Vedantu
Learn LIVE Online

Table of Contents

Straight Lines

STRAIGHT LINES

1. DISTANCE FORMULA

The distance between the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

2. SECTION FORMULA

The P(x,y) divided the line joining $A(x_1,y_1)$ and $B(x_2,y_2)$ in the ratio m : n, then ;

$$x=\frac{mx_2+nx_1}{m+n};\ y=\frac{my_2+ny_1}{m+n}$$

Note...

(i) If m/n is positive, the division is internal, but if m/n is negative, the division is external.

(ii) If P divides AB internally in the ratio m:n & Q divides AB externally in the ratio m:n then

P & Q are said to be harmonic conjugate of each other w.r.t. AB.

Mathematically, $\frac{2}{AB}=\frac{1}{AP}+\frac{1}{AQ}$ i.e. AP, AB & AQ are in H.P.

3. CENTROID, INCENTRE & EXCENTRE

If $A(x_1, y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the co-ordinates of the special points of triangle ABC are as follows :

$$\text{Centroid G} \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$\text{Incentre I} \equiv \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right) \text{ and}$$

Excentre (to A) I_1

$$\equiv \left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right) \text{ and so on.}$$

Note...

(i) Incentre divides the angle bisectors in the ratio, (b+c): a; (c+a) : b & (a+b) : c.

(ii) Incentre and excentre are harmonic conjugate of each other w.r.t. the angle bisector on which they lie.

(iii) Orthocentre, Centroid & Circumcentre are always collinear & centroid divides the line joining orthocentre & circumcentre in the ratio 2:1.

(iv) In an isosceles triangle G, O, I & C lie on the same line and in an equilateral traingle, all these four points coincide.

4. AREA OF TRIANGLE

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of triangle ABC, then its area is equal to

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}, \text{ provided the vertices are}$$

considered in the counter clockwise sense.

The above formula will give a (–)ve area if the vertices (x_i, y_i), i = 1, 2, 3 are placed in the clockwise sense.

Note...

Area of n-sided polygon formed by points $(x_1, y_1); (x_2, y_2); \ldots\ldots\ldots\ldots(x_n, y_n)$ is given by :

$$\frac{1}{2}\left(\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \ldots\ldots\ldots\ldots \begin{vmatrix} x_{n-1} & x_n \\ y_{n-1} & y_n \end{vmatrix}\right)$$

5. SLOPE FORMULA

If θ is the angle at which a straight line is inclined to the positive direction of x-axis, and $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does n't exist, but the line is parallel to the y-axis, If $\theta = 0$, then $m = 0$ and the line is parallel to the x-axis.

If $A(x_1, y_1)$ & $B(x_2, y_2)$, $x_1 \ne x_2$, are points on a straight line, then the slope m of the line is given by :

$$m = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)$$

6. CONDITION OF COLLINEARITY OF THREE POINTS

Points $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ are collinear if :

(i) $m_{AB} = m_{BC} = m_{CA}$ i.e. $\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \left(\frac{y_2 - y_3}{x_2 - x_3}\right)$

(ii) $\Delta ABC = 0$ i.e. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(iii) AC = AB + BC or AB ~ BC

(iv) A divides the line segment BC in some ratio.

7. EQUATION OF A STRAIGHT LINE IN VARIOUS FORMS

(i) Point-Slope form : $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is m and which passes through the point (x_1,y_1).

(ii) Slope-Intercept form : $y = mx + c$ is the equation of a straight line whose slope is m and which makes an intercept c on the y-axis.

(iii) Two point form : $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of a straight line which passes through the point (x_1,y_1) & (x_2,y_2)

(iv) Determinant form : Equation of line passing through (x_1,y_1) and (x_2,y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

(v) Intercept form : $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on OX & OY respectively.

(vi) Perpendicular/Normal form : $x\cos\alpha + y\sin\alpha = p$ (where $p > 0$, $0 \le \alpha < 2\pi$) is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes an angle α with positive x-axis.

(vii) Parametric form : $P(r) = (x,y) = (x_1 + r\cos\theta, y_1 + r\sin\theta)$ or $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$ is the equation of the line is parametric form, where 'r' is the parameter whose absolute value is the distance of any point (x,y) on the line from fixed point (x_1,y_1) on the line.

(viii) General Form : $ax + by + c = 0$ is the equation of a straight line in the general form. In this case, slope of line $= -\frac{a}{b}$.

8. POSITION OF THE POINT (x_1,y_1) RELATIVE OF THE LINE $ax + by + c = 0$

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax + by + c = 0$. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1,y_1) will lie on the non-origin side of $ax + by + c = 0$.

In general two points (x_1,y_1) and (x_2,y_2) will lie on same side or opposite side of $ax+by+c=0$ according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of same or opposite sign respectively.

9. THE RATIO IN WHICH A GIVEN LINE DIVIDES THE LINE SEGMENT JOINING TWO POINTS

Let the given line ax + by + c = 0 divides the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ in the ratio m:n, then

$\frac{m}{n} = -\frac{ax_1+by_1+c}{ax_2+by_2+c}$. If A and B are on the same side of the given line then m/n is negative but if A and B are on opposite sides of the given line, then m/n is positive.

10. LENGTH OF PERPENDICULAR FROM A POINT ON A LINE

The length of perpendicular from $P(x_1,y_1)$ on ax + by + c = 0 is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

11. REFLECTION OF A POINT ABOUT A LINE

(i) The image of a point (x_1,y_1) about the line ax + by + c = 0 is :

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -2\frac{ax_1+by_1+c}{a^2+b^2}$$

(ii) Similarly foot of the perpendicular from a point on the line is :

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}$$

12. ANGLE BETWEEN TWO STRAIGHT LINES IN TERMS OF THEIR SLOPES

If m_1 and m_2 are the slopes of two intersecting straight lines $(m_1m_2 \neq -1)$ and θ is the acute angle between them,

then $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$.

Note...

Let m_1, m_2, m_3 are the slopes of three line $L_1=0$; $L_2=0$; $L_3=0$ where $m_1 > m_2 > m_3$ then the interior angles of the ΔABC found by these lines are given by,

$\tan A = \frac{m_1-m_2}{1+m_1m_2}$; $\tan B = \frac{m_2-m_3}{1+m_2m_3}$; and $\tan C = \frac{m_3-m_1}{1+m_3m_1}$

13. PARALLEL LINES

(i) When two straight lines are parallel their slopes are equal. Thus any line parallel to y = mx + c is of the type y = mx + d, where d is parameter.

(ii) Two lines ax+ by+ c=0 and a′x +b′y +c′ = 0 are parallel if : $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$

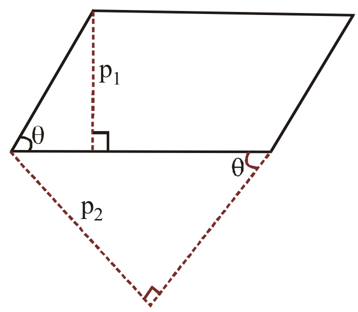
Thus any line parallel to ax+ by+ c =0 is of the type ax+ by+ k =0, where k is a parameter.

(iii) The distance between two parallel lines with equations ax + by+ c_1 =0 and

ax + by + c_2 = 0 is $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

Coefficient of x & y in both the equations must be same.

(iv) The area of the parallelogram $= \frac{p_1p_2}{\sin\theta}$, where p_1 and p_2 are



distance between two pairs of opposite sides and θ is the angle between any two adjacent sides. Note that area of the parallogram bounded by the lines $y=m_1x+c_1$, $y=m_1x+c_2$. and $y = m_2x + d_1$, $y= m_2x + d_2$ is given by $\left|\frac{(c_1-c_2)(d_1-d_2)}{m_1-m_2}\right|$

14. PERPENDICULAR LINES

(i) When two lines of slopes m_1 & m_2 are at right angles, the product of their slope is –1 i.e., $m_1m_2 = -1$. Thus any line perpendicular to $y = mx + c$ is of the form.

$y = -\frac{1}{m}x + d$, where d is any parameter.

(ii) Two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are perpendicular if $aa' + bb' = 0$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

15. STRAIGHT LINES MAKING ANGLE a WITH GIVEN LINE

The equation of lines passing through point (x_1,y_1) and making angle α with the line $y = mx + c$ are given by $(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ & $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.

16. BISECTOR OF THE ANGLES BETWEEN TWO LINES

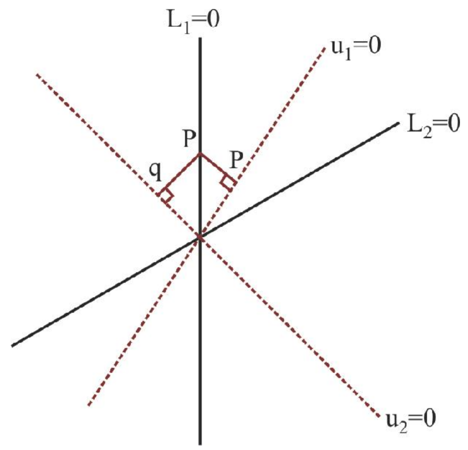
Equations of the bisectors of angles between the lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ $(ab' \neq a'b)$ are :

$$\frac{ax+by+c}{\sqrt{a^2+b^2}} = \pm\frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$$

Note...

Equation of straight lines through $P(x_1,y_1)$ & equally inclined with the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisector between these two lines & passing throught the point P.

17. METHODS TO DISCRIMINATE BETWEEN THE ACUTE BISECTOR AND THE OBTUSE ANGLE BISECTOR



(i) If θ be the angle between one of the lines & one of the bisectors, find tan θ. if $|\tan\theta| < 1$, then $2\theta < 90°$ so that this bisector is the acute angle bisector. if $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector

(ii) Let $L_1=0$ & $L_2=0$ are the given lines & $u_1=0$ and $u_2=0$ are bisectors between $L_1=0$ and $L_2=0$. Take a point P on any one of the lines $L_1=0$ or $L_2=0$ and drop perpendicular on $u_1=0$ and $u_2=0$ as shown. If.

$|p| < |q| \Rightarrow u_1$ is the acute angle bisector.

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector.

$|p| = |q| \Rightarrow$ the lines L_1 and L_2 are perpendicular.

(iii) if $aa' + bb' < 0$, while c & c′ are postive, then the angle between the lines is acute and the equation of the bisector

of this acute angle is $\frac{ax+by+c}{\sqrt{a^2+b^2}} = +\frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$

If, however, $aa' + bb' > 0$, while c and c′ are postive, then the angle between the lines is obtuse & the equation of the bisector of this obtuse angle is :

$$\frac{ax+by+c}{\sqrt{a^2+b^2}} = +\frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$$

The other equation represents the obtuse angle bisector in both cases.

18. TO DISCRIMINATE BETWEEN THE BISECTOR OF THE ANGLE CONTAINING A POINT

To discriminate between the bisector of the angle containing the origin & that of the angle not containing the origin. Rewrite the equation, $ax + by + c = 0$ & $a'x + b'y + c' = 0$ such that the constant term c, c' are positive.

Then ; $\frac{ax+by+c}{\sqrt{a^2+b^2}} = +\frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle containing origin and $\frac{ax+by+c}{\sqrt{a^2+b^2}} = -\frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle not containing the origin. In general equation of the bisector which contains the point (α, β) is.

$$\frac{ax+by+c}{\sqrt{a^2+b^2}} = \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}} \text{ or } \frac{ax+by+c}{\sqrt{a^2+b^2}} = -\frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$$

according as $a\alpha + b\beta + c$ and $a'\alpha + b'\beta + c'$ having same sign or otherwise.

19. CONDITION OF CONCURRENCY

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Alternatively : If three constants A, B and C (not all zero) can be found such that $A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

20. FAMILY OF STRAIGHT LINES

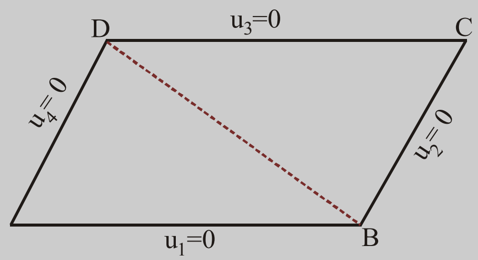
The equation of a family of straight lines passing through the points of intersection of the lines,

$L_1 \equiv a_1x + b_1y + c_1 = 0$ & $L_2 \equiv a_2x + b_2y + c_2 = 0$ is given by $L_1 + kL_2 = 0$ i.e.

$(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitray real number.

Note...

