$aty - a^2 = 0 \text{ and } x + \frac{y}{t} = 0$$

So, the fixed point is $\left(\dfrac{-a}{t^2}, \dfrac{a}{t}\right)$, therefore

$$x_2 = -\frac{a}{t^2} \quad , \quad y_2 = \frac{a}{t}$$

Clearly, $\qquad x_1x_2 = -a^2 \quad , \quad y_1y_2 = 2a^2$

$\Rightarrow \qquad x_1x_2 + y_1y_2 = a^2$

Also, $\qquad \dfrac{x_1}{x_2} = -t^4 \quad , \quad \dfrac{y_1}{y_2} = 2t^2$

or $\qquad \left(\dfrac{y_1}{y_2}\right)^2 + 4\left(\dfrac{x_1}{x_2}\right) = 0$

$\Rightarrow \ -4, \dfrac{y_1}{y_2}, \dfrac{x_1}{x_2}$ are in G.P.

Also, $y_1y_2 = 2a^2 \Rightarrow \dfrac{y_1}{2}, a, y_2$ are in G.P.

35. Equation of normal in slope form is

$$y = mx - 2am - am^3,$$

if normals meet at (h, k), then

$$am^3 - (h - 2a)m + k = 0 \qquad \text{...(i)}$$

Let $\qquad P \equiv (am_1^2, -2am_1), Q = (am_2^2, -2am_2)$

and $\qquad R \equiv (am_3^2, -2am_3)$

$\Rightarrow \qquad m_1, m_2, m_3$ are the roots of Eq. (i), then

$$m_1 + m_2 + m_3 = 0 \qquad \text{...(ii)}$$

i.e. algebraic sum of the slopes of the normals at P, Q and R vanishes.

From Eq. (ii)

$$-2am_1 - 2am_2 - 2am_3 = 0$$

i.e. algebraic sum of the ordinates of the points P, Q and R vanishes.

Also, $\qquad y$-coordinate of centroid of ΔPQR is zero

$\therefore \quad$ centroid lies on X-axis

and circle circumscribing the triangle PQR always passes through the vertex of the parabola.

36. Let $P \equiv (\lambda, \lambda + 1)$, where $\lambda \neq 0, -1$

or $\quad P \equiv (\lambda, \lambda - 1)$, where $\lambda \neq 0, 1$

The point $(\lambda, \lambda + 1)$ is on $y^2 = 4x + 1$, therefore

$$(\lambda + 1)^2 = 4\lambda + 1$$

$\Rightarrow \qquad \lambda^2 - 2\lambda = 0$

$\therefore \qquad \lambda = 2$

Therefore, the ordinate of P is 3

and the point $(\lambda, \lambda - 1)$ is on $y^2 = 4x + 1$, therefore

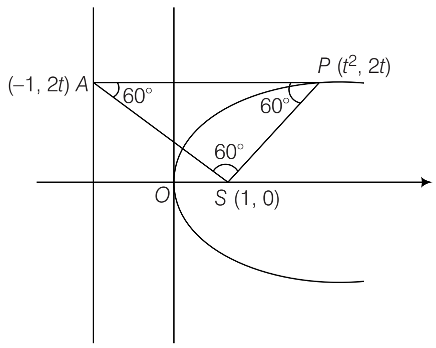
$$(\lambda - 1)^2 = 4(\lambda) + 1$$

$\Rightarrow \qquad \lambda^2 - 6\lambda = 0$

$\therefore \qquad \lambda = 6$

Therefore, the ordinate of P is 5,

37. From figure,



Slope of $AP = 0$ and slope of $AS = -t$

$\therefore \qquad \tan 60° = \left|\dfrac{0 - (-t)}{1 + 0}\right|$

$\Rightarrow \qquad \sqrt{3} = |t|$

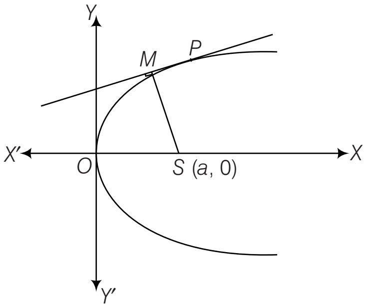
$\Rightarrow \qquad t = \pm\sqrt{3}$

$\therefore \quad$ Coordinates of P are $(3, \pm 2\sqrt{3})$

38. Let $M(\alpha, \beta)$ be the foot of the perpendicular from the focus $S(a, 0)$ on any tangent to the parabola at $P(at^2, 2at)$.

i.e. $\quad ty + x + at^2$

$\Rightarrow \quad \alpha - t\beta + at^2 = 0 \quad$ ...(i)



Since, SM is perpendicular to the tangent

$\therefore \quad \frac{1}{t} \times \frac{\beta - 0}{\alpha - a} = -1$

$\Rightarrow \quad \alpha t + \beta - at = 0 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$\alpha = 0$$

The locus of $M(\alpha, \beta)$ is the tangent at the vertex.

i.e. $\quad x = 0$

39. Given that the extremities of the latusrectum are (1, 1) and (1, − 1), then $4a = 2$ or $a = \frac{1}{2}$

So, the focus of the parabola is (1, 0).

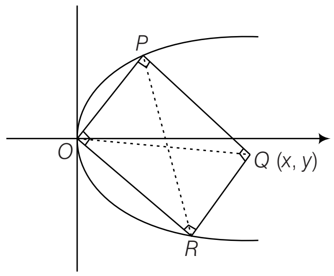
Hence, the vertex can be $\left(\frac{1}{2}, 0\right)$ or $\left(\frac{3}{2}, 0\right)$.

Therefore, the equations of the parabola can be $y^2 = 2\left(x - \frac{1}{2}\right)$ or $y^2 = 2\left(x - \frac{3}{2}\right)$

$\Rightarrow y^2 = 2x - 1$ or $y^2 = 2x - 3$.

40. Let $\quad P \equiv (at^2, 2at)$ and $R \equiv (at_1^2, 2at_1)$

$\because \quad OP \perp OR$



$\therefore$ Slope of $OP \times$ Slope of $OR = -1$

$\Rightarrow \quad \frac{2}{t} \times \frac{2}{t_1} = -1$

$\therefore \quad t_1 = -\frac{4}{t}$

Now, coordinates of R are $\left(\frac{16a}{t^2}, \frac{-8a}{t}\right)$

$\because OPQR$ is a rectangle.

$\therefore$ Mid-point of OQ = mid-point of PR

$\Rightarrow \quad x = at^2 + \frac{16a}{t^2} \quad \Rightarrow \quad \frac{x}{a} = t^2 + \frac{16}{t^2} \quad$ ... (i)

and $\quad y = 2at - \frac{8a}{t} \quad \Rightarrow \quad \frac{y}{2a} = t - \frac{4}{t} \quad$ ... (ii)

From Eqs. (i) and (ii), we get

$$\left(\frac{y}{2a}\right)^2 = t^2 + \frac{16}{t^2} - 8 = \frac{x}{a} - 8$$

$\Rightarrow \quad y^2 = 4ax - 32a^2$

or $\quad y^2 = 4a(x - 8a)$

which is equal parabola and focus $(9a, 0)$ and directrix

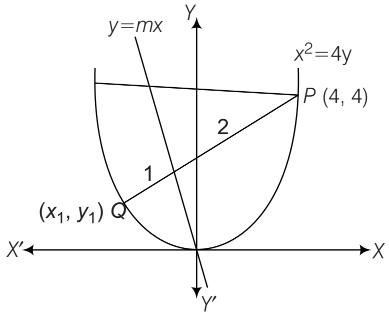
$$x - 8a = -a$$

or $\quad x - 7a = 0.$

41. Let point of intersection of the line $y = mx$ with the chord be $(\lambda, m\lambda)$, then

$\lambda = \frac{1.4 + 2.x_1}{1 + 2} \quad \Rightarrow \quad x_1 = \frac{3\lambda - 4}{2}$

and $\quad m\lambda = \frac{1.4 + 2.y_1}{1 + 2} \quad \Rightarrow \quad y_1 = \frac{3m\lambda - 4}{2}$



$\because Q(x_1, y_1)$ lies on the parabola $x^2 = 4y$, then

$$\left(\frac{3\lambda - 4}{2}\right)^2 = 4\left(\frac{3m\lambda - 4}{2}\right)$$

$\Rightarrow \quad 9\lambda^2 - 24\lambda(1 + m) + 48 = 0$

For two distinct chords $\quad D > 0$

$\Rightarrow \quad (24)^2 (1 + m)^2 - 4 \cdot 9 \cdot 48 > 0$

or $\quad (1 + m)^2 > 3$

$\Rightarrow \quad 1 + m < -\sqrt{3}$

or $\quad 1 + m > \sqrt{3}$

$\therefore \quad m < -\sqrt{3} - 1$

or $\quad m > \sqrt{3} - 1$

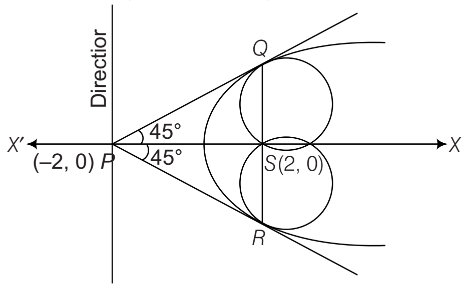
Hence , $m \in (-\infty, -\sqrt{3} - 1) \cup (\sqrt{3} - 1, \infty)$

42. The given parabola is $y^2 = 4(2)x$

$\Rightarrow \quad a = 2$

Since, $P(-2, 0)$ lies on the directrix and the axis.

$\Rightarrow$ The tangents will have slope $m = \pm 1$ and the equations are $y = x + 2$ and $y = -x - 2$.



The chord of contact of tangents is QR as $x = 2$ (i.e. L.R.)

$\because$ Common chord of two circles is X-axis.

$\because \qquad \angle QPR = 90°$

$\therefore$ Circumcentre is $S(2, 0)$ on X-axis and orthocentre is $P(-2, 0)$ on X-axis.

Centroid and incentre also lies on X-axis,

($\because$ orthocentre, centroid, circumcentre and incentre are collinear).

43. Given parabola is $(y-2)^2 = 4\left(x - \frac{1}{2}\right)$

Let $\qquad x - \frac{1}{2} = X, y - 2 = Y$

$\therefore$ Parabola is $Y^2 = 4X$

Any point on axis of parabola is $(x, 2)$ for three distinct normals $X > 2.1$

$\Rightarrow \qquad x - \frac{1}{2} > 2$ or $x > \frac{5}{2}$

$\therefore \quad x = 3, 4, 5$

Hence, points are (3, 2), (4, 2) and (5, 2).

44. The given parabola is $y^2 - 16x - 8y = 0$

$\Rightarrow \qquad (y-4)^2 = 16(x+1)$

Shifting the origin to the point $(-1, 4)$ the equation of parabola becomes $y^2 = 16x$

then the coordinates of the point (14, 7) becomes (15, 3).

$\therefore$ Equation of any normal to the parabola is $Y + tX = 8t + 4t^3$.

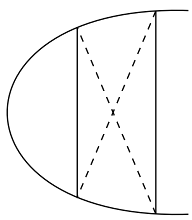
Since, it passes through (15, 3)

$\therefore \qquad 3 + 15t = 8t + 4t^3 \Rightarrow 4t^3 - 7t - 3 = 0$

or $\quad (t+1)(2t-3)(2t+1) = 0 \Rightarrow t = -1, \frac{3}{2}, -\frac{1}{2}$

$\therefore$ Corresponding points are $(4, -8)$, $(9, 12)$ and $(1, -4)$.
Hence, the coordinates of the feet of the normals w.r.t. the original system of coordinates are $(3, -4)$, $(8, 16)$ and $(0, 0)$.

45. As a circle can intersect a parabola at four points, the quadrilateral may be cyclic.



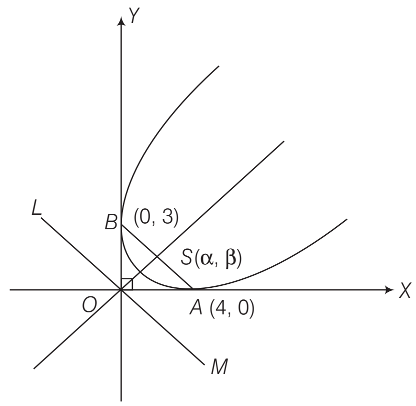
The diagonals of the quadrilateral may be equal as the quadrilateral may be an isosceles trapezium.

A rectangle cannot be inscribed in a parabola.

Sol. (Q. Nos. 46 to 48)

Let $A \equiv (4, 0)$ and $B \equiv (0, 3)$.

$\because$ OA and OB are mutually perpendicular tangents to the parabola. Therefore, O will lie on the directrix of the parabola.



Let $S(\alpha, \beta)$ be the focus of the parabola.

$\because$ Portion of a tangent to a parabola intercepted between the directrix and point of contact subtends a right angle at the focus.

$\therefore \qquad \angle OSA = \angle OSB = \frac{\pi}{2}$

Now, $OS \perp SA \Rightarrow$ Slope of $OS \times$ Slope of $SA = -1$

$\Rightarrow \qquad \left(\frac{\beta - 0}{\alpha - 0}\right) \times \left(\frac{\beta - 0}{\alpha - 4}\right) = -1$

$\Rightarrow \qquad \alpha^2 + \beta^2 - 4\alpha = 0 \qquad$ ...(i)

Again, $OS \perp SB \Rightarrow$ Slope of $OS \times$ Slope of $SB = -1$

$\Rightarrow \qquad \left(\frac{\beta - 0}{\alpha - 0}\right) \times \left(\frac{\beta - 3}{\alpha - 0}\right) = -1$

$\Rightarrow \qquad \alpha^2 + \beta^2 - 3\beta = 0 \qquad$ ...(ii)

From Eqs. (i) and (ii), we get

$4\alpha = 3\beta \qquad$ ...(iii)

Since, tangents AO and BO at A and B to parabola are at right angles, therefore AB will be a focal chord of the parabola.

Equation of AB is

$$\frac{x}{4} + \frac{y}{3} = 1 \Rightarrow \frac{\alpha}{4} + \frac{\beta}{3} = 1$$

$\Rightarrow \qquad 3\alpha + 4\beta = 12 \qquad$ ...(iv)

From Eqs. (iii) and (iv), we get

$$\alpha = \frac{36}{25} \text{ and } \beta = \frac{48}{25}$$

$\therefore$ Focus of parabola is $\left(\frac{36}{25}, \frac{48}{25}\right)$.

