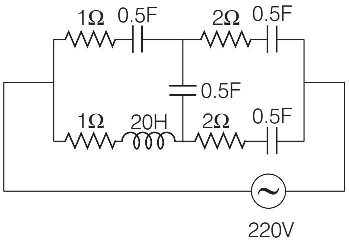
21

Alternating Current

TOPIC 1

AC Circuits and Power in AC Circuits

01 At very high frequencies, the effective impedance of the given circuit will be ....Ω.



[2021, 31 Aug Shift-II]

Ans. *(2)*

We know that, at very high frequency capacitive reactance becomes negligible i.e. short circuit (SC) and inductive reactance becomes very large i.e. open circuit (OC).

i.e. $X_C \to 0$ and $X_L \to \infty$.

Now, the circuit can be rearranged as shown in figure.

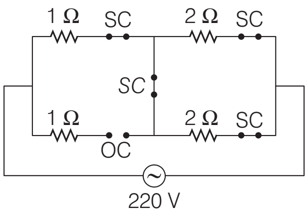


Fig. (i)

Final circuit is

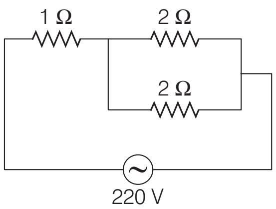


Fig. (ii)

Hence, equivalent resistance,

$$R_{eq} = 1 + \frac{2 \times 2}{2+2} = 2\,\Omega$$

Thus, correct answer is 2.

02 An AC circuit has an inductor and a resistor of resistance R in series, such that $X_L = 3R$. Now, a capacitor is added in series such that $X_C = 2R$. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5} : x$ The value of x is. **[2021, 27 Aug Shift-II]**

Ans. *(1)*

Given, resistance of resistor $= R$

Inductance, $X_L = 3R$

Capacitance $X_C = 2R$

As we know that,

Power factor, $\cos\phi = \frac{R}{Z}$

where, Z is impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For case I, an inductance connected in series with resistance

$$\cos\phi_1 = \frac{R}{Z_1} = \frac{R}{\sqrt{R^2 + (X_L - 0)^2}}$$

$$= \frac{R}{\sqrt{R^2 + (3R)^2}} = \frac{R}{R\sqrt{10}} = \frac{1}{\sqrt{10}} \quad \text{...(i)}$$

For case II, A capacitor is also connected in series with resistance

$$\therefore \quad \cos\phi_2 = \frac{R}{Z_2}$$

$$\Rightarrow \quad \cos\phi_2 = \frac{R}{\sqrt{R^2 + (3R - 2R)^2}}$$

$$= \frac{R}{\sqrt{R^2 + R^2}} = \frac{R}{R\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \text{...(ii)}$$

Now, ratio of Eqs. (ii) and (i), we get

$$\frac{\cos\phi_2}{\cos\phi_1} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{10}}} = \sqrt{\frac{10}{2}} = \sqrt{5} : 1$$

$$\therefore \quad x = 1$$

03 The alternating current is given by

$$i = \left\{ \sqrt{42} \sin\left(\frac{2\pi}{T} t\right) + 10 \right\} A$$

The rms value of this current is ...... A. **[2021, 27 Aug Shift-I]**

Ans. *(11)*

Given, equation of alternating current,

$$i = \left[\sqrt{42} \sin\left(\frac{2\pi}{T} t\right) + 10 \right] A$$

From given equation, we get

$$i = i_1 + i_2$$

where, $i_1 = \sqrt{42} \sin\left(\frac{2\pi}{T} t\right) A$ and $i_2 = 10\,A$

Now, i_1 is oscillating current, whereas i_2 is direct current and its value does not change with time.

$$(i_1)_{rms} = \frac{\sqrt{42}}{\sqrt{2}} = \sqrt{21}\,A$$

$$(i_2)_{rms} = 10\,A$$

We know that,

$$i_{rms}^2 = (i_1)_{rms}^2 + (i_2)_{rms}^2$$

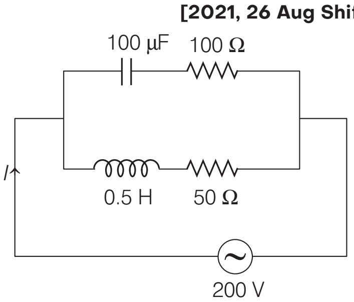
Substituting the values, we get

$$i_{rms} = \sqrt{(\sqrt{21})^2 + 10^2} = \sqrt{121}$$

$$\Rightarrow \quad i_{rms} = 11\,A$$

Thus, RMS value of given equation of current is 11 A.

04 In the given circuit the AC source has $\omega = 100$ rad s^{-1}. Considering the inductor and capacitor to be ideal, what will be the current I flowing through the circuit? **[2021, 26 Aug Shift-II]**



(a) 5.9 A (b) 4.24 A
(c) 0.94 A (d) 6 A

Ans. *(*)*

Given, angular frequency, $\omega = 100$ rad/ s

Capacitance of capacitor,
$C = 100\,\mu F = 100 \times 10^{-6}$ F

Inductance of inductor coil, $L = 0.5$ H

Resistance in upper branch, $R_1 = 100\Omega$

Resistance in lower branch, $R_2 = 50\Omega$

In the given circuit consider current in upper branch be i_1 and current flowing in lower branch be i_2. The net current flowing in circuit will be I.

Impedance of upper branch can be calculated as

$$Z_1 = \sqrt{X_C^2 + R_1^2} = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R_1^2}$$

$$= \sqrt{\left(\frac{1}{100 \times 100 \times 10^{-6}}\right)^2 + 100^2}$$

$$= \sqrt{100^2 + 100^2} = 100\sqrt{2}\ \Omega$$

Impedance of lower branch can be calculated as

$$= \sqrt{(\omega L)^2 + R_2^2}$$

$$= \sqrt{(100 \times 0.5)^2 + 50^2}$$

$$= \sqrt{50^2 + 50^2}$$

$$= 50\sqrt{2}\ \Omega$$

Current flowing in upper branch,

$$i_1 = \frac{V}{Z_1} = \frac{200}{100\sqrt{2}} = \sqrt{2}\text{A}$$

Phase of current in upper branch,

$$\cos\phi_1 = \frac{R_1}{Z_1} = \frac{100}{100\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \phi_1 = 45°$$

Thus, in upper branch, current leads voltage by 45° as capacitor is present.

Current flowing in lower branch i_2 is

$$i_2 = \frac{V}{Z_2} = \frac{200}{50\sqrt{2}} = 2\sqrt{2}\text{A}$$

Phase of current in lower branch is

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{50}{50\sqrt{2}} \Rightarrow \phi_2 = 45°$$

Thus, in lower branch current lags voltage by 45° as inductor is present.

Thus, the net current, $I = \sqrt{i_1^2 + i_2^2}$

$$I = \sqrt{(\sqrt{2})^2 + (2\sqrt{2})^2}$$

$$I = \sqrt{10} = 3.16\text{ A}$$

Thus, no option in the given question is correct.

If $I = i_1 + i_2$ is taken then

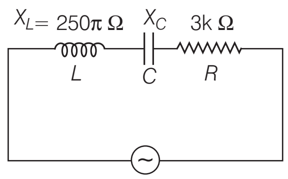
$I = \sqrt{2} + 2\sqrt{2} = 4.24$ A is obtained which is incorrect method of solution.

05 A series L-C-R circuit driven by 300 V at a frequency of 50 Hz contains a resistance $R = 3\text{k}\Omega$, an inductor of inductive reactance $X_L = 250\pi\Omega$ and an unknown capacitor. The value of capacitance to maximise the average power should be (Take, $\pi^2 = 10$) **[2021, 26 Aug Shift-I]**

(a) 4 μF (b) 25 μF
(c) 400 μF (d) 40 μF

Ans. *(a)*

The circuit diagram can be drawn as,



Average power of an L-C-R circuit is given by

$$P_{av} = V_{rms}\, i_{rms} \cos\phi$$

For P_{av} to be maximum, $\cos\phi = 1$ ...(i)

We know that,

$$\cos\phi = \frac{R}{Z},$$

where, R = resistance

and Z = impedance of L-C-R circuit.

Now, $Z = \sqrt{(X_L - X_C)^2 + R^2}$

As, $\cos\phi = 1 = \frac{R}{Z}$ [using Eq. (i)]

$$\Rightarrow \quad R = Z$$

$$\Rightarrow \quad R = \sqrt{(X_L - X_C)^2 + R^2}$$

Squaring both sides, we get

$$R^2 = (X_L - X_C)^2 + R^2$$

$$\Rightarrow \quad (X_L - X_C)^2 = 0$$

$$\Rightarrow \quad X_L = X_C$$

Since, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

and $X_L = 250\pi\Omega$ (given)

$$\Rightarrow \quad 250\pi = \frac{1}{2\pi \times 50 \times C}$$

$$\Rightarrow \quad C = \frac{1}{2\pi^2 \times 50 \times 250}$$

$$\Rightarrow \quad C = 4 \times 10^{-6}\text{ F} = 4\,\mu\text{F}$$

06 A 100 Ω resistance, a 0.1 μF capacitor and an inductor are connected in series across a 250 V supply at variable frequency. Calculate the value of inductance of inductor at which resonance will occur. Given that the resonant frequency is 60 Hz. **[2021, 27 July Shift-II]**

(a) 0.70 H (b) 70.3 mH
(c) 7.03×10^{-5} H (d) 70.3 H

Ans. *(d)*

Given,

Resistance, $R = 100\,\Omega$

Capacitance, $C = 0.1\,\mu$F

Inductance, $L = 250$ V

Resonant frequency, $f_0 = 60$ Hz

∵ We know that,

Resonant frequency, $f_0 = \frac{L}{2\pi\sqrt{LC}}$

$$L = \frac{1}{4\pi^2 f_0^2 C}$$

$$= \frac{1}{4\pi^2 \times (60)^2 \times 0.1 \times 10^{-6}}$$

$$= \frac{10^7}{4\pi^2 \times 3600}$$

$$= \frac{10^5}{4\pi^2 \times 36} = 70.3\text{ H}$$

07 A 0.07 H inductor and a 12 Ω resistor are connected in series to a 220 V, 50 Hz AC source. The approximate current in the circuit and the phase angle between current and source voltage are, respectively.

[Take, π as $\frac{22}{7}$] **[2021, 27 July Shift-I]**

(a) 8.8 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(b) 88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(c) 0.88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(d) 8.8 A and $\tan^{-1}\left(\frac{6}{11}\right)$

Ans. *(a)*

Given,

Inductance, $L = 0.07$ H

Resistance, $R = 12\,\Omega$

Voltage, $V = 220$ V

Frequency, $f = 50$ Hz

∵ We know that,

Angular frequency, $\omega = 2\pi f = 2\pi \times 50 = 100\pi$ Hz

∵ Inductive reactance, $X_L = \omega L$

$$= 100\pi \times 0.07$$

$$\Rightarrow \quad = 100 \times \frac{22}{7} \times \frac{7}{100} = 22\,\Omega$$

∵ A resistor and an inductor are connected in series,

$\therefore$ Impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 22^2}$

$= \sqrt{144 + 484}$

$= \sqrt{628} \approx 25\,\Omega$

$\therefore$ Current, $I = \frac{V}{Z} = \frac{220}{25} = 8.8\text{ A} \Rightarrow I = 8.8\text{A}$

Also, phase angle between current and source voltage can be given as

$\tan\phi = \left(\frac{X_L}{R}\right) \Rightarrow \tan\phi = \frac{22}{12}$

$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{11}{6}\right)$

08 Two circuits are shown in the figures (a) and (b). At a frequency of .... rad/s, the average power dissipated in one cycle will be same in both the circuits.

[2021, 25 July Shift-II]

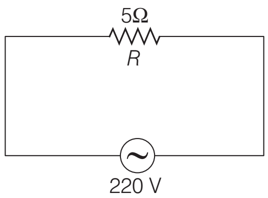


Fig. (a)

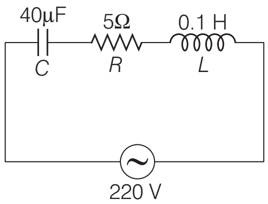


Fig. (b)

Ans. *(500)*

Given, resistance (R_a) for circuit (a) is $5\,\Omega$.

In circuit (b)

Capacitance, $C = 40\,\mu\text{F} = 40 \times 10^{-6}$ F

Resistance, $R = 5\,\Omega$

Inductance, $L = 0.1$ H

Supply voltage, $V = 220$ V

P_a and P_b be the power in circuit (a) and (b)

$\therefore \quad P_a = \frac{V^2}{R} = \frac{(220)^2}{5}$

$= \frac{484 \times 100}{5} = 484 \times 20 = 9680$ W

In R-L-C circuit,

$$P_{\text{av}} = \frac{V^2}{Z}\cos\phi$$

where, Z is impedance of circuit and $\cos\phi$ is power factor.