(i) If $u_1 = ax+by+c$, $u_2 = a'x+b'y+d$, $u_3 = ax+by+c'$, $u_4 = a'x+b'y+d'$, then $u_1 = 0$; $u_2 = 0$; $u_3 = 0$; $u_4 = 0$; form a parallegoram



The diagonal BD can be given by $u_2u_3 - u_1u_4 = 0$

Proof : Since it is the first degree equation in x & y, it is a straight line. Secondly point B satisfies $u_2 = 0$ and $u_1 = 0$ while point D satisfies $u_3 = 0$ and $u_4 = 0$. Hence the result. Similarly, the diagonal AC can be given by $u_1u_2 - u_3u_4 = 0$

(ii) The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equation are identical for some real λ and μ.

[For getting the values of λ and μ compare the coefficients of x, y & the constant terms.]

21. A PAIR OF STRAIGHT LINES THROUGH ORIGIN

(i) A homogeneous equation of degree two, "$ax^2 + 2hxy + by^2 = 0$" always represents a pair of straight lines passing through the origin if :

(a) $h^2 > ab \Rightarrow$ lines are real and distinct.

(b) $h^2 = ab \Rightarrow$ lines are coincident.

(c) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection i.e. (0,0)

(ii) If $y = m_1x$ & $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then;

$$m_1 + m_2 = -\frac{2h}{b} \text{ and } m_1m_2 = \frac{a}{b}$$

(iii) If θ is the acute angle between the pair of straight lines represented by,

$ax^2 + 2hxy + by^2 = 0$, then; $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$

(iv) The condition that these lines are :

(a) At right angles to each other is $a + b = 0$ i.e. co-efficient of x^2 + co-efficient of $y^2 = 0$

(b) Coincident is $h^2 = ab$.

(c) Equally inclined to the axis of x is $h = 0$ i.e. coeff. of $xy=0$.

Note...

A homogeneous equation of degree n represents n straight lines passing through origin.

(v) The equation to the pair of straight lines bisecting the angle between the straight lines,

$ax^2 + 2hxy + by^2 = 0$, is $\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$

22. GENERAL EQUATION OF SECOND DEGREE REPRESENTING A PAIR OF STRAIGHT LINES

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ = represents a pair of straight lines if :

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, i.e. if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

(ii) The angle θ between the two lines representing by a general equation is the same as that between two lines represented by its homogenous part only.

23. HOMOGENIZATION

The equation of a pair of straight lines joining origin to the points of intersection of the line

$L \equiv lx + my + n = 0$ and a second degree curve,

$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is

$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx+my}{-n}\right) +$$

$$2fy\left(\frac{lx+my}{-n}\right) + c\left(\frac{lx+my}{-n}\right)^2 = 0$$

The equal is obtained by homogenizing the equation of curve with the help of equation of line.

Note...

Equation of any cure passing through the points of intersection of two curves $C_1=0$ and $C_2=0$ is given by $\lambda C_1 + \mu C_2 = 0$ where λ and μ are parameters.

SOLVED EXAMPLES

Example – 1

Using distance formula show that the points (–1, 2), (5, 0) and (2, 1) are collinear.

Sol. Let the given points (–1, 2), (5, 0) and (2, 1) be denoted by A, B and C, respectively.

Now $AB = \sqrt{5-(-1)^2 + (0-2)^2} = \sqrt{36+4} = 2\sqrt{10}$

$BC = \sqrt{(2-5)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$

$CA = \sqrt{(-1-2)^2 + (2-1)^2}$

$= \sqrt{9+1} = \sqrt{10}$

It is clear that BC + CA = AB.

Hence A, B and C are collinear.

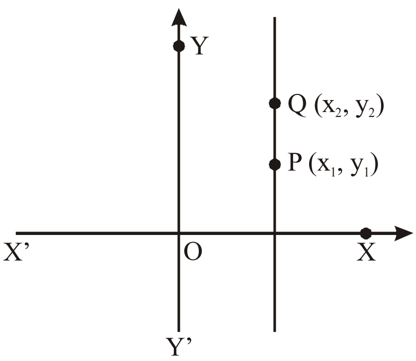
Example – 2

Find the distance between P(x_1, y_1) and Q (x_2, y_2) when (i) PQ is parallel to the y-axis. (ii) PQ is parallel to the x-axis.

Sol. (i) In this case $x_1 = x_2$.

Now $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

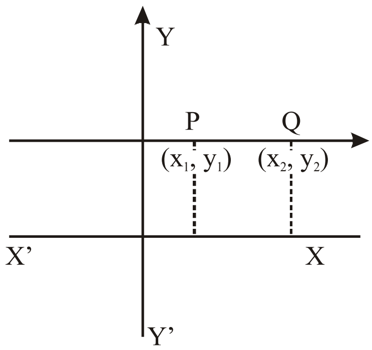
$= \sqrt{0 + (y_2 - y_1)^2} = |y_2 - y_1|$



(ii) In this case, $y_1 = y_2$

Now $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x_2 - x_1)^2 + 0} = |x_2 - x_1|$

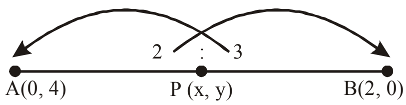


Example – 3

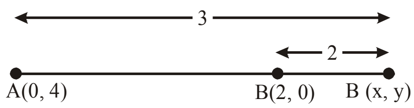
Find a point on the line-segment joining points (0, 4) and (2, 0) and dividing the line-segment (i) internally in ratio 2 : 3, (ii) externally in ratio 3 : 2.

Sol. (i) Here the points are (0, 4) and (2, 0) and the division is internal in the given ratio 2 : 3. From the figure the coordinates of point P are

$$\left(\frac{2.2+3.0}{2+3}, \frac{2.0+3.4}{2+3}\right) \text{ or } \left(\frac{4}{5}, \frac{12}{5}\right)$$



(ii) Here the ratio is 3 : 2 and the division is external as shown in figure.



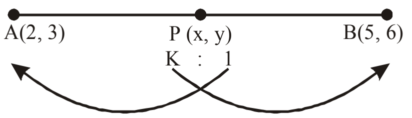
From figure, the coordinates of point P are

$$\left(\frac{3.2-2.0}{3-2}, \frac{3.0-2.4}{3-2}\right) \text{ or } (6, -8).$$

Example – 4

Find the ratio in which the line joining (2, –3) and (5, 6) is divided by (i) the x-axis, (ii) the y-axis.

Sol. Let P(x, y) divides the join of A(2, –3) and B (5, 6) in the ratio k : 1.



The co-ordinates of P are

$$\left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$$

(i) Since P lies on x-axis, so y-co-ordinate is zero.

i.e., $\frac{6k-3}{k+1} = 0, \Rightarrow 6k-3 = 0 \Rightarrow k = \frac{1}{2}$

Hence P divides in the ratio $\frac{1}{2}:1$ i.e., 1 : 2 internally.

(ii) Since P lies on y-axis, so x-co-ordinate is zero.

i.e., $\frac{5k+2}{k+1} = 0, \Rightarrow 5k+2 = 0 \Rightarrow k = \frac{-2}{5}$

Hence P divides externally in the ratio 2 : 5.

Example – 5

If the points (1, 1), (a, 0), (0, b) are collinear, show that $\frac{1}{a}+\frac{1}{b}=1$.

Sol. As the points (1, 1), (a, 0) and (0, b) are collinear, area formed by the triangle with these vertices is zero.

i.e., $\frac{1}{2}[\,1\,(0-b) + a\,(b-1) + 0\,(1-0)] = 0$

or, $-b + ab - a = 0$

or, $\frac{-b}{ab} + \frac{ab}{ab} - \frac{a}{ab} = 0$

[Dividing both sides by ab]

or, $-\frac{1}{a} + 1 - \frac{1}{b} = 0$

or, $\frac{1}{a} + \frac{1}{b} = 1$

Example – 6

Find the quation of the locus of a point equidistant from the points A(1, 3) and B(–2, 1).

Sol. Let P(x, y) be any point on the locus and let A(1, 3) and B(–2, 1) be the given points.

By the given condition.

PA = PB

or $PA^2 = PB^2$

or $(x-1)^2 + (y-3)^2 = (x+2)^2 + (y-1)^2$

or $x^2 + 1 - 2x + y^2 - 6y + 9$

$= x^2 + 4 + 4x + y^2 + 1 - 2y$

or $6x + 4y = 5$

which is the equation of the required locus that is a straight line.

Example – 7

What are the slope of the lines whose incilnations from positive direction of X–axis are :

(i) 60° (ii) 0° (iii) 150° (iv) 120°

Sol. (i) Slope = tan 60° = $\sqrt{3}$

(ii) Slope = tan 0° = 0

(iii) Slope = tan 150° = – cot 60° = $-\frac{1}{\sqrt{3}}$.

(iv) Slope = tan 120° = – cot 30° = $-\frac{1}{\sqrt{3}}$.

Example – 8

The slope of a line is double of the slope of another line. If tangent of the angle between them is 1/3, find the slopes of the lines.

Sol. Let the slopes of the two given lines be m and 2 m and θ be the angle between them

The according to questions

$\tan\theta = \frac{1}{3}$

Now $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

or $\frac{1}{3} = \frac{2m - m}{1 + 2m^2}$ or $\frac{1}{3} = \frac{m - 2m}{1 + 2m^2}$

or $1 + 2m^2 = 3m$ $\quad 1 + 2m^2 = 3m$

or $2m^2 - 2m + 1 = 0$ $\quad 2m + 3m + 1 = 0$

or $2m^2 - 2m - m + 1 = 0$

$2m^2 + 2m + m + 1 = 0$

or $2m(m - 1) - 1(m - 1) = 0$

$2m(m + 1)(+1(m + 1) = 0$

or $(m - 1)(2m - 1) = 0$

$(m + 1)(2m + 1) = 0$

or $m = 1, m = \frac{1}{2}$ $\quad m = -1, m = -\frac{1}{2}$

Hence, the slopes of the lines are

$\left(\frac{1}{2}, 1\right)$ or $\left(\frac{-1}{2}, -1\right)$

Example – 9

If the angle between two lines is π/4 and slope of one of the lines is 1/2, find the slope of the other line.

Sol. If θ be the acute angle between two lines with slope, m_1 and m_2, then

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| \quad ...(i)$$

Now $m_1 = \frac{1}{2}$, $m_2 = m$ and $\theta = \frac{\pi}{4}$

Now from (i)

$$\tan\frac{\pi}{4} = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right| \Rightarrow 1 = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right|$$

$$\Rightarrow \frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = 1 \text{ or } \frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = -1$$

$$\Rightarrow m = 3 \text{ or } m = -\frac{1}{3}$$

Hence, the slope of the other line is 3 or $-\frac{1}{3}$.

Example – 10

Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right angled triangle.

Sol. Let the vertices be A(4, 4), B(3, 5) and C(–1, –1).

Slope of AB, $m_1 = \frac{5 - 4}{3 - 4} = \frac{1}{-1} = -1$

Slope of BC, $m_2 = \frac{-1 - 5}{-1 - 3} = \frac{-6}{-4} = \frac{3}{2}$

Slope of AC, $m_3 = \frac{-1 - 4}{-1 - 4} = 1$

Now $m_1 m_3 = -1 \times (1) = -1$

Thus AB ⊥ AC or ∠ A = 90°.

Hence Δ ABC is a right angled triangle.

Example – 11

Using the concept of slope, show that (–2, –1), (4, 0) (3, 3) and (–3, 2), are vertices of a parallelogram.

Sol. Let A, B, C and D be the points (–2, –1), (4, 0) (3, 3) and (–3, 2), respectively. We have

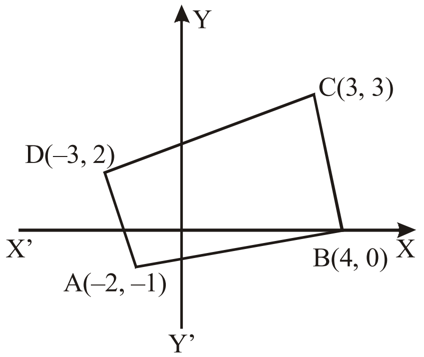
Slope of AB, $= \frac{0 - (-1)}{4 - (-2)} = \frac{1}{6}$;

Slope of BC, $= \frac{3 - 0}{3 - 4} = -3$;

Slope of CD, $= \frac{3 - 2}{3 - (-3)} = \frac{1}{6}$

Slope of DA, $\frac{2 - (-1)}{-3 - (-2)} = -3$.

