Exercise 1.5

Question :1 Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$

(v) 2π

Answer : (i) $2 - \sqrt{5}$

= 2 – 2.2360679…

= -0.2360679…

Since the number is in non-terminating non-recurring, therefore, it is an irrational number

(ii) $(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23}$

= 3

= 3/1

A number is a rational number as it can be represented in the form of p/q.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$ = 2/7

The number is a rational number as it can be represented in the form of p/q.

(iv) $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

= 0.7071067811.....

Since the number is a non-terminating and non-recurring number.

Hence, it is an irrational number

(v) $2\pi = 2 * 3.1415\ldots$

= 6.2830......

Since the number is a non-terminating and non-recurring number.

Hence, it is an irrational number

Question :2 Simplify each of the following expressions:

(i) $(3 + \sqrt{3}) (2 + \sqrt{2})$
(ii) $(3 + \sqrt{3}) (3 - \sqrt{3})$
(iii) $(\sqrt{5} + \sqrt{2})^2$
(iv) $(\sqrt{5} - \sqrt{2}) (\sqrt{5} + \sqrt{2})$

Answer :
(i) (3 + √3) (2 + √2)

= 3 (2 + √2) + √3 (2 + √2)

= 3 × 2 + 3√2 + 2√3 + √2 x √3

= 6 + 3√2 + 2√3 + √6

(ii) (3 + √3) (3 - √3)   [Since, (a + b) (a – b) = a2 – b2]

= 32 – (√3)2

= 9 – 3

= 6

(iii) $(\sqrt{5} + \sqrt{2})^2$ [Since, $(a + b)^2 = a^2 + b^2 + 2ab$]

$= (\sqrt{5})^2 + (\sqrt{2})^2 + 2 \times \sqrt{5}\times\sqrt{2}$

$= 5 + 2 + 2 \times$

$= 7 + 2\sqrt{10}$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$ [Since, $(a + b)(a - b) = a^2 - b^2$]

$= (\sqrt{5})^2 - (\sqrt{2})^2$

$= 5 - 2$

$= 3$

Question :3 Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is $\pi = \frac{c}{d}$, This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer : There is no contradiction in the statement.

When we measure any value with a scale, we only obtain an approximate value.

We never obtain an accurate value.

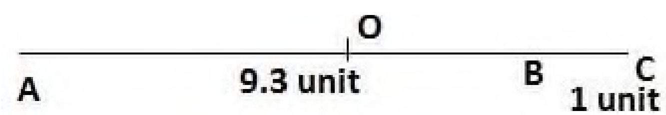
Hence, we cannot say that either c or d is irrational.

The value of π is almost equal to 22/7 or 3.142857…

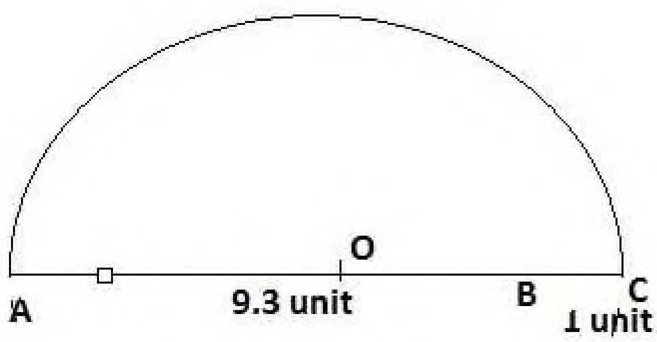
Question :4 Represent $\sqrt{9.3}$ on the number line.

Answer : Step 1: Draw a line segment AB of 9.3 unit. Then, extend it to C so that BC = 1 unit.

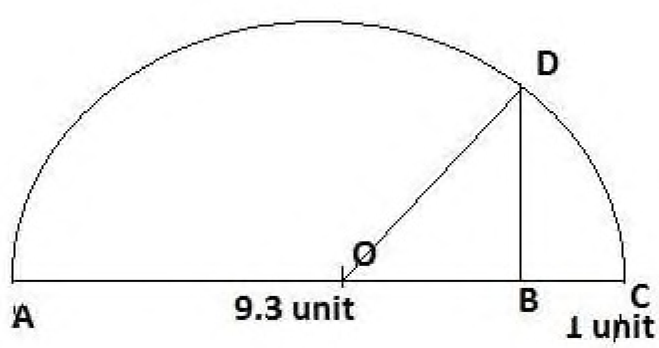
Step 2: Now, AC = 10.3 units. Find the center of AC and mark it as O



Step 3: Draw a semi-circle with radius OC and center O.



Step 4: Draw a perpendicular line BD to AC at point B intersecting the semi-circle at D. And then, join OD



Step 5: Now, OBD is a right angled triangle

Here, OD (Radius of semi-circle)

OC $= \frac{10.3}{2}$

BC = 1

Then, OB = OC – BC

Using Pythagoras theorem,

$OD^2 = BD^2 + OB^2$

$\left(\frac{10.3}{2}\right)^2 = BD^2 + \left(\frac{8.3}{2}\right)^2$

BD2 = $\left(\frac{10.3}{2}\right)^2 - \left(\frac{8.3}{2}\right)^2$

$BD^2 = \left(\frac{10.3}{2} - \frac{8.3}{2}\right)\left(\frac{10.3}{2} + \frac{8.3}{2}\right)$

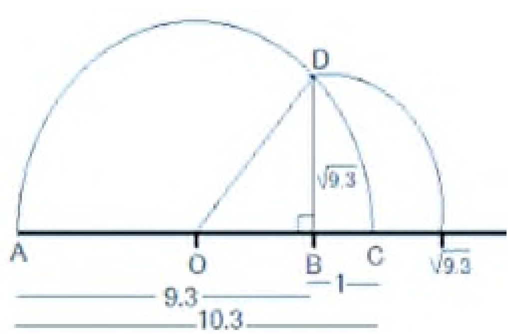
$BD^2 = 9.3$

BD= $\sqrt{9.3}$

Thus, the length of BD is $\sqrt{9.3}$

Step 6: Taking BD as radius and B as the center, construct an arc which touches the line segment.

Now, the point where it touches the line segment is at a distance of from O as shown in the figure below



Question :5 Rationalize the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(iv) $\frac{1}{\sqrt{7}-2}$

Answer: Rationalizing the denominator means that we have to remove the irrational component from the denominator of the fraction.

(i) $\frac{1}{\sqrt{7}}$

To remove the $\sqrt{7}$ from the denominator, multiply and divide the fraction with $\sqrt{7}$

$$\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

We multiplied and divided the equation by $\sqrt{7} + \sqrt{6}$ so that the denominator forms the formula of

(a + b) (a – b) = $a^2 - b^2$

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2} = \frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \sqrt{7} + \sqrt{6}$$

(iii). $\frac{1}{\sqrt{5}+\sqrt{2}}$

$= \frac{1\,(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$

$= \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv). $\frac{1}{\sqrt{7-2}}$

$= \frac{(\sqrt{7}+2)}{\sqrt{7}\times\sqrt{7}-2\times 2}$

$= \frac{\sqrt{7}+2}{7-4}$

$= \frac{\sqrt{7}+2}{3}$