$P_{\text{av}} = \frac{V^2}{Z}\frac{R}{Z} \qquad \left[\because \cos\phi = \frac{R}{Z}\right]$ ...(i)

$= \frac{V^2 R}{Z^2}$

and $\qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\Rightarrow \qquad Z^2 = R^2 + (X_L - X_C^2)$

Put in Eq. (i), we get

$$P_{\text{av}} = \frac{(220)^2 \times 5}{5^2 + (X_L - X_C)^2}$$

$\Rightarrow 5^2 + (X_L - X_C)^2 = \frac{(220)^2 \times 5}{484 \times 20}$ $\qquad [\because P_{av} = P_a]$

$= \frac{484 \times 100 \times 5}{484 \times 20} = 25$

$\Rightarrow \qquad (X_L - X_C)^2 = 0$

$\Rightarrow \qquad X_L = X_c$

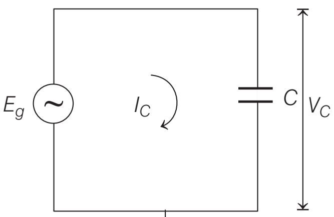
$\Rightarrow \qquad \omega L = \frac{1}{\omega C}$

$\Rightarrow \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1 \times 40 \times 10^{-6}}}$

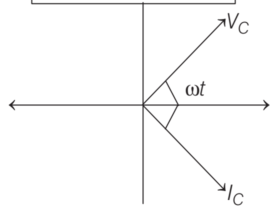
$= \frac{1}{\sqrt{4 \times 10^{-6}}} = \frac{10^3}{2}$

$= \frac{1000}{2} = 500 \text{ rad s}^{-1}$

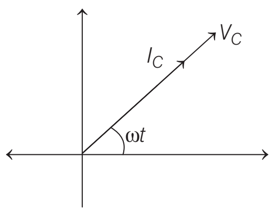
09 In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g_0}\sin\omega t$, V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is **[2021, 22 July Shift-II]**



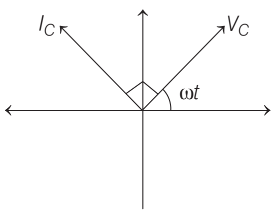
(a)



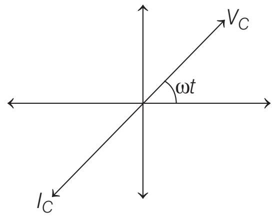
(b)



(c)



(d)



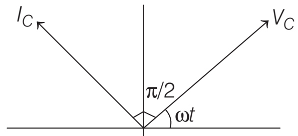
Ans. *(c)*

Given, input alternating Emf

$(E_g) = E_{g_0}\sin\omega t$

As we know that, capacitive current leads voltage by $\frac{\pi}{2}$ rad

$\therefore$ Phasor diagram will be



where, V_C and I_C are voltage and current in capacitive circuit.

10 Match List I with List II.

[2021, 22 July Shift-II]

	List I		List II
A.	$\omega L > \frac{1}{\omega C}$	(i)	Current is in phase with EMF
B.	$\omega L = \frac{1}{\omega C}$	(ii)	Current lags behind the applied EMF
C.	$\omega L < \frac{1}{\omega C}$	(iii)	Maximum current occurs
D.	Resonant frequency	(iv)	Current leads the EMF

Choose the correct answer from the options given below.

Codes

	A	B	C	D
(a)	(ii)	(i)	(iv)	(iii)
(b)	(ii)	(i)	(iii)	(iv)
(c)	(iii)	(i)	(iv)	(ii)
(d)	(iv)	(iii)	(ii)	(i)

Ans. *(a)*

Given,

(A) $\qquad \omega L > \frac{1}{\omega C}$

Since, inductive resistance > capacitive resistance, hence Inductive current lag behind applied voltage.

(B) $\qquad \omega L = \frac{1}{\omega C}$

Since, inductive resistance = capacitive resistance

Hence, this circuit behaves like a L-C-R series resonant circuit. Therefore, $Z = R$ and phase difference $= 0$.

$\therefore$ Circuit will be purely resistive and resistive current remains in phase with voltage.

(C) $\omega L < \frac{1}{\omega C}$

Since, inductive resistance < capacitive resistance, hence capacitive current lead with applied voltage.

(D) As we know that, at resonance impedance

$(Z) = R$ (resistance)

$\therefore$ Circuit current will be maximum.

Hence, option (a) is the correct.

11 For a series L-C-R circuit with $R = 100\Omega$, $L = 0.5$ mH and $C = 0.1$ pF connected across 220 V-50 Hz AC supply, the phase angle between current and supplied voltage and the nature of the circuit is **[2021, 20 July Shift-II]**

(a) 0°, resistive circuit
(b) $\approx 90^\circ$, predominantly inductive circuit
(c) 0° resonance circuit
(d) $\approx 90^\circ$, predominantly capacitive circuit

Ans. ***(d)***

According to question, there is a series L-C-R circuit.

Resistance, $R = 100\,\Omega$

Capacitance, $C = 0.1\text{ pF} = 0.1 \times 10^{-12}$ F

Inductance, $L = 0.5\text{ mH} = 0.5 \times 10^{-3}$ H

Frequency, $f = 50$ Hz

Voltage, $V = 220$ V

$\because$ Inductive reactance, $X_L = \omega L = 2\pi f L$

$= 2\pi \times 50 \times 0.5 \times 10^{-3}$

$\Rightarrow \quad X_L = 50\pi \times 10^{-3}\Omega$

$\because$ Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$\Rightarrow \quad X_C = \frac{1}{2\pi \times 50 \times 10^{-12} \times 0.1}$

$\Rightarrow \quad X_C = \frac{1}{100\pi \times \frac{1}{10} \times 10^{-12}}$

$\Rightarrow \quad X_C = \frac{10^{11}}{\pi}\Omega$

On comparing X_L with X_C, we can write

$X_C >> X_L$

and $|X_C - X_L| >> R$

which means that the circuit is predominantly capacitive circuit.

12 A series L-C-R circuit of $R = 5\Omega$, $L = 20$ mH and $C = 0.5\,\mu$F is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is ...... $\times 10^2$ W. **[2021, 20 July Shift-II]**

Ans. ***(125)***

Given, resistance, $R = 5\,\Omega$

Inductance, $L = 20$ mH

Capacitance, $C = 0.5\,\mu$F

Voltage, $V = 250$ V

$I_{\text{rms}} = \frac{V}{Z} = \frac{V}{R}$ [At resonance, $Z = R$]

Power dissipated, $P = I_{\text{rms}}^2 R$

or $P = \frac{V^2}{R} = \frac{250 \times 250}{R}$

$= \frac{250 \times 250}{5} = 12500$ J/s

$= 125 \times 10^2$ W

13 AC voltage $V(t) = 20 \sin\omega t$ volt of frequency 50 Hz is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area is 1 m^2. The amplitude of the oscillating displacement current for the applied AC voltage is ....

(Take, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m)

[2021, 20 July Shift-I]

(a) 21.14 μA (b) 83.37 μA
(c) 27.79 μA (d) 55.58 μA

Ans. ***(c)***

Given,

AC voltage, $V(t) = 20 \sin\omega t$ volt.

Frequency, $f = 50$Hz

Separation between the plates, $d = 2$ mm $= 2 \times 10^{-3}$ m

Area, $A = 1\text{ m}^2$

As, $C = \frac{\varepsilon_0 A}{d}$

where, $\varepsilon_0 =$ absolute electrical permittivity of free space

$= 8.854 \times 10^{-12}\text{ N}^{-1}\text{ kg}^2\text{m}^{-2}$

$\Rightarrow \quad C = \frac{\varepsilon_0 \times 1}{2 \times 10^{-3}}$ ...(i)

Capacitive reactance $(X_C) = \frac{1}{\omega C}$ ...(ii)

From Eqs. (i) and (ii), we get

$X_C = \frac{2 \times 10^{-3}}{2 \times 50\pi \times \varepsilon_0}$ $(\because \omega = 2\pi f)$

$= \frac{2 \times 10^{-3}}{25 \times 4\pi\varepsilon_0}$

$\Rightarrow \quad X_C = \frac{2 \times 10^{-3}}{25} \times 9 \times 10^9$

$\Rightarrow \quad X_C = \frac{18}{25} \times 10^6\,\Omega$

By using Ohm's law,

As, $I_0 = \frac{V_0}{X_C} = \frac{20 \times 25}{18} \times 10^{-6}$

$= 27.78 \times 10^{-6}$

$\Rightarrow \quad I_0 = 27.78\,\mu$A

$\therefore$ The amplitude of the oscillating displacement current for applied AC voltage will be approximately 27.79 μA.

14 In an L-C-R series circuit, an inductor 30 mH and a resistor 1Ω are connected to an AC source of angular frequency 300 rad/s. The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x} \times 10^{-3}$ F. Then, the value of x is .... **[2021, 20 July Shift-I]**

Ans. ***(3)***

Given,

Inductance, $L = 30$ mH

Resistance, $R = 1\Omega$

Angular frequency, $\omega = 300$ rad/s

We know that in L-C-R circuit,

$\tan\phi = \frac{X_C - X_L}{R}$

where, $\phi =$ phase angle $= 45°$

$X_C =$ capacitive reactance $= \frac{1}{\omega C}$

$X_L =$ inductive reactance $= \omega L$

$\Rightarrow \quad \tan 45° = \frac{X_C - X_L}{R}$

$\Rightarrow \quad X_C - X_L = R$ $[\because \tan 45° = 1]$

$\Rightarrow \quad \frac{1}{\omega C} - \omega L = R$

$\Rightarrow \quad \frac{1}{\omega C} - 300 \times 30 \times 10^{-3} = 1$

$\Rightarrow \quad \frac{1}{\omega C} = 10 \Rightarrow \omega C = \frac{1}{10}$

$\Rightarrow \quad C = \frac{1}{10\omega} \Rightarrow C = \frac{1}{10 \times 300}$

$\Rightarrow \quad C = \frac{1}{3} \times 10^{-3}$ F ...(i)

According to question, the value of capacitance is $\frac{1}{x} \times 10^{-3}$ F. So, on comparing it with Eq. (i), we can say $x = 3$.

15 In a series L-C-R circuit, the inductive reactance (X_L) is 10 Ω and the capacitive reactance (X_C) is 4 Ω. The resistance (R) in the circuit is 6 Ω. The power factor of the circuit is **[2021, 18 March Shift-II]**

(a) $\frac{1}{2}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Ans. *(c)*

Given, inductive reactance, $X_L = 10\ \Omega$

Capacitive reactance, $X_C = 4\Omega$

Resistance, $R = 6\Omega$

Therefore, power factor = $\cos\theta$

$$= \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{6}{\sqrt{6^2 + (10-4)^2}} = \frac{1}{\sqrt{2}}$$

16 In a series L-C-R resonance circuit, if we change the resistance only, from a lower to higher value, **[2021, 18 March Shift-I]**

(a) the bandwidth of resonance circuit will increase

(b) the resonance frequency will increase

(c) the quality factor will increase

(d) the quality factor and the resonance frequency will remain constant

Ans. *(a)*

Bandwidth of L-C-R series resonance circuit, $\beta = \frac{R}{L}$

As we increase the value of the resistance from lower to the higher of the circuit, the bandwidth of resonance circuit will also increase.

So, the option (a) is correct.

Resonance frequency, $\omega = \frac{1}{\sqrt{LC}}$

Since, the resonance frequency is independent of the resistance.

So, the option (b) is incorrect.

We know that,

Quality factor, $Q = \frac{\omega L}{R}$

The quality factor is inversely proportional to the resistance of the circuit. So, increasing the value of resistance, the quality factor is decreased. So, the option (c) and (d) are incorrect.

17 An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is **[2021, 18 March Shift-I]**

(a) 2.5 ms (b) 25 ms

(c) 2.5 s (d) 0.25 ms

Ans. *(a)*

Given, the frequency of the AC source, $f = 50$ Hz

Angular frequency of the circuit,

$$\omega = 2\pi f$$

$$\Rightarrow \quad \omega = 2\pi(50) \Rightarrow \omega = 100\pi$$

As we know the general expression of the current in AC circuit,

$$I = I_0 \sin\omega t$$

$$\Rightarrow \quad \frac{I_0}{\sqrt{2}} = I_0 \sin(100\ \pi t)$$

$$\Rightarrow \quad \sin(100\ \pi t) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 100\ \pi t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad 100\ \pi t = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{1}{400}\text{ sec}$$

$$\Rightarrow \quad t = 2.5 \times 10^{-3}\text{s} = 2.5\text{ms}$$

18 What happens to the inductive reactance and the current in a purely inductive circuit, if the frequency is halved ? **[2021, 17 March Shift-II]**

(a) Both inductive reactance and current will be halved.

(b) Inductive reactance will be halved and current will be doubled.

(c) Inductive reactance will be doubled and current will be halved.

(d) Both inductive reactance and current will be doubled.

Ans. *(b)*

As we know, the inductive reactance is directly proportional to the frequency of the AC circuit

i.e., $X_L = \omega L$

$\Rightarrow \quad X_L = 2\ \pi f L \qquad (\because \omega = 2\pi f)$

Here, f is the frequency of the AC circuit, L is the inductive resistance and X_L is the inductive reactance.

When the frequency of an AC circuit is halved, then the inductive reactance of the circuit is also halved.

i.e. $X_L{}' = \frac{X_L}{2}$

Using Ohm's law, $I = \frac{V}{X_L}$

When the frequency is halved, then the current

$$I' = \frac{V}{X_L{}'} \Rightarrow I' = \frac{V}{X_L/2}$$

$$I' = 2I$$

The current becomes doubled.

