28

Properties of Triangles

TOPIC 1 Properties of Triangle, Solutions of Triangles, Inscribed & Enscribed Circles, Regular Polygons

1. Let A(3, 0, –1), B(2, 10, 6) and C (1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then cos (∠GOA) (O being the origin) is equal to : **[April 10, 2019 (I)]**
(a) $\frac{1}{2\sqrt{15}}$ (b) $\frac{1}{\sqrt{15}}$
(c) $\frac{1}{6\sqrt{10}}$ (d) $\frac{1}{\sqrt{30}}$

2. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is : **[April 10, 2019 (II)]**
(a) $\frac{2}{\sqrt{3}}$ (b) $4\sqrt{3}$
(c) $2\sqrt{3}$ (d) $\frac{4}{\sqrt{3}}$

3. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is : **[April 08, 2019 (II)]**
(a) 5 : 9 : 13 (b) 4 : 5 : 6
(c) 3 : 4 : 5 (d) 5 : 6 : 7

4. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. if $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is : **[Jan. 11, 2019 (I)]**
(a) $\frac{3}{2}y$ (b) $\frac{c}{\sqrt{3}}$
(c) $\frac{c}{3}$ (d) $\frac{y}{\sqrt{3}}$

5. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value : **[Jan. 11, 2019 (II)]**
(a) (7, 19, 25) (b) (3, 4, 5)
(c) (5, 12, 13) (d) (19, 7, 25)

6. With the usual notation, in ΔABC, if ∠A + ∠B = 120°, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio ∠A : ∠B, is: **[Jan. 10, 2019 (II)]**
(a) 7 : 1 (b) 5 : 3
(c) 9 : 7 (d) 3 : 1

7. In a ΔABC, $\frac{a}{b} = 2+\sqrt{3}$ and ∠C = 60°. Then the ordered pair (∠A, ∠B) is equal to : **[Online April 10, 2015]**
(a) (45°, 75°) (b) (105°, 15°)
(c) (15°, 105°) (d) (75°, 45°)

8. ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ADB = θ, BC = p and CD = q, then AB is equal to : **[2013]**
(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (b) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$
(c) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

9. If in a triangle ABC, $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$, then cosA is equal to **[2012]**
(a) 5/7 (b) 1/5
(c) 35/19 (d) 19/35

10. In a ΔPQR, If 3 sin P + 4 cos Q = 6 and 4 sin Q + 3 cos P = 1, then the angle R is equal to : **[2012]**
(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

11. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is **[2010]**

(a) There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$

(b) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$

(c) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$

(d) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$

12. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P, then sin A, sin B, sin C are in **[2005]**

(a) G.P. (b) A. P.
(c) A.P-G.P. (d) H. P

13. In a triangle ABC, let $\angle C=\frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2(r+R)$ equals **[2005]**

(a) $b+c$ (b) $a+b$
(c) $a+b+c$ (d) $c+a$

14. The sides of a triangle are $\sin\alpha, \cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0<\alpha<\frac{\pi}{2}$. Then the greatest angle of the triangle is **[2004]**

(a) 150° (b) 90°
(c) 120° (d) 60°

15. If in a ΔABC $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$, then the sides a, b and c **[2003]**

(a) satisfy $a+b=c$ (b) are in A.P
(c) are in G.P (d) are in H.P

16. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB=\frac{\pi}{6}$ and $\angle ABE=\frac{\pi}{3}$, then the area of the ΔABC is **[2003]**

(a) $\frac{64}{3}$ (b) $\frac{8}{3}$
(c) $\frac{16}{3}$ (d) $\frac{32}{3\sqrt{3}}$

17. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a , is **[2003]**

(a) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{n}\right)$

(c) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{2n}\right)$.

18. In a triangle with sides $a, b, c, r_1>r_2>r_3$ (which are the ex-radii) then **[2002]**

(a) $a>b>c$ (b) $a<b<c$
(c) $a>b$ and $b<c$ (d) $a<b$ and $b>c$

19. The sides of a triangle are $3x+4y$, $4x+3y$ and $5x+5y$ where $x, y>0$ then the triangle is **[2002]**

(a) right angled (b) obtuse angled
(c) equilateral (d) none of these

TOPIC 2 Heights & Distances

20. A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line $x=1$ at the point A. The ray gets reflected on the line $x=1$ and meets x-axis at the point B. Then, the line AB passes through the point: **[Sep. 06, 2020 (I)]**

(a) $\left(3,-\frac{1}{\sqrt{3}}\right)$ (b) $\left(4,-\frac{\sqrt{3}}{2}\right)$

(c) $(3,-\sqrt{3})$ (d) $(4,-\sqrt{3})$

21. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45º. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30º to the horizontal plane, the angle of elevation of the top of the hill becomes 75º. Then the height of the hill (in meters) is ______. **[Sep. 06, 2020 (I)]**

22. The angle of elevation of the summit of a mountain from a point on the ground is 45º. After climbing up on km towards the summit at an inclination of 30º from the ground, the angle of elevation of the summit is found to be 60º. Then the height (in km) of the summit from the ground is: **[Sep. 06, 2020 (II)]**

(a) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(c) $\frac{1}{\sqrt{3}-1}$ (d) $\frac{1}{\sqrt{3}+1}$

23. Two vertical poles $AB = 15$ m and $CD = 10$ m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is : **[Sep. 04, 2020 (I)]**

(a) 20/3 (b) 5
(c) 10/3 (d) 6

24. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to : **[Sep. 04, 2020 (II)]**

(a) 100 (b) $200\sqrt{3}$
(c) 400 (d) $400\sqrt{3}$

25. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is : **[April 10, 2019 (I)]**

(a) $\frac{100}{3\sqrt{3}}$ (b) $10\sqrt{5}$
(c) 20 (d) 25

26. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is: **[Jan. 12, 2019 (II)]**

(a) 60 (b) 50
(c) 45 (d) 42

27. Consider a triangular plot ABC with sides AB = 7 m, BC = 5 m and CA = 6 m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is: **[Jan. 10, 2019 (I)]**