46. $\because$ Focus is (a, b).

$\therefore \qquad b - a = \beta - \alpha = \frac{12}{25}$

47. $\because$ $AS = \sqrt{\left(4 - \frac{36}{25}\right)^2 + \left(0 - \frac{48}{25}\right)^2} = \frac{16}{5}$

and $BS = \sqrt{\left(0 - \frac{36}{25}\right)^2 + \left(3 - \frac{48}{25}\right)^2} = \frac{9}{5}$

If l be the semi-latusrectum, then l = HM of AS and BS

$\therefore$ $\frac{2}{l} = \frac{5}{16} + \frac{5}{9} = \frac{125}{144}$

$\Rightarrow$ $l = \frac{288}{125}$

$\therefore$ $2l = \frac{576}{125}$

48. $\because$ Slope of $OS = \frac{\beta}{\alpha} = \frac{4}{3}$

$\therefore$ Slope of directrix $LM = -\frac{3}{4}$

$\therefore$ Equation of directrix is $y = -\frac{3}{4}x$

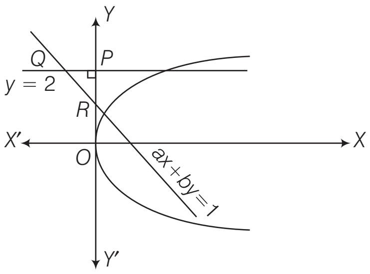
$\Rightarrow$ $3x + 4y = 0.$

Sol. (Q. Nos. 49 to 51)

Since, (a, b) lies on parabola, $y^2 = 4\lambda x$

$\therefore$ $b^2 = 4a\lambda$ ...(i)

It is clear that ΔPQR is right angled at $P(0, 2)$.



So, its circumcentre is the mid-point of Q and R, where

$Q \equiv \left(\frac{1-2b}{a}, 2\right)$ and $R \equiv \left(0, \frac{1}{b}\right)$.

$\therefore$ Circumcentre $\equiv \left(\frac{1-2b}{2a}, 1 + \frac{1}{2b}\right)$

$\therefore$ $x = \frac{1-2b}{2ab}, y = 1 + \frac{1}{2b}$

$\Rightarrow$ $2b = \frac{1}{y-1}$ and $2a = \frac{(y-2)}{x(y-1)}$ ...(ii)

From Eqs. (i) and (ii), we get

$(y-1)(y-2) = \frac{x}{8\lambda}$

$\Rightarrow$ $\left(y - \frac{3}{2}\right)^2 = \frac{1}{8\lambda}(x + 2\lambda)$

$\therefore$ Vertex is $\left(-2\lambda, \frac{3}{2}\right)$

and length of latusrectum is $\frac{1}{8\lambda}$.

49. Product of coordinates of vertex $= -2\lambda \times \frac{3}{2} = -3\lambda$

$= -3 \times 2$ $[\because \lambda = 2]$

$= -6$

50. Length of smallest focal chord = Length of latusrectum

$= \frac{1}{8\lambda}$

$= \frac{1}{8 \times \left(\frac{1}{32}\right)} = 4$ $\left[\because \lambda = \frac{1}{32}\right]$

51. Let $y - \frac{3}{2} = Y, x + 2\lambda = X$

$\therefore$ $Y^2 = \frac{1}{8\lambda}X$

Curve is symmetrical about $Y = 0$

$\Rightarrow$ $y - \frac{3}{2} = 0$

$\therefore$ $y = \frac{3}{2}$

Sol. (Q. Nos. 52 to 54)

Given parabola (P) can be written as

$(x - 2y)^2 = 32x - 4y - 16$

On adding $2(x - 2y)\lambda + \lambda^2$ both sides, it becomes

$(x - 2y + \lambda)^2 = 32x - 4y - 16 + 2(x - 2y)\lambda + \lambda^2$

$= 2(\lambda + 16)x - 4(\lambda + 1)y + \lambda^2 - 16$ ...(i)

We choose λ such that lines

$x - 2y + \lambda = 0$ and $2(\lambda + 16)x - 4(\lambda + 1)y + \lambda^2 - 16 = 0$

are perpendicular to each other. It requires

$\frac{1}{2} \times \frac{2(\lambda + 16)}{4(\lambda + 1)} = -1$

$\Rightarrow$ $\lambda + 16 = -4\lambda - 4$

$\Rightarrow$ $\lambda = -4$

Hence, Eq. (i) becomes

$(x - 2y - 4)^2 = 24x + 12y = 12(2x + y)$

$\Rightarrow$ $\left(\frac{x - 2y - 4}{\sqrt{5}}\right)^2 = \frac{12}{\sqrt{5}}\left(\frac{2x + y}{\sqrt{5}}\right) \Rightarrow Y^2 = 4\rho X$

where, $X = \frac{2x + y}{\sqrt{5}}, Y = \frac{x - 2y - 4}{\sqrt{5}}$

and $4\rho = \frac{12}{\sqrt{5}} \Rightarrow \rho = \frac{3}{\sqrt{5}}$.

52. $\because$ Equation of axis is $Y = 0$

$\Rightarrow$ $x - 2y - 4 = 0$ ...(i)

and equation of latusrectum is $X = \rho$

$\Rightarrow$ $\frac{2x + y}{\sqrt{5}} = \frac{3}{\sqrt{5}}$

$\Rightarrow$ $2x + y - 3 = 0$ ...(ii)

On solving Eqs. (i) and (ii),

focus is $(x, y) = (2, -1)$.

53. Length of latusrectum $= 4\rho = \dfrac{12}{\sqrt{5}}$

54. Equation of directrix is $X + \rho = 0$

$$\Rightarrow \quad \frac{2x+y}{\sqrt{5}} + \frac{3}{\sqrt{5}} = 0$$

$$\therefore \quad 2x + y + 3 = 0$$

Sol. (Q. Nos. 55 to 57)

Any parabola whose axis is parallel to the X-axis will be of the form

$$(y-q)^2 = 4\lambda(x-p) \quad \text{...(i)}$$

Now, $lx + my = 1$ can be written as

$$(y-q) = -\frac{l}{m}(x-p) + \left(\frac{1-mq-lp}{m}\right) \quad \text{...(ii)}$$

Eq. (ii) will touch Eq. (i), then

$$\frac{1-mq-lp}{m} = \frac{\lambda}{-\dfrac{l}{m}}$$

$$\Rightarrow \quad -l + mlq + l^2p = m^2\lambda$$

$$\Rightarrow \quad pl^2 + qlm - \lambda m^2 - l = 0 \quad \text{...(iii)}$$

But given that

$$5l^2 - 4lm + 6m^2 + 3l = 0 \quad \text{...(iv)}$$

On comparing Eqs. (iii) and (iv), we get

$$\frac{p}{5} = \frac{q}{-4} = \frac{-\lambda}{6} = \frac{-1}{3}$$

$$\Rightarrow \quad p = -\frac{5}{3}, q = \frac{4}{3}, \lambda = 2$$

So, the parabola is

$$\left(y - \frac{4}{3}\right)^2 = 8\left(x + \frac{5}{3}\right).$$

55. $\because$ Vertex is $\left(-\dfrac{5}{3}, \dfrac{4}{3}\right)$.

Here, $a = -\dfrac{5}{3}$ and $b = \dfrac{4}{3}$

$$\therefore \quad a - b = -3$$

$$\Rightarrow \quad |a-b| = 3$$

56. For focus,

$$x + \frac{5}{3} = 2 \text{ and } y - \frac{4}{3} = 0$$

$\therefore$ Coordinates of focus are $\left(\dfrac{1}{3}, \dfrac{4}{3}\right)$.

Here, $c = \dfrac{1}{3}$ and $d = \dfrac{4}{3}$

$$\therefore \quad d - c = 1$$

$$\Rightarrow \quad 2^{|d-c|} = 2$$

57. For directrix $\left(x + \dfrac{5}{3}\right) + 2 = 0$

$\Rightarrow$ $3x + 11 = 0$ given $ex + f = 0$

$$\therefore \quad e = 3, f = 11$$

Now, $\quad |e - f| = |3 - 11| = 8$

Sol. (Q. Nos. 58 to 60)

$$\because \quad C_1 : y^2 = 4x$$

$$\therefore \quad S_1 : (1, 0)$$

Let $z = x + iy$ and $z_1 = x_1 + iy_1$

If z_1 is obtained by rotating z, 120° in anti-clockwise direction, then

$$z_1 = ze^{2\pi i/3} = (x+iy)\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$$

$$= \left(-\frac{x}{2} - \frac{\sqrt{3}y}{2}\right) + i\left(\frac{\sqrt{3}}{2}x - \frac{y}{2}\right)$$

$\therefore$ Equation of curve C_2 is

$$\left(\frac{\sqrt{3}}{2}x - \frac{y}{2}\right)^2 = 4\left(-\frac{x}{2} - \frac{\sqrt{3}}{2}y\right) \quad \text{...(i)}$$

For focus $-\dfrac{x}{2} - \dfrac{\sqrt{3}}{2}y = 1$ and $\dfrac{\sqrt{3}}{2}x - \dfrac{y}{2} = 0$.

$$\therefore \quad x = -\frac{1}{2}, y = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad S_2 : \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$\because$ C_3 is reflection of C_2 with respect to $y = x$.

$$\therefore \quad S_3 : \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right).$$

58. $\because (t^2, 2t)$ are parametric form of curve C_1.

$\therefore$ From Eq. (i),

$$\frac{\sqrt{3}}{2}x - \frac{y}{2} = 2t \text{ and } -\frac{x}{2} - \frac{\sqrt{3}}{2}y = t^2,$$

we get $x = \dfrac{1}{2}(-t^2 + 2\sqrt{3}t), y = \dfrac{1}{2}(-\sqrt{3}t^2 - 2t)$

$\therefore$ Parametric coordinates of C_2 are

$$\left(\frac{1}{2}(-t^2 + 2\sqrt{3}t), \frac{1}{2}(-\sqrt{3}t^2 - 2t)\right).$$

59. Area of ΔOS_2S_3

$$= \frac{1}{2}\left|\begin{vmatrix} -\dfrac{1}{2} & -\dfrac{\sqrt{3}}{2} \\ -\dfrac{\sqrt{3}}{2} & -\dfrac{1}{2} \end{vmatrix}\right|$$

$$= \frac{1}{2}\left|\frac{1}{4} - \frac{3}{4}\right| = \frac{1}{4} \text{ sq unit}$$

60. $\Sigma x_1^2 + \Sigma y_1^2 = (1)^2 + \left(-\dfrac{1}{2}\right)^2 + \left(-\dfrac{\sqrt{3}}{2}\right)^2 + (0)^2 + \left(-\dfrac{\sqrt{3}}{2}\right)^2 + \left(-\dfrac{1}{2}\right)^2$

$$= 1 + \frac{1}{4} + \frac{3}{4} + 0 + \frac{3}{4} + \frac{1}{4} = 3$$

Sol. (Q. Nos. 61 to 63)

Since, no point of the parabola $y = x^2 + ax + 1$ is below the X-axis.

$$\therefore \quad D \le 0$$

$$\Rightarrow \quad a^2 - 4 \le 0$$

$$\Rightarrow \quad -2 \le a \le 2$$

61. The maximum value of a is 2.

The equation of the parabola is $y = x^2 + 2x + 1$.

It intersect the Y-axis at $(0, 1)$.

$\therefore$ Equation of tangent at $(0, 1)$ is

$$\frac{y+1}{2} = 0 \cdot x + (x+0) + 1$$

$$\Rightarrow \quad y = 2x + 1$$

$$\Rightarrow \quad 2x - y + 1 = 0$$

Since, $2x - y + 1 = 0$ touches the circle $x^2 + y^2 = c^2$, then

$$\frac{0 - 0 + 1}{\sqrt{4+1}} = c \qquad [\because c > 0]$$

$$\Rightarrow \quad c = \frac{1}{\sqrt{5}}$$

$$\therefore \quad 5c^2 = 1$$

62. Equation of the tangent at $(0, 1)$ to the parabola

$y = x^2 + ax + 1$ is

$$\Rightarrow \quad \frac{y+1}{2} = 0 \cdot x + \frac{a}{2}(x+0) + 1$$

$$\Rightarrow \quad y = ax + 1$$

$$\Rightarrow \quad ax - y + 1 = 0$$

As it touches the circle $x^2 + y^2 = c^2$, then

$$\frac{1}{\sqrt{(a^2+1)}} = c$$

i.e. c is maximum, when $a = 0$.

Therefore, the equation of the tangent is $y = 1$.

$\therefore$ Slope of the tangent is 0.

63. Equation of tangent is

$$y = ax + 1$$

$$\Rightarrow \quad ax - y = -1$$

$$\Rightarrow \quad \frac{x}{\left(-\frac{1}{a}\right)} + \frac{y}{(1)} = 1$$

Therefore, the area of the triangle bounded by the tangent and the axes is

$$\frac{1}{2}\left|-\frac{1}{a}\right||1| = \frac{1}{2|a|}$$

$$\therefore \quad \Delta = \frac{1}{4} \qquad [\text{for minimum area } a = 2]$$

$$\Rightarrow \quad 8\Delta = 2$$

Sol. (Q. Nos. 64 to 66)

The conic is $S \equiv x^2 + xy + y^2 - 2x - 2y + 1 = 0$

and the line is $L \equiv x + y + 1 = 0$

It is required to find equation of the parabola (P) which touches the conic $S = 0$ at those (two) points, where the line $L = 0$ intersect the conic. Obviously at these points the parabola is in double contact with the conic.