19 Match List-I with List-II **[2021, 17 March Shift-II]**

List-I	List-II
A. Phase difference between current and voltage in a purely resistive AC circuit	1. $\frac{\pi}{2}$; current leads voltage
B. Phase difference between current and voltage in a pure inductive AC circuit	2. zero
C. Phase difference between current and voltage in a pure capacitive AC circuit	3. $\frac{\pi}{2}$; current lags voltage
D. Phase difference between current and voltage in an L-C-R series circuit	4. $\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below.

	A	B	C	D
(a)	1	3	4	2
(b)	2	4	3	1
(c)	2	3	4	1
(d)	2	3	1	4

Ans. *(d)*

A. In a purely resistive AC circuit, the phase difference between the current and voltage is zero.

B. In a purely inductive AC circuit, the current lags the voltage, so the phase difference between the current and voltage is $\pi/2$.

C. In a purely capacitive AC circuit the current leads the voltage, so the phase difference between the current and voltage is $\pi/2$.

D. The phase difference between current & voltage in an L- C- R series circuit is

$$\phi = \tan^{-1}\frac{(X_C - X_L)}{R}$$

∴ The correct match is A-(2), B-(3), C-(1), D-(4).

20 An AC current is given by $I = I_1 \sin\omega t + I_2 \cos\omega t$. A hot wire ammeter will give a reading **[2021, 17 March Shift-I]**

(a) $\sqrt{\frac{I_1^2 - I_2^2}{2}}$ (b) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$

(c) $\frac{I_1 + I_2}{\sqrt{2}}$ (d) $\frac{I_1 + I_2}{2\sqrt{2}}$

Ans. *(b)*

Given, $I = I_1 \sin\omega t + I_2 \cos\omega t$ ...(i)

We know that rms value of current is given by

$$I_{rms} = \sqrt{\frac{\int I^2 dt}{T}}$$

$$\Rightarrow I_{rms} = \sqrt{\frac{\int_0^T (I_1 \sin\omega t + I_2 \cos\omega t)^2 dt}{T}}$$

[using Eq. (i)]

Squaring on both sides of the above equation, we get

$$\Rightarrow (I_{rms})^2 = \int_0^T \frac{(I_1 \sin\omega t + I_2 \cos\omega t)^2}{T} dt$$

$$\Rightarrow (I_{rms})^2 = \int_0^T \frac{I_1^2 (\sin^2\omega t + I_2 \cos^2\omega t + 2I_1 I_2 \sin\omega t \cos\omega t) dt}{T}$$

$$\Rightarrow I_{rms} = \sqrt{\frac{I_1^2}{2} + \frac{I_2^2}{2} + 0}$$

$$\Rightarrow I_{rms} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

21 A sinusoidal voltage of peak value 250 V is applied to a series *L-C-R* circuit, in which $R = 8\,\Omega$, $L = 24$ mH and $C = 60\,\mu$F. The value of power dissipated at resonant condition is x kW. The value of x to the nearest integer is .... **[2021, 16 March Shift-I]**

Ans. *(4)*

Given, $V_0 = 250$ V, $R = 8\,\Omega$, $L = 24$ mH

and $C = 60\,\mu$F

We know that, at resonance power ,

$$P = \frac{V_{rms}^2}{R}$$

$$\Rightarrow P = \frac{(250/\sqrt{2})^2}{8} \quad \left[\because V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{250}{\sqrt{2}} \text{V}\right]$$

$$= (250)^2/16$$

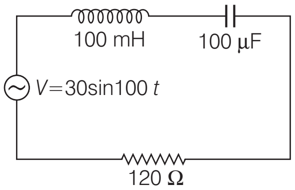
$$= \frac{62500}{16} = 3906.25 \text{ W}$$

$$\approx 4 \text{ kW}$$

$$x = 4$$

Comparing with the given value in the question i.e., x kW, the value of $x = 4$.

22 Find the peak current and resonant frequency of the following circuit (as shown in figure). **[2021, 26 Feb Shift-II]**



(a) 0.2 A and 50 Hz (b) 0.2 A and 100 Hz

(c) 2 A and 100 Hz (d) 2 A and 50 Hz

Ans. *(a)*

Given, inductance, $L = 100$ mH

$= 100 \times 10^{-3}$ H

Capacitance, $C = 100\,\mu$F

$= 100 \times 10^{-6}$ F

Resistance, $R = 120\,\Omega$

Maximum voltage, $V_0 = 30$ V

Angular frequency, $\omega = 100$ rads^{-1}

Impedance, $Z = \sqrt{(X_L - X_C)^2 + R^2}$

Here, $X_L = \omega L = 100 \times 100 \times 10^{-3} = 10\,\Omega$

$$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\,\Omega$$

$$\Rightarrow \quad Z = \sqrt{(X_C - X_L)^2 + R^2}$$

$$= \sqrt{(100 - 10)^2 + (120)^2}$$

$$= \sqrt{(90^2 + 120^2} = 153.3\,\Omega$$

As we know that, peak current,

$$I_0 = \frac{V_0}{Z} = \frac{30}{153.3} = 0.195 \text{ A} \simeq 0.2 \text{ A}$$

Resonance frequency,

$$\therefore \quad f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 100 \times 10^{-6}}}$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{10^{-5}}} = \frac{1}{2\pi \times 10^{-2}\sqrt{10^{-1}}}$$

$$= \frac{100\,\pi}{2\,\pi} = 50 \text{ Hz}$$

23 An alternating current is given by the equation $i = i_1 \sin\omega t + i_2 \cos\omega t$. The rms current will be **[2021, 26 Feb Shift-I]**

(a) $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$ (b) $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$

(c) $\frac{1}{2}(i_1^2 + i_2^2)^{1/2}$ (d) $\frac{1}{\sqrt{2}}(i_1 + i_2)$

Ans. *(a)*

Given, $i = i_1 \sin\omega t + i_2 \cos\omega t$

Let I_{rms} be the rms current.

$$\therefore \quad I_{rms} = \left(\frac{i_1^2 + i_2^2}{2}\right)^{1/2}$$

$$\Rightarrow \quad I_{rms} = \frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$$

24 In a series *L-C-R* resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be .... **[2021, 26 Feb Shift-I]**

Ans. *(400)*

Given, initial quality factor $(Q_i) = 100$

Let initial inductance $(x_{Li}) = x$

Final inductance $(x_{Lf}) = 2x$

and initial resistance $(R_i) = R$

Final resistance $(R_f) = \frac{R}{2}$

Final quality factor $= Q_f$

Since, $Q_i = \frac{X_L}{R}$

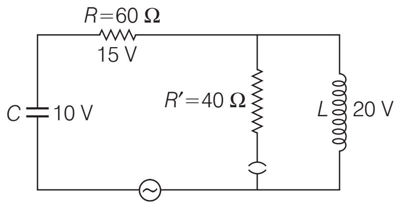
and $Q_f = \frac{2X_L}{R/2}$

$$\Rightarrow \quad Q_f = \frac{4X_L}{R} = 4Q_i = 4 \times 100$$

$$= 400$$

Hence, final quality factor will be 400.

25 The angular frequency of alternating current in an *L-C-R* circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser. **[2021, 25 Feb Shift-I]**



(a) 0.8 H and 150 μF

(b) 0.8 H and 250 μF

(c) 1.33 H and 250 μF

(d) 1.33 H and 150 μF

Ans. *(b)*

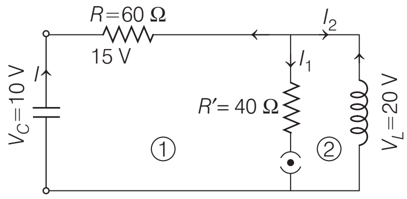
Given, angular frequency,

$\omega = 100$ rads^{-1}

$R = 60\,\Omega$, $V_R = 15$ V,

$R' = 40\,\Omega$, $V_{R'} = V_L = 20$ V

and $V_C = 10$ V



By using Ohm's law,

$V = IR \Rightarrow I = V/R$

$\Rightarrow \quad I = 15/60 = 1/4$ A ...(i)

and $\quad I_1 = \dfrac{V_{R'}}{R'} = 20/40 = 1/2$ A ...(ii)

As, $\quad X_C = \dfrac{V_C}{I} = \dfrac{10}{1/4} = 40\ \Omega$

and $\quad X_C = \dfrac{1}{\omega C}$

$\Rightarrow \quad C = \dfrac{1}{X_C \omega} = \dfrac{1}{40 \times 100}$

$= 0.25 \times 10^{-3}$ F $= 0.25$ mF $= 250\ \mu$F

By using KCL in loop 2,

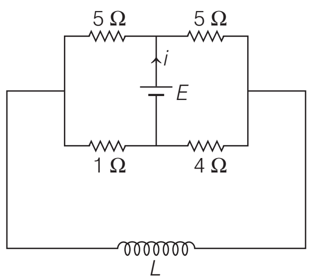
$I_2 = I - I_1$

$= 1/4 - 1/2 = -1/4$ A

$\therefore \quad X_L = \dfrac{V_L}{|I_2|} = \dfrac{20}{1/4} = 80\ \Omega \Rightarrow \omega L = 80$

$\Rightarrow \quad L = \dfrac{80}{\omega} = \dfrac{80}{100} = 0.8$ H

26 The current (i) at time $t = 0$ and $t = \infty$ respectively for the given circuit is **[2021, 25 Feb Shift-I]**



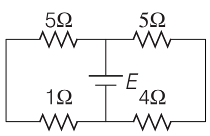
(a) $\dfrac{18E}{55}, \dfrac{5E}{18}$ (b) $\dfrac{10E}{33}, \dfrac{5E}{18}$

(c) $\dfrac{5E}{18}, \dfrac{18E}{55}$ (d) $\dfrac{5E}{18}, \dfrac{10E}{33}$

Ans. ***(d)***

As we know that at time $t = 0$, inductor acts as open circuit.

Then, the circuit becomes



Therefore,

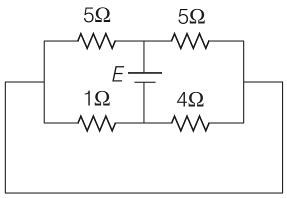
$R_{eq} = \dfrac{(5+1)(5+4)}{(5+1)+(5+4)} = \dfrac{6 \times 9}{6+9} = \dfrac{54}{15} = \dfrac{18}{5}$

By using Ohm's law,

$V = IR_{eq} \qquad [\because V = E]$

$\Rightarrow \quad I = \dfrac{E \times 5}{18} = \dfrac{5E}{18}$

At $t = \infty$, inductor will act as short circuit. It is shown below



Therefore,

$R_{eq} = \dfrac{5 \times 5}{5+5} + \dfrac{1 \times 4}{1+4} = \dfrac{25}{10} + \dfrac{4}{5} = \dfrac{5}{2} + \dfrac{4}{5}$

$= \dfrac{33}{10}\ \Omega$

and $\quad I = \dfrac{E}{\dfrac{33}{10}} = \dfrac{10\ E}{33}$

27 A transmitting station releases waves of wavelength 960 m. A capacitor of 2.56 μF is used in the resonant circuit. The self-inductance of coil necessary for resonance is .... $\times 10^{-8}$ H. **[2021, 25 Feb Shift-I]**

Ans. ***(10)***

Given, wavelength of transmission signal,

$\lambda = 960$ m

Capacitance, $C = 2.56\ \mu$F

$= 2.56 \times 10^{-6}$ F

As we know resonance frequency,

$f = \dfrac{1}{2\pi\sqrt{LC}}$

Also, frequency $(f) = \dfrac{\text{speed } (v)}{\text{wavelength } (\lambda)}$

$\therefore \quad \dfrac{v}{\lambda} = \dfrac{1}{2\pi\sqrt{LC}}$

$\Rightarrow \quad \sqrt{LC} = \dfrac{\lambda}{v \times 2\pi}$

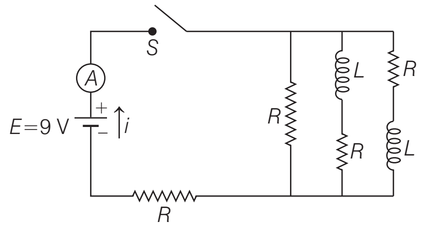
On squaring both sides, we get

$\Rightarrow LC = \dfrac{\lambda^2}{v^2 \times 4\pi^2} \Rightarrow L = \dfrac{\lambda^2}{v^2 \times 4\pi^2 \times C}$

$\Rightarrow \quad L = \dfrac{(960)^2}{(3 \times 10^8)^2 \times 4\pi^2 \times 2.56 \times 10^{-6}}$

$\therefore \quad L = 10 \times 10^{-8}$ H

28 Figure shows a circuit that contains four identical resistors with resistance $R = 2.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH and an ideal battery with electromotive force $E = 9$ V. The current i just after the switch S is closed will be **[2021, 24 Feb Shift-II]**



(a) 2.25 A (b) 3.0 A

(c) 3.37 A (d) 9 A

Ans. ***(a)***

Given, resistance, $R = 2\ \Omega$,

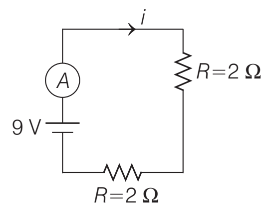
Inductance, $L = 2$ mH,

emf, $E = 9$ V

and i be the current.

$\because$ At $t = 0$ when switch is closed, inductors behave as open circuit.