(a) $\frac{3}{2}\sqrt{21}$ (b) $\frac{2}{3}\sqrt{21}$
(c) $2\sqrt{21}$ (d) $7\sqrt{3}$

28. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is : **[2018]**

(a) 50 (b) $100\sqrt{3}$
(c) $50\sqrt{2}$ (d) 100

29. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 min, for the angle of depression of the car to change from 30° to 45°, then after this, the time taken (in min) by the car to reach the foot of the tower, is. **[Online April 16, 2018]**

(a) $9(1+\sqrt{3})$ (b) $\frac{9}{2}(\sqrt{3}-1)$
(c) $18(1+\sqrt{3})$ (d) $18(\sqrt{3}-1)$

30. An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}km$ above it, is observed at an elevation of 60° from a point on the ground. If, after five seconds, its elevation from the same point, is 30°, then the speed (in km/hr) of the aeroplane is **[Online April 15, 2018]**

(a) 1500 (b) 750
(c) 720 (d) 1440

31. A tower T_1 of height 60 m is located exactly opposite to a tower T_2 of height 80 m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is **[Online April 15, 2018]**

(a) $20\sqrt{2}$ (b) $10\sqrt{2}$
(c) $10\sqrt{3}$ (d) $20\sqrt{3}$

32. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to : **[2017]**

(a) $\frac{4}{9}$ (b) $\frac{6}{7}$
(c) $\frac{1}{4}$ (d) $\frac{2}{9}$

33. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is: **[2016]**

(a) 20 (b) 5
(c) 6 (d) 10

34. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is : **[Online April 10, 2016]**

(a) 108 (b) $36\sqrt{3}$
(c) $54\sqrt{3}$ (d) 54

(which are the ex-radii) then **[2002]**

35. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is : **[2015]**

(a) $1:\sqrt{3}$ (b) $2:3$
(c) $\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}$

36. Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation at a point O on this line and all the poles are on the same side of O. If the height of the longest pole is 'h' and the distance of the foot of the smallest pole from O is 'a'; then the distance between two consecutive poles, is : **[Online April 11, 2015]**

(a) $\frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$ (b) $\frac{h\sin\alpha + a\cos\alpha}{9\sin\alpha}$

(c) $\frac{h\cos\alpha - a\sin\alpha}{9\cos\alpha}$ (d) $\frac{h\sin\alpha - a\cos\alpha}{9\cos\alpha}$

37. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is $45°$. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to $30°$. Then the speed (in m/s) of the bird is **[2014]**

(a) $20\sqrt{2}$ (b) $20\left(\sqrt{3}-1\right)$

(c) $40\left(\sqrt{2}-1\right)$ (d) $40\left(\sqrt{3}-\sqrt{2}\right)$

38. The angle of elevation of the top of a vertical tower from a point P on the horizontal ground was observed to be α. After moving a distance 2 metres from P towards the foot of the tower, the angle of elevation changes to β. Then the height (in metres) of the tower is: **[Online April 11, 2014]**

(a) $\frac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$ (b) $\frac{\sin\alpha\sin\beta}{\cos(\beta-\alpha)}$

(c) $\frac{2\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$ (d) $\frac{\cos(\beta-\alpha)}{\sin\alpha\sin\beta}$

39. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD =7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is **[2008]**

(a) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}m$ (b) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

(c) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)m$ (d) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}m$

40. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB\ (=a)$ subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A and B is 30°. The height of the tower is **[2007]**

(a) $a/\sqrt{3}$ (b) $a\sqrt{3}$

(c) $2a/\sqrt{3}$ (d) $2a\sqrt{3}$

41. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is **[2004]**

(a) 60m (b) 30 m

(c) 40m (d) 20 m

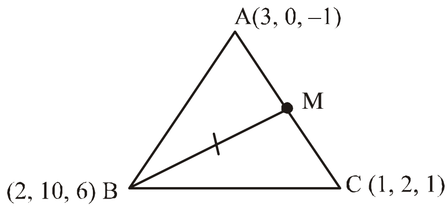
42. The upper $\frac{3}{4}$ th portion of a vertical pole subtends an angle $\tan^{-1}\frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is **[2003]**

(a) 80 m (b) 20 m

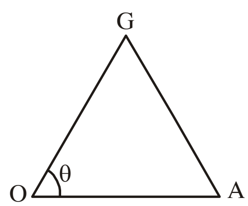
(c) 40 m (d) 60 m.

Hints & Solutions

1. **(b)** G is the centroid of ΔABC.



$$\Rightarrow G \equiv \left(\frac{3+2+1}{3}, \frac{0+10+2}{3}, \frac{-1+6+1}{3}\right) \equiv (2, 4, 2)$$



$OG = \sqrt{4+16+4}$, $OA = \sqrt{9+1}$, $AG = \sqrt{1+16+9}$

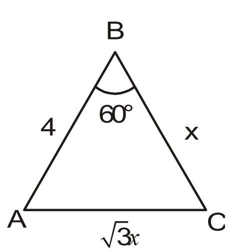
$$\therefore \cos\theta = \frac{(OG)^2 + (OA)^2 - (AG)^2}{2(OG)(OA)} = \frac{24+10-26}{2\sqrt{24}\sqrt{10}}$$

$$= \frac{8}{2\sqrt{8\times3\times2\times5}} = \frac{4}{4\sqrt{15}} = \frac{1}{\sqrt{15}}$$

2. **(c)** Given that A, B, C, are in A.P. $\Rightarrow 2B = A + C$

Now, $A + B + C = \pi \Rightarrow B = \dfrac{\pi}{3}$

Area $= \dfrac{1}{2}(4x)\sin 60° = \sqrt{3}x$



Now $\cos 60° = \dfrac{16 + x^2 - 3x^2}{8x}$

$\Rightarrow 4x = 16 - 2x^2 \Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow x = 2$ [$\because x$ can't be negative]

Hence, area $= 2\sqrt{3}$ sq. cm

3. **(b)** Let the sides of triangle are $a > b > c$ where

Given $A = 2C$

$\because A + B + C = \pi$ and $A = 2C$

$\Rightarrow B = \pi - 3C$ ...(i)