$\therefore$ The equation of any such conic is $\phi \equiv S + \lambda L^2 = 0$

$$\Rightarrow (x^2 + xy + y^2 - 2x - 2y + 1) + \lambda(x + y + 1)^2 = 0 \qquad \text{...(i)}$$

$$\Rightarrow (1+\lambda)x^2 + (1+2\lambda)xy + (1+\lambda)y^2 + 2(\lambda-1)x + 2(\lambda-1)y + \lambda + 1 = 0$$

It will be a parabola, if $h^2 = ab$

$$\Rightarrow \quad \frac{1}{4}(1+2\lambda)^2 = (1+\lambda)^2$$

$$\Rightarrow \quad 1 + 4\lambda + 4\lambda^2 = 4 + 8\lambda + 4\lambda^2$$

$$\therefore \quad \lambda = -\frac{3}{4}$$

Hence, from Eq. (i), the required parabola is

$$(x^2 + xy + y^2 - 2x - 2y + 1) - \frac{3}{4}(x + y + 1)^2 = 0$$

$$\Rightarrow \quad x^2 - 2xy + y^2 - 14x - 14y + 1 = 0 \qquad \text{...(ii)}$$

64. Comparing parabola (ii) with

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$\therefore$ $a = 1, h = -1, b = 1, g = -7, f = -7, c = 1$

Now, $|a + b + c + f + g + h| = |1 + 1 + 1 - 7 - 7 - 1| = 12$

65. The parabola (P) can be written as

$$(x-y)^2 = (14x + 14y - 1)$$

$$\Rightarrow \quad \left(\frac{x-y}{\sqrt{2}}\right)^2 = 7\sqrt{2}\left(\frac{14x + 14y - 1}{\sqrt{(14)^2 + (14)^2}}\right) \qquad \text{...(iii)}$$

$\therefore$ Length of latusrectum is $7\sqrt{2}$.

66. For vertex,

$$\frac{x-y}{\sqrt{2}} = 0, \frac{14x + 14y - 1}{\sqrt{(14)^2 + (14)^2}} = 0 \qquad [\text{from Eq. (iii)}]$$

$$\Rightarrow \quad x = y, 14x + 14y = 1$$

$$\therefore \quad x = y = \frac{1}{28}$$

$$\text{Vertex is } \left(\frac{1}{28}, \frac{1}{28}\right) = (a, b) \qquad [\text{given}]$$

$$\therefore \quad |a - b| = 0$$

Sol. (Q. Nos. 67 to 69)

$\because$ $y = 3x$ is tangent to the parabola

$$2y = ax^2 + b \qquad \text{...(i)}$$

$$\therefore \quad 2(3x) = ax^2 + b$$

[substitute the value of $y = 3x$ in $2y = ax^2 + b$]

$$\Rightarrow \quad ax^2 - 6x + b = 0$$

$$D = 0$$

[$\because$ $y = 3x$ is tangent to $2y = ax^2 + b$]

$$\therefore \quad 36 - 4ab = 0$$

$$\Rightarrow \quad ab = 9 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$2y = ax^2 + \frac{9}{a} \qquad \text{...(iii)}$$

67. $\because$ $\dfrac{a+b}{2} \ge \sqrt{ab} = 3$ [from Eq. (ii)]

$$\Rightarrow \quad a + b \ge 6$$

$\therefore$ Minimum value of $a + b$ is 6.

68. $\because$ (2, 6) is the point of contact.

From Eq. (iii), we get

$$12 = 4a + \frac{9}{a}$$

$\Rightarrow \quad 4a^2 - 12a + 9 = 0$

$\Rightarrow \quad (2a - 3)^2 = 0$

$\therefore \quad 2a = 3$

69. For $b = 18$

From Eq. (ii), $\quad a = \frac{1}{2}$

From Eq. (iii), $\quad 2y = \frac{x^2}{2} + 18 \quad$ ...(iv)

On solving $y = 3x$ and Eq. (iv), we get

$$6x = \frac{x^2}{2} + 18$$

$\Rightarrow \quad x^2 - 12x + 36 = 0$

$\Rightarrow \quad (x - 6)^2 = 0$

$\therefore \quad x = 6$, then $y = 3x = 18$

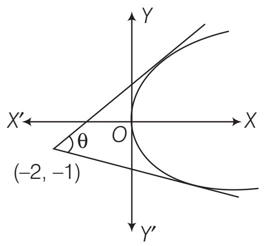
$\therefore$ Point of contact is (6, 18).

70. Equation of tangent in terms of slope (m) of the parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$.

$\because$ Point of intersection of tangents is $(-2, \; -1)$, then

$$-1 = -2m + \frac{1}{m}$$

$\Rightarrow \quad 2m^2 - m - 1 = 0$



Let m_1, m_2 be slopes of the tangents, then

$$m_1 - m_2 = \frac{\sqrt{D}}{a} = \frac{\sqrt{(1+8)}}{2} = \frac{3}{2}$$

and $\quad m_1 m_2 = -\frac{1}{2}$

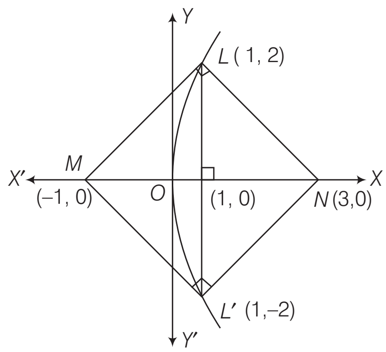
$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{3/2}{1 - 1/2}\right| = 3$

71. Let S be the focus and point of intersection of tangents at P and Q is R.

$\because \quad (SR)^2 = SP \cdot SQ = 4 \times 9 = 36$

$\therefore \quad SR = 6$

72. The four lines form a square. The tangents at $L(1, 2)$ and $L'(1, -2)$ are $x - y + 1 = 0$ and $x + y + 1 = 0$. They intersect at $M(-1, 0)$.



The area of the square $MLNL'$ is $(ML)^2 = (1 + 1)^2 + (2 - 0)^2 = 8$ sq units

$\therefore \quad \lambda = 8$

73. Parabola is $y^2 = x$

$\therefore \quad a = \frac{1}{4}$

The normal at t is

$$tx + y = \frac{t^3}{4} + \frac{t}{2}$$

It passes through $(a, 0)$.

$\therefore \quad ta = \frac{t^3}{4} + \frac{t}{2}$

$\Rightarrow \quad t^2 = 4\left(a - \frac{1}{2}\right) \Rightarrow a > \frac{1}{2}$

$\therefore \quad t_1 = 2\sqrt{\left(a - \frac{1}{2}\right)}, t_2 = -2\sqrt{\left(a - \frac{1}{2}\right)}$

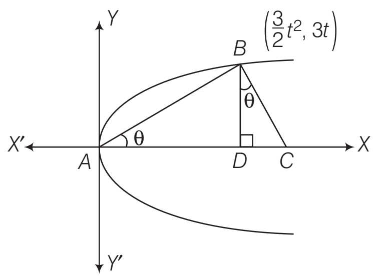
The normals are perpendicular $\Rightarrow t_1 t_2 = -1$

$\Rightarrow \quad -4\left(a - \frac{1}{2}\right) = -1$

$\therefore \quad 4a = 3$

74. Let B as $\left(\frac{3}{2}t^2, 3t\right)$ $\quad \left[\text{Here, } a = \frac{3}{2}\right]$

and let $BD \perp AC$ and $\angle DAB = \theta$



$\Rightarrow \quad \tan\theta = \frac{BD}{AD} = \frac{3t}{\frac{3}{2}t^2} = \frac{2}{t}$

$\therefore$ Projection of BC on the axis $= DC$

$$= BD \tan\theta = 3t\left(\frac{2}{t}\right) = 6 \text{ units}$$

$\therefore \quad \lambda = 6$

75. The parabolas are $y = x^2 - 9$ and $y = \lambda x^2$.

$\Rightarrow \quad x^2 - 9 = \lambda x^2 \quad \Rightarrow \quad x^2(1-\lambda) = 9$

$\Rightarrow \quad x^2 = \dfrac{9}{1-\lambda}$

$\therefore \quad x = \pm \dfrac{3}{\sqrt{(1-\lambda)}}$

Now, from the symmetry about Y-axis,

$$AB = 2a = \frac{6}{\sqrt{(1-\lambda)}} \quad \Rightarrow \quad a = \frac{3}{\sqrt{(1-\lambda)}}$$

$\Rightarrow \quad a^2(1-\lambda) = 9 \quad \Rightarrow \quad \lambda a^2 + 9 = a^2$

$\Rightarrow \quad a^2 + \mu = a^2$ [given]

$\therefore \quad \mu = 9$

76. Given, $y^2 - 8x < 0$, $x^2 + y^2 < 16$ and $x > 0$.

For $x = 1$,

$y^2 < 8$ and $y^2 < 15 \Rightarrow y^2 < 8$

$\therefore \quad y = 0, \pm 1, \pm 2$

$\therefore$ 5 points.

For $x = 2$,

$y^2 < 16, y^2 < 12 \Rightarrow y^2 < 12$

$\therefore \quad y = 0, \pm 1, \pm 2, \pm 3$

$\therefore$ 7 points

For $x = 3$,

$y^2 < 24, y^2 < 7 \Rightarrow y^2 < 7$

$\therefore \quad y = 0, \pm 1, \pm 2$

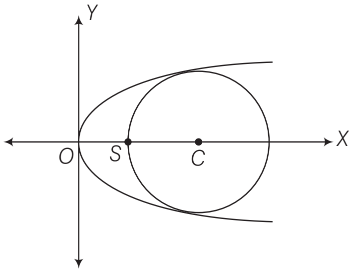
$\therefore$ 5 points

Hence, total points is 17.

$\therefore \quad n = 17$

Sum of digits of n is 8.

77. Focus of the parabola $y^2 = 4x$ is $S(1, 0)$.



Let radius of circle be r.

$\therefore$ Centre of circle is $(1 + r, 0)$.

$\Rightarrow$ Equation of circle is $(x - 1 - r)^2 + y^2 = r^2$

$\Rightarrow \quad (x - 1 - r)^2 + 4x = r^2$ [$\because y^2 = 4x$]

$\Rightarrow \quad x^2 + 2(2 - 1 - r)x + 2r + 1 = 0$

$\Rightarrow \quad x^2 + 2(1 - r)x + 2r + 1 = 0$

It would have same roots due to symmetry.

$\therefore \quad D = 0$

$\Rightarrow \quad 4(1 - r)^2 - 4 \cdot 1 \cdot (2r + 1) = 0$

$\therefore \quad r = 0, 4$

Hence, $\quad r = 4$ [$\because r \neq 0$]

78. For maximum number of common chords, the circle and the parabola must intersect at four points.

Now, solving the given curves, we have

$$(x - 6)^2 + 4x = r^2$$

$\Rightarrow \quad x^2 - 8x + 36 - r^2 = 0$

The curves touch, if $\quad D = 0$

$\Rightarrow \quad 64 - 4 \cdot 1 \cdot (36 - r^2) = 0$

$\Rightarrow \quad r^2 = 20$

$\therefore \quad r = 2\sqrt{5}$

Hence, the least integral value of r for which the curves intersect is 5.

79. Given parabola is

$$x^2 = 4(y - 1)$$

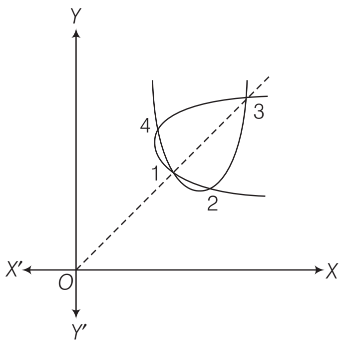
$\therefore$ Focus is (0, 2).

Now, the shortest intercept of the line on the parabola which passes through the focus is latusrectum. The axis of the given parabola is the Y-axis.

Therefore, the latusrectum is parallel to the X-axis.

$\therefore$ Slope = 0

80. (A) The given parabolas are symmetrical about the line $y = x$ as shown in the figure



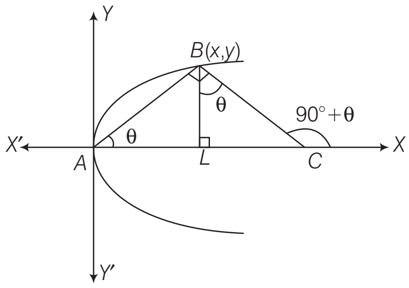
They intersect each other at four distinct points.

Hence, the number of common chords is

$$^4C_2 = \frac{4 \cdot 3}{1 \cdot 2} = 6$$

Which is perfect number [$\because 1 \times 2 \times 3 = 1 + 2 + 3 = 6$]

(B) $\because \quad \tan\theta = \dfrac{y}{x}$ ...(i)



LC = Projection of BC on X-axis

$= BL \tan\theta = y \tan\theta$

$= y \times \dfrac{y}{x}$ [from Eq. (i)]

$= \dfrac{y^2}{x}$

$= 4$ [$\because y^2 = 4x$]

(C) Normals to $y^2 = 4ax$ and $x^2 = 4by$ in terms of m are

$y = mx - 2am - am^3$ and $y = mx + 2b + \dfrac{b}{m^2}$.

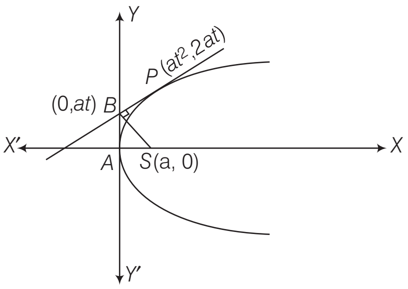
For common normal,

$$2b + \frac{b}{m^2} = -2am - am^3$$

or $$am^5 + 2am^3 + 2bm^2 + b = 0$$

It is clear that at most five common normals.

(D) Let middle point of P and B be (h, k), then $2h = at^2$



and $$2k = 3at$$

or $$2h = a\left(\frac{2k}{3a}\right)^2$$

or $$2k^2 = 9ah$$

$\therefore$ Locus of mid-point is

$$2y^2 = 9ax$$

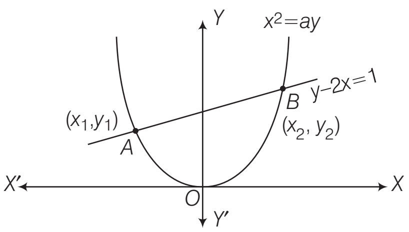
$\therefore$ Length of latusrectum $= \dfrac{9a}{2}$

$= \dfrac{9}{2} \times 2$ [$\because a = 2$]

$= 9$

81. (A) Given parabola is $x^2 = ay$ ...(i)

and the given line is $y - 2x = 1$ ...(ii)



On solving Eqs. (i) and (ii), we get

$$x^2 = a(2x + 1) \Rightarrow x^2 - 2ax - a = 0$$

Let coordinates of A and B are (x_1, y_1) and (x_2, y_2) respectively, then

$$|x_1 - x_2| = \frac{\sqrt{D}}{a} = \frac{\sqrt{(4a^2 + 4a)}}{1} = 2\sqrt{(a^2 + a)} \quad \text{...(iii)}$$

Also A, B lie on $y - 2x = 1$

$\therefore$ $y_1 - 2x_1 = 1$ and $y_2 - 2x_2 = 1$

or $y_2 - y_1 = 2(x_2 - x_1)$ ...(iv)

$\therefore$ Length of $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{5}\,|x_1 - x_2|$ [from Eq. (iv)]

$= 2\sqrt{5}\,\sqrt{(a^2 + a)}$ [from Eq. (iii)]

Given, $2\sqrt{5}\,\sqrt{(a^2 + a)} = \sqrt{40}$

$\Rightarrow$ $a^2 + a = 2$

$\Rightarrow$ $a^2 + a - 2 = 0$

$\Rightarrow$ $(a + 2)(a - 1) = 0$

$\Rightarrow$ $a = -2, 1$

(B) If tangents are drawn from (x_1, y_1) to the parabola $y^2 = 4ax$ and angle between tangents is θ, then

$$|\tan\theta| = \frac{\sqrt{(y_1^2 - 4ax_1)}}{|x_1 + a|}$$

Here, $x_1 = 0, y_1 = 2, \theta = \dfrac{3\pi}{4}$, then

$$|-1| = \frac{2}{|0 + a|}$$

$\Rightarrow$ $|a| = 2$

$\therefore$ $a = \pm 2$

Aliter :

Observe that one tangent is the Y-axis, the other tangent is at $\theta = \dfrac{\pi}{4}$ and its equation is $ty = x + at^2$

for $t = 1$, we get $y = x + a$

$\Rightarrow a = 2$ from the symmetry $a = -2$ is also possible.

