1 MARK

1. If the line L in the xy plane has half the slope and twice the y-intercept of the line $y = \frac{2}{3}x + 4$, then an equation for L is

(1) $y = \frac{1}{3}x + 8$ (2) $y = \frac{4}{3}x + 2$

(3) $y = \frac{1}{3}x + 4$ (4) $y = \frac{4}{3}x + 4$

2. A parabola $y = ax^2 + bx + c$ has vertex (4, 2). If (2,0) is on the parabola, then abc equals

(1) –12 (2) –6 (3) 0 (4) 12

3. The slope of the line $\frac{x}{3} + \frac{y}{2} = 1$ is

(1) $-\frac{3}{2}$ (2) $-\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

4. Let [t] denote the greatest integer $\le$ t where $t \ge 0$ and $S = \{(x,y) : (x - T)^2 + y^2 \le T^2$ where $T = t = (t)\}$. (t) = fractional part function. Then we have

(1) The point (0,0) does not belong to S for any t

(2) $0 \le$ Area S $< \pi$ for all t

(3) S is contained in the first quadrant for all $t \ge 5$

(4) The center of S for any t is on the line y = x

5. A town's population increased by 1,200 people, and then this new population decreased by 11%. The town now had 32 less people than it did before the 1,200 increase. What is the original population?

(1) 1,200 (2) 11,200 (3) 9,968 (4) 10,000

6. For which real values of m are the simultaneous equations $y = mx + 3$, $y = (2m - 1)x + 4$ satisfied by at least one pair of real numbers (x,y)?

(1) all m (2) all $m \ne 0$

(3) all $m \ne 1/2$ (4) all $m \ne 1$

7. Let P be an interior point of circle K other than the center of K. Form all chords of K which pass through P, and determine their midpoints. The locus of these midpoint is -

(1) a circle with one point deleted

(2) a circle if the distance from P to the center of K is less than one half the radius of K; otherwise a circular arc of less than 360°

(3) a semicircle with one point deleted

(4) a circle

8. The number of distinct pairs (x,y) of real numbers satisfying both of the following equations :

$x = x^2 + y^2$

$y = 2xy$ is-

(1) 0 (2) 1 (3) 2 (4) 3

9. If r is positive and the line whose equation is x + y = r is tangent to the circle whose equation is $x^2 + y^2 = r$ then r equals -

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) $\sqrt{2}$

10. If sinx + cosx = 1/5 and $0 \le x < \pi$, then tan x is

(1) $-\frac{4}{3}$ (2) $-\frac{3}{4}$ (3) $\frac{3}{4}$ (4) $\frac{4}{3}$

11. If $a = \frac{1}{2}$ and (a + 1) (b + 1) = 2, then the radian measure of Arctan a + Arctan b equals :-

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{5}$

12. A man walks x miles due west, turns 150° to his left and walks 3 miles in the new direction. If he finshes at a point $\sqrt{3}$ miles from his starting point, then x is :-

(1) $\sqrt{3}$ (2) $3\sqrt{3}$ (3) 3/2 (4) 3

13. The lines L and K are symmetric to each other with respect to the line y = x. If the equation of line L is y = ax + b with $a \ne 0$ and $b \ne 0$. then the equation of K is y =

(1) $\frac{1}{a}x + b$ (2) $-\frac{1}{a}x + b$

(3) $-\frac{1}{a}x - \frac{b}{a}$ (4) $\frac{1}{a}x - \frac{b}{a}$

14. The number of pairs of positive integers (x,y) which satisfy the equation $x^2 + y^2 = x^3$ is :-

(1) 0 (2) 1

(3) 2 (4) not finite

2 MARKS

1. A point P lies in the same plane as a given square of side 1. Let the vertices of the square, taken counter clockwise, be A, B, C and D. Also let the distance from P and A, B and C respectively, be u, v and w. What is the greatest distance that P can be from D if $u^2 + v^2 = w^2$?