$\because a, b, c$ are in A.P. $\Rightarrow a + c = 2b$

$\Rightarrow \sin A + \sin C = 2 \sin B$ ...(ii)

$\Rightarrow \sin A = \sin(2C)$ and $\sin B = \sin 3C$

From (ii),

$\sin 2C + \sin C = 2 \sin 3C$

$\Rightarrow (2\cos C + 1)\sin C = 2 \sin C\,(3 - 4\sin^2 C)$

$\Rightarrow 2\cos C + 1 = 6 - 8(1 - \cos^2 C)$

$\Rightarrow 8\cos^2 C - 2\cos C - 3 = 0$

$\Rightarrow \cos C = \dfrac{3}{4}$ or $\cos C = -\dfrac{1}{2}$

$\because$ C is acute angle

$\Rightarrow \cos C = \dfrac{3}{4} \Rightarrow \sin C = \dfrac{\sqrt{7}}{4}$

and $\sin A = 2 \sin C \cos C = 2 \times \dfrac{\sqrt{7}}{4} \times \dfrac{3}{4} = \dfrac{3\sqrt{7}}{8}$

$\sin B = \dfrac{3\sqrt{7}}{4} - \dfrac{4\sqrt{7}}{4} \times \dfrac{7}{16} = \dfrac{5\sqrt{7}}{16}$

$\Rightarrow \sin A : \sin B : \sin C :: a : b : c$ is $6 : 5 : 4$

4. **(b)** Let two sides of triangle are a and b.

$a + b = x$

$ab = y$

$x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow (a + b - c)(a + b + c) = ab$

$\Rightarrow 2(s - c)(2s) = ab$

$\Rightarrow 4s(s - c) = ab$

$\Rightarrow \dfrac{s(s-c)}{ab} = \dfrac{1}{4}$

$\Rightarrow \cos^2 \dfrac{c}{2} = \dfrac{1}{4}$

$\Rightarrow \cos c = -\dfrac{1}{2} \Rightarrow c = 120°$

$\therefore$ Area of triangle is,

$\Delta = \dfrac{1}{2}ab(\sin 120°) = \dfrac{\sqrt{3}}{4}ab$

$\because \quad R=\frac{abc}{4\Delta}$

$\because \quad \mathrm{R}=\frac{abc}{\sqrt{3}\,ab}=\frac{c}{\sqrt{3}}$

5. **(a)** Let $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}=k$ (Say).

$\therefore \quad b+c=11k,\ c+a=12k,\ a+b=13k$

$\therefore \quad a+b+c=18k$

$\therefore \quad a=7k,\ b=6k$ and $c=5k$

$\therefore \quad \cos A=\frac{36k^2+25k^2-49k^2}{2.30k^2}=\frac{1}{5}$

and $\cos B=\frac{49k^2+25k^2-36k^2}{2.35k^2}=\frac{19}{35}$

and $\cos C=\frac{49k^2+36k^2-25k^2}{2.42k^2}=\frac{5}{7}$

$\therefore \quad \cos A:\cos B:\cos C=7:19:25$

$\therefore \quad \frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$

Hence, required ordered triplet is (7, 19, 25).

6. **(a)** $\angle A+\angle B=120°$ ...(1)

$\Rightarrow \quad \angle C=180°-120°=60°$

$\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$

$=\frac{2}{2\sqrt{3}}(\cot 30°)=\frac{1}{\sqrt{3}}\times\sqrt{3}=1$

$\Rightarrow \quad \frac{\angle A-\angle B}{2}=\frac{\pi}{4}$ ($\angle$ is angle)

$\Rightarrow \quad \angle A-\angle B=90°$ ...(2)

From eqn (1) and (2)

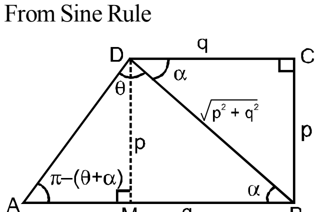
$\angle A=105°,\ \angle B=15°$

Then, $\angle A:\angle B=7:1$

7. **(b)** $\frac{\sin \mathrm{A}}{\sin \mathrm{B}}=2+\sqrt{3}$

$\frac{\sin(105°)}{\sin(15°)}=2+\sqrt{3} \quad \frac{\cos 15°}{\sin 15°}=2+\sqrt{3}$

8. **(a)** From Sine Rule



$\frac{\mathrm{AB}}{\sin\theta}=\frac{\sqrt{p^2+q^2}}{\sin(\pi-(\theta+\alpha))}$

$\mathrm{AB}=\frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha}$

$=\frac{(p^2+q^2)\sin\theta}{q\sin\theta+p\cos\theta}$

$\left(\because \cos\alpha=\frac{q}{\sqrt{p^2+q^2}}\text{ and }\sin\alpha=\frac{\mathrm{p}}{\sqrt{\mathrm{p}^2+q^2}}\right)$

9. **(b)** In a triangle ABC.

Let $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}=K$

$\Rightarrow b+c=11K,\ c+a=12K,\ a+b=13K$

On solving these equations, we get

$a=7K,\ b=6K,\ c=5K$

Now we know,

$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{36K^2+25K^2-49K^2}{2(6K)(5K)}=\frac{1}{5}$

10. **(b)** Given that $3\sin P+4\cos Q=6$ ...(i)

$4\sin Q+3\cos P=1$ ...(ii)

Squaring and adding (i) & (ii) we get

$9\sin^2 P+16\cos^2 Q+24\sin P\cos Q$

$+16\sin^2 Q+9\cos^2 P+24\sin Q\cos P$

$=36+1=37$

$\Rightarrow \quad 9(\sin^2 P+\cos^2 P)+16(\sin^2 Q+\cos^2 Q)$

$+24(\sin P\cos Q+\cos P\sin Q)=37$

$\Rightarrow \quad 9+16+24\sin(P+Q)=37$

$[\because \sin^2\theta+\cos^2\theta=1$ and $\sin A\cos B+\cos A\sin B=\sin(A+B)]$