(C) Let the other end be at $(at^2, 2at)$

So, mid-point is $\left(\dfrac{at^2 + a}{2}, \dfrac{2at + 2a}{2}\right)$

which satisfy $x + y = 1$

or $$\frac{at^2 + a}{2} + at + a = 1$$

$\Rightarrow$ $at^2 + 2at + 3a - 2 = 0$

Since, two distinct chords are possible, so $D > 0$.

$\therefore$ $4a^2 - 4a(3a - 2) > 0$

$\Rightarrow$ $-8a^2 + 8a > 0$

$\Rightarrow$ $8a(a - 1) < 0$

$\therefore$ $0 < a < 1$

or $0 < 4a < 4$

or $0 <$ Length of latusrectum < 4

$\therefore$ Length of latusrectum can be 1 or 2 or 3 from the given values.

(D) The given parabola is

$$x^2 - ay + 3 = 0$$

or $$x^2 = a\left(y - \frac{3}{a}\right)$$

Let $x = X, y - \frac{3}{a} = Y$

Then, the parabola is

$$X^2 = aY$$

For focus $X = 0, Y = \frac{a}{4}$

$\Rightarrow$ $x = 0, y - \frac{3}{a} = \frac{a}{4}$

$\therefore$ Focus is $\left(0, \frac{3}{a} + \frac{a}{4}\right)$ given focus is $(0, 2)$

$\therefore$ $\frac{3}{a} + \frac{a}{4} = 2$

$\Rightarrow$ $a^2 - 8a + 12 = 0$

$\Rightarrow$ $(a - 6)(a - 2) = 0$

$\therefore$ $a = 2, 6$

Here, $a_1 = 6, a_2 = 2$

$\therefore$ $\frac{a_1}{a_2} = 3$

82. (A) Points $(1, 2)$ and $(-2, 1)$ satisfy both the curves.

(B) Equation of tangent at $(t^2, 2t)$ on $y^2 = 4x$ is

$$ty = x + t^2$$

It passes through the point $(2, 3)$, then

$$3t = 2 + t^2$$

$\Rightarrow$ $t^2 - 3t + 2 = 0$

or $(t - 1)(t - 2) = 0$

or $t = 1$ or 2

The point of contact is $(1, 2)$ or $(4, 4)$.

(C) Let $P(\sqrt{5}\cos\theta, \sqrt{5}\sin\theta)$, then the chord of contact of the parabola $y^2 = 4x$ w.r.t. P is

$$y \cdot \sqrt{5}\sin\theta = 2(x + \sqrt{5}\cos\theta)$$

or $y = \frac{2x}{\sqrt{5}\sin\theta} + 2\cot\theta$

On comparing with $y = 2(x - 2)$, then

$\sqrt{5}\sin\theta = 1$ and $\cot\theta = -2$

or $\sqrt{5}\sin\theta = 1$ and $\sqrt{5}\cos\theta = -2$

Hence, coordinates of P are $(-2, 1)$.

(D) Let coordinates of Q be $(t^2, 2t)$.

Now, the area of ΔOPQ is

$$\frac{1}{2}\left|\begin{vmatrix} t^2 & 2t \\ 4 & -4 \end{vmatrix}\right| = 6 \quad \text{[given]}$$

$\Rightarrow$ $2t^2 + 4t = \pm 6$

or $t^2 + 2t \pm 3 = 0$

$\therefore$ $t^2 + 2t - 3 = 0$ $[\because t^2 + 2t + 3 \neq 0]$

$\Rightarrow$ $(t + 3)(t - 1) = 0$

Then, $t = 1$ or -3

Hence, the point Q are $(1, 2)$ or $(9, -6)$.

83. Equation of tangent to $y^2 = 4x$ is

$$y = mx + \frac{1}{m}$$

it is also tangent to $x^2 = 4y$, then

$$x^2 = 4\left(mx + \frac{1}{m}\right)$$

$\Rightarrow$ $x^2 - 4mx - \frac{4}{m} = 0$

It discriminant $= 0$

$\Rightarrow$ $16m^2 + \frac{16}{m} = 0$

$\Rightarrow$ $m^3 = -1$

$\therefore$ $m = -1$

$\therefore$ Equation of common tangent is $x + y + 1 = 0$.

Statement II is also true and it is correct explanation of Statement I.

84. The vertex of $(y + 2)^2 = 2(x - 1)$ is $(1, -2)$ and equation of axis is $y = -2$.

Here, $4a = 2$

$\therefore$ $a = \frac{1}{2} \Rightarrow 3a = \frac{3}{2}$

$\therefore$ Required point is $\left(1 + \frac{3}{2}, -2\right)$, i.e. $\left(\frac{5}{2}, -2\right)$.

Hence, both statements are true and Statement II is correct explanation for Statement I.

85. $\because$ $y = mx + c$ ...(i)

and $y^2 = 4ax$ ...(ii)

From Eqs. (i) and (ii),

$$y^2 = 4a\left(\frac{y - c}{m}\right)$$

$\Rightarrow$ $my^2 - 4ay + 4ac = 0$

If line Eq. (i) intersect the parabola $y^2 = 4ax$ at one point, then

$$B^2 = 4AC$$

$\Rightarrow$ $(-4a)^2 = 4 \cdot m \cdot 4ac$

$\Rightarrow$ $c = \frac{a}{m}$

From Eq. (i), equation of tangent is

$$y = mx + \frac{a}{m}$$

$\therefore$ Statement I and Statement II are individual true and Statement II is correct explanation of Statement I.

86. $\because$ $\sqrt{ax} + \sqrt{by} = 1$ ...(i)

On squaring both sides, then

$$ax + by + 2\sqrt{abxy} = 1$$

$\Rightarrow$ $(ax + by - 1) = -2\sqrt{abxy}$

Again, on squaring both sides, we get

$$a^2x^2 + b^2y^2 + 1 + 2abxy - 2ax - 2by + 1 = 4abxy$$

$\Rightarrow a^2x^2 - 2abxy + b^2y^2 - 2ax - 2by + 1 = 0$

Now, comparing with

$$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + C = 0$$

$\therefore \quad A = a^2, H = -ab, B = b^2, G = -a, F = -b, C = 1$

$\therefore \quad \Delta = ABC + 2FGH - AF^2 - BG^2 - CH^2$

$= a^2b^2 - 2a^2b^2 - a^2b^2 - a^2b^2 - a^2b^2$

$= -4a^2b^2 \neq 0$

and $H^2 = AB$

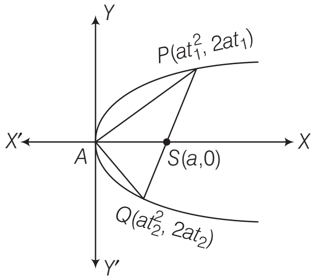
Hence, Eq. (i) represent a parabola.

$\therefore$ Statement I is true and Statement II is false.

87. $\because$ Slope of $AP = m_1 = \dfrac{2at_1 - 0}{at_1^2 - 0} = \dfrac{2}{t_1}$

and slope of $AQ = m_2 = \dfrac{2at_2 - 0}{at_2^2 - 0} = \dfrac{2}{t_2}$

$\because \quad m_1m_2 = \dfrac{4}{t_1t_2} \neq -1$



Also, P, S, Q are collinear, then

$$\frac{2at_1 - 0}{at_1^2 - a} = \frac{0 - 2at_2}{a - at_2^2}$$

$\Rightarrow \quad t_1 - t_1t_2^2 = -t_1^2t_2 + t_2$

$\Rightarrow \quad (t_1 - t_2) + t_1t_2(t_1 - t_2) = 0$

$\because \quad t_1 - t_2 \neq 0, 1 + t_1t_2 = 0$

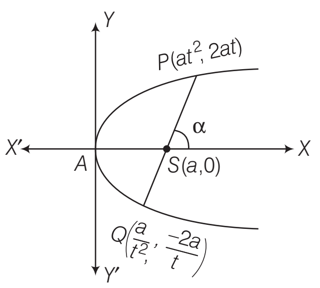
$\therefore \quad t_1t_2 = -1$

Hence, Statement I is false and Statement II is true.

88. Length of focal chord $PQ = a\left(t + \dfrac{1}{t}\right)^2$ ...(i)

and $\tan\alpha = \dfrac{2at + \dfrac{2a}{t}}{at^2 - \dfrac{a}{t^2}} = \dfrac{2t}{t^2 - 1} = \dfrac{2\left(\dfrac{1}{t}\right)}{1 - \left(\dfrac{1}{t}\right)^2}$

$\therefore \quad \dfrac{1}{t} = \tan(\alpha/2)$



From Eq. (i)

$$PQ = a(\cot(\alpha/2) + \tan(\alpha/2))^2$$

$$= a\left(\frac{1}{\sin(\alpha/2)\cos(\alpha/2)}\right)^2$$

$$= a\left(\frac{2}{\sin\alpha}\right)^2 = 4a \operatorname{cosec}^2 \alpha$$

For $\alpha = 60°$, $4a = 8$ and

$PQ = 8 \operatorname{cosec}^2 60° = 8\left(\dfrac{2}{\sqrt{3}}\right)^2 = 32/3$

$\therefore$ Statement I is true and Statement II is false.

89. $\because \quad x + y = \lambda \Rightarrow y = \lambda - x$ ...(i)

and $\quad y = x - x^2$ ...(ii)

From Eqs. (i) and (ii), we get

$$\lambda - x = x - x^2$$

$\Rightarrow \quad x^2 - 2x + \lambda = 0$

$\because$ Eq. (i) touch the parabola Eq. (ii), then

$$B^2 - 4AC = 0$$

$\Rightarrow \quad (-2)^2 - 4 \cdot 1 \cdot \lambda = 0$

$\therefore \quad \lambda = 1$

$\Rightarrow$ Statement I is true.

From Statement II,

$$(x - 1)^2 = x - x^2$$

$\Rightarrow \quad 2x^2 - 3x + 1 = 0$

$\therefore$ Discriminant $= (-3)^2 - 4 \cdot 2 \cdot 1 = 1 \neq 0$

$\therefore$ Statement II is false.

90. $\because$ $3x + 4y + 5 = 0$ and $4x + 3y + 2 = 0$ are not perpendicular to each other.

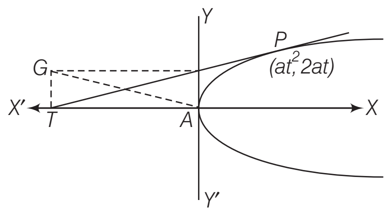
$\therefore$ Latusrectum $\neq 4$

$\therefore$ Statement I is false and Statement II is true.

91. Let $P(at^2, 2at)$ be any point on the parabola $y^2 = 4ax$.

Then, tangent at $P(at^2, 2at)$ is $ty = x + at^2$.

Since, tangent meet the axis of parabola in T and tangent at the vertex in Y



$\therefore$ Coordinates of T and Y are $(-at^2, 0)$ and $(0, at)$ respectively.

Let coordinates of G be (x_1, y_1).

Since, $TAYG$ is rectangle.

$\because$ Mid-points of diagonals TY and GA is same.

$\Rightarrow \quad \dfrac{x_1 + 0}{2} = \dfrac{-at^2 + 0}{2}$

$\Rightarrow \quad x_1 = -at^2$ ...(i)

and $\dfrac{y_1+0}{2}+\dfrac{0+at}{2} \Rightarrow y_1 = at$ ...(ii)

Eliminating t from Eqs. (i) and (ii), we get

$$x_1 = -a\left(\frac{y_1}{a}\right)^2$$

$\Rightarrow$ $y_1^2 = -ax_1$

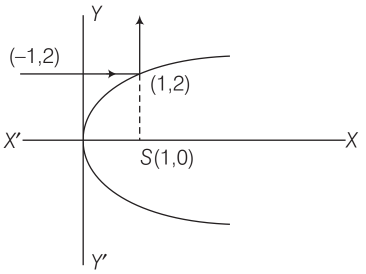
$\Rightarrow$ $y_1^2 + ax_1 = 0$

Hence, the locus of $G(x_1, y_1)$ is $y^2 + ax = 0$.

92. Equation of incident ray parallel to axis of parabola (X-axis) is $y = \lambda$, which pass through $(-1, 2)$, then $2 = \lambda$.

$\therefore$ Equation of incident ray is $y = 2$

Incident ray strikes the parabola $y^2 = 4x$ at (1, 2).



The reflected ray passes through the focus (1, 0).

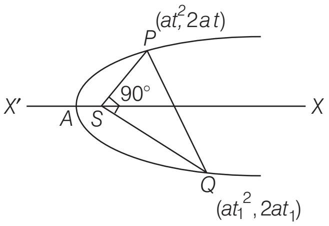
Hence, the equation of the reflected ray is $x = 1$.

93. Let PQ be a normal chord to a parabola at $P(at^2, 2at)$.

Since, the ordinate and abscissa of P are equal.

$\Rightarrow$ $at^2 = 2at, t \neq 0$

$\therefore$ $t = 2$



Since, normal at $P\,(at^2, 2at)$ meet the parabola at $Q\,(at_1^2, 2at_1)$.