(1) $1+\sqrt{2}$ (2) $2\sqrt{2}$

(3) $2+\sqrt{2}$ (4) $3\sqrt{2}$

2. A point (x, y) is to be chosen in the coordinate plane so that it is equally distant from the x-axis, the y-axis and the line x + y = 2. Then x is

(1) $\sqrt{2}-1$

(2) 1/2

(3) $2-\sqrt{2}$

(4) not uniquely determined

3. Find the largest value of y/x for pairs of real numbers (x, y) which satisfy $(x-3)^2 + (y-3)^2 = 6$.

(1) $3+2\sqrt{2}$ (2) $2+\sqrt{3}$

(3) $3\sqrt{3}$ (4) 6

4. An arbitrary circle can intersect the graph of y = sin x in

(1) at most 2 points (2) at most 4 points

(3) at most 6 points (4) more than 16 points

5. Consider the graphs of $y = Ax^2$ and $y^2 + 3 = x^2 + 4y$, where A is a positive constant and x and y are real variables. In how many points do the two graphs intersect ?

(1) exactly 4

(2) exactly 2

(3) at least 1 but the number varies for different positive values of A

(4) 0 for at least one positive value of A

(5) none of these

6. It is desired to construct a right triangle in the coordinate plane so that its legs are parallel to the x and y axes and so that the medians to the midpoints of the legs lie on the lines y = 3x + 1 and y = mx + 2. The number of different constant m for which such a triangle exists is

(1) 0 (2) 1 (3) 2 (4) 3

7. Let c be constant. The simultaneous equations x – y = 2, cx + y = 3 have a solution (x,y) inside Quadrant I if and only if

(1) c = – 1 (2) c > –1

(3) c < 3/2 (4) –1 < c < 3/2

8. ABC is a triangle : A = (0,0), B = (36,15) and both the coordinates of C are integers. What is the minimum area ~ ABC can have ?

(1) 1/2 (2) 1 (3) 3/2 (4) 13/2

9. You plot weight (y) against height (x) for three of your friends and obtain the points $(x_1,y_1),(x_2,y_2)(x_3,y_3)$. If $x_1 < x_2 < x_3$ and $x_3 - x_2 = x_2 - x_1$, which of the following is necessarily the slope of the line which best fits the data ? "Best fits" means that the sum of the squares of the vertical distances from the data points to the line is smaller than for any other line.

(1) $\dfrac{y_3-y_1}{x_3-x_1}$ (2) $\dfrac{(y_2-y_1)-(y_3-y_2)}{x_3-x_1}$

(3) $\dfrac{2y_3-y_1-y_2}{2x_3-x_1-x_2}$ (4) $\dfrac{y_2-y_1}{x_2-x_1}+\dfrac{y_3-y_2}{x_3-x_2}$

10. If (a,b) and (c,d) are two points on the line whose equation is y = mx + k, then the distance between (a,b) and (c,d) in terms of a,c and m is -

(1) $|a-c|\sqrt{1+m^2}$ (2) $|a+c|\sqrt{1+m^2}$

(3) $\dfrac{|a-c|}{\sqrt{1+m^2}}$ (4) $|a-c|(1+m^2)$

11. There are two spherical balls of different sizes lying in two corners of a rectangular room, each touching two walls and the floor. If there is a point on each ball which is 5 inches from each wall which that ball touches and 10 inches from the floor, then the sum of the diameters of the balls is-

(1) 20 inches (2) 30 inches

(3) 40 inches (4) 60 inches

12. A vertical line divides the triangle with vertices (0,0), (1,1) and (9,1) in the xy - plane into two regions of equal area. The equation of the line is x =

(1) 2.5 (2) 3.0 (3) 3.5 (4) 4.0

1 MARK

1. $y = \frac{2}{3}x + 4$, slope $= \frac{2}{3}$

put $x = 0$ to find y – intercept

$\Rightarrow$ y – intercept = 4

$\Rightarrow$ y – intercept of required line = 8

$\Rightarrow$ slope of required line $= \frac{1}{2}\left(\frac{2}{3}\right) = \frac{1}{3}$

$y = mx + c$

$\Rightarrow y = \frac{1}{3}x + c$

$\because$ y – intercept = 8 $\Rightarrow c = 8$

$\Rightarrow y = \frac{1}{3}x + 8$ is the equation of our requried line.