$\therefore$ Effective circuit will be



By using Ohm's law, $V = i\ R_{eq}$

$\Rightarrow \quad i = V/R_{eq}$

where, R_{eq} is equivalent resistance of series resistors,

i.e., $\quad R_{eq} = R + R = 2R = 2 \times 2 = 4\ \Omega$

$\therefore \quad i = \dfrac{9}{4} = 2.25$ A

29 A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28 Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is .... [Take, $\pi = 3.14$] **[2021, 24 Feb Shift-I]**

Ans. ***(2000)***

Given, $L = 2 \times 10^{-4}$ H, $R = 6.28\ \Omega$,

$f_0 = 10$ MHz $= 10 \times 10^6$ Hz

$\therefore$ Quality factor $= \omega_0 \dfrac{L}{R} = 2\pi f_0 \dfrac{L}{R}$

$= 2\pi \times 10 \times 10^6 \times \dfrac{2 \times 10^{-4}}{6.28}$

$= 2 \times 10^3 = 2000$

30 In an AC-circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is **[2020, 2 Sep Shift-II]**

(a) $2R^2$ (b) Zero
(c) R (d) $R\sqrt{2}$

Ans. *(c)*

Given, inductor, capacitor and resistors are connected in series and have same reactance i.e., $X_L = X_C = R$

Since, impedance $(Z) = \sqrt{R^2 + (X_L - X_C)^2}$

$\therefore \quad Z = \sqrt{R^2 + (R - R)^2} = R$

31 An inductance coil has a reactance of $100\ \Omega$. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is **[2020, 2 Sep Shift-II]**

(a) 1.1×10^{-2} H (b) 1.1×10^{-1} H
(c) 5.5×10^{-5} H (d) 6.7×10^{-7} H

Ans. *(a)*

Given, reactance, $X_L = 100\ \Omega$

Frequency of AC, $f = 1000$ Hz

As voltage leads current by 45°, so there must be some resistance in the coil otherwise

$$\Delta\phi = 90°.$$

Using, $\quad \tan\phi = \dfrac{X_L}{R}$

We have, $\tan 45° = \dfrac{X_L}{R}$ or $X_L = R$

$\therefore$ Reactance of circuit containing resistance and inductance is

$$\sqrt{X_L^2 + R^2} = 100\ \Omega$$

$$\Rightarrow \sqrt{X_L^2 + X_L^2} = 100\ \Omega$$

or $\quad X_L = 50\sqrt{2}\ \Omega$

But $\quad X_L = L\omega$

$$\Rightarrow \quad L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{50\sqrt{2}}{2\pi \times 1000}$$

$$\Rightarrow \quad L = 1.125 \times 10^{-2} \text{ H}$$

Hence, correct option is (a).

32 A 750 Hz, 20 V (rms) source is connected to a resistance of $100\ \Omega$, an inductance of 0.1803 H and a capacitance of $10\ \mu$F all in series combination. The time in which the resistance (heat capacity 2 J/°C) will get heated by 10°C is close to. (Assume no loss of heat to the surroundings) **[2020, 3 Sep Shift-I]**

(a) 418 s (b) 245 s (c) 365 s (d) 348 s

Ans. *(d)*

Given, frequency, $f = 750$ Hz

$V_{rms} = 20\text{V}, R = 100\ \Omega, L = 0.1803$ H

and $\quad C = 10\ \mu$F

So, impedance of L-C-R circuit is

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{(100)^2 + \left(2\pi \times 750 \times 0.1803 - \frac{1}{2\pi \times 750 \times 10 \times 10^{-6}}\right)^2}$$

$\approx 834\ \Omega$

Now, power lost by L-C-R circuit, which occurs in resistance is given by

$$P = V_{rms} I_{rms} \cos\phi$$

As, $\cos\phi = \dfrac{R}{Z}$ and $I_{rms} = \dfrac{V_{rms}}{Z}$

$$\therefore \quad P = \frac{V_{rms}^2 \cdot R}{Z^2} = \frac{(20)^2 \times 100}{(834)^2} = 0.0575 \text{ Js}^{-1}$$

Heat developed in resistor,

$$H = Pt = S(\Delta\theta)$$

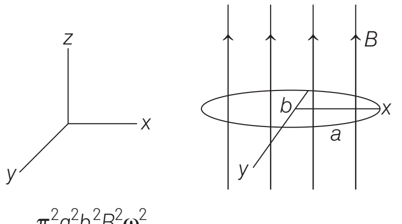
where, $t =$ time, $S =$ heat capacity $= 2$ J/° C

$\Delta\theta =$ temperature change $= 10$ °C

$$\Rightarrow \quad t = \frac{S\Delta\theta}{P} = \frac{2 \times 10}{0.0575} = 348 \text{ s}$$

Hence, option (d) is correct.

33 An elliptical loop having resistance R, of semi-major axis a and semi-minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the X-axis with angular frequency ω, then the average power loss in the loop due to joule's heating is **[2020, 3 Sep Shift-I]**



(a) $\dfrac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$ (b) zero

(c) $\dfrac{\pi abB\omega}{R}$ (d) $\dfrac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$

Ans. *(a)*

Average power developed which eventually gets lost as heat due to resistance of loop is given by

$$P_{avg} = \frac{V_{rms}^2}{R} = \frac{(V_{max}/\sqrt{2})^2}{R} = \frac{V_{max}^2}{2R}$$

Also, $V_{max} = NBA\omega$

In given loop, $N = 1, A = \pi ab$

So, $P_{avg} = \dfrac{(BA\omega)^2}{2R} = \dfrac{B^2 \pi^2 a^2 b^2 \omega^2}{2R}$

Hence, option (a) is correct.

34 An AC circuit has $R = 100\ \Omega$, $C = 2\ \mu$F and $L = 80$ mH connected in series. The quality factor of the circuit is **[2020, 6 Sep Shift-I]**

(a) 2 (b) 0.5 (c) 20 (d) 400

Ans. *(a)*

Given that, $R = 100\ \Omega, C = 2\ \mu\text{F}, L = 80$ mH

For a series L-C-R AC circuit,

Quality factor, $\phi = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

$$= \frac{1}{100}\sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}}$$

$$\phi = 2$$

35 In a series L-R circuit, power of 400W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value of C as $\left(\dfrac{n}{3\pi}\right)\mu$F, then value of n is .... **[2020, 6 Sep Shift-II]**

Ans. *(400)*

Given, for L - R circuit

$V_{rms} = 250$ V, $f = 50$ Hz, $P = 400$ W

and $\quad \cos\phi = 0.8$

As, we know, power is given by

$$P = V_{rms} i_{rms} \cos\phi$$

$$\Rightarrow \quad P = V_{rms} \frac{V_{rms}}{Z} \cos\phi$$

$$\Rightarrow \quad Z = \frac{V_{rms}^2}{P} \cos\phi$$

$$\Rightarrow \quad Z = \frac{(250)^2}{400} \times 0.8 = 125\ \Omega$$

Also, $\cos\phi = \dfrac{R}{Z} \Rightarrow 0.8 = \dfrac{R}{125} \Rightarrow R = 100\ \Omega$

Impedance, $\quad Z = \sqrt{R^2 + X_L^2}$

$$\Rightarrow \quad 125 = \sqrt{(100)^2 + X_L^2}$$

$$X_L = 75\ \Omega$$

Now, to obtain power factor unity, X_C must be equal to X_L.

i.e. $\quad X_C = X_L = 75\ \Omega$

$$\Rightarrow \quad \frac{1}{\omega C} = 75 \Rightarrow C = \frac{1}{\omega \times 75} = \frac{1}{2\pi \times 50 \times 75}$$

$\Rightarrow \quad C = \frac{1}{7500\pi}F$ or $C = \frac{400}{3\pi}\mu F$

Given, $C = \frac{n}{3\pi}\mu F$

On comparing the both, we get

$n = 400$

36 An *L-C-R* circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant *b*, the correct equivalence would be **[2020, 7 Jan Shift-I]**

(a) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

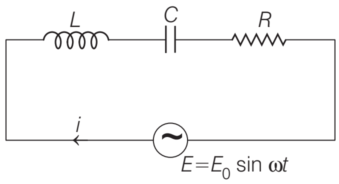
(b) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

(c) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(d) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

Ans. ***(b)***

For an *L-C-R* circuit,



By KVL, we have

$$-L\frac{di}{dt} - \frac{q}{C} - iR + E = 0$$

Above can be rearranged as

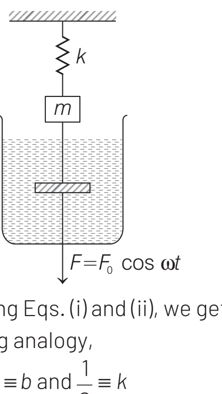
$$L\frac{d^2q}{dt^2} + R\cdot\frac{dq}{dt} + \frac{1}{C}q = E_0 \sin\omega t \quad \text{...(i)}$$

Now, for a damped harmonic oscillator, we have

$$ma = -kx - bv + F_0 \cos\omega t$$

Rearranging above equation, we have

$$m\frac{d^2x}{dt^2} + b\cdot\frac{dx}{dt} + kx = F_0 \cos\omega t \quad \text{...(ii)}$$



On comparing Eqs. (i) and (ii), we get the following analogy,

$$L \equiv m, R \equiv b \text{ and } \frac{1}{C} \equiv k$$

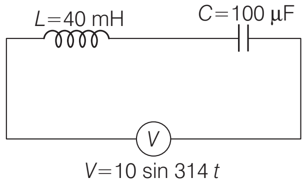
or $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}$ and $R \leftrightarrow b$

37 In *L-C* circuit, the inductance $L = 40$ mH and capacitance $C = 100\mu F$. If a voltage $V(t) = 10\sin(314t)$ is applied to the circuit, the current in the circuit is given as **[2020, 9 Jan Shift-II]**

(a) $0.52\cos 314\,t$ (b) $0.52\sin 314\,t$

(c) $10\cos 314\,t$ (d) $5.2\cos 314\,t$

Ans. ***(a)***



Impedance of given ciruit,

$$Z = \sqrt{(X_C - X_L)^2} = X_C - X_L$$

$$= \frac{1}{\omega C} - \omega L$$

$$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$$

$$= 19.28\,\Omega$$

As $X_C > X_L$, circuit is captive, hence current in circuit leads emf by $\frac{\pi}{2}$ radians.

Current in circuit is given by

$$I = I_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= \frac{V_{max}}{Z}\cos\omega t$$

$$= \frac{10}{19.28} \times \cos 314t$$

or $I = 0.52\cos 314t$

38 An alternating voltage $V(t) = 220\sin 100\pi t$ volt is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is **[2019, 8 April Shift-I]**

(a) 5 ms (b) 2.2 ms

(c) 7.2 ms (d) 3.3 ms

Ans. ***(d)***

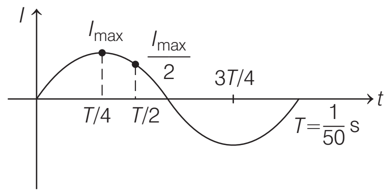
In an AC resistive circuit, current and voltage are in phase.

So, $\quad I = \frac{V}{R}$

$\Rightarrow \quad I = \frac{220}{50}\sin(100\pi t) \quad \text{...(i)}$

∴ Time period of one complete cycle of current is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{1}{50}\text{ s}$$



So, current reaches its maximum value at

$$t_1 = \frac{T}{4} = \frac{1}{200}\text{ s}$$

When current is half of its maximum value, then from Eq. (i), we have

$$I = \frac{I_{max}}{2} = I_{max}\sin(100\pi t_2)$$

$$\Rightarrow \sin(100\pi t_2) = \frac{1}{2} \Rightarrow 100\pi t_2 = \frac{5\pi}{6}$$

So, instantaneous time at which current is half of maximum value is $t_2 = \frac{1}{120}$ s

Hence, time duration in which current reaches half of its maximum value after reaching maximum value is

$$\Delta t = t_2 - t_1 = \frac{1}{120} - \frac{1}{200} = \frac{1}{300}\text{ s} = 3.3\text{ ms}$$

39 A circuit connected to an AC source of emf $e = e_0 \sin(100t)$ with *t* in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf *e* and current *i*. Which of the following circuits will exhibit this? **[2019, 8 April Shift-II]**