$\therefore$ $t_1 = -t - \dfrac{2}{t}$

or $t_1 = -2 - 1$

or $t_1 = -3$

$\therefore$ Coordinates of P and Q are $(4a, 4a)$ and $(9a, -6a)$, respectively.

$\therefore$ Slope of $SP = \dfrac{4a-0}{4a-a} = \dfrac{4}{3} = m_1$ (say)

and slope of $SQ = \dfrac{-6a-0}{9a-a} = -\dfrac{3}{4} = m_2$ (say)

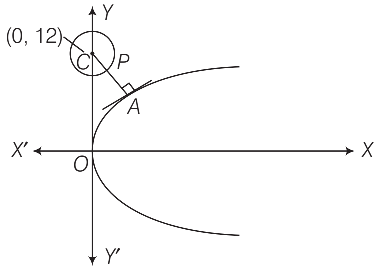
$\therefore$ $m_1m_2 = -1$

Hence, SP and SQ are perpendicular to each other

i.e. $\angle PSQ = 90^\circ$

94. The centre and radius of the given circle are (0, 12) and 4, respectively.

Now, the shortest distance always occurs along the common normal.



Let $A \equiv (t^2, 2t)$

Equation of normal at A is $y + tx = 2t + t^3$,

which passes through (0, 12), then

$$12 + 0 = 2t + t^3 \Rightarrow t^3 + 2t - 12 = 0$$

or $(t-2)(t^2+2t+6) = 0$

$\therefore$ $t = 2$ $[\because t^2 + 2t + 6 \neq 0]$

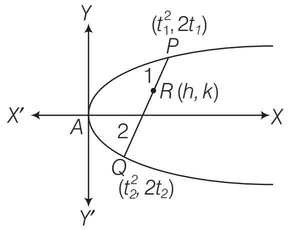
Coordinates of A are (4, 4).

$\therefore$ Shortest distance $= AP = AC - CP = \sqrt{80} - 4 = 4(\sqrt{5}-1)$

95. Let $P\,(t_1^2, 2t_1)$ and $Q\,(t_2^2, 2t_2)$ be extremities of the chord with slope 2.

$\therefore$ $\dfrac{2t_1 - 2t_2}{t_1^2 - t_2^2} = 2$

$\Rightarrow$ $t_1 + t_2 = 1$ ...(i)



Let $R\,(h, k)$ be coordinates of the point which divides PQ in the ratio 1 : 2, then

$$h = \frac{2t_1^2 + t_2^2}{3}$$

and $k = \dfrac{4t_1 + 2t_2}{3}$

$\Rightarrow$ $3h = 2t_1^2 + (1-t_1)^2$

and $3k = 4t_1 + 2(1-t_1)$ [from Eq. (i)]

or $3h = 3t_1^2 - 2t_1 + 1$

and $3k = 2t_1 + 2$

Eliminating t_1

$\therefore$ $3h = 3\left(\dfrac{3k-2}{2}\right)^2 - 2\left(\dfrac{3k-2}{2}\right) + 1$

$\Rightarrow$ $9k^2 - 16k - 4h + 8 = 0$

$\Rightarrow$ $k^2 - \dfrac{16k}{9} - \dfrac{4h}{9} + \dfrac{8}{9} = 0$

$\Rightarrow \quad \left(k - \frac{8}{9}\right)^2 = \frac{4}{9}\left(h - \frac{2}{9}\right)$

$\therefore$ Locus of $R\,(h, k)$ is $\left(y - \frac{8}{9}\right)^2 = \frac{4}{9}\left(x - \frac{2}{9}\right)$,

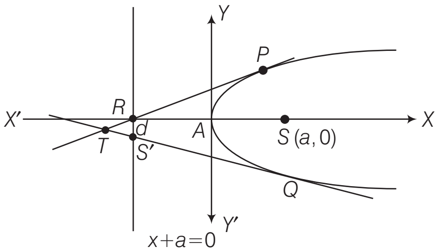
which is a parabola with vertex $\left(\frac{8}{9}, \frac{2}{9}\right)$.

96. Let coordinates of P and Q on the parabola $y^2 = 4ax$ are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$.

Equation of tangents at P and Q are

$$t_1 y = x + at_1^2 \quad \text{and} \quad t_2 y = x + at_2^2$$

Let these tangents meet $x + a = 0$ at R and S, then coordinates of R and S' are $\left(-a, \frac{a(t_1^2 - 1)}{t_1}\right)$ and $\left(-a, \frac{a(t_2^2 - 1)}{t_2}\right)$, respectively.



Given, $|RS'| = d$

$$\therefore \quad \left| a\left(\frac{t_1^2 - 1}{t_1}\right) - a\left(\frac{t_2^2 - 1}{t_2}\right) \right| = d$$

$$\Rightarrow \quad \left| \frac{a}{t_1 t_2}\, t_1^2 t_2 - t_2 - t_1 t_2^2 + t_1 \right| = d$$

$$\Rightarrow \quad \left| \frac{a}{t_1 t_2}\,((t_1 - t_2)(1 + t_1 t_2)) \right| = d$$

$$\Rightarrow \quad \left| \frac{a\sqrt{\{(t_1 + t_2)^2 - 4t_1 t_2\}}\,(1 + t_1 t_2)}{t_1 t_2} \right| = d$$

Let the point of intersection of tangents at P at Q is T then

$$T = (at_1 t_2, a(t_1 + t_2))$$

Now, let $\quad T = (h, k)$

$$\therefore \quad h = at_1 t_2 \quad \text{and} \quad k = a(t_1 + t_2)$$

$$\therefore \quad t_1 t_2 = \frac{h}{a} \quad \text{and} \quad t_1 + t_2 = \frac{k}{a} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\left| \frac{a\sqrt{\left(\frac{k^2}{a^2} - \frac{4h}{a}\right)}\left(1 + \frac{h}{a}\right)}{\frac{h}{a}} \right| = d$$

$$\Rightarrow \quad (k^2 - 4ah)(h + a)^2 = h^2 d^2$$

Hence, required locus is

$$(y^2 - 4ax)(x + a)^2 = d^2 x^2.$$

Aliter :

Let point of intersection of tangents at P and Q is $T(h, k)$ then equation of pair of tangents TP and TQ is

$$SS_1 = T^2$$

$$\Rightarrow \quad (y^2 - 4ax)(k^2 - 4ah) = [ky - 2a(x + h)]^2 \qquad \text{...(i)}$$

Let the pair of tangents cut the directrix $x + a = 0$ are in R and S, then substitute $x = -a$ in Eq. (i), then we get

$$hy^2 - k(h - a)y + a\{(h + a)^2 - k^2\} = 0$$

Now let coordinates of R and S be

$$(-a, y_1) \quad \text{and} \quad (-a, y_2).$$

$$\therefore \quad y_1 + y_2 = \frac{k(h - a)}{h}$$

and $\quad y_1 y_2 = \dfrac{a\{(h + a)^2 - k^2\}}{h}$

but given $\quad |RS| = d$

$$\Rightarrow \quad (RS)^2 = d^2$$

$$\Rightarrow \quad (y_1 - y_2)^2 = d^2$$

$$\Rightarrow \quad (y_1 + y_2)^2 - 4y_1 y_2 = d^2$$

$$\Rightarrow \quad \frac{k^2}{h^2}(h - a)^2 - \frac{4a}{h}\{(h + a)^2 - k^2\} = d^2$$

$$\Rightarrow \quad k^2\{(h - a)^2 + 4ah\} - 4ah(h + a)^2 = d^2 h^2$$

$$\Rightarrow \quad (k^2 - 4ah)(h + a)^2 = d^2 h^2$$

Hence, locus of $T(h, k)$ is

$$(y^2 - 4ax)(x + a)^2 = d^2 x^2.$$

97. Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ be two points on the parabola $y^2 = 4ax$

The equations of tangents at P and Q are

$$t_1 y = x + at_1^2$$

and $\quad t_2 y = x + at_2^2$

$$\therefore \quad \tan\theta_1 = \frac{1}{t_1}, \tan\theta_2 = \frac{1}{t_2}$$

Equations of circles with OP and OQ as diameters are

$$(x - 0)(x - at_1^2) + (y - 0)(y - 2at_1) = 0$$

and $\quad (x - 0)(x - at_2^2) + (y - 0)(y - 2at_2) = 0$ respectively.

i.e. $\quad x^2 + y^2 - axt_1^2 - 2at_1 y = 0$

and $\quad x^2 + y^2 - axt_2^2 - 2at_2 y = 0$

then, point of intersection of circles are $O\,(0, 0)$

and $\quad R\left(\dfrac{-4at_1 t_2}{(t_1 + t_2)^2 + 4}, \dfrac{2at_1 t_2(t_1 + t_2)}{(t_1 + t_2)^2 + 4}\right)$

Since, OR makes an angle ϕ with the X-axis. Therefore,

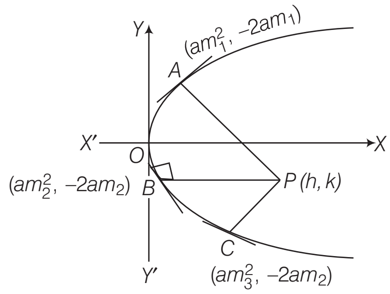
$$\tan\phi = -\left(\frac{t_1 + t_2}{2}\right)$$

Now, $\quad \cot\theta_1 + \cot\theta_2 + 2\tan\phi = t_1 + t_2 - (t_1 + t_2)$

$$= 0$$

98. Any normal of the parabola $y^2 = 4x$ with slope m is

$$y = mx - 2m - m^3$$



It passes through P, then

$$k = mh - 2m - m^3$$

$\Rightarrow m^3 + (2 - h)m + k = 0$ ...(i)

Thus, $m_1m_2m_3 = -k$

$\alpha m_3 = -k$ $(\because m_1m_2 = \alpha)$

$\Rightarrow \quad m_3 = -\dfrac{k}{\alpha}$

$\because$ m_3 is a root of Eq. (i), then

$$-\frac{k^3}{\alpha^3} + (2-h)\left(-\frac{k}{\alpha}\right) + k = 0$$

$\Rightarrow \quad k^3 + (2-h)k\alpha^2 - k\alpha^3 = 0$

$\therefore$ Locus of $P(h, k)$ is

$$y^3 + (2 - x)y\alpha^2 - y\alpha^3 = 0$$

$\Rightarrow \quad y^2 + (2-x)\alpha^2 - \alpha^3 = 0$ $[\because y \neq 0]$

(P does not lie on the axis of the parabola)

$\Rightarrow \quad y^2 = \alpha^2 x - 2\alpha^2 + \alpha^3$

If it is a part of the parabola $y^2 = 4x$

then, $\alpha^2 = 4$

and $-2\alpha^2 + \alpha^3 = 0$

$\Rightarrow \quad \alpha^2(\alpha - 2) = 0$

$\Rightarrow \quad \alpha - 2 = 0, \alpha \neq 0$

$\therefore \quad \alpha = 2$

99. Let $P(at^2, 2at)$ be any point on $y^2 = 4ax$. Then, vertex $A(0,0)$. The equation of tangent at P is

$ty = x + at^2$ ...(i)

Tangent at P will be normal to the circle, AP is a chord whose mid-point is $\left(\dfrac{at^2}{2}, at\right)$ and slope is $\dfrac{2}{t}$.

$\therefore$ Equation of the line passing through mid-point of AP and perpendicular to AP is

$$y - at = -\frac{t}{2}\left(x - \frac{at^2}{2}\right)$$

$\Rightarrow \quad tx + 2y = \dfrac{at^3}{2} + 2at$ ...(ii)

Eqs. (i) and (ii) both pass through (x_1, y_1) which is the centre of the circle

$ty_1 = x_1 + at^2$ ...(iii)

and $2tx_1 + 4y_1 = at^3 + 4at$ ...(iv)

Multiplying Eq. (iii) by t and subtracting Eq. (iv), we have

$t^2y_1 + t(4a - 3x_1) - 4y_1 = 0$ ...(v)

Also, from Eq. (iii), $at^2 - ty_1 + x_1 = 0$ ...(vi)

Eliminating t from Eqs. (v) and (vi), we get

$$\frac{t^2}{x_1(4a - 3x_1) - 4y_1^2} = \frac{t}{-4ay_1 - x_1y_1} = \frac{1}{-y_1^2 - a(4a - 3x_1)}$$

On simplyfying, we get

$$2y_1^2(2y_1^2 + x_1^2 - 12ax_1) = ax_1(3x_1 - 4a)^2$$

Hence, required locus is

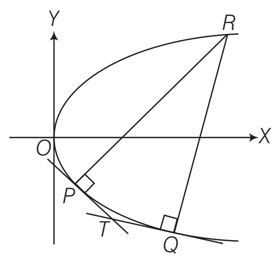
$$2y^2(2y^2 + x^2 - 12ax) = ax(3x - 4a)^2.$$

100. Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ be two points on the parabola $y^2 = 4ax$ such that the normals at P and Q intersect at a point $R(aT^2, 2aT)$ on the parabola $y^2 = 4ax$, then

$$T = -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2}, \text{ then } t_1t_2 = 2$$

$\because$ Tangents at P and Q intersect at $T(at_1t_2, a(t_1 + t_2))$

i.e. $T(2a, a(t_1 + t_2))$.



Also, the coordinates of R being the point of intersection of normals at P and Q are

$(2a + a(t_1^2 + t_2^2 + t_1t_2), -at_1t_2(t_1 + t_2))$

$\Rightarrow \quad (4a + a(t_1^2 + t_2^2), -2a(t_1 + t_2))$

Now, $\angle TPR = \angle TQR = 90°$

$\Rightarrow \quad \angle TPR + \angle TQR = 180°$

$\Rightarrow$ Quadrilateral $PTQR$ is a cyclic quadrilateral, therefore centre of the circle circumscribing the ΔTPQ is at the mid-point of TR.