Ans. (1)

2. Vertex of a parabola is $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

$\Rightarrow \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right) = (4, 2)$

$\Rightarrow \frac{-b}{2a} = 4$ and $c - \frac{b^2}{4a} = 2$

$\Rightarrow b = -8a$ and $c - \frac{(-8a)^2}{4a} = 2$

$\Rightarrow c - \frac{64a^2}{4a} = 2$

$\Rightarrow c - 16a = 2$

$\because$ (2, 0) lies on parabola $y = ax^2 + bx + c$

$\Rightarrow 0 = a(2)^2 + 2(b) + c$

$\Rightarrow 0 = 4a + 2b + c$

$\Rightarrow 0 = 4a + 2(-8a) + (16a + 2)$

$\Rightarrow 0 = 4a + 2 \Rightarrow a = -\frac{1}{2}$

$\Rightarrow b = -8a = -8\left(-\frac{1}{2}\right) = 4$

and $C = 16a + 2 = 16\left(-\frac{1}{2}\right) + 2 = -6$

$\Rightarrow abc = \left(-\frac{1}{2}\right)4\ (-6) = 12$

Ans. (4)

3. $\frac{y}{2} = -\frac{x}{3} + 1$

$\Rightarrow y = -\frac{2}{3}x + 2$

slope $= -\frac{2}{3}$

Ans. (2)

4. $T = \{t\}$

$\Rightarrow T \in [0,1)$

$(x - T)^2 + y^2 \le T^2$

radius of circle = T

Area $= \pi T^2$

$\Rightarrow 0 \le$ Area $< \pi$

$(\because\ 0 \le T^2 < 1)$

Ans. (2)

5. Let the original population be x

$\Rightarrow (x + 1200)\ \frac{89}{100} = x - 32$

$\Rightarrow 89x + 1200 \times 89 = 100x - 3200$

$\Rightarrow 11x = 1200 \times 89 + 3200$

$\Rightarrow 11x = 110000$

$\Rightarrow x = 10000$

6. $y = mx + 3$

$y = (2m - 1)x + 4$

For no solution

$\frac{1}{1} = \frac{m}{2m-1} \ne \frac{3}{4}$

$\Rightarrow 2m - 1 = m$ and $4m \neq 3(2m - 1)$

$\Rightarrow m = 1$ and $4m \neq 6m - 3$

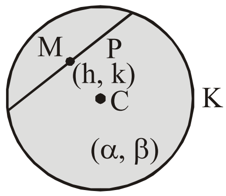
$\Rightarrow m = 1$ and $2m \neq 3$

$\Rightarrow m \neq \dfrac{3}{2}$

$\Rightarrow$ for m = 1 there is no solution because at m = 1 these two equation forms two parallel lines.

$\Rightarrow$ for all $m \neq 1$ there is at least one solution.

7.



Let M(b, k) be the mid points of all the chords. Let the Co-ordinates of given point $P(x_1, y_1)$ and $C(\alpha, \beta)$

$\because CM \perp MP$

m_1 = slope of CM = $\dfrac{k-\beta}{h-\alpha}$

m_2 = slope of MP = $\dfrac{k-y_1}{h-x_1}$

$m_1 \cdot m_2 = -1$ ($\because CM \perp MP$)

$$\left(\frac{k-\beta}{h-\alpha}\right)\left(\frac{k-y_1}{h-x_1}\right) = -1$$

$\Rightarrow (k - \beta)(k - y_1) = -(h - \alpha)(h - x_1)$

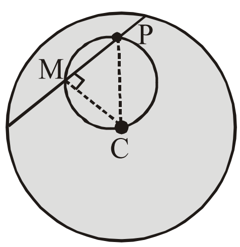
$\Rightarrow (k - \beta)(k - y_1) + (h - \alpha)(h - x_1) = 0$

replacing (h, k) by (x, y) to find its locus :-

$\Rightarrow (y - \beta)(y - y_1) + (x - \alpha)(x - x_1) = 0$

This is a diametric from of a circle with (α, β) and (x_1, y_1) as end points of the diameter.