(a) *RC* circuit with $R = 1\,k\Omega$ and $C = 1\mu F$

(b) *RL* circuit with $R = 1\,k\Omega$ and $L = 1\,mH$

(c) *RC* circuit with $R = 1\,k\Omega$ and $C = 10\,\mu F$

(d) *RL* circuit with $R = 1\,k\Omega$ and $L = 10$ mH

Ans. ***(c)***

Given, phase difference, $\phi = \frac{\pi}{4}$

As we know, for *R-L* or *R-C* circuit,

$$\tan\phi = \frac{\text{Capacitive reactance } (X_C) \text{ or inductive reactance } (X_L)}{\text{Resistance } (R)}$$

$$\tan\frac{\pi}{4} = \frac{X_C \text{ or } X_L}{R}$$

$$1 = \frac{X_C \text{ or } X_L}{R}$$

$\Rightarrow \quad R = X_C$ or X_L

Also, given $e = e_0 \sin(100\,t)$

Comparing the above equation with general equation of emf, i.e. $e = e_0 \sin\omega t$, we get

$$\omega = 100 \text{ rad/s} = 10^2 \text{ rad/s}$$

Now, checking option wise,

For *R-C* circuit, with

$$R = 1k\Omega = 10^3\,\Omega \text{ and } C = 1\mu F = 10^{-6} F$$

So, $X_C = \frac{1}{\omega C} = \frac{1}{10^2 \times 10^{-6}} = 10^4\ \Omega$

$\Rightarrow \quad R \neq X_C$

For R-L circuit, with

$R = 1\text{k}\Omega = 10^3\Omega$ and $L = 1\text{ mH} = 10^{-3}\text{H}$

So, $X_L = \omega L = 10^2 \times 10^{-3} = 10^{-1}\Omega$

$\Rightarrow \quad R \neq X_L$

For R-C circuit, with

$R = 1\text{k}\Omega = 10^3\Omega$

and $C = 10\mu\text{F} = 10 \times 10^{-6}\text{F} = 10^{-5}\text{F}$

So, $X_C = \frac{1}{10^2 \times 10^{-5}} = 10^3\Omega \Rightarrow R = C$

For R-L circuit, with

$R = 1\text{k}\Omega = 10^3\Omega$

and $L = 10\text{ mH} = 10 \times 10^{-3}\text{H} = 10^{-2}\text{H}$

$X_L = 10^2 \times 10^{-2} = 1\Omega \Rightarrow \quad R \neq X_L$

Alternate Solution

Since, $\tan\frac{\pi}{4} = 1 = \frac{X_C \text{ or } X_L}{R}$

$\therefore$ For R-C circuit, we have

$$1 = \frac{1}{C\omega R} \quad \text{or} \quad \omega = \frac{1}{CR} \qquad \text{...(i)}$$

Similarly, for R-L circuit, we have

$$1 = \frac{\omega L}{R} \Rightarrow \omega = \frac{R}{L} \qquad \text{...(ii)}$$

It is given in the question that,

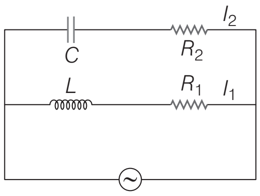
$$\omega = 100 \text{ rad/s}$$

Thus, again by substituting the given values of R, C or L option wise in the respective Eqs. (i) and (ii), we get that only for option (c),

$$\omega = \frac{1}{CR} = \frac{1}{10 \times 10^{-6} \times 10^3}$$

or $\quad \omega = 100$ rad/s

40



In the above circuit, $C = \frac{\sqrt{3}}{2}\mu\text{F}$, $R_2 = 20\,\Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\,\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path is I_2. The voltage of AC source is given by $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is **[2019, 12 Jan Shift-II]**

(a) 30° (b) 60°

(c) 0° (d) 90°

Ans. *(a)*

Phase difference between I_2 and V, i.e. $C - R_2$ circuit is given by

$$\tan\phi = \frac{X_C}{R_2} \Rightarrow \tan\phi = \frac{1}{C\omega R_2}$$

Substituting the given values, we get

$$\tan\phi = \frac{1}{\frac{\sqrt{3}}{2} \times 10^{-6} \times 100 \times 20} = \frac{10^3}{\sqrt{3}}$$

$\therefore \phi_1$, is nearly 90°.

Phase difference between I_1 and V, i.e. in $L - R_1$ circuit is given by

$$\tan\phi_2 = -\frac{X_L}{R_1} = -\frac{L\omega}{R}$$

Substituting the given values, we get

$$\tan\phi_2 = -\frac{\frac{\sqrt{3}}{10} \times 100}{10} = -\sqrt{3}$$

As, $\quad \tan\phi_2 = -\sqrt{3}$

$\therefore \quad \phi_2 = 120°$

Now, phase difference between I_1 and I_2 is

$$\Delta\phi = \phi_2 - \phi_1 = 120° - 90° = 30°$$

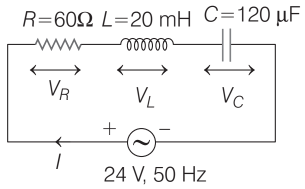
41 A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is **[2019, 9 Jan Shift-II]**

(a) 3.39×10^3 J (b) 5.65×10^2 J

(c) 2.26×10^3 J (d) 5.17×10^2 J

Ans. *(d)*

The given series R-L-C circuit is shown in the figure below.



Here,

V_R = potential across resistance (R)

V_L = potential across inductor (L) and

V_C = potential across capacitor (C).

Impedance of this series circuit is,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{...(i)}$$

$\because \quad X_L = \omega L = (2\pi f)(L)$

$= 2\pi \times 50 \times 20 \times 10^{-3}\ \Omega$

$X_L = 6.28\ \Omega \qquad$ ...(ii)

and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$= \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = \frac{250}{3\pi}\ \Omega \qquad \text{...(iii)}$$

and $X_L - X_C = \left(6.28 - \frac{250}{3\pi}\right)$

$= -20.23\ \Omega \qquad$ ...(iv)

RMS value of current in circuit is,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{24}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_{\text{rms}} = \frac{24}{\sqrt{60^2 + (-20.23)^2}} = \frac{24}{63.18}$$

$I_{\text{rms}} = 0.379$ A

Therefore, energy dissipated is

$= I_{\text{rms}}^2 \times R \times t$

$E = (0.379)^2 \times 60 \times 60$

or $\quad = 517.10 = 5.17 \times 10^2$ J

42 In an AC circuit, the instantaneous emf and current are given by $e = 100\sin 30t$, $i = 20\sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of AC, the average power consumed by the circuit and the wattless current are, respectively **[JEE Main 2018]**

(a) 50, 10 (b) $\frac{1000}{\sqrt{2}}$, 10

(c) $\frac{50}{\sqrt{2}}$, 0 (d) 50, 0

Ans. *(b)*

Given, $e = 100\sin 30t$

and $\quad i = 20\sin\left(30t - \frac{\pi}{4}\right)$

$\therefore$ Average power,

$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$

$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos\frac{\pi}{4} = \frac{1000}{\sqrt{2}}$ watt

Wattless current is,

$I = I_{\text{rms}} \sin\phi$

$= \frac{20}{\sqrt{2}} \times \sin\frac{\pi}{4} = \frac{20}{2} = 10$ A

$\therefore \quad P_{\text{av}} = \frac{1000}{\sqrt{2}}$ watt

and $\quad I_{\text{wattless}} = 10$ A

43 For an R-L-C circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the current exhibits resonance. The quality factor, Q is given by **[JEE Main 2018]**

(a) $\frac{\omega_0 L}{R}$ (b) $\frac{\omega_0 R}{L}$ (c) $\frac{R}{\omega_0 C}$ (d) $\frac{CR}{\omega_0}$

Ans. *(a)*

Sharpness of resonance of a resonant L-C-R circuit is determined by the ratio of resonant frequency with the selectivity of circuit. This ratio is also called "Quality Factor" or Q-factor.

$$Q\text{-factor} = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

44 An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to **[JEE Main 2016]**

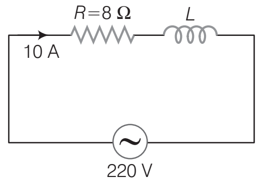
(a) 80 H (b) 0.08 H
(c) 0.044 H (d) 0.065 H

Ans. ***(d)***

Given, $I = 10$ A, $V = 80$ V,

$$R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega \text{ and } \omega = 50 \text{ Hz}$$

For AC circuit, we have



$$I = \frac{V}{\sqrt{8^2 + X_L^2}} \Rightarrow 10 = \frac{220}{\sqrt{64 + X_L^2}}$$

$$\Rightarrow \sqrt{64 + X_L^2} = 22$$

Squaring on both sides, we get

$$64 + X_L^2 = 484$$

$$\Rightarrow X_L^2 = 484 - 64 = 420$$

$$X_L = \sqrt{420} \Rightarrow 2\pi \times \omega L = \sqrt{420}$$

Series inductor on an arc lamp,

$$L = \frac{\sqrt{420}}{(2\pi \times 50)} = 0.065 \text{ H}$$

45 In an AC circuit, the voltage applied is $E = E_0 \sin\omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by **[AIEEE 2007]**

(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P =$ zero
(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2}\, E_0 I_0$

Ans. ***(b)***

For given circuit, current is lagging the voltage by $\frac{\pi}{2}$, so circuit is purely inductive and there is no power consumption in the circuit. The work done by battery is stored as magnetic energy in the inductor.

46 In a series resonant L-C-R circuit, the voltage across R is 100 V and $R = 1\text{k}\Omega$ with $C = 2\mu$F. The resonant frequency ω is 200 rad/s. At resonance, the voltage across L is **[AIEEE 2006]**

(a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

Ans. ***(c)***

At resonance, $\omega L = 1/\omega C$

Current flowing through the circuit,

$$I = \frac{V_R}{R} = \frac{100}{1000} = 0.1 \text{ A}$$

So, voltage across L is given by

$$V_L = I X_L = I\omega L$$

But $\omega L = 1/\omega C$

$$\therefore \quad V_L = \frac{I}{\omega C} = V_C = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250$$

V

47 A circuit has a resistance of 12 Ω and an impedance of 15 Ω. The power factor of the circuit will be **[AIEEE 2005]**

(a) 0.8 (b) 0.4 (c) 1.25 (d) 0.125

Ans. ***(a)***

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$$

48 The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of **[AIEEE 2005]**

(a) 4μF (b) 8μF (c) 1μF (d) 2μF

Ans. ***(c)***

Given, $L = 10$ H, $f = 50$ Hz

For maximum power,

$$X_C = X_L \quad [\because \text{resonance condition}]$$

or $$\frac{1}{\omega C} = \omega L \text{ or } C = \frac{1}{\omega^2 L}$$

$$\therefore \quad C = \frac{1}{4\pi^2 \times 50 \times 50 \times 10}$$

or $$C = 0.1 \times 10^{-5} \text{ F} = 1\mu\text{F}$$

49 The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit ? **[AIEEE 2005]**

(a) C alone (b) R, L
(c) L, C (d) L alone

Ans. ***(c)***

(a) In a circuit having C alone, the voltage lags the current by $\frac{\pi}{2}$.

(b) In circuit containing R and L, the voltage leads the current by $\frac{\pi}{2}$.

(c) In L-C circuit, the phase difference between current and voltage can have any value between 0 to $\frac{\pi}{2}$ depending on the values of L and C.

(d) In a circuit containing L alone, the voltage leads the current by $\pi/2$.

50 Alternating current cannot be measured by DC ammeter because **[AIEEE 2004]**

(a) AC cannot pass through DC ammeter
(b) AC changes direction
(c) average value of current for complete cycle is zero
(d) DC ammeter will get damaged

Ans. ***(c)***

The full cycle of alternating current consists of two half cycles. For one half, current is positive and for second half, current is negative.

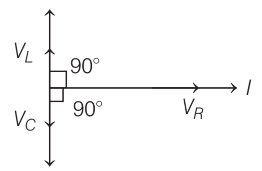
Therefore, for an AC cycle, the net value of current average out to zero. While the DC ammeter, read the average value. Hence, the alternating current cannot be measured by DC ammeter.

51 In an L-C-R series AC circuit, the voltage across each of the components L, C and R is 50 V. The voltage across the L-C combination will be **[AIEEE 2004]**

(a) 50 V (b) $50\sqrt{2}$ V
(c) 100 V (d) zero

Ans. ***(d)***

In an L-C-R series AC circuit, the voltage across inductor L leads the current by 90° and the voltage across capacitor C lags behind the current by 90°. [$\because$ $V = V_L \sim V_C = 50 - 50 = 0$]



Hence, the voltage across L-C combination will be zero.

52 In an L-C-R circuit, capacitance is changed from C to $2C$. For the resonant frequency to remains unchanged, the inductance should be changed from L to **[AIEEE 2004]**

(a) $4L$ (b) $2L$ (c) $L/2$ (d) $L/4$

Ans. ***(c)***

In the condition of resonance,

$$X_L = X_C \text{ or } \omega L = \frac{1}{\omega C} \quad \text{...(i)}$$

Since, resonance frequency remains unchanged, so

$$\sqrt{LC} = \text{constant or } LC = \text{constant}$$

$$\therefore \quad L_1C_1 = L_2C_2$$

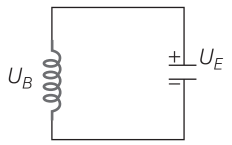
$$\text{or} \quad L \times C = L_2 \times 2C \text{ or } L_2 = L/2$$

53 In an oscillating L-C circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is **[AIEEE 2003]**

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) $\frac{Q}{\sqrt{2}}$ (d) Q

Ans. ***(c)***

In an L-C circuit, the energy oscillates between inductor (in the magnetic field) and capacitor (in the electric field).



$U_{E_{max}}$ [Maximum energy stored in capacitor]

$$= \frac{Q^2}{2C}$$

$U_{B_{max}}$ [Maximum energy stored in inductor]

$$= \frac{Li^2}{2}$$

where, I is the current at this time.