We note that the slope of AB and BC are different and therefore points A, B and C are not collinear. Also, points A, D and C are not collinear. Thus, the given points form a quadrilateral.



Also, the slope of AB = Slope of CD, i.e., AB is parallel to CD and the slope of BC = Slope of DA, i.e., BC is parallel to DA. Thus, opposite sides of the quadrilateral ABCD are parallel.

Hence, the given four points constitute the vertices of a parallelogram.

[redacted]

A line is drawn passing through P(h, k) and parallel to X-axis. If the area of triangle formed by this line and lines y = x and x + y = 2 is $4h^2$, then find the locus of P(h, k).

Sol. Given. Area of $\Delta ABC = 4h^2$

$\Rightarrow \quad \frac{1}{2} \,|\,[1\,.\,(k-k) + 1.(2-k-k) + 1.\,(k^2 - k\,(2-k)\,|= 4h^2$

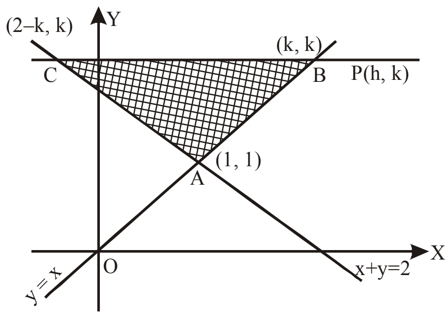
or $\quad 2 - 2k + k^2 + 2k + k^2 = \pm\, 8h^2$

or $\quad 2k^2 - 4k + 2 = \pm\, 8h^2$

or $\quad k^2 - 2k + 1 = \pm\, 4h^2$

or $\quad (k-1)^2 = \pm\, 4h^2$

$\Rightarrow \quad k - 1 = \pm\, 2h$



Hence locus of P(h, k) is a pair of straight lines y – 1 = 2x and y – 1 = –2x.

Example – 13

Determine the equation of the line passing through the point (–1, –2) and having slope 4/7.

Sol. We are given that

$x_1 = -1,\ y_1 = -2$ and $m = 4/7$

Substituting these values in the point-slope form of the equation of a line, we get

$y - (-2) = 4/7\ [\,x - (-1)]$

or $\quad 7(y+2) = 4(x+1)$ or $7y = 4x - 10$

which is the required equation of the line.

Example – 14

Find the equation of the line which satisfies the given conditions :

(i) Passing through (2, 2) and inclined to x-axis at 45°

(ii) Intersecting x-axis at a distance of 3 units to the left of the origin with slope –2.

(iii) Intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30° with the positive direction of x-axis.

Sol. (i) The slope of the given line = tan 45° = 1

The required equation of line is

$$\frac{y-2}{x-2} = 1$$

$\Rightarrow \quad y - 2 = x - 2 \Rightarrow x - y = 0.$

(ii) Since the line intersects the x-axis at a distance of 3 units to the left of the origin i.e., it passes through (–3, 0). Also its slope = –2 [Given]

The required equation of line is

$$\frac{y-0}{x-(-3)} = -2$$

$\Rightarrow \quad y = -2(x+3) \quad \Rightarrow 2x + y + 6 = 0.$

(iii) Since the line intersects the y-axis at a distance of 2 units above the origin i.e., it passes through (0, 2).

Also its slope = tan 30° = $\frac{1}{\sqrt{3}}$.

The required equation of line is

$$\frac{y-2}{x-0} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \sqrt{3}y - 2\sqrt{3} = x \quad$ i.e., $x - \sqrt{3}y + 2\sqrt{3} = 0$.

Example – 15

Find the ratio in which the line segment joining the points (2, 3) and (4, 5) is divided by the line joining the points (6, 8) and (–3, –2).

Sol. The equation of the line joining the points (6, 8) and (–3, –2) is

$$y-8=\frac{-2-8}{-3-6}(x-6)$$

$$\Rightarrow \quad y-8=\frac{10}{9}(x-6)$$

$\Rightarrow$ 9y – 72 = 10x – 60

$\Rightarrow$ 10x – 9y + 12 = 0 ...(i)

Let this line divided the join of (2, 3) and (4, 5) at the point P in the ratio of k : 1.

Then the coordinates of P are

$$\left(\frac{4k+2}{k+1}, \frac{5k+3}{k+1}\right)$$

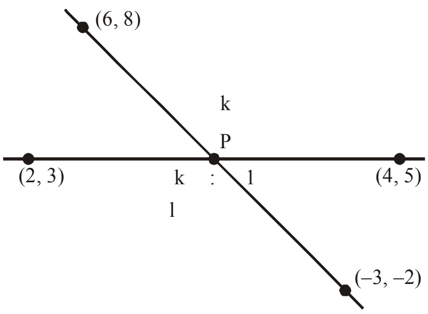
Now the point P lies on the line (i),

$$\text{i.e.,} \quad 10.\left(\frac{4k+2}{k+1}\right)-9\left(\frac{5k+3}{k+1}\right)+12=0$$

$\Rightarrow$ 40k + 20 – 45k – 27 + 12k + 12 = 0

$\Rightarrow$ 7k = –5

$\Rightarrow$ k = –5/7



Since the value of k is negative, the line is divided externally.

The required ratio is 5 : 7 externally.

Example – 16

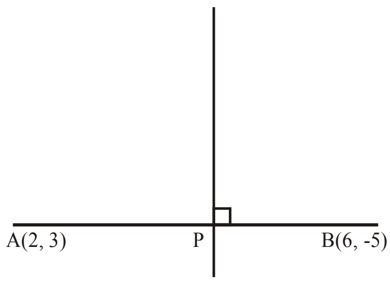
Find the equation of the perpendicular bisector of the line segment joining the points A(2, 3) and B (6, –5).

Sol. The slope of AB is given by

$$m=\frac{-5-3}{6-2}=-2 \qquad \left[\text{Using } m=\frac{y_2-y_1}{x_2-x_1}\right]$$

The slope of a line ⊥ to

$$AB=-\frac{1}{m}=\frac{1}{2}$$



Let P be the mid-point of AB. Then the coordinates of P are

$$\left(\frac{2+6}{2}, \frac{3-5}{2}\right) \text{ i.e., } (4, -1)$$

Thus, the required line passes through P(4, –1) and has slope 1/2. So its equation is

y + 1 = 1/2 (x – 4)

[Using $y - y_1 = m(x - x_1)$]

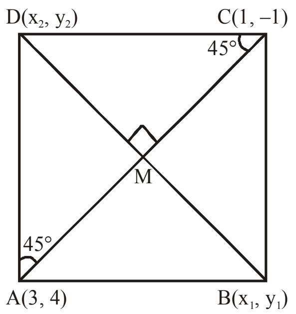
$$\Rightarrow \quad x-2y-6=0.$$

Example – 17

Two opposite vertices of a square are (3, 4) and (1, –1). Find the coordinates of the other vertices.

Sol. Let squares has the vertices A(3, 4), B(x_1, y_1), C(1, –1) and D(x_2, y_2).

Slope of AC = $\frac{4-(-1)}{3-1} = \frac{5}{2}$



Let m be the slope of a line making angle of 45° with AC

Hence, $\tan 45° = \left|\frac{m-\frac{5}{2}}{1+m.\frac{5}{2}}\right|$

$\Rightarrow \quad \frac{m-\frac{5}{2}}{1+\frac{5m}{2}} = \pm 1$

$\Rightarrow$ 2m – 5 = ± (2 + 5m)

$\Rightarrow$ 2m – 5 = 2 + 5m

$\Rightarrow$ 3m = 7

$\Rightarrow$ m = –7/3

and 2m – 5 = –(2 + 5m)

$\Rightarrow$ 7m = 3

$\Rightarrow$ m = 3/7

Let slope of AD be –7/3.

$\Rightarrow$ Slope of CD = 3/7

Equation of AD is y – 4 = –7/3 (x – 3)

$\Rightarrow$ 7x + 3y – 33 = 0 ...(i)

Equation of CD is y + 1 = 3/7 (x – 1)

$\Rightarrow$ 3x – 7y – 10 = 0 ...(ii)

Solving (i) and (ii), we get

$$\frac{x}{-30-231} = \frac{y}{-99+70} = \frac{1}{-49-9}$$

$\Rightarrow \quad x = \frac{-261}{-58} = \frac{9}{2}$

and $\quad y = \frac{-29}{-58} = \frac{1}{2}$

Coordinates of D are $\left(\frac{9}{2}, \frac{1}{2}\right)$

M is mid-point of BD and AC both

Hence $\left(\frac{x_1+9/2}{2}, \frac{y_1+1/2}{2}\right) = \left(\frac{3+1}{2}, \frac{4+(-1)}{2}\right)$

$\Rightarrow \quad \frac{2x_1+9}{4} = 2, \frac{2y_1+1}{4} = \frac{3}{2}$

$\Rightarrow \quad x_1 = \frac{1}{2}, \quad y_1 = \frac{5}{2}$

Coordinates of B are $\left(\frac{-1}{2}, \frac{5}{2}\right)$

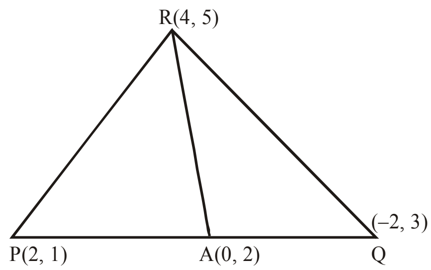
Example – 18

The vertices of triangle PQR are P(2, 1), Q (–2, 3) and R(4, 5). Find equation of the median through the vertex R.

Sol. Given : P (2, 1), Q (–2, 3) and R (4, 5).

Mid point of PQ is

$$\left(\frac{2+(-2)}{2}, \frac{1+3}{2}\right) = (0, 2)$$



Now we have to find the equation of median passing through R i.e., of line AR

Now equation of line in 2-point form is

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$$

$\Rightarrow \quad y - 2 = \frac{5-2}{4-0}(x-0)$

$\Rightarrow \quad y - 2 = \frac{3}{4}(x-2)$

$\Rightarrow$ 4y – 8 = 3x

3x = 4y – 8

$\Rightarrow$ 3x– 4y + 8 = 0

Example – 19

Find the equation of the line passing (–3, 5) and perpendicular to the line through the points (2, 5) and (–3, 6).

Sol. The required line passes through A(–3, 5) and is perpendicular to the line through the points (2, 5) and (–3, 6).

$$m_{BC} = \frac{6-5}{-3-2} = -\frac{1}{5}$$

Slope of required line = 5.

Now equation of line in point-slope form is

$y - y_1 = m(x - x_1)$

$y - 5 = 5(x + 3)$

$y - 5 = 5x + 15$

$5x - y + 20 = 0.$

Example – 20

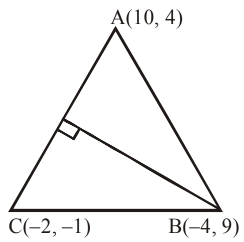
Given the triangle with vertices A(10, 4), B(–4, 9), C(–2, –1). Find the equation of the altitude through B.

Sol. The given vertices are A(10, 4), B(–4, 9), C(–2, –1).

$$\text{Slope of AC} = \frac{-1-4}{-2-10} = \frac{-5}{-12} = \frac{5}{12}$$

The altitude through B is perpendicular of AC. Let m be the slope of the altitude through B.

$$\text{Hence } m \times \frac{5}{12} = -1 \Rightarrow m = -\frac{12}{5}$$



The equation of the altitude through B(–4, 9) is

$$y - 9 = -\frac{12}{5}(x - (-4))$$

[Using $y - y_1 = m(x - x_1)$]

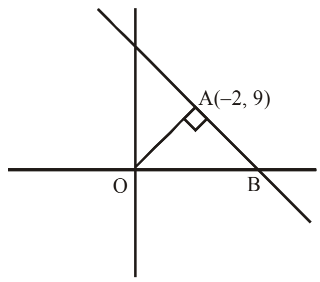
$\Rightarrow$ $5y - 45 = -12x - 48$

$\Rightarrow$ $12 + 5y + 3 = 0$

Example – 21

The perpendicular from the origin to a line meets it at the point (–2, 9). Find the equation of the line.

Sol. The perpendicular from the origin O(0, 0) meets the line AB at A(–3, 9)



$$\text{Hence, } m_{OA} = \frac{9}{-2} = -\frac{9}{2}$$

$$\Rightarrow \quad m_{AB} = \frac{2}{9}$$

Now, equation of the line will be

$$y - 9 = \frac{2}{9}(x + 2)$$

$2x - 9y + 85 = 0$

Example – 22

Find the equations of the lines which cut off intercepts on the axes whose sum and product are 1 and –6, respectively.