Alternate solution :-



As, $CM \perp MP \Rightarrow$ M lies on a circle with CP as its diamter.

8. $y = 2xy$

$\Rightarrow y - 2xy = 0 \Rightarrow y(1 - 2x) = 0$

This equation is satisfied when either

$y = 0$ or $1 - 2x = 0$

$\Rightarrow y = 0$ or $x = \dfrac{1}{2}$

Also, given $x = x^2 + y^2$

for $y = 0$, $x = x^2 + 0 \Rightarrow x^2 - x = 0$

$\Rightarrow x(x - 1) = 0$

$\Rightarrow x = 0$ or 1

$\Rightarrow$ (0, 0) and (0, 1) satisfies both the equations.

for $x = \dfrac{1}{2}$, $\quad x = x^2 + y^2$

$$\Rightarrow \frac{1}{2} = \left(\frac{1}{2}\right)^2 + y^2$$

$$\Rightarrow y^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\Rightarrow y = \pm \frac{1}{2}$$

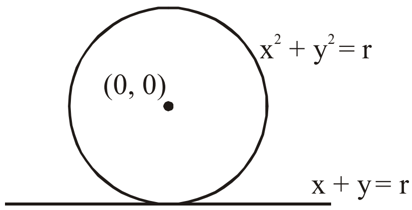
$\Rightarrow \left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2}, -\dfrac{1}{2}\right)$ also satisfies both the equations.

$\Rightarrow$ Total solutions = (0, 0), (0, 1)

$$\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, -\frac{1}{2}\right)$$

$\Rightarrow$ Number of solutions = 4

Ans. (4)

9. 

$x^2 + y^2 = r$ is a circle with centre (0, 0) and radius $= \sqrt{r}$

$\because$ x + y = r is tangent to this circle

$\Rightarrow$ Distance from centre = radius of circle.

$\Rightarrow \dfrac{|0+0-r|}{\sqrt{1^2+1^2}} = \sqrt{r}$

$\Rightarrow$ r $= \sqrt{2}.\sqrt{r}$

$\Rightarrow \sqrt{r} = \sqrt{2} \Rightarrow r = 2$

10. $\sin x + \cos x = \dfrac{1}{5}$

$\Rightarrow \cos x = \dfrac{1}{5} - \sin x$

$\Rightarrow 5\cos x = 1 - 5\sin x$

Squaring both sides :-

$(5\cos x)^2 = (1 - 5\sin x)^2$

$\Rightarrow 25(1 - \sin^2 x) = 25\sin^2 x - 10\sin x + 1$

$\Rightarrow 50\sin^2 x - 10\sin x - 24 = 0$

$\Rightarrow 50\sin^2 x - 40\sin x + 30\sin x - 24 = 0$

$\Rightarrow 10\sin x(5\sin x - 4) + 6(5\sin x - 4) = 0$

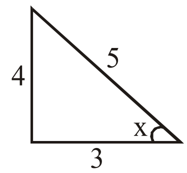
$\Rightarrow (10\sin x + 6)(5\sin x - 4) = 0$

$\Rightarrow \sin x = \dfrac{-6}{10}$ or $\dfrac{4}{5}$

$\Rightarrow \sin x = \mathbf{\dfrac{-3}{5}}$ **or** $\mathbf{\dfrac{4}{5}}$

rejected because $x \in [0, \pi)$

$\Rightarrow \sin x = \dfrac{4}{5}$



If $x \in \left(0, \dfrac{\pi}{2}\right)$

$\cos x = \dfrac{3}{5}$

$\Rightarrow \sin x + \cos x = \dfrac{4}{5} + \dfrac{3}{5} = \dfrac{8}{5} \neq \dfrac{1}{5}$

$\Rightarrow$ x does not belongs to first quadrant.