Let its coordinates be (h, k), then

$$2h = 2a + 4a + a(t_1^2 + t_2^2)$$

$\Rightarrow \quad \dfrac{2h - 6a}{a} = (t_1 + t_2)^2 - 2t_1t_2$

$\Rightarrow \quad \dfrac{2h - 2a}{a} = (t_1 + t_2)^2$ $[\because t_1t_2 = 2]$ ...(i)

and $2k = a(t_1 + t_2) - 2a(t_1 + t_2)$

$\Rightarrow \quad -\dfrac{2k}{a} = (t_1 + t_2)$ ...(ii)

From Eqs. (i) and (ii), then $\left(\frac{2h-2a}{a}\right) = \left(-\frac{2k}{a}\right)^2$

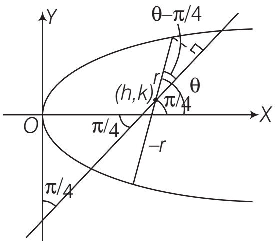
$$\Rightarrow \qquad 2k^2 = a\,(h-a)$$

Hence, locus of (h, k) is $\qquad 2y^2 = a\,(x-a)$.

101. Let the equation of straight line (h, k) as its mid-point,

then, $$\frac{x-h}{\cos\theta} = \frac{y-k}{\sin\theta} = r \qquad \text{...(i)}$$

Any point on the line Eq. (i) is $(h + r\cos\theta,\ k + r\sin\theta)$.



Solving with the equation of parabola

$$y^2 = 4ax, \text{ we get}$$

$$(k + r\sin\theta)^2 = 4a(h + r\cos\theta)$$

$$\Rightarrow\ r^2\sin^2\theta + 2r(k\sin\theta - 2a\cos\theta) + k^2 - 4ah = 0 \qquad \text{...(ii)}$$

which is quadratic in r.

The roots of the quadratic equation will be equal but of opposite sign as (h, k) is the mid-point.

$$\text{Sum of roots} = -\frac{2(k\sin\theta - 2a\cos\theta)}{\sin^2\theta} = 0$$

$$\therefore \qquad k\sin\theta - 2a\cos\theta = 0$$

$$\therefore \qquad \tan\theta = \frac{2a}{k}$$

Now, from Eq. (ii),

$$r^2\sin^2\theta + (k^2 - 4ah) = 0$$

$$\Rightarrow \qquad r^2 \cdot \frac{4a^2}{(4a^2 + k^2)} + (k^2 - 4ah) = 0 \qquad \text{...(iii)}$$

Length of the chord will be $2r$. Angle between the two lines will be $(\theta - \pi/4)$ and the projection of the chord on the given line will be $2r\cos(\theta - \pi/4) = c$

$$\Rightarrow \qquad \frac{2r}{\sqrt{2}}(\cos\theta + \sin\theta) = c$$

$$\Rightarrow \qquad \frac{2r}{\sqrt{2}}\left(\frac{k + 2a}{\sqrt{4a^2 + k^2}}\right) = c$$

$$\Rightarrow \qquad 2r^2(k + 2a)^2 = c^2(4a^2 + k^2)$$

$$\Rightarrow \qquad \frac{2r^2}{(4a^2 + k^2)} = \frac{c^2}{(k + 2a)^2} \qquad \text{...(iv)}$$

From Eqs. (iii) and (iv), we get

$$\frac{2a^2c^2}{(k + 2a)^2} + (k^2 - 4ah) = 0$$

$$\Rightarrow \qquad (k^2 - 4ah)(k + 2a)^2 + 2a^2c^2 = 0$$

Hence, the locus of the middle points is

$$(y^2 - 4ax)(y + 2a)^2 + 2a^2c^2 = 0.$$

102. Equation of normal at $(am^2, -2am)$ is

$$y = mx - 2am - am^3$$

If the three normals of P, Q, R meet at (h, k), then

$$am^3 + m(2a - h) + k = 0$$

$$\therefore \qquad \Sigma m_1 = 0$$

$$\Sigma m_1 m_2 = \frac{(2a - h)}{a},$$

$$m_1 m_2 m_3 = \frac{-k}{a}$$

$$P \equiv (am_1^2, -2am_1)$$

$$Q \equiv (am_2^2, -2am_2)$$

and $\quad R \equiv (am_3^2, -2am_3)$

Equation of PQ is

$$-y\,(m_1 + m_2) = 2(x + am_1m_2) \qquad \text{...(i)}$$

and equation of diameter through R is

$$y = -2am_3 \qquad \text{...(ii)}$$

Point of intersection of Eqs. (i) and (ii) is on the directrix and hence it must be $(-a, -2am_3)$ and it satisfies Eq. (ii), then

$$2am_3(m_1 + m_2) = 2(-a + am_1m_2)$$

$$\Rightarrow \qquad m_3(-m_3) = -1 + m_1m_2$$

$$\Rightarrow \qquad m_1m_2 = 1 - m_3^2$$

$\therefore$ Equation of PQ becomes

$$-y(0 - m_3) = 2(x + a - am_3^2)$$

$$\Rightarrow \qquad 2am_3^2 + m_3y - 2(x + a) = 0 \qquad [m_3 \text{ is parameter}]$$

Its envelope is given by the discriminant of this quadratic equated to zero.

$$\therefore \qquad (y)^2 - 4\cdot 2a\cdot\{-2\cdot(x + a)\} = 0$$

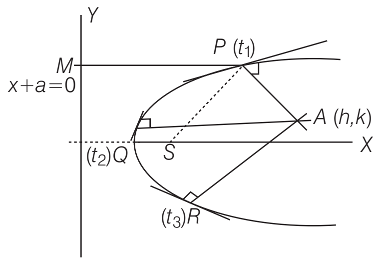
$$\Rightarrow \qquad y^2 + 16a\,(x + a) = 0$$

103. Equation of normal at 't' is $y = -tx + 2at + at^3$.

Let A be (h, k), then $\quad k = -th + 2at + at^3$

or $$at^3 - t(h - 2a) - k = 0 \qquad \text{...(i)}$$

Let the coordinates of P, Q, R are $(at_1^2, 2at_1), (at_2^2, 2at_2), (at_3^2, 2at_3)$ respectively, then from Eq. (i),



$$t_1 + t_2 + t_3 = 0$$

$$\Rightarrow \qquad t_1t_2 + t_2t_3 + t_3t_1 = -\left(\frac{h - 2a}{a}\right)$$

and $$t_1t_2t_3 = \frac{k}{a}$$

Since, $SP = PM = a + at_1^2$

Similarly, $SQ = a + at_2^2$

and $SR = a + at_3^2$

$\therefore$ $SP \cdot SQ \cdot SR = a(1 + t_1^2) \cdot a(1 + t_2^2) \cdot a(1 + t_3^2)$

$= a^3\{1 + (t_1^2 + t_2^2 + t_3^2) + (t_1^2t_2^2 + t_2^2t_3^2 + t_3^2 t_1^2) + (t_1^2 t_2^2 t_3^2)\}$

$= a^3 \{1 + (t_1 + t_2 + t_3)^2 - 2(t_1t_2 + t_2t_3 + t_3t_1)$

$+ (t_1t_2 + t_2t_3 + t_3t_1)^2 - 2t_1t_2t_3(t_1 + t_2 + t_3) + (t_1t_2t_3)^2\}$

$$= a^3\left\{1 + 0 + 2\left(\frac{h - 2a}{a}\right) + \left(\frac{h - 2a}{a}\right)^2 - 0 + \frac{k^2}{a^2}\right\}$$

$$= a^3\left\{\left(\frac{h - 2a}{a} + 1\right)^2 + \frac{k^2}{a^2}\right\}$$

$= a\{(h - a)^2 + (k - 0)^2\} = a(SA)^2$

104. Equation of tangent at $P(at^2, 2at)$ is

$$ty = x + at^2 \Rightarrow x - ty + at^2 = 0 \quad \text{...(i)}$$

which is also tangent to the circle

$$x^2 + y^2 = a^2 / 2 \quad \text{...(ii)}$$

then, length of perpendicular from centre of Eq. (ii) to Eq. (i) radius of the circle

$\Rightarrow$ $\dfrac{|at^2|}{\sqrt{1 + t^2}} = \dfrac{a}{\sqrt{2}}$

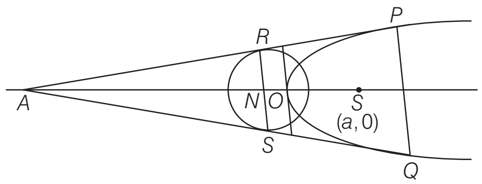
$\Rightarrow$ $2t^4 = (1 + t^2)$

$\Rightarrow$ $(t^2 - 1)(2t^2 + 1) = 0$

$\therefore$ $2t^2 + 1 \neq 0 \quad \therefore \quad t^2 - 1 = 0$

then, $t = \pm 1$

then, coordinates of P and Q are $(a, 2a)$ and $(a, -2a)$ respectively.



$\therefore$ $PQ = 4a$

$\therefore$ Equation of tangent at $P(a, 2a)$ is

$$x - y + a = 0 \quad \text{...(iii)}$$

[from Eq. (i)]

Let R be (x_1, y_1)

then, equation of tangent at $R(x_1, y_1)$ on Eq. (ii) is

$$xx_1 + yy_1 = \frac{a^2}{2} \quad \text{...(iv)}$$

Hence, Eqs. (iii) and (iv) are identical.

$\therefore$ $\dfrac{x_1}{1} = \dfrac{y_1}{-1} = -\dfrac{a}{2}$

$\therefore$ $(x_1, y_1) = \left(-\dfrac{a}{2}, \dfrac{a}{2}\right)$

Hence coordinates of S are $\left(-\dfrac{a}{2}, \dfrac{a}{2}\right)$.

$\therefore$ $RS' = a$

Hence, quadrilateral $PQRS'$ is trapezium whose area

$$= \frac{1}{2}(PQ + RS') \times \left(a + \frac{a}{2}\right)$$

$$= \frac{1}{2} \times (4a + a) \times \frac{3a}{2} = \frac{15a^2}{4} \text{ sq units.}$$

Aliter :

Here, centre of the circle is the vertex of the parabola and both circle and parabola are symmetrical about axis of parabola. In this case the point of intersection of common tangents must lie on the directrix and axis of the parabola.

i.e. $A(-a, 0)$.

Chord of contact of circle w.r.t. $A(-a, 0)$ is

$$x(-a) + y \cdot 0 = \frac{a^2}{2}$$

$\therefore$ $x = -\dfrac{a}{2}$

$\therefore$ Coordinates of R are $\left(-\dfrac{a}{2}, \dfrac{a}{2}\right)$ and chord of contact of parabola w.r.t. $A(-a, 0)$ is

$$y \cdot 0 = 2a(x - a)$$

i.e. $x = a$

$\therefore$ Coordinates of P is $(a, 2a)$

$\therefore$ Area of quadrilateral

$PQRS' = 2\{\text{Area of } \Delta PAS - \text{Area of } \Delta RAN\}$

$$= 2\left\{\frac{1}{2} \cdot 2a \cdot 2a - \frac{1}{2} \cdot \frac{a}{2} \cdot \frac{a}{2}\right\}$$

$$= 4a^2 - \frac{a^2}{4} = \frac{15a^2}{4} \text{ sq units}$$

105. Let parabola be $y^2 = 4ax$ and let $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ and $R(at_3^2, 2at_3)$ are three points on the parabola.

$\because$ Tangents at P, Q and R on parabola $y^2 = 4ax$

are $t_1y = x + at_1^2$, $t_2y = x + at_2^2$

and $t_3y = x + at_3^2$

Slopes of these tangents are $\dfrac{1}{t_1}, \dfrac{1}{t_2}, \dfrac{1}{t_3}$

but given slopes are in HP.

$\therefore$ t_1, t_2, t_3 are in AP if d is the common difference. Then,

$t_2 - t_1 = d$, $t_3 - t_2 = d$

and $t_3 - t_1 = 2d$ ...(i)

Let the tangents at Q and R meet at P', R and P meet at Q', P and Q meet at R'.

$\therefore$ $P' = \{at_2t_3, a(t_2 + t_3)\}$

$Q' = \{at_3t_1, a(t_3 + t_1)\}$

and $R' = \{at_1t_2, a(t_1 + t_2)\}$

$\therefore$ Area of $\Delta P'Q'R' = \frac{1}{2}\begin{vmatrix} at_2t_3 & a(t_2+t_3) & 1 \\ at_3t_1 & a(t_3+t_1) & 1 \\ at_1t_2 & a(t_1+t_2) & 1 \end{vmatrix}$

$= \frac{1}{2}\{at_2t_3\{a(t_3+t_1)-(t_1+t_2)\}+\ldots+\ldots\}$

$= \frac{1}{2}a^2\Sigma\, t_2t_3(t_3-t_2)$

$= \frac{1}{2}a^2(t_1-t_2)(t_2-t_3)(t_3-t_1)$

$= \frac{1}{2}a^2(-d)(-d)(2d)$ [by using Eq. (i)]

$= a^2d^3$, which is constant.

Remark

$\Sigma t_1^2(t_2-t_3) = \Sigma\, t_2t_3(t_3-t_2) = (t_1-t_2)(t_2-t_3)(t_3-t_1)$

Corollary Area of triangle of $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ and $R(at_3^2, 2at_3)$ is $a^2(t_1-t_2)(t_2-t_3)(t_3-t_1) = \Delta$ (say)

It is clear from just previous example the area of the triangle inscribed in a parabola is twice the area of the triangle formed by the tangents at the vertices.

106. Equation of tangent at (1, 7) to $y = x^2 + 6$

$\Rightarrow \quad \frac{1}{2}(y+7) = x\cdot 1 + 6$

$\Rightarrow \quad y = 2x + 5$ ...(i)

This tangent also touches the circle.

$x^2 + y^2 + 16x + 12y + c = 0$ ...(ii)

Now, solving Eqs. (i) and (ii), we get

$x^2 + (2x+5)^2 + 16x + 12(2x+5) + c = 0$

$\Rightarrow \quad 5x^2 + 60x + 85 + c = 0$

Since, roots are equal, so

$B^2 - 4AC = 0$

$\Rightarrow \quad (60)^2 - 4\times 5\times(85+c) = 0$

$\Rightarrow \quad 85 + c = 180$

$\Rightarrow \quad 5x^2 + 60x + 180 = 0$

$\Rightarrow \quad x = -\frac{60}{10} = -6 \quad \Rightarrow \quad y = -7$

Hence, point of contact is $(-6, -7)$.

107. $\because P \equiv (1,0)$, let $Q \equiv (h, k)$

such that $\quad k^2 = 8h$ ...(i)

Let (α, β) be the mid-point of PQ.