For the given instant, $U_E = U_B$

$$\text{i.e.,} \quad \frac{q^2}{2C} = \frac{LI^2}{2} \quad \text{...(i)}$$

From energy conservation,

$$U_E + U_B = U_{E_{max}} = U_{B_{max}}$$

$$\frac{q^2}{2C} + \frac{1}{2}LI^2 = \frac{Q^2}{2C}$$

$$\Rightarrow \quad \frac{2q^2}{2C} = \frac{Q^2}{2C} \quad \text{[from Eq. (i)]}$$

$$\text{or} \quad q = \frac{Q}{\sqrt{2}}$$

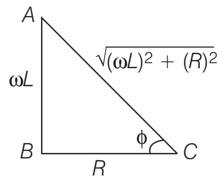
54 The power factor of an AC circuit having resistance R and inductance L (connected in series) and an angular velocity ω is **[AIEEE 2002]**

(a) $\frac{R}{\omega L}$ (b) $\frac{R}{(R^2 + \omega^2 L^2)^{1/2}}$

(c) $\frac{\omega L}{R}$ (d) $\frac{R}{(R^2 - \omega^2 L^2)^{1/2}}$

Ans. ***(b)***

From the relation, $\tan \phi = \frac{\omega L}{R}$



Power factor,

$$\cos \phi = \frac{1}{\sqrt{1 + \tan^2 \phi}}$$

$$\Rightarrow \cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

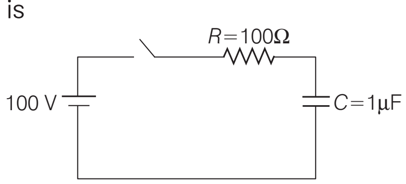
$$= \frac{1}{\sqrt{1 + (\omega L/R)^2}} = \frac{1}{\sqrt{1 + \tan^2 \phi}}$$

$$= \frac{1}{\sqrt{1 + (\omega L/R)^2}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

TOPIC 2

Growth and Decay of Current

55 A capacitor of capacitance $C = 1\mu F$ is suddenly connected to a battery of 100 V through a resistance $R = 100\Omega$. The time taken for the capacitor to be charged to get 50 V is



[Take, $\ln 2 = 0.69$]

[2021, 27 July Shift-I]

(a) 1.44×10^{-4} s (b) 3.33×10^{-4} s

(c) 0.69×10^{-4} s (d) 0.30×10^{-4} s

Ans. ***(c)***

From the given figure in question, it can be seen that a capacitor of capacitance, $C = 1\mu F$ is connected to a battery of 100 V through a resistance, $R = 100\,\Omega$.

$\because$ We know that in RC- circuit

$$V = V_0\left(1 - e^{-\left(\frac{t}{RC}\right)}\right)$$

where, V = voltage across the capacitor,

V_0 = supply voltage,

t = elapsed time, since the application of voltage

and RC = time constant of the RC-charging circuit.

$$\Rightarrow \quad 50 = 100\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow \quad \frac{1}{2} = 1 - e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad \frac{1}{2} - 1 = -e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad -\frac{1}{2} = -e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad 2 = e^{\frac{t}{RC}} \quad \text{...(i)}$$

Taking log on both sides of Eq. (i), we get

$$\ln 2 = \ln\left(e^{\frac{t}{RC}}\right)$$

$$\Rightarrow \quad \ln 2 = \frac{t}{RC}$$

$$\Rightarrow \quad t = RC \ln 2$$

$$= 100 \times 10^{-6} \times (0.69)$$

$$\Rightarrow \quad t = 0.69 \times 10^{-4} \text{ s}$$

56 An inductor of 10 mH is connected to a 20 V battery through a resistor of 10 kΩ and a switch. After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after $1\mu s$ is $\frac{x}{100}$ mA. Then, x is equal to .... (Take, $e^{-1} = 0.37$)

[2021, 25 July Shift-I]

Ans. ***(74)***

Given,

inductance of inductor, $L = 10$ mH $= 10 \times 10^{-3}$ H

Supply voltage, $V = 20$ V

Resistor of resistance $(R) = 10\text{ k}\Omega = 10 \times 10^3\,\Omega$

Time, $t = 1\mu s = 1 \times 10^{-6}$ s

As we know that,

In case of circuit having inductor and resistor,

$$I = I_0 e^{-t/\tau} \quad \text{...(i)}$$

where, I = decayed current,

I_0 = peak current

t = time taken

and $\tau =$ time constant $= \frac{L}{R}$

Substituting in Eq. (i), we get

$$I = I_0 e^{\frac{-tR}{L}} = \frac{V}{R} e^{-t\frac{R}{L}}$$

$$\Rightarrow \quad I = \frac{20}{10 \times 10^3} e^{-\frac{1\times10^{-6}\times10\times10^3}{10\times10^{-3}}}$$

$$= 2\times10^{-3} e^{-1} = 2e^{-1} \text{mA}$$

$$= 2\times0.37 \text{ mA} = 0.74 \text{ mA}$$

$$= 74/100 \text{ mA}$$

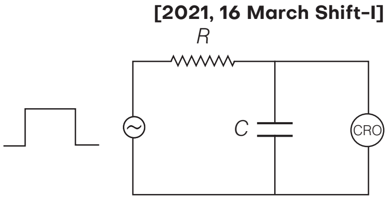
Comparing with given value, i.e.

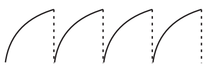
$I = \frac{x}{100}$ mA, we get

$x = 74$

57 An R-C circuit as shown in the figure is driven by an AC source generating a square wave. The output wave pattern monitored by CRO would look close to

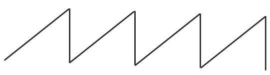
[2021, 16 March Shift-I]



(a) 

(b) 

(c) 

(d) 

Ans. *(c)*

When square wave is applied at the input, then

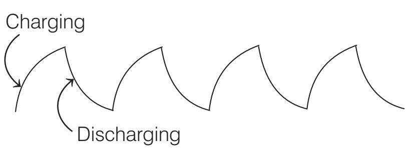
For charging, the capacitor

$$Q_1 = Q(1 - e^{\frac{-t}{RC}})$$

Similarly, for discharging the capacitor,

$$Q_2 = Q_{max}(e^{\frac{-t}{RC}})$$

In this manner, charging and discharging exponentially with time will keep on happening alternatively. Therefore, the output wave pattern monitored by CRO would look close to



58 A series L-R circuit is connected to a battery of emf V. If the circuit is switched ON at $t = 0$, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is

[2020, 4 Sep Shift-II]

(a) $\frac{L}{R}\ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$ (b) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$

(c) $\frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$ (d) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$

Ans. *(d)*

Let t be the required time at which the energy stored in inductor becomes $\left(\frac{1}{n}\right)$ times of its maximum value.

According to question, for given L-R circuit,

$$U = \frac{1}{n} U_{\max}$$

$$\frac{1}{2}LI^2 = \frac{1}{n} \times \frac{1}{2} LI^2_{\max}$$

$$\frac{1}{2}L\left[I_{\max}\left(1 - e^{\frac{-Rt}{L}}\right)\right]^2 = \frac{1}{n} \times \frac{1}{2}LI^2_{\max}$$

$$\frac{1}{2}LI^2_{\max}\left(1 - e^{\frac{-Rt}{L}}\right)^2 = \frac{1}{n} \times \frac{1}{2}LI^2_{\max}$$

$$\left(1 - e^{\frac{-Rt}{L}}\right)^2 = \frac{1}{n}$$

$$\Rightarrow \quad 1 - e^{\frac{-Rt}{L}} = \left(\frac{1}{n}\right)^{\frac{1}{2}}$$

$$e^{\frac{-Rt}{L}} = 1 - \frac{1}{\sqrt{n}}$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}} = \frac{\sqrt{n}-1}{\sqrt{n}}$$

Taking natural log on both sides, we get

$$\ln(e^{\frac{-Rt}{L}}) = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

$$-\frac{Rt}{L}\ln(e) = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

$$-\frac{Rt}{L} \times 1 = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

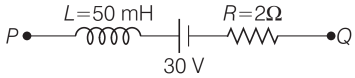
$$\Rightarrow \quad t = \frac{-L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

$$t = \frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)^{-1}$$

$$\Rightarrow \quad t = \frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$$

Hence, option (d) is correct.

59 A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at the rate of 10^2 As^{-1}. The value of the potential difference $V_P - V_Q$ (in volt) at that instant, is ....



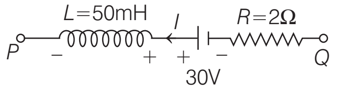
[2020, 6 Sep Shift-I]

Ans. *(33)*

Given that, at any instant,

$I = 1$ A

$\frac{dI}{dt} = -10^2$ A/s (as current is decreasing)



Applying KVL from point P to point Q,

$$V_P + \frac{LdI}{dt} - 30 + IR = V_Q$$

$$V_P - V_Q = 30 - 1\times2 - 50\times10^{-3}\times(-10^2)$$

$$= 33 \text{ V}$$

60 An emf of 20 V is applied at time $t = 0$ to a circuit containing in series 10 mH inductor and 5Ω resistor. The ratio of the currents at time $t = \infty$ and at $t = 40$ s is close to (Take, $e^2 = 7.389$) **[2020, 7 Jan Shift-II]**

(a) 1.06 (b) 1.15 (c) 1.46 (d) 0.84

Ans. *(a)*

In an L-R circuit, current growth occurs as

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{...(i)}$$

where, $I =$ instantaneous current,

$I_0 =$ maximum current

$=$ current at $t = \infty = \frac{E}{R}$,

$R =$ resistance of circuit,

$L =$ inductance of circuit

and $t =$ instantaneous time.

Here, $R = 5\,\Omega$, $L = 10$ mH $= 10\times10^{-3}$ H,

$E = 20$ V, $t = 40$ s

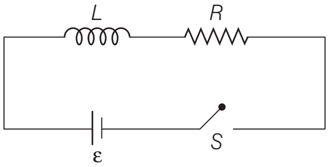
So, substituting these values in Eq. (i), we get

$$\frac{I}{I_0} = \left(1 - e^{-\frac{5}{10\times10^{-3}}\times40}\right) = 1 - e^{-2\times10^4}$$

$$\Rightarrow \quad \frac{I_0}{I} = \frac{1}{1 - e^{-2\times10^4}} \approx 1$$

$$\therefore \quad \frac{I_0}{I} \approx 1.06 \text{ (nearest option)}$$

61



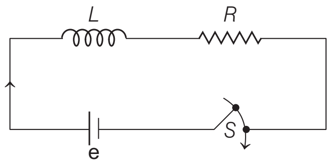
As shown in the figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at $t=0$. The total charge that flows from the battery, between $t=0$ and $t=t_c$ (t_c is the time constant of the circuit) is **[2020, 8 Jan Shift-II]**

(a) $\frac{\varepsilon L}{R^2}\left(1-\frac{1}{e}\right)$ (b) $\frac{\varepsilon L}{R^2}$

(c) $\frac{\varepsilon R}{eL^2}$ (d) $\frac{\varepsilon L}{eR^2}$

Ans. *(d)*

In an L-R circuit as shown in the figure, instantaneous current at time t.



$$i = I(1-e^{-\frac{R.t}{L}})$$

where, $\frac{L}{R} = t_c$ = time constant of the circuit.

Now, charge that flows from $t=0$ to $t=t_c$ from battery,

$$q = \int_0^{t_c} i dt = I\int_0^{t_c}(1-e^{-\frac{R}{L}.t})dt$$

$$= I\left[t - \frac{1}{-\frac{R}{L}}.e^{\frac{-R}{L}.t}\right]_0^{t_c} = I\left[t + \frac{L}{R}.e^{\frac{-R}{L}.t}\right]_0^{t_c}$$

$$= I\left[\left(t_c + \frac{L}{R}e^{-\frac{R}{L}t_c}\right) - \left(\frac{L}{R}e^0\right)\right]$$

$$= I\left[\frac{L}{R} + \frac{L}{R}e^{-1} - \frac{L}{R}\right] = I.\frac{L}{R}.e^{-1}$$

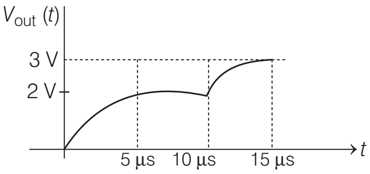
Here, I is maximum current which occurs at $t=\infty$ and its value is,

$$I = \frac{\varepsilon}{R}$$

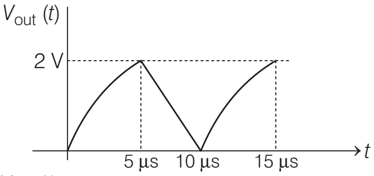
So, $q = \frac{\varepsilon}{R}.\frac{L}{R}.\frac{1}{e} = \frac{\varepsilon L}{eR^2}$

62 For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_{out}(t)$, across the capacitor is correctly depicted by **[2020, 6 Sep Shift-I]**

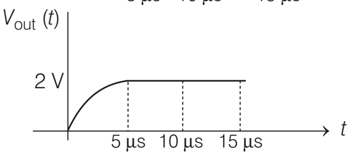
(a)



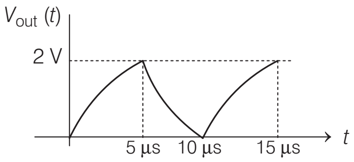
(b)



(c)

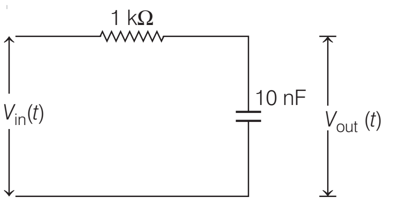


(d)



Ans. *(a)*

The question clearly involves R-C circuit with DC input, which does not remains constant. For first 5μs, capacitor gets charged as voltage is 5V and for next 5μs, capacitor gets discharged as voltage is zero. Redrawing the given circuit in simple manner



Time constant, $\tau = RC$

$$= (1\times10^3)\times(10\times10^{-9}) = 10\mu s$$

At $t = 5\mu s$,

Voltage across capacitor,

$$V_{out} = V_m(1-e^{-t/\tau})$$

$$= 5\left(1 - e^{-\frac{5\mu s}{10\mu s}}\right) \quad [\because V_m = 5\,V]$$

$$= 5(1-e^{-0.5}) = 1.96\,V \simeq 2\,V$$

So, the capacitor is charged to voltage of 2V, i.e. $V_c = 2V$

For next 5μs, $V_{in} = 0$, that means capacitor is discharging.