If $x \in \left(\dfrac{\pi}{2}, \pi\right) \Rightarrow \cos x = \dfrac{-3}{5}$

$\Rightarrow \sin x + \cos x = \dfrac{4}{5} - \dfrac{3}{5} = \dfrac{1}{5}$

Hence, x is an angle of second quadrant

$\Rightarrow \tan x = \dfrac{-4}{3}$

11. $a = \dfrac{1}{2}$ and $(a + 1)(b + 1) = 2$

$\Rightarrow \left(\dfrac{1}{2} + 1\right)(b + 1) = 2$

$\Rightarrow \dfrac{3}{2}(b+1) = 2 \Rightarrow b + 1 = \dfrac{4}{3}$

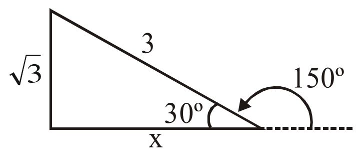
$\Rightarrow b = \dfrac{4}{3} - 1 = \dfrac{1}{3}$

$\Rightarrow \tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{3}\right)$

$= \tan^{-1}\left(\dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}.\frac{1}{3}}\right)$

$= \tan^{-1}\left(\dfrac{5/6}{5/6}\right) = \tan^{-1}(1)$

$= \dfrac{\pi}{4}$

12. 

By cosine rule :-

$$\cos 30^\circ = \frac{x^2+3^2-(\sqrt{3})^2}{2.x.3}$$

$$\Rightarrow \frac{\sqrt{3}}{2} \Rightarrow \frac{x^2+9-3}{6x}$$

$$\Rightarrow 3\sqrt{3}x = x^2+6$$

$$\Rightarrow x^2-3\sqrt{3}x+6=0$$

$$x = \frac{3\sqrt{3}\pm\sqrt{(3\sqrt{3})^2-4.6}}{2(1)}$$

$$\Rightarrow \frac{3\sqrt{3}\pm\sqrt{27-24}}{2}$$

$$x = \frac{3\sqrt{3}\pm\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{4\sqrt{3}}{2} \text{ or } \frac{2\sqrt{3}}{2}$$

$$\Rightarrow x = 2\sqrt{3} \text{ or } \sqrt{3}$$

$\Rightarrow$ Answer - (1)

13. $\because$ L and K are symmetric with respect to line $y = x \Rightarrow$ interchange $x \leftrightarrow y$ in equation of line L to find equation of K.

Given - L : $y = ax + b$

$\Rightarrow$ K : $x = ay + b$

$\Rightarrow$ $ay = x - b$

$$\Rightarrow y = \frac{1}{a}x - \frac{b}{a}$$

Ans. (4)

14. $y^2 = x^3 - x^2$

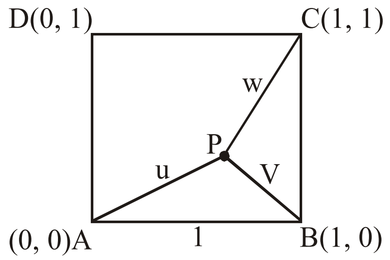
$\Rightarrow y^2 = x^2(x - 1)$

$\Rightarrow x > 1$ (Because L.H.S = R.H.S ≥ 0)

$\Rightarrow$ There are infinite integral values of x greater than 1 at which y is also an integer.

2 MARK

1.



Given, $u^2 + v^2 = w^2$

Let the coordinates of P be (α, β)

AP = $u = \sqrt{\alpha^2 + \beta^2}$

PB = $v = \sqrt{(\alpha - 1)^2 + \beta^2}$

PC = $w = \sqrt{(\alpha - 1)^2 + (\beta - 1)^2}$

$\because u^2 + v^2 = \omega^2$

$\alpha^2 + \beta^2 + (\alpha - 1)^2 + \beta^2 = (\alpha - 1)^2 + (\beta - 1)^2$

$\Rightarrow \alpha^2 + 2\beta^2 = \beta^2 - 2\beta + 1$

$\Rightarrow \alpha^2 + \beta^2 + 2\beta = 1$

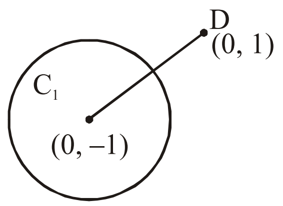
$\Rightarrow \alpha^2 + \beta^2 + 2\beta + 1 = 1 + 1$