Sol. Let the intercept on x-axis and y-axis be a and b, respectively, so that

$a + b = 1$ ...(i)

and $ab = -6$ ...(ii)

Eliminating a from (i) and (ii), we get

$\Rightarrow$ $-b^2 + b = -6$

$\Rightarrow$ $b^2 - b - 6 = 0$

$$\Rightarrow \quad b = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2}$$

i.e. either $b = 3$

or $b = -2$

when $b = -2, a = 3$

and when $b = 3, a = -2$.

Thus, by the intercept from the equations of the lines are

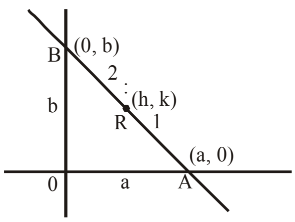
$$\frac{x}{3} + \frac{y}{-2} = 1 \quad \text{or} \quad \frac{x}{-2} + \frac{y}{3} = 1$$

i.e., $2x - 3y - 6 = 0$ or $3x - 2y + 6 = 0$

Example – 23

Point R(h, k) divides a line-segment between the axes in the ratio 1 : 2. Find the equation of the line.

Sol. As shown in the figure AB is the required line and A is (a, 0) and B is (0, b). R divides AB in the ratio 1 : 2.



Thus $h = \dfrac{2a + 0}{1 + 2} = \dfrac{2a}{3}$

and $k = \dfrac{2 \times 0 + b}{1 + 2} = \dfrac{b}{3}$

Therefore $a = \dfrac{3h}{2}$ and $b = 3k$.

Now using intercept form, equation of the line AB is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{or} \quad \frac{x}{\frac{3h}{2}} + \frac{y}{3k} = 1$$

$$\frac{2x}{h} + \frac{y}{k} = 3$$

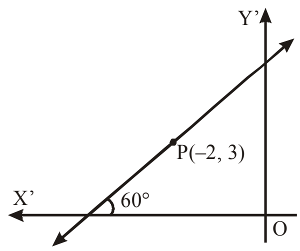
$2kx + hy = 3kh$

(Required equation)

Example – 24

Find the equation of a line which passes through the point (–2, 3) and makes angle 60° with the positive direction of x-axis.

Sol. The equation of the line in symmetric form is



$$\frac{y - y_1}{\sin\theta} = \frac{x - x_1}{\cos\theta}$$

Substituting $x_1 = -2$, $y_1 = 3$ and $\theta = 60°$, we get

$$\frac{y - 3}{\sin 60°} = \frac{x - (-2)}{\cos 60°}$$

or $$\frac{y - 3}{\frac{\sqrt{3}}{2}} = \frac{x + 2}{\frac{1}{2}}$$

or $\sqrt{3}x - y + 3 + 2\sqrt{3} = 0$.

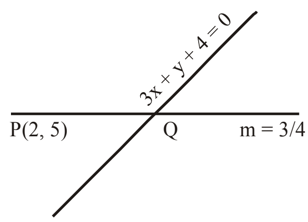
This is the required equation of the line.

Example – 25

Find the distance of the point (2, 5) from the line $3x + y + 4 = 0$, measured parallel is a line having slope 3/4.

Sol. Let PQ be the distance of the given point P(2, 5) from the line $3x + y + 4 = 0$, measured parallel to a line having slope 3/4.

Let $\tan\theta = \dfrac{3}{4}$.



As θ lies in the first quadrant

We have $\sin\theta = \tan\theta\cos\theta$

$$= \frac{\tan\theta}{\sec\theta} = \frac{3/4}{\sqrt{1+9/16}} = \frac{3}{5}$$

and $\cos\theta = \cos\theta.\frac{1}{\sin\theta}.\sin\theta$

$$= \cot\theta\sin\theta = \frac{4}{3}\times\frac{3}{5} = \frac{4}{5}$$

Using $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$, the equation of PQ is

$$\frac{x-2}{4/5} = \frac{y-5}{3/5} = r$$

or $x = 2+\frac{4}{5}r,\quad y = 5+\frac{3}{5}r$

Let the coordinates of Q be $\left(2+\frac{4}{5}r,\ 5+\frac{3}{5}r\right)$

This point lies on $3x+y+4=0$.

$\Rightarrow\ 3\left(2+\frac{4}{5}r\right)+\left(5+\frac{3}{5}r\right)+4=0$

$\Rightarrow\ 3r=-15$

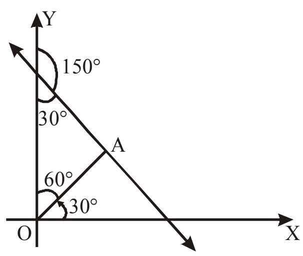
$\Rightarrow\ r=-5$

Required distance PQ = 5 units.

Example – 26

The length of the perpendicular from the origin to a line is 7 units and the line makes an angle of 150° with the positive direction of y-axis. Find the equation of the line.

Sol. Here $p = 7$ and $\omega = 30°$



Equation of the required line is

$x\cos 30° + y\sin 30°\ 7$

$\Rightarrow\ x\frac{\sqrt{3}}{2}+y.\frac{1}{2}=7$

$\Rightarrow\ \sqrt{3}x+y=14$

Example – 27

Two sides of a square lie on the lines $x + y = 1$ and $x + y + 2 = 0$. What is its area?

Sol. Clearly the length of the side of the square is equal to the distance between the parallel lines

$x + y - 1 = 0$ ...(i)

and $x + y + 2 = 0$ ...(ii)

Putting $x = 0$ in (i), we get $y = 1$. So, (0, 1) is a point on line (i).

Now, distance between the parallel lines

= length of the $\perp$ from (0, 1) to $x + y + 2 = 0$

$$= \left|\frac{0+1+2}{\sqrt{1^2+1^2}}\right| = \left|\frac{3}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$$

Thus, the length of the side of the square is $\frac{3}{\sqrt{2}}$

and hence its area $= \left(\frac{3}{\sqrt{2}}\right)^2 = \frac{9}{2}$.

Example – 28

Find the distance between parallel lines :

(i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

(ii) $l(x + y) + p = 0$ and $lx + ly - r = 0$

Sol. (i) $15x + 8y - 34 = 0$

and $15x + 8y + 31 = 0$

Now $A = 15$, $B = 8$, $C_1 = -34$, $C_2 = 31$

Distance between parallel lines $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$

$$= \frac{|-34-31|}{\sqrt{(15)^2+8^2}} = \frac{65}{17}$$

(ii) $l(x + y) + p = 0$

$lx + ly - r = 0.$

Now $A = l, B = l, C_1 = p, C_2 = -r$

Distance between parallel lines $d = \dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$

$$= \frac{|p+r|}{\sqrt{l^2 + l^2}} = \frac{|p+r|}{\sqrt{2}l} = \frac{1}{\sqrt{2}}\left|\frac{p+r}{l}\right|.$$

Example – 29

Find the equation of the bisector of the obtuse angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$.

Sol. Firstly make the constant terms (c_1, c_2) positive

$3x - 4y + 7 = 0$ and $-12x - 5y + 2 = 0$

$\because$ $a_1a_2 + b_1b_2 = (3)(-12) + (-4)(-5) = -36 + 20 = -16$

$\therefore$ $a_1a_2 + b_1b_2 < 0$

Hence "–" sign gives the obtuse bisector.

$\therefore$ Obtuse bisector is $\dfrac{(3x - 4y + 7)}{\sqrt{(3)^2 + (-4)^2}} = -\dfrac{(-12x - 5y + 2)}{\sqrt{(-12)^2 + (-5)^2}}$

$\Rightarrow$ $13(3x - 4y + 7) = -5(-12x - 5y + 2)$

$\Rightarrow$ $21x + 77y - 101 = 0$

is the obtuse angle bisector.

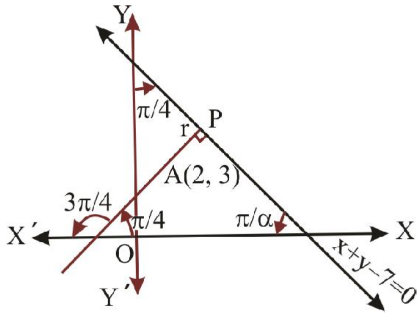
Example – 30

A line through (2, 3) makes an angle $\dfrac{3\pi}{4}$ with the negative direction of x–axis. Find the length of the line segment cut off between (2, 3) and the line $x + y - 7 = 0$.

Sol. $\because$ Line makes an angle $\dfrac{3\pi}{4}$ with the negative direction of x–axis.

$\therefore$ Line makes an angle $\dfrac{\pi}{4}$ with the positive direction of x–axis.

$\therefore$ The equation of the line through (1, 2) in parametric form is



$$\frac{x-2}{\cos\left(\frac{\pi}{4}\right)} = \frac{y-3}{\sin\left(\frac{\pi}{4}\right)} = r$$

i.e., $\dfrac{x-2}{\frac{1}{\sqrt{2}}} = \dfrac{y-3}{\frac{1}{\sqrt{2}}} = r$ ... (1)

$\therefore$ $x = 2 + \dfrac{r}{\sqrt{2}}$ and $y = 3 + \dfrac{r}{\sqrt{2}}$

Let the line (1) meet the line $x + y - 7 = 0$ in P

$\therefore$ Co-ordinates of P $\left(2 + \dfrac{r}{\sqrt{2}}, 3 + \dfrac{r}{\sqrt{2}}\right)$ lies on

$x + y - 7 = 0$

then $2 + \dfrac{r}{\sqrt{2}} + 3 + \dfrac{r}{\sqrt{2}} - 7 = 0$

or $\dfrac{2r}{\sqrt{2}} = 2$

or $r = \sqrt{2}$

$AP = \sqrt{2}$

Example – 31

The family of lines x (a + 2b) + y (a + 3b) = a + b passes through the point for all values of a and b. Find the point.

Sol. The given equation can be written as

$$a(x+y-1)+b(2x+3y-1)=0$$

which is equation of a line passing through the point of intersection of the lines x + y – 1 = 0 and 2x + 3y – 1 = 0. The point of intersection of these lines is (2, –1). Hence the given family of lines passes through the point (2, –1) for all values of a and b.

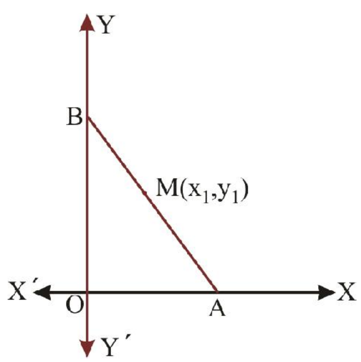
Example – 32

A variable straight line through the point of intersection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ meets the co-ordinate axes in A and B. Show that the locus of the mid point of AB is the curve 2xy (a + b) = ab (x + y).

Sol. Any line through the point of intersection of given lines is

$$\left(\frac{x}{a}+\frac{y}{b}-1\right)+\lambda\left(\frac{x}{b}+\frac{y}{a}-1\right)=0$$

$$x\left(\frac{1}{a}+\frac{\lambda}{b}\right)+y\left(\frac{1}{b}+\frac{\lambda}{a}\right)=(1+\lambda)$$



$$\Rightarrow \quad x\left(\frac{b+a\lambda}{ab}\right)+y\left(\frac{a+b\lambda}{ab}\right)=(1+\lambda)$$

$$\Rightarrow \quad \frac{x}{\left\{\frac{ab(1+\lambda)}{b+a\lambda}\right\}}+\frac{y}{\left\{\frac{ab(1+\lambda)}{a+b\lambda}\right\}}=1$$

This meets the x–axis at $A\equiv\left(\frac{ab(1+\lambda)}{b+a\lambda},0\right)$

and meets the y–axis at $B\equiv\left(0,\frac{ab(1+\lambda)}{a+b\lambda}\right)$

Let the mid point of AB is M (x_1, y_1) then

$$x_1=\frac{ab(1+\lambda)}{2(b+a\lambda)} \text{ and } y_1=\frac{ab(1+\lambda)}{2(a+b\lambda)}$$

$$\therefore \quad \frac{1}{x_1}+\frac{1}{y_1}=\frac{2(b+a\lambda)}{ab(1+\lambda)}+\frac{2(a+b\lambda)}{ab(1+\lambda)}$$

$$=\frac{2}{ab(1+\lambda)}(b+a\lambda+a+b\lambda)$$

$$=\frac{2}{ab(1+\lambda)}(a+b)(1+\lambda)$$

$$\Rightarrow \quad \frac{(x_1+y_1)}{x_1y_1}=\frac{2(a+b)}{ab}$$

$$\Rightarrow \quad 2x_1y_1(a+b)=ab(x_1+y_1)$$

Hence the locus of mid point of AB is

$$2xy(a+b)=ab(x+y).$$

Example – 33

Find the equations of the straight lines passing through the point (2, 3) and inclined at π/4 radians to the line $2x+3y=5$.