$\therefore \quad \alpha = \frac{h+1}{2}, \beta = \frac{k+0}{2}$

$\Rightarrow \quad h = 2\alpha - 1, k = 2\beta$

From Eq. (i), we get

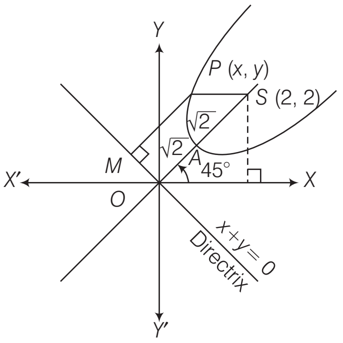
$(2\beta)^2 = 8(2\alpha - 1)$

$\Rightarrow \quad \beta^2 = 4\alpha - 2$

$\Rightarrow \quad \beta^2 - 4\alpha + 2 = 0$

$\therefore$ Required locus is $y^2 - 4x + 2 = 0$.

108. Coordinates of S are $(2\sqrt{2}\cos 45°, 2\sqrt{2}\sin 45°)$ i.e. (2, 2).



$\therefore \quad SP = PM$

$\Rightarrow \quad (SP)^2 = (PM)^2$

$\Rightarrow \quad (x-2)^2 + (y-2)^2 = \left[\frac{(x+y)}{\sqrt{2}}\right]^2$

$\Rightarrow \quad 2(x^2 + y^2 - 4x - 4y + 8) = x^2 + y^2 + 2xy$

$\Rightarrow \quad x^2 + y^2 - 2xy - 8x - 8y + 16 = 0$

$\therefore \quad (x-y)^2 = 8(x+y-2)$

109. Equation of tangent to $y = x^2$ is

$y = mx - \frac{1}{4}m^2$ ...(i)

Equation of tangent to $(x-2)^2 = -y$ is

$y = m(x-2) + \frac{1}{4}m^2$ ...(ii)

$\because$ Eqs. (i) and (ii) are identical.

$\Rightarrow \quad m = 0$ or 4

$\therefore$ Common tangents are $y = 0$ and $y = 4x - 4 = 4(x-1)$.

110. Given parabola is

$y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ ...(i)

For vertex $\frac{dy}{dx} = 0 \Rightarrow x = -\frac{3}{4a}$

Substitute $x = -\frac{3}{4a}$ in Eq. (i), we get

$y = -\frac{35a}{16}$

$\therefore$ Coordinates of vertex are $\left(-\frac{3}{4a}, -\frac{35a}{16}\right)$.

For locus let $x = -\frac{3}{4a}$ and $y = -\frac{35a}{16}$.

$\therefore \quad xy = \frac{105}{64}$, which is the required locus.

111. $\because \quad y = x^2 - 5x + 6$

$\therefore$ Equation of tangent at (2, 0) is

$\frac{y+0}{2} = x\cdot 2 - \frac{5}{2}(x+2) + 6$

$\Rightarrow \quad y = -x + 2$ ...(i)

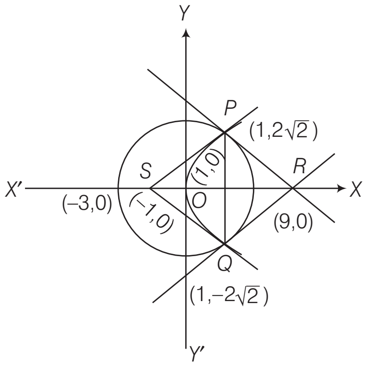
and equation of tangent at (3, 0) is

$\Rightarrow \qquad y = x - 3$ ...(ii)

$\because$ Eqs. (i) and (ii) are perpendicular.

$\therefore$ Angle between tangents is $\pi / 2$.

112. (i) Coordinates of P and Q are $(1, 2\sqrt{2})$ and $(1, -2\sqrt{2})$.



$$\text{Area of } \Delta PQR = \frac{1}{2} \cdot 4\sqrt{2} \cdot 8 = 16\sqrt{2}$$

$$\text{Area of } \Delta PQS = \frac{1}{2} \cdot 4\sqrt{2} \cdot 2 = 4\sqrt{2}$$

$\therefore$ Ratio of area of ΔPQS and ΔPQR is 1 : 4.

(ii) Equation of circumcircle of ΔPRS is

$$(x + 1)(x - 9) + y^2 + \lambda y = 0$$

It will pass through $(1, 2\sqrt{2})$, then

$$-16 + 8 + \lambda 2\sqrt{2} = 0 \Rightarrow \lambda = \frac{8}{2\sqrt{2}} = 2\sqrt{2}$$

Equation of circumcircle is

$$x^2 + y^2 - 8x + 2\sqrt{2}y - 9 = 0$$

Hence, radius is $3\sqrt{3}$.

Aliter :

Let $\angle PSR = \theta \Rightarrow \sin\theta = \frac{2\sqrt{2}}{2\sqrt{3}}$

$\Rightarrow \qquad PR = 6\sqrt{2} = 2R \cdot \sin\theta \Rightarrow R = 3\sqrt{3}$.

(iii) Radius of incircle is $r = \frac{\Delta}{s}$.

As $\qquad \Delta = 16\sqrt{2}$

$\therefore \qquad s = \frac{6\sqrt{2} + 6\sqrt{2} + 4\sqrt{2}}{2} = 8\sqrt{2}$

$\therefore \qquad r = \frac{16\sqrt{2}}{8\sqrt{2}} = 2$

113. $y = -\frac{x^2}{2} + x + 1 \Rightarrow y - \frac{3}{2} = -\frac{1}{2}(x - 1)^2$

$\Rightarrow$ It is symmetric about $x = 1$.

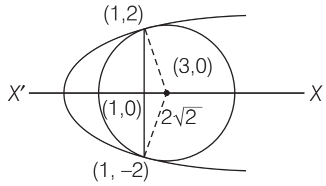
Hence, both statement are true and Statement II is correct explanation of Statement I.

114. $\because$ Point of intersection of two perpendicular tangents to the parabola lies on directrix of the parabola.

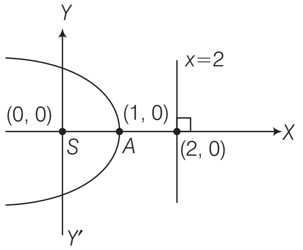
$\therefore$ Equation of directrix is $x + 2 = 0$.

So, point is $(-2, 0)$.

115. The circle and the parabola touch each other at $x = 1$, i.e. at the points (1, 2) and (1, − 2) as shown in figure.

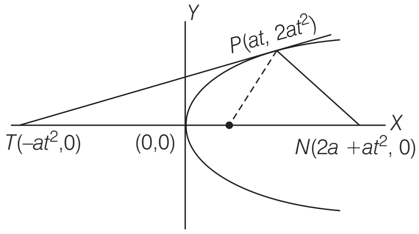


116. Vertex is (1,0).



117. $\qquad G \equiv (h, k)$

$\Rightarrow \qquad h = \frac{2a + at^2}{3}, k = \frac{2at}{3}$



$\Rightarrow \qquad \left(\frac{3h - 2a}{a}\right) = \frac{9k^2}{4a^2}$

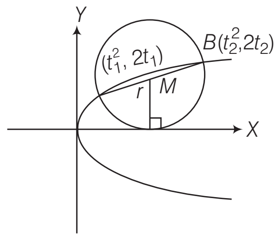
$\therefore$ Required parabola is

$$\frac{9y^2}{4a^2} = \frac{(3x - 2a)}{a} = \frac{3}{a}\left(x - \frac{2a}{3}\right)$$

$\Rightarrow \qquad y^2 = \frac{4a}{3}\left(x - \frac{2a}{3}\right)$

$\therefore$ Vertex $\equiv \left(\frac{2a}{3}, 0\right)$; Focus $\equiv (a, 0)$.

118. Slope of $AB = \frac{2t_2 - 2t_1}{(t_2^2 - t_1^2)} = \frac{2}{(t_2 + t_1)}$ ...(i)



$$M = \text{Mid-point of } AB = \left(\frac{t_1^2 + t_2^2}{2}, t_1 + t_2\right)$$

$\therefore \qquad r = |t_1 + t_2| \Rightarrow t_1 + t_2 = \pm r$

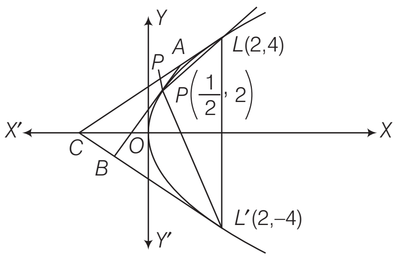
Now, from Eq. (i),

$$\text{slope of } AB = \pm \frac{2}{r}.$$

119. The locus of perpendicular tangent is directrix, i.e., $x + 1 = 0$ or $x = -1$.

120. $\Delta_1 =$ Area of $\Delta PLL'$

$$= \frac{1}{2} \times 8 \times \left(2 - \frac{1}{2}\right) = 6 \text{ sq unitss}$$



Now, equation of AB is $y = 2x + 1$,
equation of AC is $y = x + 2$ and
equation of BC is $y = -x - 2$
On solving above equations, we get

$A(1, 3)$, $B(-1, -1)$ and $C(-2, 0)$

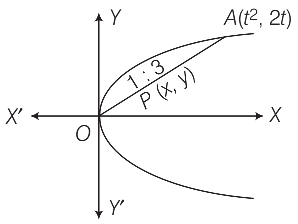
$$\therefore \quad \Delta_2 = \frac{1}{2} \begin{Vmatrix} 1+2 & 3-0 \\ -1+2 & -1-0 \end{Vmatrix} = 3 \text{ sq units}$$

$$\therefore \quad \frac{\Delta_1}{\Delta_2} = 2$$

121. Let $A(x, y) = A(t^2, 2t)$ be any point on the parabola $y^2 = 4x$, then

$$x = \frac{t^2}{4} \quad \text{...(i)}$$

and
$$y = \frac{2t}{4} \quad \text{...(ii)}$$



From Eqs. (i) and (ii), we get

$$x = y^2$$

122. The equation of normal to

$$y^2 = 4x \text{ is } y = mx - 2m - m^3 \quad \text{...(i)}$$

As it passes through (9, 6), then

$$6 = 9m - 2m - m^3$$

$$\Rightarrow \quad m^3 - 7m + 6 = 0$$

$$\Rightarrow (m-1)(m-2)(m+3) = 0$$

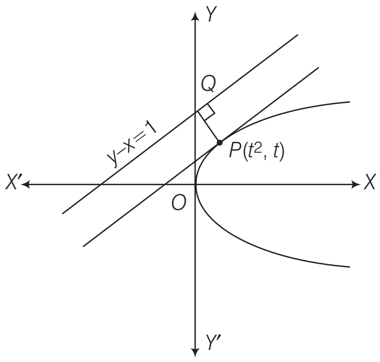
$$\Rightarrow \quad m = 1, 2, -3$$

From Eq. (i), equations of normals are

$$y = x - 3,\ y = 2x - 12,\ y = -3x + 33$$

$$\Rightarrow \ y - x + 3 = 0,\ y - 2x + 12 = 0,\ y + 3x - 33 = 0$$

123. The shortest distance between $y = x - 1$ and $y^2 = x$ is along the normal of $y^2 = x$.



Let $P(t^2, t)$ be any point on $y^2 = x$.

$\therefore$ Tangent at P is $y = \frac{x}{2t} + \frac{t}{2}$.

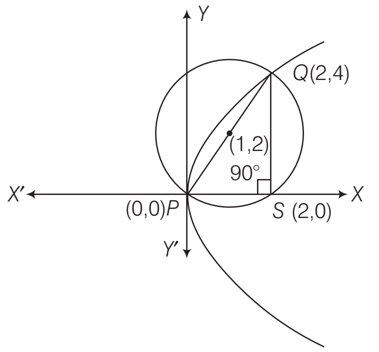
$\therefore$ Slope of tangent $= \frac{1}{2t}$

and tangent at P is parallel to $y - x = 1$

$$\therefore \quad \frac{1}{2t} = 1 \Rightarrow t = \frac{1}{2} \Rightarrow P\left(\frac{1}{4}, \frac{1}{2}\right)$$

$$\text{Hence, shortest distance} = PQ = \frac{\left|\frac{1}{2} - \frac{1}{4} - 1\right|}{\sqrt{(1+1)}} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

124. We observe that both parabola $y^2 = 8x$ and circle $x^2 + y^2 - 2x - 4y = 0$ pass through origin say $P(0, 0)$.



Let Q be the point $(2t^2, 4t)$, then it will satisfy the equation of circle.

$$\therefore \quad (2t^2)^2 + (4t)^2 - 2(2t^2) - 4(4t) = 0$$

$$\Rightarrow \quad 4t^4 + 12t^2 - 16t = 0$$

$$\Rightarrow \quad t(t-1)(t^2 + t - 4) = 0 \Rightarrow t = 0 \text{ or } 1$$

For $t = 0$, we get point P, therefore $t = 1$ gives point Q as (2, 4).
Here, $P(0, 0)$ and $Q(2, 4)$ are end points of diameter of the given circle and focus of the parabola is the point $S(2, 0)$.

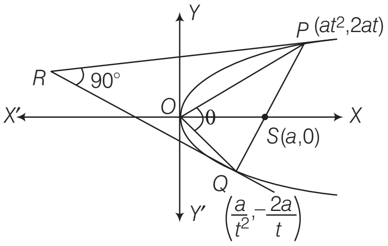
$$\therefore \quad \angle PSQ = 90^\circ$$

Hence, area of $\Delta PQS = \frac{1}{2} \times 2 \times 4 = 4$ sq units.

Sol. (Q. Nos. 125 and 126)

$\because$ PQ is the focal chord of $y^2 = 4ax$.

$\therefore$ Coordinates of P and Q are $(at^2, 2at)$ and $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$.



Tangents at P and Q are

$$ty = x + at^2 \text{ and } ty = xt^2 + a$$

which intersect each other at $R\left(-a, a\left(t - \frac{1}{t}\right)\right)$.