$\Rightarrow \quad \sin(P+Q)=\frac{1}{2}$

$\Rightarrow \quad P+Q=\frac{\pi}{6}\ or\ \frac{5\pi}{6}$

$\Rightarrow \quad R=\frac{5\pi}{6}$ or $\frac{\pi}{6}$ $\quad (\because P+Q+R=\pi)$

If $\quad R=\frac{5\pi}{6}$ then $0<Q,\ P<\frac{\pi}{6}$

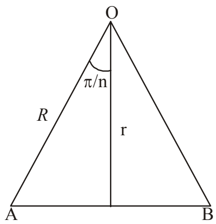
$\Rightarrow \quad \cos Q<1$ and $\sin P<\frac{1}{2}$

$\Rightarrow \quad 3\sin P+4\cos Q<\frac{11}{2}$

But given that $3\sin P+4\sin Q=6$

So, $R=\frac{\pi}{6}$

11. **(b)** Let O is centre of polygon of n sides and AB is one of the side, then by figure



$$\cos\frac{\pi}{n}=\frac{r}{R}$$

$$\Rightarrow \quad \frac{r}{R}=\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}$$

for $n = 3, 4, 6$ respectively.

12. **(b)** Let altitudes from A, B and C be p_1, p_2 and p_3 resp.

$$\therefore \quad \Delta=\frac{1}{2}p_1a=\frac{1}{2}p_2b=\frac{1}{2}p_3b$$

Given that, p_1, p_2, p_3, are in H.P.

$$\Rightarrow \quad \frac{2\Delta}{a},\frac{2\Delta}{b},\frac{2\Delta}{c} \text{ are in H.P.}$$

$$\Rightarrow \quad \frac{1}{a},\frac{1}{b},\frac{1}{c} \text{ are in H.P.}$$

$\Rightarrow$ a, b, c are in A.P.

By sine formula

$\Rightarrow$ $K \sin A, K \sin B, K \sin C$ are in AP

$\Rightarrow$ $\sin A, \sin B, \sin C$ are in A.P.

13. **(b)** We know that for the circle circumscribing a right triangle, hypotenutse is the diameter

$\because \quad \angle C = 90°$

$$\therefore \quad 2R = c \Rightarrow R=\frac{c}{2}$$

$$\text{also } r=\frac{\Delta}{s}=\frac{\frac{1}{2}\times a\times b}{\frac{a+b+c}{2}}$$

$$\Rightarrow r=\frac{ab}{a+b+c}$$

$$\therefore \quad 2r+2R=\frac{2ab}{a+b+c}+c=\frac{2ab+ac+bc+c^2}{a+b+c}$$

$$=\frac{2ab+ac+bc+a^2+b^2}{a+b+c} \qquad (\because c^2=a^2+b^2)$$

$$=\frac{(a+b)^2+(a+b)c}{a+b+c}=(a+b)$$

14. **(c)** Let $a = \sin\alpha$, $b = \cos\alpha$ and

$$c=\sqrt{1+\sin\alpha\cos\alpha}$$

Clearly a and $b < 1$ but $c > 1$ as $\sin\alpha > 0$ and $\cos\alpha > 0$

$\therefore$ c is the greatest side and greatest angle is C.

$$\text{We know that, } \cos C=\frac{a^2+b^2-c^2}{2ab}$$

$$=\frac{\sin^2\alpha+\cos^2\alpha-1-\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}=-\frac{1}{2}$$

$\therefore C = 120°$

15. **(b)** Given that, $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$

$a[\cos C + 1] + c[\cos A + 1] = 3b$

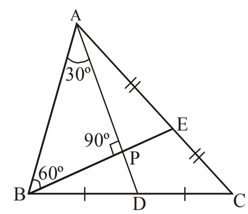
$(a + c) + (a \cos C + c \cos B) = 3b$

We know that, $b = a \cos C + c \cos B$

$a + c + b = 3b$ or $a + c = 2b$

or a, b, c are in A.P.

16. **(d)**



We know that median divides each other in ratio 2 :1

$$AP=\frac{2}{3}AD=\frac{8}{3};\quad PD=\frac{4}{3};\quad \text{Let } PB=x$$

$$\tan 60^o=\frac{8/3}{x} \text{ or } x=\frac{8}{3\sqrt{3}}$$

$$\text{Area of } \Delta ABD=\frac{1}{2}\times 4\times\frac{8}{3\sqrt{3}}=\frac{16}{3\sqrt{3}}$$

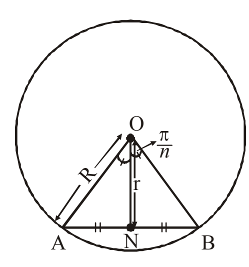
$$\therefore \text{Area of } \Delta ABC=2\times\frac{16}{3\sqrt{3}}=\frac{32}{3\sqrt{3}}$$

[$\because$ Median of a Δ divides it into two Δ's of equal area.]

17. **(c)** We know that, $\tan\left(\frac{\pi}{n}\right)=\frac{a}{2r};\ \sin\left(\frac{\pi}{n}\right)=\frac{a}{2R}$

$$\Rightarrow \quad r=\frac{a}{2}\cot\frac{\pi}{n};\ R=\frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$$

$$r+R=\frac{a}{2}\left[\cot\frac{\pi}{n}+\operatorname{cosec}\frac{\pi}{n}\right]$$



$$=\frac{a}{2}\left[\frac{\cos\frac{\pi}{n}+1}{\sin\frac{\pi}{n}}\right]=\frac{a}{2}\left[\frac{2\cos^2\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}\right]=\frac{a}{2}\cot\frac{\pi}{2\pi}$$

18. **(a)** We know that, $r_1 = \dfrac{\Delta}{s-a}, r_2 = \dfrac{\Delta}{s-b}$ and $r_3 = \dfrac{\Delta}{s-c}$

Given that,

$r_1 > r_2 > r_3 \Rightarrow \dfrac{\Delta}{s-a} > \dfrac{\Delta}{s-b} > \dfrac{\Delta}{s-c}$;