Sol. Let the line 2x + 3y = 5 make an angle θ with positive x–axis.

Then $\tan\theta = -\frac{2}{3}$

Now $\tan\theta.\tan\frac{\pi}{4} = -\frac{2}{3}\times 1 = -\frac{2}{3} \neq \pm 1$

Slopes of required lines are

$$\tan\left(\theta+\frac{\pi}{4}\right) \text{ and } \tan\left(\theta-\frac{\pi}{4}\right)$$

∴

$$\tan\left(\theta+\frac{\pi}{4}\right) = \frac{\tan\theta+\tan\left(\frac{\pi}{4}\right)}{1-\tan\theta\tan\left(\frac{\pi}{4}\right)} = \frac{\left(-\frac{2}{3}\right)+1}{1-\left(-\frac{2}{3}\right)(1)} = \frac{1}{5}$$

$$\text{and } \tan\left(\theta-\frac{\pi}{4}\right) = \frac{\tan\theta-\tan\left(\frac{\pi}{4}\right)}{1+\tan\theta\tan\left(\frac{\pi}{4}\right)} = \frac{\left(-\frac{2}{3}\right)-1}{1+\left(-\frac{2}{3}\right)(1)} = -5$$

∴ Equations of required lines are

$$y-3 = \frac{1}{5}(x-2) \text{ and } y-3 = -5(x-2)$$

i.e., $x-5y+13=0$ and $5x+y-13=0$

Example – 34

Find the equations to the straight lines passing through the point (2, 3) and equally inclined to the lines $3x-4y-7=0$ and $12x-5y+6=0$.

Sol. Let m be the slope of the required line. Then its equation is

$$y-3 = m(x-2) \quad \text{...(1)}$$

It is given that line (1) is equally inclined to the lines

$3x-4y-7=0$ and $12x-5y+6=0$ then

$$\Rightarrow \left(\frac{\frac{3}{4}-m}{1+\frac{3}{4}m}\right) = -\left(\frac{\frac{12}{5}-m}{1+\frac{12}{5}m}\right)$$

$$\left(\begin{array}{l}\text{slope of } 3x-4y-7=0 \text{ is } \frac{3}{4} \\ \text{and slope of } 12x-5y+6=0 \text{ is } \frac{12}{5}\end{array}\right)$$

$$\Rightarrow \left(\frac{3-4m}{4+3m}\right) = -\left(\frac{12-5m}{5+12m}\right)$$

$$\Rightarrow (3-4m)(5+12m)+(4+3m)(12-5m)=0$$

$$\Rightarrow 63m^2-32m-63=0$$

$$\Rightarrow (7m-9)(9m+7)=0$$

$$\therefore \quad m = \frac{9}{7}, -\frac{7}{9}$$

Putting these values of m in (1) we obtain the equations of required lines as $9x-7y+3=0$ and $x+9y-41=0$.

Example – 35

The ends AB of a straight line segment of constant length c slide upon the fixed rectangular axes OX and OY respectively. If the rectangle OAPB be completed, then show that the locus of the foot of perpendicular drawn from P to AB is $x^{2/3} + y^{2/3} = c^{2/3}$.

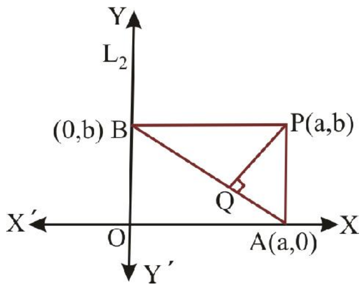
Sol. Let $A \equiv (a, 0)$, $B \equiv (0, b)$ then $P \equiv (a, b)$

Since AB = c

$$\sqrt{a^2 + b^2} = c$$

or $a^2 + b^2 = c^2$ ... (1)

and let $Q \equiv (x_1, y_1)$



$\because$ $PQ \perp AB$

$\therefore$ slope of PQ × slope of AB = – 1

$$\Rightarrow \left(\frac{b - y_1}{a - x_1}\right) \times \left(\frac{b - 0}{0 - a}\right) = -1$$

$\Rightarrow$ $ax_1 - by_1 = a^2 - b^2$ ... (2)

$\because$ Equation of AB is $\frac{x}{a} + \frac{y}{b} = 1$

but Q lies on AB then $\frac{x_1}{a} + \frac{y_1}{b} = 1$

$\Rightarrow$ $bx_1 + ay_1 = ab$ ... (3)

From (2) and (3), we get

$$x_1 = \frac{a^3}{a^2 + b^2},\ y_1 = \frac{b^3}{a^2 + b^2}$$

Now, $$x_1^{2/3} + y_1^{2/3} = \frac{(a^2 + b^2)}{(a^2 + b^2)^{2/3}}$$

$= (a^2 + b^2)^{1/3} = c^{2/3}$ [from (1)]

Hence required locus is $x^{2/3} + y^{2/3} = c^{2/3}$.

Example – 36

Find the equation of straight lines through point (2, 3) and having an intercept of length 2 units between the straight lines $2x + y = 3$, $2x + y = 5$.

Sol. Given lines are parallel and distance between them < 2

Given lines are

$2x + y = 3$ ... (1)

and $2x + y = 5$ ... (2)

Equation of any line through (2, 3) is

$y - 3 = m(x - 2)$

or $y = mx - 2m + 3$ ... (3)

Let line (3) cut lines (1) and (2) at A and B respectively.

Solving (1) and (3), we get

$$A \equiv \left(\frac{2m}{m + 2}, \frac{6 - m}{m + 2}\right)$$

and solving (2) and (3), we get

$$B \equiv \left(\frac{2m + 2}{m + 2}, \frac{m + 6}{m + 2}\right)$$

According to question AB = 2

$\Rightarrow$ $(AB)^2 = 4$

$$\Rightarrow \left(\frac{2}{m + 2}\right)^2 + \left(\frac{2m}{m + 2}\right)^2 = 4$$

$\Rightarrow$ $1 + m^2 = m^2 + 4m + 4$ ... (4)

Case I : When m is finite (line is not perpendicular to x–axis) then from (4).

$1 = 4m + 4$

$\therefore$ $m = -\frac{3}{4}$

Case II : When m is infinite (line is perpendicular to x–axis) then from (4),

$$\frac{1}{m^2} + 1 = 1 + \frac{4}{m} + \frac{4}{m^2}$$

$0 + 1 = 1 + 0 + 0$

$1 = 1$ which is true

Hence $m \to \infty$ acceptable.

Hence equation of the required lines are

$$y - 3 = -\frac{3}{4}(x - 2) \text{ and } \frac{y - 3}{\infty} = x - 2 \Rightarrow x - 2 = 0$$

i.e., $3x + 4y = 18$ and $x - 2 = 0$

Example – 37

If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the nth power of the other, then prove that $(ab^n)^{\frac{1}{n+1}} + (a^n b)^{\frac{1}{n+1}} + 2h = 0.$

Sol. Let m and m^n be the slopes of the lines represented by

$$ax^2 + 2hxy + by^2 = 0$$

then $\quad m + m^n = -\frac{2h}{b} \quad$ ... (1)

and $\quad m.m^n = \frac{a}{b}$

or $\quad m^{n+1} = \frac{a}{b}$

or $\quad m = \left(\frac{a}{b}\right)^{\frac{1}{(n+1)}} \quad$ ... (2)

Substituting the value of m from (2) in (1), then

$$\left(\frac{a}{b}\right)^{\frac{1}{(n+1)}} + \left(\frac{a}{b}\right)^{\left(\frac{n}{n+1}\right)} = -\frac{2h}{b}$$

$$\Rightarrow \quad a^{\frac{1}{n+1}} . b^{\frac{n}{n+1}} + a^{\frac{n}{n+1}} . b^{\frac{1}{n+1}} + 2h = 0$$

Corollary : If slope of one line is square of the other, then put n = 2 then

$$(ab^2)^{\frac{1}{3}} + (a^2 b)^{\frac{1}{3}} = -2h$$

Cubing both sides then

$$ab^2 + a^2 b + 3(ab^2)^{\frac{1}{3}} (a^2 b)^{\frac{1}{3}} \left((ab^2)^{\frac{1}{3}} + (a^2 b)^{\frac{1}{3}}\right) = -8h^3$$

$$\Rightarrow \quad ab\,(a + b) + 3ab\,(-2h) = -\,8h^3$$

$$\Rightarrow \quad \frac{(a+b)}{h} + \frac{8h^2}{ab} = 6.$$

Example – 38

Show that the difference of tangent of the angles made by two straight lines $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ with the axis of '*x*' is 2.

Sol. The given equation is

$$x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0 \quad ...(1)$$

and general equation of second degree

$$ax^2 + 2hxy + by^2 = 0 \quad ...(2)$$

Comparing (1) and (2), we get

$a = \tan^2\theta + \cos^2\theta$

$h = -\tan\theta$

and $\quad b = \sin^2\theta$

Let separate lines of (2) are

$y = m_1 x$ and $y = m_2 x$

where $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$

therefore, $\quad m_1 + m_2 = -\frac{2h}{b} = \frac{2\tan\theta}{\sin^2\theta}$

and $\quad m_1 m_2 = \frac{\tan^2\theta + \cos^2\theta}{\sin^2\theta}$

$$\therefore \quad |m_1 - m_2| = \sqrt{(m_1 + m_2)^2 - 4m_1 m_2}$$

$$\Rightarrow \quad |\tan\theta_1 - \tan\theta_2| = \sqrt{\frac{4\tan^2\theta}{\sin^4\theta} - \frac{4(\tan^2\theta + \cos^2\theta)}{\sin^2\theta}}$$

$$= \frac{2}{\sin^2\theta}\sqrt{\tan^2\theta - \sin^2\theta\,(\tan^2\theta + \cos^2\theta)}$$

$$= \frac{2\sin\theta}{\sin^2\theta}\sqrt{(\sec^2\theta - \tan^2\theta - \cos^2\theta)}$$

$$= \frac{2\sin\theta}{\sin^2\theta}\sqrt{(1 - \cos^2\theta)} = \frac{2}{\sin\theta}\sin\theta$$

$$= 2$$

Example – 39

If $\alpha < \pi/4$, Find the acute angle between the lines

$(x^2 + y^2)\sin^2\alpha = (x\cos\beta - y\sin\beta)^2$

Sol. Given equation is

$$(x^2 + y^2)\sin^2\alpha = (x\cos\beta - y\sin\beta)^2$$

$$x^2(\sin^2\alpha - \cos^2\beta) + 2xy\sin\beta\cos\beta + y^2(\sin^2\alpha - \sin^2\beta) = 0 \quad \ldots (1)$$

The general equation of second degree is

$$ax^2 + 2hxy + by^2 = 0 \quad \ldots (2)$$

Comparing (1) and (2), we get

$a = \sin^2\alpha - \cos^2\beta,\ h = \sin\beta\cos\beta,\ b = \sin^2\alpha - \sin^2\beta$

Let the angle between the lines representing by (1) is θ

$$\therefore \quad \tan\theta = 2\frac{\sqrt{h^2 - ab}}{|a+b|}$$

$$= 2\frac{\sqrt{\sin^2\beta\cos^2\beta - (\sin^2\alpha - \cos^2\beta)(\sin^2\alpha - \sin^2\beta)}}{|\sin^2\alpha - \cos^2\beta + \sin^2\alpha - \sin^2\beta|}$$

$$= 2\frac{\sqrt{\{\sin^2\beta\cos^2\beta - \sin^4\alpha + \sin^2\alpha\sin^2\beta + \sin^2\alpha\cos^2\beta - \sin^2\beta\cos^2\beta\}}}{|(2\sin^2\alpha - 1)|}$$

$$= 2\frac{\sqrt{\sin^2\alpha(1 - \sin^2\alpha)}}{|-\cos 2\alpha|}$$

$$= \frac{2\sin\alpha\cos\alpha}{|-\cos 2\alpha|} = \tan 2\alpha$$

$\theta = 2\alpha$

Example – 40

Find the equation of the bisectors of the angle between the lines represented by $3x^2 - 5xy - 4y^2 = 0$.