$\Rightarrow \alpha^2 + (\beta + 1)^2 = 2$

replacing (α, β) by (x, y)

$\Rightarrow x^2 + (y + 1)^2 = 2$

$\Rightarrow$ P lies on a circle with centre $C_1(0, -1)$ and radius = $\sqrt{2}$

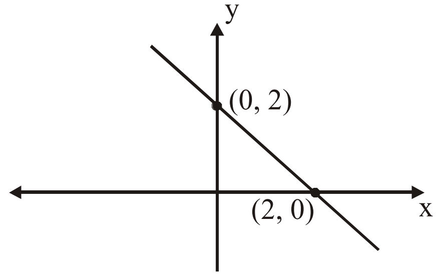


Maximum distance of P from D = $C_1D + r$

$= \sqrt{0^2 + (2)^2} + \sqrt{2}$

$= 2 + \sqrt{2}$

2.



Let the coordinates of point be (r, r)

$\frac{|r + r - 2|}{\sqrt{1^2 + 1^2}} = r$

$2\,|r - 1| = \sqrt{2}r$

$4(r^2 - 2r + 1) = 2r^2$

$\Rightarrow 2r^2 - 4r + 2 = r^2$

$\Rightarrow r^2 - 4r + 2 = 0$

$\Rightarrow r = \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm \sqrt{8}}{2}$

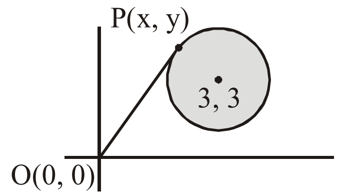
$\Rightarrow r = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$

$\Rightarrow$ Coordinates of P are

$(2 + \sqrt{2}, 2 + \sqrt{2})$ or $(2 - \sqrt{2}, 2 - \sqrt{2})$

Hence, Answer (4)

3.



Let P(x, y) lies on the circle.

$\frac{y - 0}{x - 0}$ is the slope of line OP.

$\Rightarrow$ For $\frac{y}{x}$ to be maximum OP should be a tangent to the given circle.

Let the equation of OP be y = mx

Condition for tangency :-

Distance from centre (3, 3) = radius = $\sqrt{6}$

$\frac{|3 - 3m|}{\sqrt{1 + m^2}} = \sqrt{6}$

$9(1 - m)^2 = 6(1 + m^2)$

$3(m^2 - 2m + 1) = 2(1 + m^2)$

$m^2 - 6m + 1 = 0$

$\Rightarrow m = \frac{6 \pm \sqrt{36 - 4}}{2}$

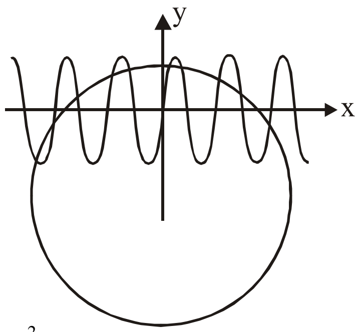
$m = \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm 4\sqrt{2}}{2}$

$m = 3 + 2\sqrt{2}$ or $3 - 2\sqrt{2}$

Maximum m = $3 + 2\sqrt{2}$

4. As the radius of the circle becomes large it will cut the graph of y = sinx at infinite number of points.

$\Rightarrow$ Infinite Solutions



5. $y = Ax^2$

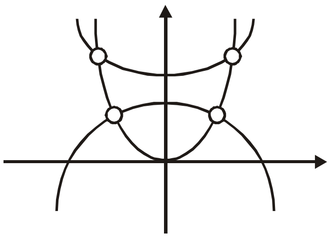
$y^2 + 3 = x^2 + 4y$

$y^2 - 4y + 3 + 1 = x^2 + 1$

$\Rightarrow (y - 2)^2 = x^2 + 1$

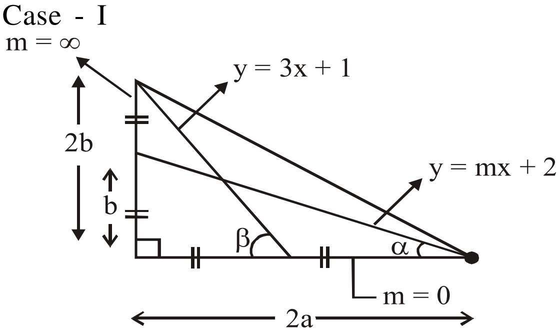
$\Rightarrow (y - 2)^2 - x^2 = 1$

This equation represents a hyperbola.