$\Rightarrow \quad s-a < s-b < s-c$

$\Rightarrow \quad -a < -b < -c \Rightarrow a > b > c$

19. **(b)** Let $a = 3x + 4y$, $b = 4x + 3y$ and $c = 5x + 5y$

as $x, y > 0$, $c = 5x + 5y$ is the largest side

$\therefore$ C is the largest angle . Now

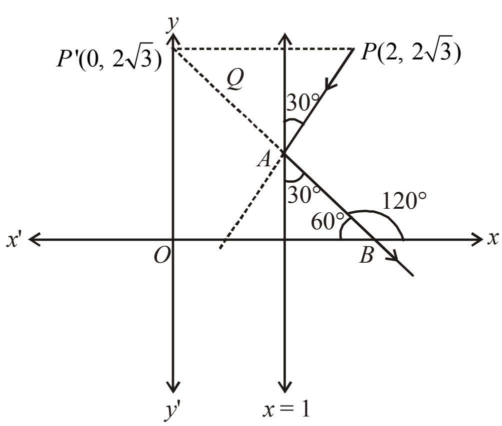
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{(3x+4y)^2 + (4x+3y)^3 - (5x+5y)^2}{2(3x+4y)(4x+3y)}$$

$$= \frac{-2xy}{2(3x+4y)(4x+3y)} < 0$$

$\therefore$ C is obtuse angle $\Rightarrow \Delta$ABC is obtuse angled

20. **(c)**



Slope of $AB = \tan 120° = -\sqrt{3}$

$\therefore$ Equation of line AB (i.e. BP') :

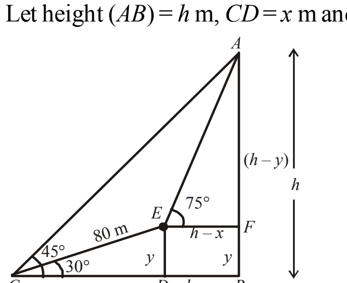
$$y - 2\sqrt{3} = -\sqrt{3}(x - 0)$$

$$\Rightarrow \sqrt{3}x + y = 2\sqrt{3}$$

$\therefore$ Point $(3, -\sqrt{3})$ lies on line AB.

21. **(80)**

Let height $(AB) = h$ m, $CD = x$ m and $ED = y$ m



In rt. ΔCDE,

$$\sin 30° = \frac{y}{80} \Rightarrow y = 40$$

$$\cos 30° = \frac{x}{80} \Rightarrow x = 40\sqrt{3}$$

Now, in ΔAEF,

$$\tan 75° = \frac{h-y}{h-x}$$

$$\Rightarrow (2+\sqrt{3}) = \frac{h-40}{h-40\sqrt{3}}$$

$$\Rightarrow (2+\sqrt{3})(h-40\sqrt{3}) = h-40$$

$$\Rightarrow 2h - 80\sqrt{3} + \sqrt{3}h - 120 = h - 40$$

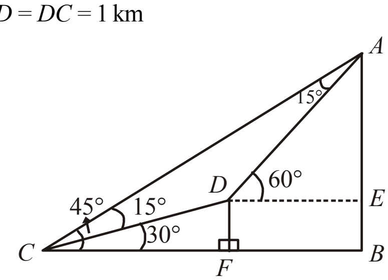
$$\Rightarrow h + \sqrt{3}h = 80 + 80\sqrt{3}$$

$$\Rightarrow (\sqrt{3}+1)h = 80(\sqrt{3}+1)$$

$\therefore h = 80$ m

22. **(c)** $\because \angle DCA = \angle DAC = 30°$

$\therefore AD = DC = 1$ km



In ΔDEA,

$$\frac{AE}{AD} = \sin 60° \Rightarrow AE = \frac{\sqrt{3}}{2} \text{ km}$$

In ΔCDF, $\sin 30° = \dfrac{DF}{CD} \Rightarrow DF = \dfrac{1}{2}$ km

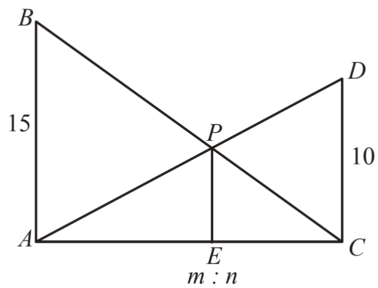
$\therefore EB = DF = \dfrac{1}{2}$ km

$\therefore$ Height of mountain $= AE + EB$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \left(\frac{\sqrt{3}+1}{2}\right) \text{ km}$$

$$= \frac{1}{\sqrt{3}-1} \text{ km}$$

23. **(d)**



Let $PE \perp AC$ and $\frac{AE}{EC} = \frac{m}{n}$

$\because \Delta AEP \sim \Delta ACD,\ \frac{m}{PE} = \frac{m+n}{10}$

$\Rightarrow PE = \frac{10m}{m+n}$ ...(i)

$\because \Delta CEP \sim \Delta CAB,\ \frac{n}{PE} = \frac{m+n}{15}$

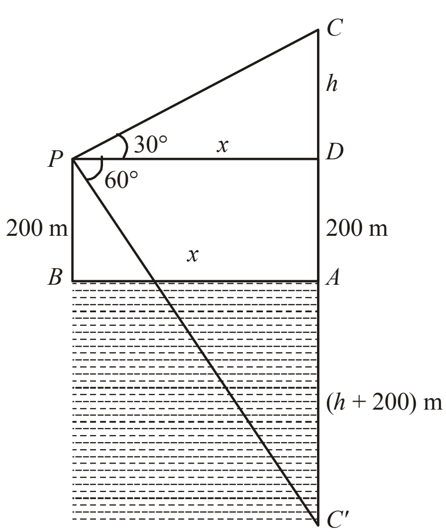
$\Rightarrow PE = \frac{15n}{m+n}$ ...(ii)

From (i) and (ii),

$10m = 15n \Rightarrow m = \frac{3}{2}n$

So, $PE = 6$

24. **(c)**



Here in ΔPCD,

$\sin 30° = \frac{h}{PC} \Rightarrow PC = 2h$ ...(i)

$\tan 30° = \frac{h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = \sqrt{3}h$ ...(ii)

Now, in right $\Delta PC'D$

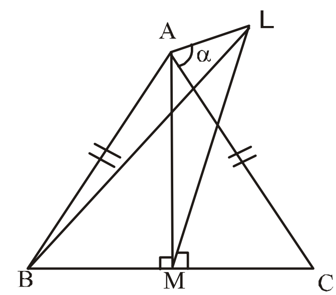
$\tan 60° = \frac{h+400}{x}$

$\Rightarrow \sqrt{3}x = h + 400 \Rightarrow 3h = h + 400$ [From (ii)]

$\Rightarrow h = 200$

So, $PC = 400$ m [From (i)]

25. **(3)** Let the height of the vertical tower situated at the mid point of BC be h.