Sol. Given equation is

$$3x^2 - 5xy - 4y^2 = 0 \quad \ldots (1)$$

Comparing it with the equation

$$ax^2 + 2hxy + by^2 = 0 \quad \ldots (2)$$

then $\quad a = 3,\ h = -\dfrac{5}{2},\ b - 4$

Hence equation of bisectors of the angle between the pair of the lines (1) is

$$\frac{x^2 - y^2}{3 + 4} = \frac{xy}{-5/2}$$

$$\Rightarrow \quad \frac{x^2 - y^2}{7} = \frac{2xy}{-5}$$

$$\Rightarrow \quad 5x^2 + 14xy - 5y^2 = 0$$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Distance Formula

1. If the distance between the points (x, 2) and (3, 4) is 2, then the value of x is

(a) 2 (b) 1

(c) 3 (d) 4

2. The point whose abscissa is equal to its ordinate and which is equidistant from A (5, 0) and B (0, 3) is

(a) (1, 1) (b) (2, 2)

(c) (3, 3) (d) (4, 4)

3. The points (1, 1), (–2, 7) and (3, –3)

(a) form a right angled triangle

(b) form an isosceles triangle

(c) are collinear

(d) none of the above

4. The points A (2a, 4a), B (2a, 6a) and $C\,(2a+\sqrt{3}a,\,5a)$ (when a > 0) are vertices of

(a) an obtuse angled triangle

(b) an equilateral triangle

(c) an isosceles obtuse angled triangle

(d) a right angled triangle

Section Formula

5. The coordinates of point which divides the line segment joining points A (0, 0) and B (9, 12) in the ratio 1 : 2, are

(a) (–3, 4) (b) (3, 4)

(c) (3, –4) (d) none of these

6. The ratio in which the line segment joining the points (3, –4) and (–5, 6) is divided by the x-axis, is

(a) 2 : 3 (b) 3 : 2

(c) 6 : 4 (d) none of these

7. A point which divides the joint of (1, 2) and (3, 4) externally in the ratio 1 : 1

(a) lies in the first quadrant

(b) lies in the second quadrant

(c) lies in third quadrant

(d) cannot be found

8. C is a point on the line segment joining A(–3, 4) and B (2, 1) such that AC = 2 BC then coordinates of C are

(a) $\left(\frac{1}{3}, 2\right)$ (b) $\left(2, \frac{1}{3}\right)$

(c) (2, 7) (d) (7, 2)

9. The ratio in which the line x + y = 4 divides the line joining the points (–1, 1) and (5, 7) is

(a) 1 : 2 (b) 2 : 1

(c) 3 : 2 (d) 3 : 1

Area of Δ, Collinerity, I, O, G, C

10. If the vertices of a triangle are (1, 2), (4, –6) and (3, 5), then its area is

(a) $\frac{23}{2}$ sq unit (b) $\frac{25}{2}$ sq unit

(c) 12 sq unit (d) none of these

11. The area of the quadrilateral with vertices at (4, 3), (2, –1), (–1, 2), (–3, 2) is

(a) 18 (b) 36

(c) 54 (d) none of these

12. If the points (a, –1), (2, 1) and (4, 5) are collinear. Then, the value of a is

(a) 1 (b) 2

(c) 3 (d) 4

13. If (1, 4) is the centroid of a triangle and two vertices are (4, –3) and (–9, 7), then third vertices is

(a) (8, 7) (b) (7, 8)

(c) (8, 8) (d) (6, 8)

14. The vertices of a triangle are (6, 0), (0, 6) and (6, 6). The distance between its circumcentre and centroid is

(a) 2 (b) $\sqrt{2}$

(c) 1 (d) $2\sqrt{2}$

15. If orthocentre and circumcentre of a triangle are respectively (1, 1) and (3, 2), then the coordinates of its centroid are

(a) $\left(\frac{7}{3}, \frac{5}{3}\right)$ (b) $\left(\frac{5}{3}, \frac{7}{3}\right)$

(c) (7, 5) (d) none of these

Locus and Shiffing of Origin

16. The locus of a point, whose abscissa and ordinate are always equal to

(a) $x + y = 0$ (b) $x - y = 0$

(c) $x + y = 1$ (d) $x + y + 1 = 0$

17. The locus of the equation $xy = 0$ is

(a) a straight line

(b) a pair of perpendiculr lines

(c) a hyperbola

(d) None of these

18. The locus of the equation $x^2 - 5x + 6 = 0$ is

(a) the empty set

(b) a set containing two distinct points

(c) a pair of parallel lines

(d) none of these

19. The locus represented by the point (x, y) with coordinates $x = 5\cos\theta + 3\sin\theta$ & $y = 5\sin\theta - 3\cos\theta$ is

(a) $x^2 - y^2 = 34$ (b) $x^2 + y^2 = 16$

(c) $x^2 + y^2 = 34$ (d) none of these

20. The locus of a point which moves such that the line segments having end points (2, 0) and (–2, 0) subtend a right angle at that point

(a) $x + y = 4$ (b) $x^2 + y^2 = 4$

(c) $x^2 - y^2 = 4$ (d) none of these

21. If O is the origin and Q is a variable point of $y^2 = x$. Find the locus of the mid-point of OQ.

(a) $2y^2 = x$ (b) $y^2 = x$

(c) $y^2 = 2x$ (d) none of these

22. If the axes are transformed from origin to the point (–2, 1), then new coordinates of (4, –5) are

(a) (2, 6) (b) (6, 4)

(c) (6, –6) (d) (2, –4)

23. Keeping the origin constant axes are rotated at an angle 30° in negative direction, if then new coordinates of the point are (2, 1) then its coordinate are with respect to old axis are

(a) $\left(\frac{2\sqrt{3}+1}{2}, \frac{2\sqrt{3}}{2}\right)$ (b) $\left(\frac{2\sqrt{3}+1}{2}, \frac{-2+\sqrt{3}}{2}\right)$

(c) $\left(\frac{2+\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$ (d) none of these

Slope of Line

24. Slopes of a line is not defined if the line is

(a) parallel to X-axis

(b) parallel to the lines $x - y = 0$

(c) parallel to the lines $x + y = 0$

(d) parallel to Y-axis

25. If A (–2, 1), B (2, 3) and C (–2, –4) are three points, then the angle between BA and BC is :

(a) $\tan^{-1}\left(\frac{3}{2}\right)$ (b) $\tan^{-1}\left(\frac{2}{3}\right)$

(c) $\tan^{-1}\left(\frac{7}{4}\right)$ (d) none of these

26. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are the vertices of :

(a) an obtuse angled triangle

(b) an acute angled triangle

(c) an isosceles triangle

(d) none

All forms of Line

27. The equation of a line through the point (–4, –3) and parallel to x-axis, is

(a) $y = 3$ (b) $y = -3$

(c) $y = 0$ (d) $y = 4$

28. If the foot of the perpendicular from the origin to a straight line is at the point (3, –4). Then, the equation of the line is

(a) $3x - 4y = 25$ (b) $3x - 4y + 25 = 0$

(c) $4x + 3y - 25 = 0$ (d) $4x - 3y + 25 = 0$

29. The equation of the line which passes through the point (1, –2) and cuts off equal intercept from the axes is

(a) $x + y = 1$ (b) $x - y = 1$

(c) $x + y + 1 = 0$ (d) $x - y - 2 = 0$

30. Two points (a, 0) and (0, b) are joined by a straight line. Another point on this line is

(a) (3a, –2b) (b) (a^2, ab)

(c) (–3a, 2b) (d) (a, b)

31. A line is drawn through the points (3, 4) and (5, 6). If the line is extended to a point whose ordinate is –1, then the abscissa of that point is

(a) 0 (b) –2

(c) 1 (d) 2

32. The equation of the line which passes through the point (3, 4) and the sum of its intercept on the axes is 14 is

(a) $4x - 3y = 24, x - y = 7$

(b) $4x + 3y = 24, x + y = 7$

(c) $4x + 3y + 24 = 0, x + y + 7 = 0$

(d) $4x - 3y + 24 = 0, x - y + 7 = 0$

33. If the intercept made by the line between the axes is bisected at the point (x_1, y_1), then its equation is

(a) $\frac{x}{x_1} + \frac{y}{y_1} = 2$ (b) $\frac{x}{x_1} + \frac{y}{y_1} = 1$

(c) $\frac{x}{x_1} + \frac{y}{y_1} = \frac{1}{2}$ (d) none of these

34. The line joining two points A (2, 0); B (3, 1) is rotated about A in the anticlockwise direction through an angle of 15°. The equation of the line in the new position is :

(a) $x - \sqrt{3}y - 2 = 0$ (b) $x - 2y - 2 = 0$

(c) $\sqrt{3}x - y - 2\sqrt{3} = 0$ (d) none

35. Equation of a straight line passing through the origin and making with x–axis an angle twice the size of the angle made by the line $y = 0.2x$ with the x–axis, is :

(a) $y = 4.0x$ (b) $y = (5/12)x$

(c) $6y - 5x = 0$ (d) none of these

36. The line passing through (0, 1) and perpendicular to the line $x - 2y + 11 = 0$ is

(a) $2x - y + 1 = 0$ (b) $2x - y + 3 = 0$

(c) $2x + y - 1 = 0$ (d) $2x + y - 2 = 0$

37. If a line is drawn through the origin and parallel to the line $x - 2y + 5 = 0$, then its eq. is

(a) $x - 2y - 5 = 0$ (b) $2x + y = 0$

(c) $x + 2y = 0$ (d) $x - 2y = 0$

38. The line passing through (1, 1) and parallel to the line $2x - 3y + 5 = 0$

(a) $3x + 2y = 5$ (b) $2x - 3y + 1 = 0$

(c) $3x - 2y = 1$ (d) $2x + 3y = 0$

39. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is

(a) –4 (b) 4/3

(c) –4/3 (d) None of these

40. The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150° with the positive direction of y axis. The equation of the line is :

(a) $\sqrt{3}x + y = 14$ (b) $\sqrt{3}x - y = 14$

(c) $\sqrt{3}x + y + 14 = 0$ (d) $\sqrt{3}x - y + 14 = 0$

41. The equation of a line parallel to ax + by + c = 0 and passing through the point (c, d) is

(a) $a(x + c) - b(y + d) = 0$

(b) $a(x + c) + b(y + d) = 0$

(c) $a(x - c) + b(y - d) = 0$

(d) none of these

42. The equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is (2, –1). Length of its side is

(a) $\sqrt{\frac{1}{2}}$ (b) $\sqrt{\frac{3}{2}}$

(c) $\sqrt{\frac{2}{3}}$ (d) $\sqrt{2}$

43. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. Then, the point O divides the segment PQ in the ratio

(a) 1 : 2 (b) 3 : 4

(c) 2 : 1 (d) 4 : 3

44. If the vertices P, Q, R of a ΔPQR are rational points, which of the following point(s) of the Δ PQR is not always rational?