$\Rightarrow$ Number of solutions = 4

6. Case - I



Let the lengths of the legs be 2a and 2b.

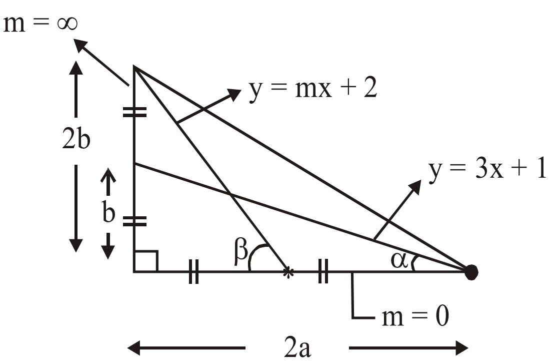
$\Rightarrow \tan\alpha = \frac{b}{2a} = m \Rightarrow \frac{b}{a} = 2m$

and $\tan\beta = \frac{2b}{a} = 3$

$\Rightarrow 2(2m) = 3$

$\Rightarrow m = \frac{3}{4}$

Case - II



Here, $\tan\alpha = \frac{b}{2a} = 3 \Rightarrow \frac{b}{a} = 6$

and $\tan\beta = \frac{2b}{a} = m$

$\Rightarrow 2(6) = m \Rightarrow m = 12$

Hence, two possible values of m exists.

7. $x - y = 2$ .... (1)

$cx + y = 3$ .... (2)

On solving these equation :-

$c(2 + y) + y = 3$

$\Rightarrow 2c + cy + y = 3$

$\Rightarrow y(c + 1) = 3 - 2c \Rightarrow y = \frac{3-2c}{1+c} > 0$

$\Rightarrow \frac{2c-3}{1+c} < 0$

$\Rightarrow c \in \left(-1, \frac{3}{2}\right)$

$\because x = 2 + y = 2 + \frac{3-2c}{1+c}$

$\Rightarrow x = \frac{2(1+c)+3-2c}{1+c}$

$\Rightarrow x = \frac{5}{1+c} > 0$

$\Rightarrow c > -1$

$\Rightarrow c > -1$ and $c \in (-1, 3/2)$

On taking intersection :-

$\Rightarrow c \in \left(-1, \frac{3}{2}\right)$

8. Let the co-ordinates of C be (x, y)

$\Rightarrow \text{Area} = \frac{1}{2}\left|\begin{vmatrix} x & y & 1 \\ 0 & 0 & 1 \\ 36 & 15 & 1 \end{vmatrix}\right|$

$= \frac{1}{2}\left|(-15x + 36y)\right|$

$= \frac{3}{2}\left|(-5x + 12y)\right|$

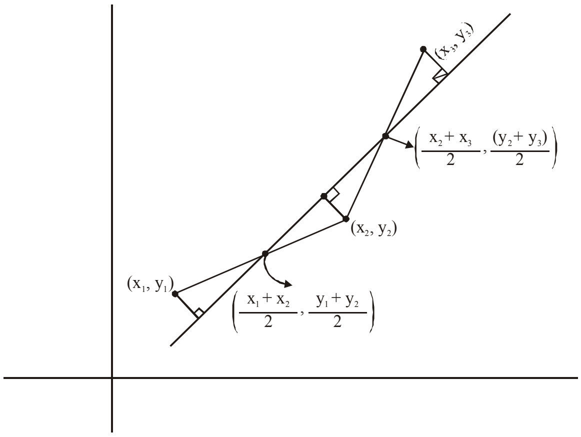
$\because$ (x, y) are integers

$\Rightarrow$ Minimum value of $|-5x + 12y|$

Can be 1 at x = 5 and y = 2

$\Rightarrow$ Minimum area = $\frac{3}{2}$

9.