In ΔALM,

$\cot A = \frac{AM}{LM}$

$\Rightarrow 3\sqrt{2} = \frac{AM}{h} \Rightarrow AM = 3\sqrt{2}h$

In ΔBLM,

$\cot B = \frac{BM}{LM} \Rightarrow \sqrt{7} = \frac{BM}{h} \Rightarrow BM = \sqrt{7}h$

In ΔABM by Pythagoras theorem

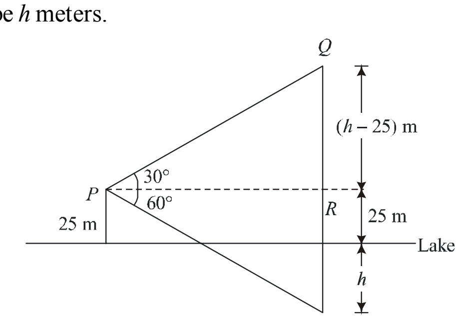
$AM^2 + MB^2 = AB^2$

$\therefore\ AM^2 + MB^2 = (100)^2$

$\Rightarrow 18h^2 + 7h^2 = 100 \times 100$

$\Rightarrow h^2 = 4 \times 100 \Rightarrow h = 20$

26. **(2)** Let height of the cloud from the surface of the lake be h meters.



$\therefore$ In ΔPRQ:

$$\tan 30° = \frac{h-25}{PR}$$

$\therefore \quad PR = (h-25)\sqrt{3} \qquad$ ...(i)

and in ΔPRS : $\tan 60° = \dfrac{h+25}{PR}$

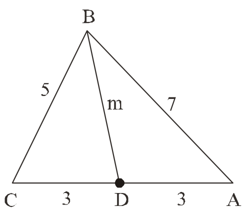
$$PR = \frac{h+25}{\sqrt{3}} \qquad \text{...(ii)}$$

Then, from eq. (i) and (ii),

$$(h-25)\sqrt{3} = \frac{h+25}{\sqrt{3}}$$

$\therefore \quad h = 50\,\text{m}$

27. **(b)** Let the height of the lamp-post is h.



By Appolonius Theorem,

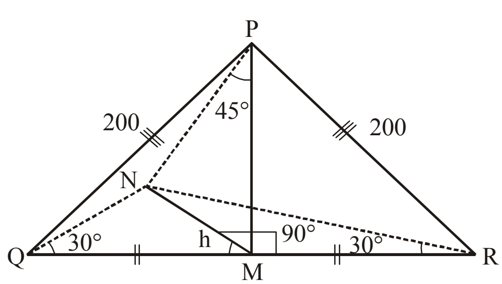
$$2\left(BD^2 + \left(\frac{AC}{2}\right)^2\right) = BC^2 + AB^2$$

$$\Rightarrow \quad 2(m^2 + 3^2) = 25 + 49 \Rightarrow m = 2\sqrt{7}$$

$$\tan 30° = \frac{h}{BD}$$

$$\Rightarrow \quad h = 2\sqrt{7} \times \frac{1}{\sqrt{3}} = \frac{2\sqrt{21}}{3}$$

28. **(d)**



Let height of tower MN = h

In ΔQMN we have

$$\tan 30° = \frac{MN}{QM}$$

$\therefore \quad QM = \sqrt{3}h = MR \qquad$ ...(1)

Now in ΔMNP

MN = PM $\qquad$ ...(2)

In ΔPMQ we have :

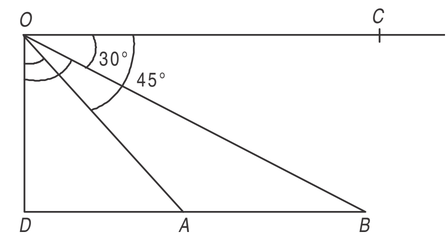
$$MP = \sqrt{(200)^2 - (\sqrt{3}\,h)^2}$$

$\therefore \quad$ From (2), we get :

$$\sqrt{(200)^2 - (\sqrt{3}\,h)^2} = h \Rightarrow h = 100\text{m}$$

29. **(a)** Here; $\angle DOA = 45°$; $\angle DOB = 60°$

Now, let height of tower = h.



In ΔDOA, $\tan(\angle DOA) = \dfrac{DA}{OD}$

$\Rightarrow \quad \tan 45° = \dfrac{DA}{h} \qquad \Rightarrow h = DA$

Now, in Δ DOB

$$\tan(\angle DOB) = \frac{BD}{OD}$$

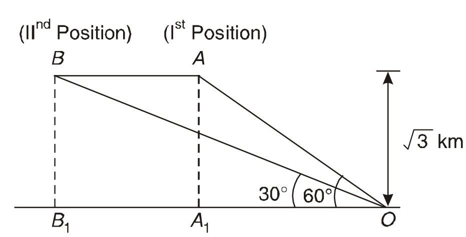
$\Rightarrow \tan 60° = \dfrac{BD}{h} \qquad \Rightarrow BD = \sqrt{3}\,h.$

$\therefore$ speed for the distance BA $= \dfrac{BD - AD}{18} = \dfrac{(\sqrt{3}-1)\,h}{18}$

$\therefore$ required time taken

$$= \frac{AD}{\text{speed}} = \frac{h \times 18}{(\sqrt{3}-1)\,h} = \frac{18}{\sqrt{3}-1} = 9(\sqrt{3}+1)$$

30. **(d)** For Δ OA, A, $OA_1 = \dfrac{\sqrt{3}}{\tan 60°} = 1$ km.