So, output voltage,

$$V_{out} = V_C\, e^{-t/\tau} = 2e^{-\frac{5\mu s}{10\mu s}} = 2e^{-0.5} = 1.21\,V$$

Now, for next 5μs,

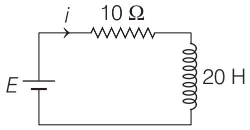
$$V_{out} = 5(1-e^{-t/\tau}) + 1.21e^{-t/\tau}$$

$$= 5 - 3.79e^{-t/\tau}$$

$$= 5 - 3.79\,e^{-\frac{5\mu s}{10\mu s}} = 2.71\,V \simeq 3\,V$$

The given values of output voltage are most closely resembles to that in graph (a).

63 A 20 H inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is **[2019, 8 April Shift-I]**

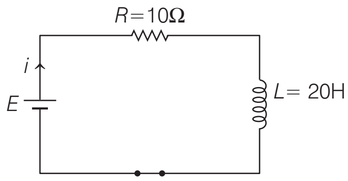


(a) $\frac{2}{\ln 2}$ (b) $\frac{1}{2}\ln 2$

(c) $2\ln 2$ (d) $\ln 2$

Ans. *(c)*

Given circuit is a series L-R circuit



In an L-R circuit, current increases is

$$i = \frac{E}{R}(1-e^{-\frac{R}{L}\cdot t})$$

Now, energy stored in inductor is

$$U_L = \frac{1}{2}Li^2$$

where, L = self inductance of the coil and energy dissipated by resistor is

$$U_R = i^2R$$

Given, rate of energy stored in inductor is equal to the rate of energy dissipation in resistor. So, after differentiating, we get

$$iL\frac{di}{dt} = i^2R \Rightarrow \frac{di}{dt} = \frac{R}{L}i$$

$$\Rightarrow \quad \frac{E}{R}\cdot\frac{R}{L}e^{-\frac{R}{L}t} = \frac{R}{L}\cdot\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow \quad 2e^{-\frac{R}{L}t} = 1 \Rightarrow e^{-\frac{R}{L}t} = \frac{1}{2}$$

Taking log on both sides, we have

$$\Rightarrow \quad \frac{-R}{L}t = \ln\left(\frac{1}{2}\right) \Rightarrow \frac{R}{L}t = \ln 2$$

$$\Rightarrow \quad t = \frac{L}{R}\ln 2 = \frac{20}{10}\ln 2 \Rightarrow t = 2\ln 2$$

64 A coil of self inductance 10 mH and resistance $0.1\,\Omega$ is connected through a switch to a battery of internal resistance $0.9\,\Omega$. After the switch is closed, the time taken for the current to attain 80% of the saturation value is [Take, $\ln 5 = 1.6$]

[2019, 10 April Shift-II]

(a) 0.002 s (b) 0.324 s
(c) 0.103 s (d) 0.016 s

Ans. *(d)*

Key Idea In an *L-R* circuit, current during charging of inductor is given by

$$i = i_0\left(1 - e^{-\frac{R}{L}\cdot t}\right)$$

where, i_0 = saturation current.

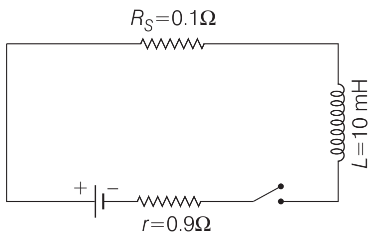
In given circuit,

Inductance of circuit is

$L = 10\text{ mH} = 10 \times 10^{-3}\text{ H}$

Resistance of circuit is

$$R = (R_s + r) = 0.1 + 0.9 = 1\,\Omega$$



Now, from

$$i = i_0\left(1 - e^{-\frac{R}{L}\cdot t}\right) \quad \ldots(i)$$

Given, $i = 80\%$ of i_0

$$\Rightarrow \quad i = \frac{80\, i_0}{100} = 0.8\, i_0$$

Substituting the value of i in Eq. (i), we get

$$0.8 = 1 - e^{-\frac{R}{L}\cdot t}$$

$$\Rightarrow \quad e^{-\frac{R}{L}t} = 0.2 \Rightarrow e^{\frac{R}{L}t} = 5$$

$$\Rightarrow \quad \ln(e)^{\frac{R}{L}t} = \ln 5 \Rightarrow \frac{R}{L}t = \ln 5$$

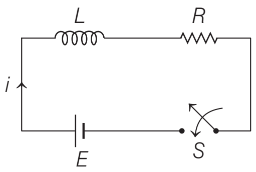
$$\Rightarrow \quad t = \frac{L}{R}\cdot \ln(5) = \frac{10 \times 10^{-3}}{1} \times \ln(5)$$

$$= 10 \times 10^{-3} \times 1.6$$

$$= 1.6 \times 10^{-2}\text{ s} = 0.016\text{ s}$$

65 Consider the *L-R* circuit shown in the figure. If the switch *S* is closed at $t = 0$, then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is

[2019, 12 April Shift-II]



(a) $\frac{2.7\,EL}{R^2}$ (b) $\frac{EL}{2.7\,R^2}$ (c) $\frac{7.3\,EL}{R^2}$ (d) $\frac{EL}{7.3\,R^2}$

Ans. *(b)*

In an *L-R* circuit, current during charging is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

where, $I_0 = \frac{E}{R}$ = saturation current.

So, we have $\frac{dq}{dt} = I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$$\Rightarrow \quad dq = I_0\left(1 - e^{-\frac{R}{L}t}\right)dt$$

So, charge q that passes through battery from time $t = 0$ to $t = \frac{L}{R}$ is obtained by integrating the above equation within the specified limits, i.e.

$$q = \int_0^Q dq = \int_{t=0}^{t=\frac{L}{R}} I_0\left(1 - e^{-\frac{R}{L}t}\right)dt$$

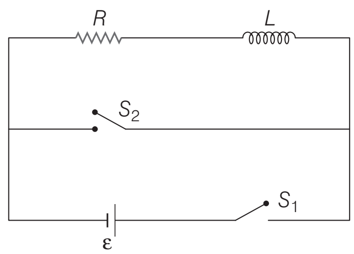
$$= I_0\left[\left(t - \frac{1}{\left(-\frac{R}{L}\right)}\cdot e^{-\frac{R}{L}t}\right)\right]_0^{\frac{L}{R}}$$

$$= \frac{E}{R}\left[\left\{\frac{L}{R} + \frac{L}{Re'}\right\} - \left\{0 + \frac{L}{R}\right\}\right]$$

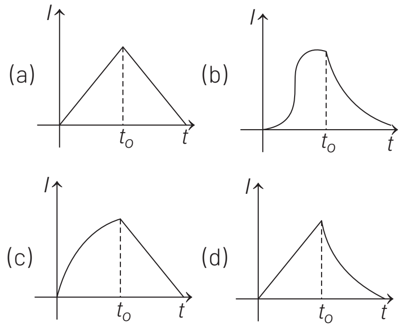
$$= \frac{E}{R} \times \frac{L}{Re} = \frac{EL}{R^2 e}$$

$$\Rightarrow \quad Q = \frac{EL}{2.7\,R^2} \quad [\because e \approx 2.72]$$

66 In the circuit shown,



The switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't' is given by



[2019, 11 Jan Shift-I]

Ans. *(b)*

Initially in the given *RL* circuit with a source, when S_1 is closed and S_2 is open at $t \le t_0$.

$$I_1 = \frac{V}{R}\left[1 - \exp\left(\frac{-R}{L}t\right)\right]$$

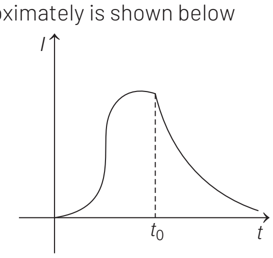
In this case, inductor *L* is charging.

When switch S_2 is closed and S_1 is open (after $t > t_0$), the inductor will be discharged through resistor.

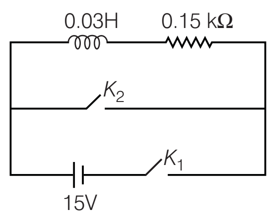
In this case $(t > t_0)$,

$$I_2 = \frac{V}{R}\exp\left[-\frac{R}{L}(t - t_0)\right]$$

Thus, the variation of I with t approximately is shown below



67 An inductor ($L = 0.03$ H) and a resistor ($R = 0.15\,\text{k}\Omega$) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ms, the current in the circuit will be ($e^5 \cong 150$) **[JEE Main 2015]**



(a) 100 mA (b) 67 mA
(c) 6.7 mA (d) 0.67mA

Ans. *(d)*

Key Idea After long time inductor behaves as short-circuit.

At $t=0$, the inductor behaves as short-circuited. The current

$$I_0 = \frac{E_0}{R} = \frac{15\,\text{V}}{0.15\,\text{k}\Omega} = 100\ \text{mA}$$

As K_2 is closed, current through the inductor starts decay, which is given at any time t as

$$I = I_0\, e^{\frac{-tR}{L}} = (100\ \text{mA})\, e^{\frac{-t\times 15000}{3}}$$

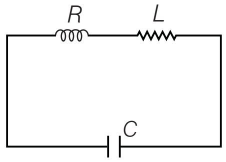
At $\quad t = 1\text{ms}$

$$I = (100\ \text{mA})\, e^{-\frac{1\times10^{-3}\times15\times10^{3}}{3}}$$

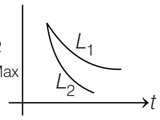
$$I = (100\ \text{mA})\, e^{-5} = 0.6737\ \text{mA}$$

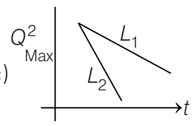
or $\quad I = 0.67\ \text{mA}$

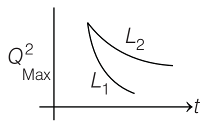
68 An LCR circuit is equivalent to a damped pendulum. In an LCR circuit, the capacitor is charged to Q_0 and then connected to the L and R as shown below.

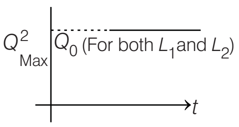


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L, then which of the following represents this graph correctly? (plots are schematic and not drawn to scale) **[JEE Main 2015]**

(a) 

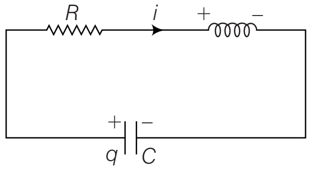
(c) 

(b) 

(d) 

Ans. *(a)*

Considert the LCR circuit at any time t



Now, applying KVL

We have $\quad \dfrac{q}{C} - iR - \dfrac{Ldi}{dt} = 0$

As current is decreasing with time we can write $i = -\dfrac{dq}{dt}$

$$\Rightarrow \quad \frac{q}{C} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$$

or $$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

This equation is equivalent to that of a damped oscillator

Thus, we can write the solution as

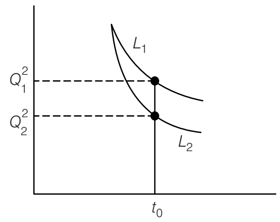
$$Q_{\max}(t) = Q_0 \cdot e^{-Rt/2L}$$

or $\quad Q^2_{\max} = Q^2_0\, e^{-\frac{Rt}{L}}$

As $L_1 > L_2$ damping is faster for L_2

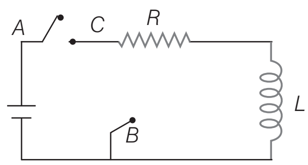
Aliter Inductance is inertia of circuit. It means inductance opposes the flow of charge, more inductance means decay of charge is slow.

In option (a), in a given time to, $Q_1^2 > Q_2^2$.



So, $L_1 > L_2$. Hence, option (a) is correct.

69 In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly point C is disconnected from point A and connected to point B at time $t=0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to **[JEE Main 2014]**

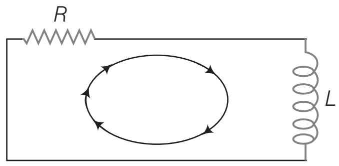


(a) $\dfrac{e}{1-e}$ (b) 1

(c) -1 (d) $\dfrac{1-e}{e}$

Ans. *(c)*

After connecting C to B hanging the switch, the circuit will act like L-R discharging circuit.