As R lies on the line $y = 2x + a, a > 0$

$$\therefore \quad a\left(t - \frac{1}{t}\right) = -2a + a$$

$$\Rightarrow \quad t - \frac{1}{t} = -1$$

125. $\because$ Slope of $OP = \frac{2}{t}$ and slope of $OQ = -2t$

$$\therefore \quad \tan\theta = \left|\frac{\frac{2}{t} + 2t}{1 - 4}\right| = \frac{2}{3}\left|t + \frac{1}{t}\right|$$

$$= \frac{2}{3}\sqrt{\left(t - \frac{1}{t}\right)^2 + 4} = \frac{2}{3}\sqrt{5} \qquad \left[\because t - \frac{1}{t} = -1\right]$$

$$\because \quad \theta > 90°$$

$$\therefore \quad \tan\theta = -\frac{2}{3}\sqrt{5}$$

126. $PQ = a\left(t + \frac{1}{t}\right)^2 = a\left\{\left(t - \frac{1}{t}\right)^2 + 4\right\}$

$$= a(1 + 4) = 5a \qquad \left[\because t - \frac{1}{t} = -1\right]$$

127. Equation of tangent of $y^2 = 4x$ in terms of slope is

$$y = mx + \frac{1}{m} \qquad \text{...(i)}$$

$\because$ Line Eq. (i) touches $x^2 = -32y$

$$\Rightarrow \quad x^2 = -32\left(mx + \frac{1}{m}\right)$$

$$\Rightarrow \quad x^2 + 32mx + \frac{32}{m} = 0 \qquad \text{...(ii)}$$

For touching roots of Eq. (ii) are equal.

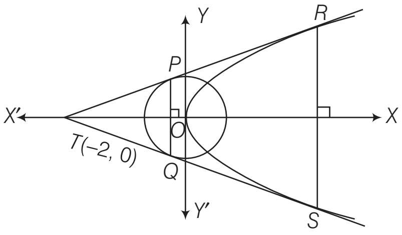
$$\therefore \quad D = 0$$

$$\Rightarrow \quad (32m)^2 = 4 \cdot 1 \cdot \left(\frac{32}{m}\right)$$

$$\Rightarrow \quad m^3 = \frac{1}{8}$$

$$\therefore \quad m = 1/2$$

128. Let the tangent to, $y^2 = 8x$ be $y = mx + \frac{2}{m}$.



If it is common tangent to parabola and circle, then $y = mx + \frac{2}{m}$ is a tangent to $x^2 + y^2 = 2$.

$$\therefore \quad \frac{\frac{2}{m}}{\sqrt{(1 + m^2)}} = \sqrt{2}$$

$$\Rightarrow \quad \frac{4}{m^2(1 + m^2)} = 2$$

$$\Rightarrow \quad m^4 + m^2 - 2 = 0$$

$$\Rightarrow \quad (m^2 + 2)(m^2 - 1) = 0$$

$$\therefore \quad m = \pm 1$$

$\therefore$ Required tangents are $y = x + 2$ and $y = -x - 2$.

Their common point is $T(-2, 0)$.

Chord of contact PQ to circle is

$$x \cdot (-2) + y \cdot 0 = 2$$

$$\Rightarrow \quad x = -1$$

Hence, coordinates of P and Q are $(-1, 1)$ and $(-1, -1)$ and chord of contact RS to parabola is

$$y \cdot 0 = 4(x - 2)$$

$$\Rightarrow \quad x = 2$$

Hence, coordinates of R and S are $(2, 4)$ and $(2, -4)$.

$\therefore$ Area of trapezium $PQRS = \frac{1}{2}(2 + 8) \times 3 = 15$ sq units

Sol. (Q. Nos. 129 and 130)

$\because$ PQ is a focal chord, then $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$.

Also, $\quad QR \parallel PK \Rightarrow m_{QR} = m_{PK}$

$$\Rightarrow \quad \frac{2ar + \frac{2a}{t}}{ar^2 - \frac{a}{t^2}} = \frac{0 - 2at}{2a - at^2}$$

$$\Rightarrow \quad \frac{2}{r - \frac{1}{t}} = \frac{-2t}{2 - t^2}$$

$$\left[\because r + \frac{1}{t} \neq 0, \text{ otherwise } Q \text{ will coincide with } R\right]$$

$$\Rightarrow \quad 2 - t^2 = -rt + 1$$

$$\therefore \quad r = \frac{t^2 - 1}{t}$$

129. $r = \frac{t^2 - 1}{t}$

130. Tangent at P is $\quad ty = x + at^2 \quad$ ...(i)

Normal at S is $\quad y + sx = 2as + as^3 \quad$ ...(ii)

Putting the value of x from Eq. (i) in Eq. (ii), then

$$y + s(ty - at^2) = 2as + as^3$$

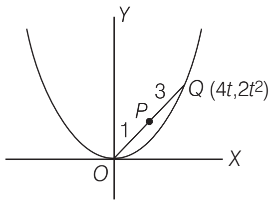
$$\Rightarrow \quad y + (st)y - a(st)t = 2as + as^3$$

$$\Rightarrow \quad y + y - at = \frac{2a}{t} + \frac{a}{t^3} \quad [\because st = 1]$$

$$\Rightarrow \quad 2y = a\left(t + \frac{2}{t} + \frac{1}{t^3}\right)$$

$$\therefore \quad y = \frac{a(t^2 + 1)^2}{2t^3}$$

131. Let any point Q on $x^2 = 8y$ is $(4t, 2t^2)$ and given $P(h, k)$ divides OQ in the ratio 1:3 (internally).



Then, $\quad h = \frac{4t}{4} = t$ and $k = \frac{2t^2}{4} \Rightarrow 2k = h^2$

$\therefore$ Required locus of P is $x^2 = 2y$.

132. End points of latusrectum of

$$y^2 = 4x \text{ are } (1, \pm 2).$$

Equation of normal to $y^2 = 4x$ at (1, 2) is

$$y - 2 = -\frac{2}{2}(x - 1)$$

$$\Rightarrow \quad x + y - 3 = 0$$

As it is tangent to circle $(x - 3)^2 + (y + 2)^2 = r^2$

$$\therefore \quad \frac{|3 - 2 - 3|}{\sqrt{(1 + 1)}} = r \Rightarrow r^2 = 2$$

133. Let $(t^2, 2t)$ be any point on $y^2 = 4x$. Let (h, k) be image of $(t^2, 2t)$ with respect to the line $x + y + 4 = 0$, then

$$\frac{h - t^2}{1} = \frac{k - 2t}{1} = \frac{-2(t^2 + 2t + 4)}{1 + 1}$$

$$\Rightarrow \quad h = -(2t + 4) \text{ and } k = -(t^2 + 4)$$

$$\Rightarrow \quad (k + 4) = -\left(\frac{h + 4}{-2}\right)^2$$

$$\Rightarrow \quad (h + 4)^2 = -4(k + 4)$$

Locus of (h, k) is $(x + 4)^2 = -4(y + 4)$.

$\therefore$ Curve C is $(x + 4)^2 = -4(y + 4)$

Now, intersection of C with $y = -5$, then

$$(x + 4)^2 = -4(-5 + 4) = 4$$

$$\therefore \quad x + 4 = \pm 2 \Rightarrow x = -6, -2$$

$\therefore$ $A(-6, -5)$ and $B(-2, -5)$

$$\therefore \quad AB = 4$$

134. Let $P\left(\frac{t_1^2}{2}, t_1\right)$ and $Q\left(\frac{t_2^2}{2}, t_2\right)$ such that $t_1 > 0$

[$\because$ P lies in first quadrant]

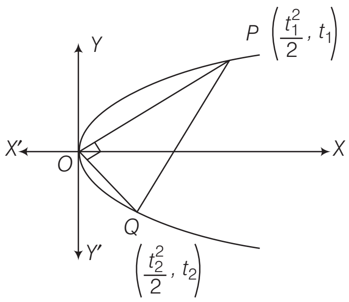
$\because$ Circle with PQ as diameter passes through the vertex $O(0, 0)$ of the parabola.

$$\therefore \quad \angle POQ = 90^\circ$$

$\Rightarrow$ Slope of $OP \times$ Slope of $OQ = -1$

$$\Rightarrow \quad \frac{2}{t_1} \times \frac{2}{t_2} = -1$$

$$\Rightarrow \quad t_1 t_2 = -4 \quad [\because t_2 < 0]$$



Now, area of $\Delta OPQ = 3\sqrt{2}$

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} \frac{t_1^2}{2} & t_1 \\ \frac{t_2^2}{2} & t_2 \end{vmatrix} = 3\sqrt{2}$$

$$\Rightarrow \quad \frac{1}{4} t_1 t_2 (t_1 - t_2) = \pm 3\sqrt{2}$$

$$\Rightarrow \quad t_1 - t_2 = \pm 3\sqrt{2} \quad [\because t_1 t_2 = -4]$$

$$\Rightarrow \quad t_1 + \frac{4}{t_1} = \pm 3\sqrt{2}$$

$$\text{or} \quad t_1 + \frac{4}{t_1} = 3\sqrt{2} \quad [\because t_1 > 0]$$

$$\Rightarrow \quad t_1^2 - 3\sqrt{2}t_1 + 4 = 0$$

$$\therefore \quad t_1 = \frac{3\sqrt{2} \pm \sqrt{2}}{2} = 2\sqrt{2}, \sqrt{2}.$$

$\therefore$ Point P can be $(4, 2\sqrt{2})$ or $(1, \sqrt{2})$.

135. Let $P(2t^2, 4t)$ and $C(0, -6)$.

$$\therefore \quad (CP)^2 = 4t^4 + (4t + 6)^2 = z \quad \text{(say)}$$

$$\therefore \quad \frac{dz}{dt} = 0$$

$$\Rightarrow \quad 16t^3 + 2(4t + 6) \cdot 4 = 0$$

$$\Rightarrow \quad t^3 + 2t + 3 = 0$$

$$\Rightarrow \quad (t + 1)(t^2 - t + 3) = 0$$

$$\therefore \quad t = -1$$

$$\Rightarrow \quad P(2, -4)$$

Equation of circle is

$$(x - 2)^2 + (y + 4)^2 = (2 - 0)^2 + (-4 + 6)^2$$

$$\Rightarrow \quad x^2 + y^2 - 4x + 8y + 12 = 0$$

136. $\because C_1 : x^2 + y^2 = 3$ and parabola $x^2 = 2y$, then

$$y^2 + 2y - 3 = 0 \Rightarrow y = 1, -3$$

$\therefore \quad P(\sqrt{2}, 1)$ [$\because P$ lies in first quadrant]

Now, tangent at $P(\sqrt{2}, 1)$ on the circle C_1 is

$$x\sqrt{2} + y = 3$$

Let Q_2 or $Q_3(0, \lambda)$

$$\therefore \quad \frac{|0 + \lambda - 3|}{\sqrt{(2+1)}} = 2\sqrt{3}$$

$$\Rightarrow \quad |\lambda - 3| = 6$$

$$\therefore \quad \lambda = 9 \text{ or } -3$$

$\Rightarrow Q_2(0, -3)$ and $Q_3(0, 9)$.

Alternate (a) $Q_2Q_3 = 12$

Alternate (b) $R_2R_3 =$ Length of external common tangent

$$= \sqrt{(Q_2Q_3)^2 - (2\sqrt{3} + 2\sqrt{3})^2}$$

$$= \sqrt{(144 - 48} = 4\sqrt{6}$$

Alternate (c) Area of $\Delta OR_2R_3 = \frac{1}{2} \times R_2R_3 \times \frac{|0 + 0 - 3|}{\sqrt{(2+1)}}$

$$= \frac{1}{2} \times 4\sqrt{6} \times \frac{3}{\sqrt{3}} = 6\sqrt{2}$$

Alternate (d) Area of $\Delta PQ_2Q_3 = \frac{1}{2} \times Q_2Q_3 \times \sqrt{2}$

$$= \frac{1}{2} \times 12 \times \sqrt{2} = 6\sqrt{2}$$

137. Let $P(t^2, 2t)$, $S(2, 8)$ and $r = \sqrt{(4 + 64 - 64)} = 2$

We know that, shortest distance between two curves lies along their common normal. The common normal will pass through centre of circle.

$\therefore$ Slope of PS = Slope of normal to the parabola $y^2 = 4x$ at $P(t^2, 2t)$

$$\Rightarrow \quad \frac{2t - 8}{t^2 - 2} = -t \text{ or } t^3 = 8 \Rightarrow t = 2$$

$\therefore P(4, 4)$

Alternate (a) $\quad SP = \sqrt{(2-4)^2 + (8-4)^2} = 2\sqrt{5}$

Alternate (b) $\quad SQ = r = 2$

$$\therefore \quad \frac{SQ}{QP} = \frac{SQ}{SP - SQ} = \frac{2}{2\sqrt{5} - 2}$$

$$= \frac{1}{(\sqrt{5} - 1)} \times \frac{(\sqrt{5} + 1)}{(\sqrt{5} + 1)} = \frac{\sqrt{5} + 1}{4}$$

$$\Rightarrow \quad SQ : QP = (\sqrt{5} + 1) : 4$$

Alternate (c) Equation of normal at $P(4, 4)$ is

$$y - 4 = -\frac{4}{2}(x - 4)$$

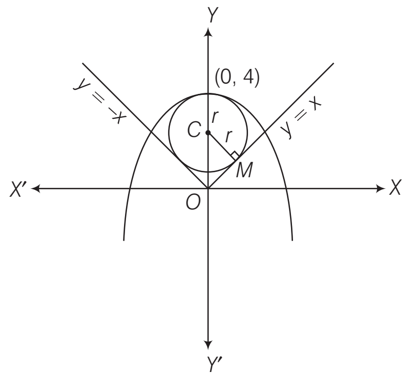
$$\Rightarrow \quad y - 4 = -2x + 8$$

$$\Rightarrow \quad 2x + y = 12$$

$\therefore$ Intercept on X-axis is 6.

Alternate (d) Slope of tangent at Q = Slope of tangent at $P = \frac{1}{2}$

138. Centre of circle



$$C \equiv (0, 4 - r)$$

$$\because \quad CM = r$$

$$\therefore \quad \frac{|10 - (4 - r)|}{\sqrt{2}} = r$$

$$\Rightarrow \quad 4 - r = r\sqrt{2}$$

or $$r = \frac{4}{\sqrt{2} + 1}$$

$$= 4(\sqrt{2} - 1)$$

139. (a) Equation of chord of parabola $y^2 = 16x$ whose mid-point (h, k) is

$$T = S_1$$

or $$ky - 8(x + h) = k^2 - 16h$$

or $$8x - ky = 8h - k^2 \quad \text{...(i)}$$

Now comparing Eq. (i) and $2x + y = p$, then

$$\frac{8}{2} = \frac{-k}{1} = \frac{8h - k^2}{p}$$

$$\Rightarrow \quad k = -4 \text{ and } 4p = 8h - k^2$$

or $\quad k = -4$ and $p = 2h - 4$

Hence, $\quad p = 2, h = 3, k = -4$