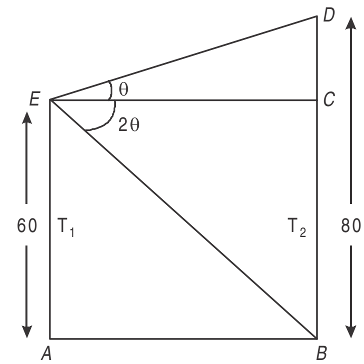


For Δ OB_1, B, $OB_1 = \dfrac{\sqrt{3}}{\tan 30°} = 3$ *km.*

As, a distance of 3 – 1 = 2 km is covered in 5 seconds. Therefore the speed of the plane is

$$\frac{2 \times 3600}{5} = 1440 \text{ km/hr}$$

31. **(d)** Let the distance between T_1 and T_2 be x



From the figure

$EA = 60$ m (T_1) and $DB = 80$ m (T_2)

$\angle DEC = \theta$ and $\angle BEC = 2\theta$

Now in ΔDEC,

$$\tan\theta = \frac{DC}{AB} = \frac{20}{x}$$

and in ΔBEC,

$$\tan 2\theta = \frac{BC}{CE} = \frac{60}{x}$$

We know that

$$\tan 2\theta = \frac{2\tan\theta}{1-(\tan\theta)^2}$$

$$\Rightarrow \frac{60}{x} = \frac{2\left(\frac{20}{x}\right)}{1-\left(\frac{20}{x}\right)^2}$$

$$\Rightarrow x^2 = 1200 \quad \Rightarrow x = 20\sqrt{3}$$

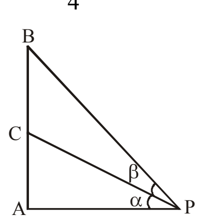
32. **(d)** Since $AP = 2AB \Rightarrow \frac{AB}{AP} = \frac{1}{2}$ ...(i)

Let $\angle APC = \alpha$

$$\therefore \quad \tan\alpha = \frac{AC}{AP} = \frac{1}{2}\frac{AB}{AP} = \frac{1}{4}$$

($\because$ C is the mid point) $\left(\therefore AC = \frac{1}{2}AB\right)$

$$\Rightarrow \tan\alpha = \frac{1}{4}$$



As $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$$\Rightarrow \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{1}{2}$$

$$\left[\begin{array}{l}\because \tan(\alpha+\beta) = \frac{AB}{AP} \\ \tan(\alpha+\beta) = \frac{1}{2}\ [\text{From}(1)]\end{array}\right]$$

$$\Rightarrow \frac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}\tan\beta} = \frac{1}{2} \quad \therefore \tan\beta = \frac{2}{9}$$

33. **(b)** $\tan 30° = \frac{h}{x+a}$

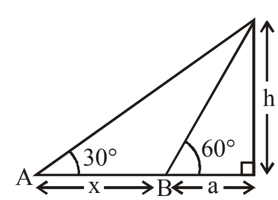
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+a} \Rightarrow \sqrt{3}h = x+a$$

...(1)

$$\tan 60° = \frac{h}{a} \Rightarrow \sqrt{3} = \frac{h}{a}$$

$$\Rightarrow h = \sqrt{3}a$$

...(2)



From (1) and (2)

$3a = x + a \Rightarrow x = 2a$

Here, the speed is uniform

So, time taken to cover x = 2 (time taken to cover a)

$\therefore$ Time taken to cover $a = \frac{10}{2}$ minutes = 5 minutes

34. **(d)** Let $AP = x$

$BP = y$

$$\tan 45° = \frac{H}{x} \Rightarrow H = x$$

$$\tan 30° = \frac{H}{y} \Rightarrow y = \sqrt{3}H$$

$$x^2 + (54\sqrt{2})^2 = y^2$$

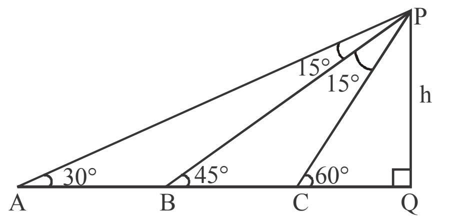
$$H^2 + (54\sqrt{2})^2 = 3H^2$$

$$(54\sqrt{2})^2 = 2H^2$$

$$54\sqrt{2} = \sqrt{2}H$$

$$54 = H$$

35. (c)



$\because$ PB bisects $\angle APC$, therefore

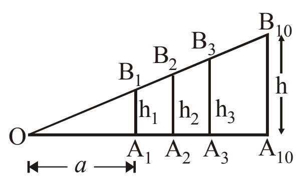
$AB : BC = PA : PC$

Also in ΔAPQ, $\sin 30° = \frac{h}{PA} \Rightarrow PA = 2h$

and in ΔCPQ, $\sin 60° = \frac{h}{PC} \Rightarrow PC = \frac{2h}{\sqrt{3}}$

$\therefore$ $AB : BC = 2h : \frac{2h}{\sqrt{3}} = \sqrt{3} : 1$

36. (a)



$\Delta OA_1B_1, \Delta OA_2B_2, \Delta OA_3B_3, \ldots\ldots, \Delta OA_{10}B_{10}$ all are similar triangles.

$$\Rightarrow \frac{h_1}{a_1} = \frac{h_2}{a_2} = \frac{h_3}{a_3} = \ldots\ldots = \frac{h_{10}}{a_{10}} = \tan\alpha .$$

Since, $h_{10} = h = a_{10} \tan\alpha$ ...(1)

and $a_1 = a \Rightarrow h_1 = a \tan\alpha$ ...(2)