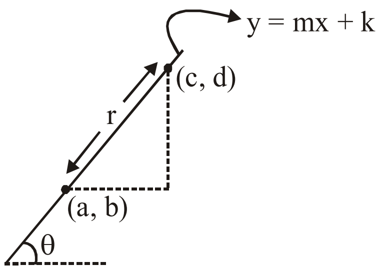
$x_3 - x_2 = x_2 - x_1$

$x_2 = \dfrac{x_1 + x_3}{2}$

$$m = \frac{\dfrac{y_2+y_3}{2} - \dfrac{y_1+y_2}{2}}{\dfrac{x_2+x_3}{2} - \dfrac{x_1+x_2}{2}}$$

$m = \dfrac{y_1 - y_3}{x_1 - x_3}$ Answer - (1)

10.



Let the distance be r.

$c = a \pm r\cos\theta \Rightarrow \pm r\cos\theta = c - a$

$\Rightarrow \pm r = (c - a)\sec\theta$

$\Rightarrow \pm r = (c - a)\sqrt{1+\tan^2\theta}$

$\Rightarrow r = |c - a|\sqrt{1+m^2}$

$(\because m = \tan\theta)$ **Answer :- (1)**

11. Let us take the sphere into a 3 - D Cartesian plane where the corner is represented by the origin O(0, 0, 0), where the 2 walls represent the x – z plane and y – z plane and the floor is represented by the x – y plane. Let r be the radius of the sphere.

Centre of the sphere = (r, r, r)

$\Rightarrow$ Equation of the sphere =

$(x - r)^2 + (y - r)^2 + (z - r)^2 = r^2$

Now, given that a point on the sphere is 5, 5 10 units from the 2 walls and floor. Hence, (5, 5, 10) satisfies the equation of sphere.

$\Rightarrow (5 - r)^2 + (5 - r)^2 + (10 - r)^2 = r^2$

$\Rightarrow 2(r^2 - 10r + 25) + (r^2 - 20r + 100) = r^2$

$\Rightarrow 2r^2 - 40r + 150 = 0$

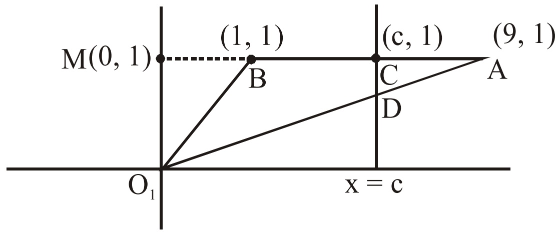
$\Rightarrow r^2 - 20r + 75 = 0$

$\Rightarrow (r - 15)(r - 5) = 0$

$\Rightarrow r - 5, 15$

$\therefore$ Sum of diameters = (5 + 15)2 = 40

12.



$\because \Delta OMA \sim \Delta DCA$

$\Rightarrow \dfrac{OM}{DC} = \dfrac{MA}{CA}$

$\Rightarrow \dfrac{1}{DC} = \dfrac{9}{9-c} \Rightarrow DC = \dfrac{9-c}{9}$

Given Area of OAB = 2 Area of ACD

$\Rightarrow \dfrac{1}{2} \times 8 \times 1 = 2\left(\dfrac{1}{2}(9-c)\dfrac{(9-c)}{9}\right)$

$(\because$ Area = $\dfrac{1}{2} \times \text{base} \times \text{height})$

$\Rightarrow 4 = \dfrac{(9-c)^2}{9}$

$\Rightarrow c^2 - 18c + 81 = 36$

$\Rightarrow c^2 - 18c + 45 = 0$

$\Rightarrow (c - 15)(c - 3) = 0$

$\Rightarrow c = 3$ or 15 $\rightarrow$ Rejected

$\Rightarrow c = 3$