Applying Kirchhoff's loop equation,

$$V_R + V_L = 0 \Rightarrow V_R = -V_L$$

$$\therefore \quad \frac{V_R}{V_L} = -1$$

$$e = \int Bv\,dx \qquad [\because v = \omega x]$$

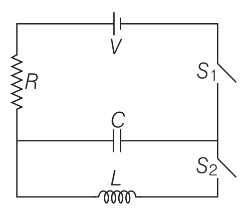
$$\Rightarrow \quad e = \int_{2l}^{3l} (\omega x)\cdot B \cdot dx$$

Using $\displaystyle\int x^n dx = \frac{x^{n+1}}{n+1}$

$$\Rightarrow \quad e = B\omega\left[\frac{x^2}{2}\right]_{2l}^{3l}$$

$$e = B\omega\frac{[(3l)^2 - (2l)^2]}{2} = \frac{5Bl^2\omega}{2}$$

70 In an L-C-R circuit as shown below, both switches are open initially. Now, switch S_1 is closed and S_2 kept open (q is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct? **[JEE Main 2013]**



(a) Work done by the battery is half of the energy dissipated in the resistor

(b) At $t = \tau$, $q = CV/2$

(c) At $t = 2\tau$, $q = CV(1-e^{-2})$

(d) At $t = \dfrac{\tau}{2}$, $q = CV(1-e^{-1})$

Ans. *(c)*

For charging capacitor, q is given as

$$q = q_0(1 - e^{-t/\tau}) = CV(1 - e^{-t/\tau})$$

At $\quad t = 2\tau$,

$$q = CV(1 - e^{\frac{-2\tau}{\tau}})$$

$$\Rightarrow \quad q = CV(1 - e^{-2})$$

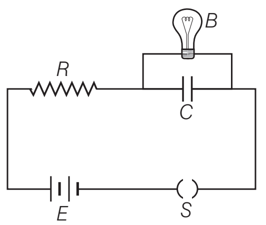
71 A resistor R and $2\mu F$ capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed (take $\log_{10} 2.5 = 0.4$) **[AIEEE 2011]**

(a) $1.7 \times 10^5 \Omega$ (b) $2.7 \times 10^6 \Omega$

(c) $3.3 \times 10^7 \Omega$ (d) $1.3 \times 10^4 \Omega$

Ans. *(b)*

Neon bulb is filled with gas, so its resistance is infinite, hence no current flows through it.



Now, $V_c = E(1 - e^{-t/RC})$

$$\Rightarrow \quad 120 = 200(1 - e^{-t/RC})$$

$$\Rightarrow e^{-t/RC} = \frac{2}{5} \Rightarrow t = RC \ln 2.5$$

$$\Rightarrow \quad R = \frac{t}{C \ln 2.5} = \frac{t}{2.303 C \log 2.5}$$

$$= 2.7 \times 10^6 \Omega$$

72 Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then, the ratio t_1 / t_2 will be **[AIEEE 2010]**

(a) 1 (b) 1/2 (c) 1/4 (d) 2

Ans. *(c)*

Energy stored in capacitor,

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2C}(q_0 e^{-t/\tau})^2$$

$$= \frac{q_0^2}{2C} e^{-2t/\tau} \text{[where, } \tau = CR]$$

$$U = U_i e^{-2t/\tau}, \frac{1}{2}U_i = U_i e^{-2t_1/\tau}$$

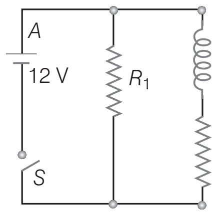
$$\frac{1}{2} = e^{-2t_1/\tau} \Rightarrow t_1 = \frac{\tau}{2}\ln 2$$

Now, $q = q_0 e^{-t/\tau}, \frac{1}{4}q_0 = q_0 e^{-t_2/\tau}$

$$t_2 = \tau \ln 4 = 2\tau \ln 2$$

$$\therefore \quad \frac{t_1}{t_2} = \frac{1}{4}$$

73 An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 4\Omega$ and $R_2 = 2\Omega$ are connected to battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is **[AIEEE 2009]**

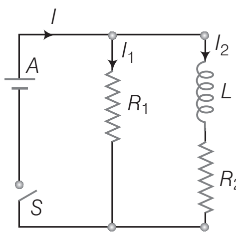


(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V

(c) $6(1 - e^{-t/0.2})$V (d) $12e^{-5t}$ V

Ans. *(d)*

$$I_1 = \frac{E}{R_1} = \frac{12}{4} = 3\text{ A}$$



$\because$ Potential drop $= E - I_2 R_2$

$I_2 = I_0(1 - e^{-T/t_c})$

[current as a function of time]

$$\Rightarrow \quad I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\text{ A}$$

and $\quad t_c = \frac{L}{R_2} = \frac{400 \times 10^{-3}}{2} = 0.2$

$$I_2 = 6(1 - e^{-t})$$

Potential drop across

$$L = E - R_2 I_2 = 12 - 2 \times 6(1 - e^{-5t})$$

$$= 12e^{-5t}$$

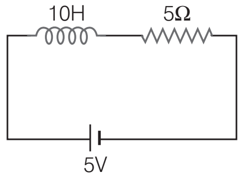
74 An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. After 2 s, the connection is made, the current flowing (in ampere) in the circuit is

(a) $(1 - e)$ (b) e **[AIEEE 2007]**

(c) e^{-1} (d) $(1 - e^{-1})$

Ans. *(d)*

Rise of current in L-R circuit is given by



$$I = I_0(1 - e^{-t/\tau})$$

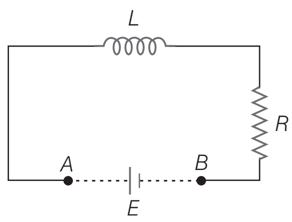
where, $\quad I_0 = \frac{E}{R} = \frac{5}{5} = 1\text{ A}$

Now, $\quad \tau = \frac{L}{R} = \frac{10}{5} = 2\text{ s}$

After 2s *i.e.*, at $t = 2$ s,

Rise of current, $I = (1 - e^{-1})$ A

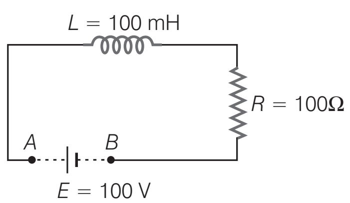
75 An inductor ($L = 100$ mH), a resistor ($R = 100\Omega$) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time, the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 millisecond after the short circuit is **[AIEEE 2006]**



(a) $1/e$ A (b) e A (c) 0.1 A (d) 1 A

Ans. *(a)*

This is a combined example of growth and decay of current in an L-R circuit.



The current through circuit just before shorting the battery,

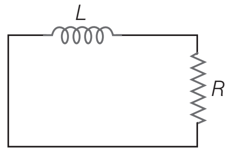
$$I_0 = \frac{E}{R} = \frac{100}{100} = 1\text{ A}$$

[as inductor would be shorted in steady state]

After this decay of current starts in the circuit, according to the equation,

$$I = I_0 e^{-t/\tau}$$

where, $\tau = L/R$



$$I = 1 \times e^{-(1\times10^{-3})/(100\times10^{-3}/100)} = \left(\frac{1}{e}\right) \text{A}$$

$[\because t = 1 \text{ millisecond} = 1\times10^{-3} \text{ s}$ and $L = 100\times10^{-3} \text{ H}]$

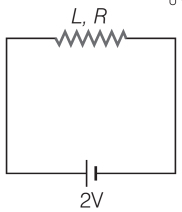
76 A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in **[AIEEE 2005]**

(a) 0.05 s (b) 0.1 s
(c) 0.15 s (d) 0.3 s

Ans. *(b)*

The current at any instant is given by

$$I = I_0(1 - e^{-Rt/L})$$



$$\Rightarrow \quad \frac{I_0}{2} = I_0(1 - e^{-Rt/L})$$

or $\quad \frac{1}{2} = (1 - e^{-Rt/L})$

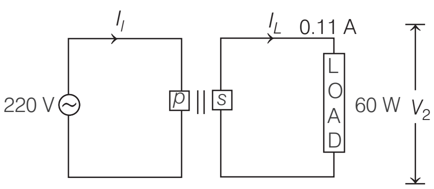
or $\quad e^{-Rt/L} = \frac{1}{2}$ or $\frac{Rt}{L} = \ln 2$

$$\therefore t = \frac{L}{R}\ln 2 = \frac{300\times10^{-3}}{2}\times 0.693$$

$$= 150\times0.693\times10^{-3} = 0.10395 \text{ s} = 0.1 \text{ s}$$

TOPIC 3
AC Generator and Transformer

77 For the given circuit, comment on the type of transformer used. **[2021, 16 March Shift-II]**



(a) Auxilliary transformer
(b) Auto transformer
(c) Step-up transformer
(d) Step down transformer

Ans. *(c)*

Voltage across secondary coil,

$$= \frac{\text{Power across load}}{\text{Current passing through load}}$$

$$\Rightarrow \quad V_2 = \frac{P}{I_L} = \frac{60}{0.11} \Rightarrow V_2 = 545.45 \text{ V}$$

Voltage across primary coil, $V_1 = 220\text{V}$

$$\Rightarrow \quad V_2 > V_1$$

It means that step-up transformer is used.

78 An electrical power line, having a total resistance of 2Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately **[2020, 5 Sep Shift-I]**

(a) 91% (b) 85%
(c) 96% (d) 72%

Ans. *(c)*

Given, $P = 1 \text{ kW} = 1000 \text{ W}$

$V = 220 \text{ V}, R = 2\Omega$

Current, $I = \frac{P}{V} = \frac{1000}{220} \text{ A}$

Power loss, $P_{\text{loss}} = I^2 R = \left(\frac{1000}{220}\right)^2 \times 2$

$= 41.32 \text{ W}$

Now, the efficiency of transmission line,

$$\eta = \left(\frac{P}{P + P_{\text{loss}}}\right)\times 100$$

$$= \left(\frac{1000}{1000 + 41.32}\right)\times 100$$

$$\eta = 96.03\% \approx 96\%$$

Hence, correct option is (c).

79 A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are **[2019, 10 April Shift-I]**

(a) 440 V and 5 A (b) 220 V and 20 A
(c) 220 V and 10 A (d) 440 V and 20 A

Ans. *(a)*

Given,

Number of turns in primary, $N_1 = 300$

Number of turns in secondary, $N_2 = 150$

Output power, $P_2 = 2.2 \text{ kW} = 2.2\times10^3 \text{ W}$

Current in secondary coil, $I_2 = 10 \text{ A}$

Output power, $P_2 = I_2 V_2$

$$\Rightarrow \quad V_2 = \frac{P_2}{I_2} = \frac{2.2\times10^3}{10} = 220 \text{ V} \quad \text{...(i)}$$

We know that,

$$\frac{N_1}{N_2} = \frac{\text{Input voltage}}{\text{Output voltage}}$$

$$= \frac{V_1}{V_2} \Rightarrow V_1 = \left(\frac{N_1}{N_2}\right)V_2$$

$$\Rightarrow \quad V_1 = \left(\frac{300}{150}\right)\times(220 \text{ V}) \quad [\text{using Eq. (i)}]$$

$$V_1 = 440 \text{ V} \quad \text{...(ii)}$$

Again, $\frac{V_1}{V_2} = \frac{I_2}{I_1}$

$$\Rightarrow \quad I_1 = \left(\frac{V_2}{V_1}\right)I_2 = \frac{220}{440}\times10$$

[using Eqs. (i) and (ii)]

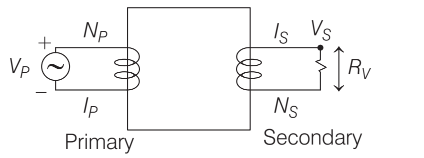
$$\Rightarrow \quad I_1 = 5 \text{ A}$$

80 A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be **[2019, 9 Jan Shift-II]**

(a) 45 A (b) 50 A
(c) 25 A (d) 35 A

Ans. *(a)*

For a transformer, there are two circuits which have N_p and N_s (number of coil turns), I_p and I_S (currents) respectively as shown below.



Here, input voltage, $V_p = 2300 \text{ V}$

Number of turns in primary coil, $N_P = 4000$

Output voltage, $V_S = 230$ volt

Output power, $P_S = V_S \cdot I_S$

Input power, $P_P = V_P I_P$

$\therefore$ The efficiency of the transformer is

$$\eta = \frac{\text{Output (secondary) power}}{\text{Input (primary) power}}$$

$$\Rightarrow \quad \eta = \frac{V_S \cdot I_S}{V_P \cdot I_P}\times 100$$

$$\Rightarrow \qquad \eta = \frac{(230)(I_S)}{(2300)(5)} \times 100$$

$$90 = \frac{230\, I_S}{(2300) \times 5} \times 100$$

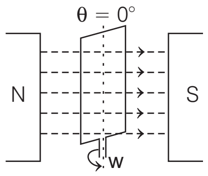
$$\Rightarrow \qquad I_S = 45 \text{ A}$$

81 In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is **[AIEEE 2006]**

(a) $NABR\omega$
(b) NAB
(c) $NABR$
(d) $NAB\omega$

Ans. ***(d)***

The emf generated would be maximum when flux (cutting) would be maximum *i.e.*, angle between area vector of coil and magnetic field is 0°. The emf generated is given by (as a function of time)



$$e = NBA\omega \cos \omega t$$

$$\Rightarrow \quad e_{max} = NAB\omega$$

$$[\because \cos \omega t = \cos \theta = 1 \Rightarrow \theta = 0°]$$

82 The core of any transformer is laminated so as to **[AIEEE 2003]**

(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage

Ans. ***(a)***

The core of transformer is laminated to reduce energy loss due to eddy currents because induction is reduced by laminating.

83 In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A, then that in the secondary is **[AIEEE 2002]**

(a) 4 A
(b) 2 A
(c) 6 A
(d) 10 A

Ans. ***(b)***

Given, $I_p = 4$ A, $N_p = 140$ and $N_s = 280$

From the formula,

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

or $$\frac{4}{I_s} = \frac{280}{140}$$

So, $$I_s = 2 \text{ A}$$