$\Rightarrow$ h = (a + 9d) tan α where d is distance between poles

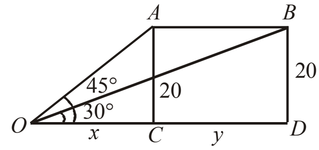
$(\because a_{10} = a + 9d)$

$\Rightarrow h = a\tan\alpha + 9d\tan\alpha$

$$\Rightarrow \frac{h - a\tan\alpha}{9\tan\alpha} = d \Rightarrow \frac{h - \frac{a\sin\alpha}{\cos\alpha}}{9\frac{\sin\alpha}{\cos\alpha}} = d$$

$$\Rightarrow d = \frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$$

37. (b) Given that height of pole $AB = 20$ m



Let O be the point on the ground such that $\angle AOC = 45°$

Let $OC = x$ and $CD = y$

In right ΔAOC, $\tan 45° = \frac{20}{x}$ ...(i)

In right ΔBOD, $\tan 30° = \frac{20}{x+y}$ ...(ii)

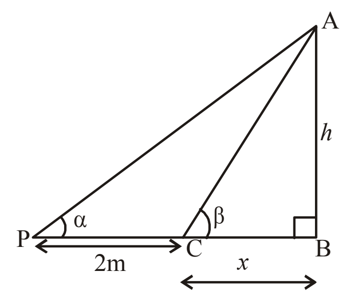
From (i) and (ii), we have $x = 20$ and $\frac{1}{\sqrt{3}} = \frac{20}{x+y}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{20+y} \Rightarrow 20 + y = 20\sqrt{3}$$

So, $y = 20(\sqrt{3}-1)$ m and time = 1 s (Given)

Hence, speed $= 20(\sqrt{3}-1)$ m/s

38. (a) Let AB be the tower of height 'h'.



Given : In ΔABP

$\tan\alpha = \frac{AB}{PB}$

or $\frac{\sin\alpha}{\cos\alpha} = \frac{h}{x+2}$

$\Rightarrow (x+2)\sin\alpha = h\cos\alpha$

$$\Rightarrow h = \frac{x\sin\alpha + 2\sin\alpha}{\cos\alpha} \quad \ldots(1)$$

Now, In ΔABC, $\tan\beta = \frac{AB}{BC}$

$$\Rightarrow \frac{\sin\beta}{\cos\beta} = \frac{h}{x} \Rightarrow x = \frac{h\cos\beta}{\sin\beta} \quad \ldots(2)$$

Putting the value of x in eq. (2) to eq. (1), we get

$$h = \frac{\frac{h\cos\beta\sin\alpha}{\sin\beta} + \frac{2\sin\alpha}{1}}{\cos\alpha}$$

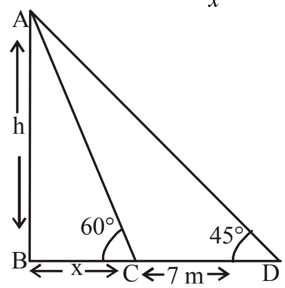
$$\Rightarrow h = \frac{h\cos\beta.\sin\alpha + 2\sin\alpha\sin\beta}{\sin\beta.\cos\alpha}$$

$\Rightarrow h(\sin\beta.\cos\alpha - \cos\beta.\sin\alpha) = 2\sin\alpha.\sin\beta$

$\Rightarrow h[\sin(\beta - \alpha)] = 2\sin\alpha.\sin\beta$

$$\Rightarrow h = \frac{2\sin\alpha.\sin\beta}{\sin(\beta-\alpha)}$$

39. **(b)** In right, ΔABC $\tan 60° = \dfrac{h}{x} = \sqrt{3}$



$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \text{....(i)}$$

In right, $\Delta ABD = \tan 45° = \dfrac{h}{x+7} = 1$

$$\Rightarrow \quad h = x + 7 \Rightarrow h - \frac{h}{\sqrt{3}} = 7 \qquad \text{[From (i)]}$$

$$\Rightarrow \quad h = \frac{7\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \quad h = \frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$$

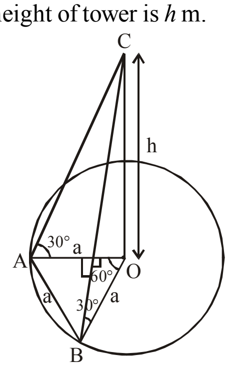
40. **(a)** In the ΔAOB given that $\angle AOB = 60°$ and $OA = OB =$ radius

$\therefore \quad \angle OBA = \angle OAB = 60°$

$\therefore \quad \Delta AOB$ is a equilateral triangle.

$\Rightarrow \quad OA = OB = AB = a$

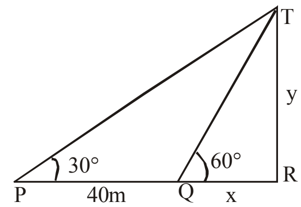
Let the height of tower is h m.



In ΔOAC, $\tan 30° = \dfrac{h}{a} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{a}$

$$\Rightarrow h = \frac{a}{\sqrt{3}}$$

41. **(d)**



In right ΔQTR

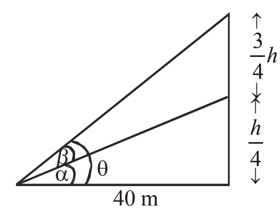
$\tan 60° = \dfrac{y}{x} \Rightarrow y = \sqrt{3}x$ ....(1)

In right ΔPTR

$\tan 30° = \dfrac{y}{x+40} \Rightarrow y = \dfrac{x+40}{\sqrt{3}}$ ....(2)

From (1) and (2), $\sqrt{3}x = \dfrac{x+40}{\sqrt{3}} \Rightarrow x = 20m$

42. **(c)**



$\theta = \alpha + \beta, \beta = \tan^{-1}\left(\dfrac{3}{5}\right)$

or $\quad \beta = \theta - \alpha$

$$\Rightarrow \quad \tan\beta = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta . \tan\alpha}$$

$$\text{or} \quad \frac{3}{5} = \frac{\dfrac{h}{40} - \dfrac{h}{160}}{1 + \dfrac{h}{40} . \dfrac{h}{160}}$$

$h^2 - 200h + 6400 = 0$

$\Rightarrow \quad h = 40$ or 160 metre

$\therefore \quad$ possible height $= 40$ metre