(a) centroid (b) incentre

(c) circumcentre (d) orthocentre

45. A line has slope – 3/4, positive y-intercept and forms a triangle of area 24 sq. units with coordinate axes. Then, the equation of the line is

(a) $3x+4y+24=0$ (b) $3x+4y-24=0$

(c) $3x+4y-25=0$ (d) $3x+4y+25=0$

46. A non-horizontal line passing through the point (4, –2) and whose distance from the origin is 2 units is

(a) $3x+4y-10=0$ (b) $x+y-2=0$

(c) $4x+3y-10=0$ (d) $2x+3y-2=0$

Angle b/w Lines

47. The lines $2x-3y=5$ and $6x-9y-7=0$ are

(a) perpendicular

(b) parallel

(c) interesting but not perpendicular

(d) coincident

48. The acute angle between the lines $y=2x$ and $y=-2x$ is

(a) 45° (b) less than 60°

(c) greater than 60° (d) None of these

49. The sides AB, BC, CD and DA of a quadrilateral have the equations $x+2y=3$, $x=1$, $x-3y=4$, $5x+y+12=0$ respectively, then the angle between the diagonals AC and BD is :

(a) 60° (b) 45°

(c) 90° (d) none of these

Line making angle with given line

50. The equation of two equal sides of an isosceles triangle are $7x-y+3=0$ and $x+y-3=0$ and its third side is passes through the point (1, –10). The equation of the third side is

(a) $x-3y-31=0$ but not $3x+y+7=0$

(b) neither $3x+y+7=0$ nor $x-3y-31=0$

(c) $3x+y+7=0$ or $x-3y-31=0$

(d) $3x+y+7=0$ but not $x-3y-31=0$

51. The equation of two straight lines through (7, 9) and making an angle of 60° with the line $x-\sqrt{3}y-2\sqrt{3}=0$ is :

(a) $x=7,\ x+\sqrt{3}y=7+9\sqrt{3}$

(b) $x=\sqrt{3},\ x+\sqrt{3}y=7+9\sqrt{3}$

(c) $x=7,\ x-\sqrt{3}y=7+9\sqrt{3}$

(d) $x=\sqrt{3},\ x-\sqrt{3}y=7+9\sqrt{3}$

52. The equations of the lines through (–1, –1) and making angles 45° with the line $x+y=0$ are

(a) $x+1=0,\ x-y=0$

(b) $x-y=0,\ y+1=0$

(c) $x+y+2=0,\ y+1=0$

(d) $x+1=0,\ y+1=0$

Foot of ⊥ and image of point in a line

53. Assuming that the line $x-3y+4=0$ is working as a mirror for the point (1, 2) then the coordinates of that image is

(a) $\left(\frac{1}{5},\frac{2}{5}\right)$ (b) $\left(\frac{2}{5},\frac{3}{5}\right)$

(c) $\left(\frac{3}{5},\frac{6}{5}\right)$ (d) $\left(\frac{6}{5},\frac{7}{5}\right)$

54. The image of the point A (1, 2) by the line mirror $y=x$ is the point B and the image of B by the line mirror $y=0$ is the point (α, β) then :

(a) $\alpha=1, \beta=-2$ (b) $\alpha=0, \beta=0$

(c) $\alpha=2, \beta=-1$ (d) none of these

55. The co-ordinates of the foot of perpendicular from (a, 0) on the line $y=mx+\frac{a}{m}$ are

(a) $\left(0,-\frac{a}{m}\right)$ (b) $\left(\frac{a}{m},0\right)$

(c) $\left(0,\frac{a}{m}\right)$ (d) None of these

Family of Lines

56. The lines $x + 2y - 3 = 0$, $2x + y - 3 = 0$ and the line l are concurrent. If the line l passes through the origin, then its equation

(a) $x - y = 0$ (b) $x + y = 0$

(c) $x + 2y = 0$ (d) $2x + y = 0$

57. The equation of a line through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$ is

(a) $2x + y - 5 = 0$ (b) $2x - y + 5 = 0$

(c) $2x + y - 10 = 0$ (d) $2x - y - 10$

58. For the family of straight lines $bx + ay = ab$, which one is not correct.

(a) It will be x–axis if $a \neq 0, b = 0$

(b) It will represent concurrent lines passing through fixed point $(a, 0), a \neq 0$

(c) It will represent parallel lines if b/a is fixed, $a \neq 0$

(d) It will represent y–axis if $a \neq 0, b = 0$

59. The equation of the straight line which passes through the intersection of the lines $x - y - 1 = 0$ and $2x - 3y + 1 = 0$ and is parallel to x-axis, is

(a) $y = 3$ (b) $y = -3$

(c) $x + y = 3$ (d) none of these

60. The equation of the straight line which passes through the point of intersection of the straight lines $x + 2y = 5$ and $3x + 7y = 17$ and is perpendicular to the straight line $3x + 4y = 10$, is

(a) $4x + 3y + 2 = 0$ (b) $4x - y + 2 = 0$

(c) $4x - 3y - 2 = 0$ (d) $4x - 3y + 2 = 0$

61. A line passes through the point of intersection of the lines $100x + 50y - 1 = 0$ and $75x + 25y + 3 = 0$ and makes equal intercepts on the axes. Its equation is

(a) $25x + 25y - 1 = 0$ (b) $5x - 5y + 3 = 0$

(c) $25x + 25y - 4 = 0$ (d) $25x - 25y + 6 = 0$

Perpendicular Distance & Distance b/w 11 Lines

62. The distance between the lines $5x - 12y + 65 = 0$ and $5x - 12y - 39 = 0$ is

(a) 2 (b) 8

(c) –2 (d) None of these

63. The number of lines that are parallel to $2x + 6y - 7 = 0$ and have an intercept 10 units between the coordinate axes is

(a) 1 (b) 2

(c) 4 (d) infinitely many

64. The lines $8x + 4y = 1$, $8x + 4y = 5$, $4x + 8y = 3$, $4x + 8y = 7$ form a

(a) rhombus (b) rectangle

(c) square (d) none of these

Miscs

65. Given the four lines with equations $x + 2y - 3 = 0$, $2x + 3y - 4 = 0$, $3x + 4y - 5 = 0$, $4x + 5y - 6 = 0$, then these lines

(a) are concurrent

(b) are the sides of a quadrilateral

(c) are the sides of a parallelogram

(d) none of these

66. The area of triangle formed by the lines $y = x$, $y = 2x$ and $y = 3x + 4$ is

(a) 4 (b) 7

(c) 9 (d) 8

67. The incentre of the triangle formed by the lines $y = 15$, $12y = 5x$ and $3x + 4y = 0$ is

(a) (8, 1) (b) (–1, 8)

(c) (1, 8) (d) None of these

68. The sides AB, BC, CD and DA of a quadrilateral have the equations $x + 2y = 3$, $x = 1$, $x - 3y = 4$, $5x + y + 12 = 0$ respectively, then the angle between the diagonals AC and BD is :

(a) 60° (b) 45°

(c) 90° (d) none of these

69. In a ΔABC, if A is the point (1, 2) and equations of the median through B and C are respectively $x + y = 5$ and $x = 4$, then B is

(a) (1, 4) (b) (7, –2)

(c) (4, 1) (d) (–2, 7)

70. Let A(2, –3) and B(–2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on $2x + 3y = 1$, then the locus of the vertex C is the line

(a) $3x + 2y = 5$ (b) $2x - 3y = 7$

(c) $2x + 3y = 9$ (d) $2x - 2y = 3$

71. If $P \equiv (1, 0)$, $Q \equiv (-1, 0)$ and $R \equiv (2, 0)$ be three given points, then the locus of the point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$ is

(a) a straight line parallel to x-axis

(b) a circle passing through the origin

(c) a circle with the centre at the origin

(d) a straight line paralle to y-axis

72. If $x + 2y = 3$ is a line and A (–1, 3); B (2, –3); C (4, 9) are three points, then :

(a) A is on one side and B, C are on other side of the line

(b) A, B are on one side and C is on other side of the line

(c) A, C on one side and B is on other side of the line

(d) All three points are on one side of the line

73. The area of the parallelogram formed by the lines $4y - 3x = 1, 4y - 3x - 3 = 0, 3y - 4x + 1 = 0, 3y - 4x + 2 = 0$ is

(a) 3/8 (b) 2/7

(c) 1/6 (d) none of these

74. A straight line L is drawn perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L and co-ordinates axes is 5, then the equation of line, is :

(a) $x + 5y = \pm 5$ (b) $x + 5y = \pm\sqrt{2}$

(c) $x + 5y = \pm 5\sqrt{2}$ (d) none of these

75. The point (–1, 1) and (1, –1) are symmetrical about the line

(a) $y + x = 0$ (b) $y = x$

(c) $x + y = 1$ (d) none of these

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. A triangle with vertices (4, 0), (–1, –1), (3, 5) is **(2002)**

(a) isosceles and right angled

(b) isosceles but not right angled

(c) right angled but not isosceles

(d) neither right angled nor isosceles

2. The incentre of the triangle with vertices $(1, \sqrt{3})$, (0, 0) and (2, 0) is **(2002)**

(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$

(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

3. Three straight lines $2x + 11y - 5 = 0$, $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$ **(2002)**

(a) form a triangle

(b) are only concurrent

(c) are concurrent with one line bisecting the angle between the other two

(d) None of the above

4. A straight line through the point (2, 2) intersects the lines $\sqrt{3}x + y = 0$ and $\sqrt{3}x - y = 0$ at the points A and B. The equation to the line AB so that the ΔOAB is equilateral, is **(2002)**

(a) $x - 2 = 0$ (b) $y - 2 = 0$

(c) $x + y - 4 = 0$ (d) None of these

5. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is **(2003)**

(a) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

(b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

(c) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

(d) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

6. Locus of centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, – b cos t) and (1, 0), where t is a parameter, is **(2003)**

(a) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$

(b) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(c) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

(d) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

7. Let A(2, –3) and B (–2, 1) be the vertices of a ΔABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line **(2004)**

(a) $2x + 3y = 9$ (b) $2x - 3y = 7$

(c) $3x + 2y = 5$ (d) $3x - 2y = 3$

8. The equation of the straight line passing through the point (4, 3) and making intercepts on the coordinate axes, whose sum is –1, is **(2004)**

(a) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

(d) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

9. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is **(2005)**

(a) above the x-axis at a distance of (2/3) from it

(b) above the x-axis at a distance of (3/2) from it

(c) below the x-axis at a distance of (2/3) from it

(d) below the x-axis at a distance of (3/2) from it

10. If non-zero numbers a, b, c are in HP, then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point. That point is **(2005)**

(a) $\left(1, -\frac{1}{2}\right)$ (b) (1, –2)

(c) (–1, –2) (d) (–1, 2)

11. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2), then the centroid of the triangle is **(2005)**

(a) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (b) $\left(1, \frac{7}{3}\right)$

(c) $\left(-\frac{1}{3}, \frac{7}{3}\right)$ (d) $\left(-1, \frac{7}{3}\right)$

12. A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is **(2006)**

(a) $3x - 4y + 7 = 0$ (b) $4x + 3y = 24$

(c) $3x + 4y = 25$ (d) $x + y = 7$

13. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}, x > 0$ and $y = 3x, x > 0$, then a belongs to **(2006)**

(a) $(3, \infty)$ (b) $\left(\frac{1}{2}, 3\right)$

(c) $\left(-3, -\frac{1}{2}\right)$ (d) $\left(0, \frac{1}{2}\right)$

14. Let A(h, k), B(1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by **(2007)**

(a) {1, 3} (b) {0, 2}

(c) {–1, 3} (d) {–3, –2}

15. Let P = (–1, 0), Q = (0, 0) and $R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is **(2007)**

(a) $\sqrt{3}x + y = 0$ (b) $x + \frac{\sqrt{3}}{2}y = 0$

(c) $\frac{\sqrt{3}}{2}x + y = 0$ (d) $x + \sqrt{3}y = 0$

16. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines xy = 0, then m is **(2007)**

(a) $-\frac{1}{2}$ (b) –2

(c) ± 1 (d) 2

17. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept – 4. Then, a possible value of k is **(2008)**

(a) –4 (b) 1

(c) 2 (d) –2

18. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for **(2009)**

(a) exactly one value of p

(b) exactly two values of p

(c) more than two values of p

(d) no value of p

19. Three distinct points A, B and C given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then, the circumcentre of the ΔABC is at the point **(2009)**

(a) $\left(\frac{5}{4}, 0\right)$ (b) $\left(\frac{5}{2}, 0\right)$

(c) $\left(\frac{5}{3}, 0\right)$ (d) (0, 0)

20. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L & K is **(2010)**

(a) $\frac{23}{\sqrt{15}}$ (b) $\sqrt{17}$

(c) $\frac{17}{\sqrt{15}}$ (d) $\frac{23}{\sqrt{17}}$

21. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then, the set of all possible values of a in the interval **(2011)**

(a) $(-1, 1]$ (b) $(0, \infty)$
(c) $(1, \infty)$ (d) $(-1, \infty)$

22. If A (2, –3) and B (–2, 1) are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is **(2011)**

(a) $2x - 3y = 1$ (b) $x - y = 1$
(c) $2x + 3y = 1$ (d) $2x + 3y = 3$

23. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals. **(2012)**

(a) $\frac{29}{5}$ (b) 5
(c) 6 (d) $\frac{11}{5}$

24. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **(2013)**

(a) $y=x+\sqrt{3}$ (b) $\sqrt{3}y=x-\sqrt{3}$
(c) $y=\sqrt{3}x-\sqrt{3}$ (d) $\sqrt{3}y=x-1$

25. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is. **(2013)**

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$
(c) $1+\sqrt{2}$ (d) $1-\sqrt{2}$

26. Let PS be the median of the triangle with vertices P(2, 2), Q(6, –1) and R(7, 3). The equation of the line passing through (1, –1) and parallel to PS is **(2014)**

(a) $2x - 9y - 11 = 0$ (b) $4x - 7y - 11 = 0$
(c) $2x + 9y + 7 = 0$ (d) $4x + 7y + 3 = 0$

27. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then : **(2014)**

(a) $3bc + 2ad = 0$ (b) $2bc - 3ad = 0$
(c) $2bc + 3ad = 0$ (d) $3bc - 2ad = 0$

28. Given three points P, Q R with P (5, 3) and R lies on the x-axis. If equation of RQ is $x - 2y = 2$ and PQ is parallel to the x-axis, then the centroid of ΔPQR lies on the line:

(2014/Online Set–1)

(a) $2x + y - 9 = 0$ (b) $2 - 2y + 1 = 0$
(c) $5x - 2y = 0$ (d) $2x - 5y = 0$

29. The base of an equilateral triangle is along the line given by $3x + 4y = 9$. If a vertex of the triangle is (1, 2), then the length of a side of the triangle is:

(2014/Online Set–2)

(a) $\frac{2\sqrt{3}}{15}$ (b) $\frac{4\sqrt{3}}{15}$
(c) $\frac{4\sqrt{3}}{5}$ (d) $\frac{2\sqrt{3}}{5}$

30. Let A(2, 3, 5), B(-1, 3, 2) and $C(\lambda, 5, \mu)$ be the vertices of a ΔABC. If the median through A is equally inclined to the coordinate axes, then: **(2014/Online Set–2)**

(a) $5\lambda - 8\mu = 0$ (b) $8\lambda - 5\mu = 0$
(c) $10\lambda - 7\mu = 0$ (d) $7\lambda - 10\mu = 0$

31. If a line intercepted between the coordinate axis is trisected at a point A(4, 3), which is nearer to x-axis, then its equation is: **(2014/Online Set–3)**

(a) $4x - 3y = 7$ (b) $3x + 2y = 18$
(c) $3x + 8y = 36$ (d) $x + 3y = 13$

32. If the three distinct lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4ay + a = 0$ are concurrent, then the point (a, b) lies on a: **(2014/Online Set–3)**

(a) circle (b) hyperbola
(c) straight line (d) parabola

33. The circumcentre of a triangle lies at the origin and its centroid is the mid-point of the line segment joining the points $(a^2 + 1, a^2 + 1)$ and $(2a, -2a)$, $a \neq 0$. Then for any a, the orthocentre of this triangle lies on the line:

(2014/Online Set–4)

(a) $y - 2ax = 0$ (b) $y - (a^2 + 1)\ x = 0$
(c) $y + x = 0$ (d) $(a - 1)^2 x - (a + 1)^2 y = 0$

34. If a line L perpendicular to the line 5x - y = 1, and the area of the triangle formed by the line L and the coordinate axes is 5, then the distance of line L from the line x + 5y = 0 is: **(2014/Online Set–4)**

(a) $\frac{7}{\sqrt{5}}$ (b) $\frac{5}{\sqrt{13}}$

(c) $\frac{7}{\sqrt{13}}$ (d) $\frac{5}{\sqrt{7}}$

35. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is: **(2015)**

(a) 820 (b) 780

(c) 901 (d) 861

36. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0, K \in R$, is a: **(2015)**

(a) circle of radius $\sqrt{2}$.

(b) circle of radius $\sqrt{3}$

(c) straight line parallel to x – axis.

(d) straight line parallel to y – axis.

37. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) : **(2015/Online Set–1)**

(a) form an acute angled triangle.

(b) lie on a straight line

(c) form an obtuse angled triangle

(d) form a right angled triangle.

38. Two sides of a rhombus are along the lines, x – y + 1 = 0 and 7x – y – 5 = 0. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus ? **(2016)**

(a) (–3, –8) (b) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

(c) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (d) (–3, –9)

39. If a variable line drawn through the intersection of the lines $\frac{x}{3} + \frac{y}{4} = 1$ and $\frac{x}{4} + \frac{y}{3} = 1$, meets the coordinate axes at A and B, $(A \neq B)$, then the locus of the midpoint of AB is : **(2016/Online Set–1)**

(a) $6xy = 7(x + y)$

(b) $4(x + y)^2 - 28(x + y) + 49 = 0$

(c) $7xy = 6(x + y)$

(d) $14(x + y)^2 - 97(x + y) + 168 = 0$

40. The point (2, 1) is translated parallel to the line L : x – y = 4 by $2\sqrt{3}$ units. If the new point Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is : **(2016/Online Set–1)**

(a) $x + y = 2 - \sqrt{6}$ (b) $x + y = 3 - 3\sqrt{6}$

(c) $x + y = 3 - 2\sqrt{6}$ (d) $2x + 2y = 1 - \sqrt{6}$

41. A ray of light is incident along a line which meets another line, 7x – y + 1 = 0, at the point (0, 1). The ray is then reflected from this point along the line, y + 2x = 1. Then the equation of the line of incidence of the ray of light is : **(2016/Online Set–2)**

(a) $41x - 38y + 38 = 0$ (b) $41x + 25y - 25 = 0$

(c) $41x + 38y - 38 = 0$ (d) $41x - 25y + 25 = 0$

42. A straight line through origin O meets the lines 3y = 10 – 4x and 8x + 6y + 5 = 0 at points A and B respectively. Then O divides the segment AB in the ratio : **(2016/Online Set–2)**

(a) 2 : 3 (b) 1 : 2

(c) 4 : 1 (d) 3 : 4

43. Let k be an integer such that the triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point: **(2017)**

(a) $\left(2, -\frac{1}{2}\right)$ (b) $\left(1, \frac{3}{4}\right)$

(c) $\left(1, -\frac{3}{4}\right)$ (d) $\left(2, \frac{1}{2}\right)$

44. A tangent to the curve, $y = f(x)$ at P (x, y) meets x-axis at A and y-axis at B. If AP : BP = 1 : 3 and $f(1) = 1$, then the curve also passes through the point : **(2017/Online Set–2)**

(a) $\left(\frac{1}{3}, 24\right)$ (b) $\left(\frac{1}{2}, 4\right)$

(c) $\left(2, \frac{1}{8}\right)$ (d) $\left(3, \frac{1}{28}\right)$

45. A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30º with the positive direction of the x-axis, then the sum of the x-coordinates of the vertices of the square is : **(2017/Online Set–2)**

(a) $2\sqrt{3}-1$ (b) $2\sqrt{3}-2$

(c) $\sqrt{3}-2$ (d) $\sqrt{3}-1$

46. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60º. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is : **(2017/Online Set–2)**

(a) 12.5 (b) 13.2

(c) 12 (d) 13

47. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is **(2018)**

(a) $3x+2y=6xy$ (b) $3x+2y=6$

(c) $2x+3y=xy$ (d) $3x+2y=xy$

48. In a triangle ABC, coordinates of A are (1, 2) and the equations of the medians through B and C are respectively, $x+y=5$ and $x=4$. Then area of ΔABC (in sq. units) is : **(2018/Online Set–1)**

(a) 12 (b) 4

(c) 5 (d) 9

49. The sides of a rhombus ABCD are parallel to the lines, $x-y+2=0$ and $7x-y+3=0$. If the diagonals of the rhombus intersect at P(1, 2) and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is :

(2018/Online Set–2)

(a) $\frac{5}{2}$ (b) $\frac{7}{4}$

(c) 2 (d) $\frac{7}{2}$

50. The foot of the perpendicular drawn from the origin, on the line, $3x+y=\lambda(\lambda\neq0)$ is P. If the line meets x-axis at A and y-axis at B, then the ratio BP : PA is:

(2018/Online Set–2)

(a) 1 : 3 (b) 3 : 1

(c) 1 : 9 (d) 9 : 1

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. The bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$, is :

(a) $11x + 3y - 9 = 0$ (b) $21x + 77y - 101 = 0$

(c) $11x - 3y + 9 = 0$ (d) none of these

2. The equation of the bisector of the angle between two lines $3x - 4y + 12 = 0$ and $12x - 5y + 7 = 0$ which contains the points $(-1, 4)$ is :

(a) $21x + 27y - 121 = 0$

(b) $21x - 27y + 121 = 0$

(c) $21x + 27y + 191 = 0$

(d) $\frac{-3x + 4y - 12}{5} = \frac{12x - 5y + 7}{13}$

3. The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$. The equation of the other line is :

(a) $3x + 3y - 1 = 0$ (b) $x - 3y + 2 = 0$

(c) $5x + 5y - 3 = 0$ (d) none

4. Find the angle between the lines represented by the equation $x^2 - 2pxy + y^2 = 0$

(a) $\sec^{-1}(p)$ (b) $\tan^{-1}(p)$

(c) $\cos^{-1}(p)$ (d) none of these

5. The distance between the lines represented by the equation,

$x^2 + 2\sqrt{2}xy + 2y^2 + 4x + 4\sqrt{2}y + 1 = 0$ is :

(a) $\frac{4}{\sqrt{3}}$ (b) 4

(c) 2 (d) $2\sqrt{3}$

6. If sum and product of the slopes of lines represented by $4x^2 + 2hxy - 7y^2 = 0$ is equal then h is equal to :

(a) –6 (b) –2

(c) –4 (d) 4

7. If θ is the angle between the lines represented by $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$, where λ is a real number, then $\operatorname{cosec}^2 \theta$ equals :

(a) 9 (b) 3

(c) 10 (d) 100

8. The difference of the slopes of the lines,

$x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ is

(a) –2 (b) 1/2

(c) 2 (d) 1

9. If pairs of straight lines, $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair then :

(a) $pq = -1/2$ (b) $pq = -2$

(c) $pq = -1$ (d) $p/q = -1$

10. Find the condition that the pair of straight lines joining the origin to the intersections of the line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ may be at right angles.

(a) $2a^2 = c^2(1 + m^2)$ (b) $2a^2 = c^2(1 - m^2)$

(c) $2c^2 = a^2(1 + m^2)$ (d) $2c^2 = a^2(1 - m^2)$

11. The vertices of ΔOBC are respectively (0, 0), (–3, –1) and (–1, –3). The equation of line parallel to BC and at a distance 1/2 from O which intersects OB and OC is :

(a) $2x + 2y + \sqrt{2} = 0$ (b) $2x - 2y + \sqrt{2} = 0$

(c) $2x + 2y - \sqrt{2} = 0$ (d) none of these

12. If the sum of the distance of a point from two perpendicular lines in a plane is 1 then it locus is :

(a) square (b) circle

(c) a straight line (d) two intersecting line

13. Given the points A (0, 4) and B (0,–4), the equation of the locus of the point P (x, y) such that $|AP - BP| = 6$ is :

(a) $9x^2 - 7y^2 + 63 = 0$ (b) $9x^2 - 7y^2 - 63 = 0$

(c) $7x^2 - 9y^2 + 63 = 0$ (d) $7x^2 - 9y^2 - 63 = 0$

14. A triangle ABC with vertices A(–1, 0), B (–2, 3/4) and C (–3, –7/6) has its orthocentre H. Then the orthocentre of triangle BCH will be :

(a) (–3 –2) (b) (1, 3)

(c) (–1, 2) (d) none of these

15. A variable straight line passes through a fixed point (a, b) intersecting the co-ordinates axes at A and B. If 'O' is the origin then the locus of the centroid of the triangle OAB is:

(a) $bx + ay - 3xy = 0$ (b) $bx + ay - 2xy = 0$

(c) $ax + by - 3xy = 0$ (d) $ax + by - 2xy = 0$

16. Two vertices of a triangle are (4, –3) and (–2, 5). If the othocentre of the triangle is at (1, 2), then the coordinates of the third vertex are

(a) (33, 130) (b) (33, 26)

(c) (–33, 26) (d) (33, –26)

17. Drawn from the origin are two mutually perpendicular straight lines forming an isosceles triangle together with the straight line, $2x + y = a$. Then the area of the triangle is:

(a) $\frac{a^2}{2}$ (b) $\frac{a^2}{3}$

(c) $\frac{a^2}{5}$ (d) none

18. The point $(a^2, a + 1)$ is a point in the angle between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin if:

(a) $a \geq 1$ or $a \leq -3$ (b) $a \in (-3, 0) \cup (1/3, 1)$

(c) $a \in (0, 1)$ (d) none of these

19. Area of the quadrilateral formed by the lines $|x| + |y| = 2$ is :

(a) 8 (b) 6

(c) 4 (d) none

20. The orthocentre of the triangle ABC is 'B' and the circumcentre is 'S' (a, b). If A is the origin then the co–ordinates of C are :

(a) (2a, 2b) (b) $\left(\frac{a}{2}, \frac{b}{2}\right)$

(c) $\left(\sqrt{a^2 + b^2}, 0\right)$ (d) none

21. The equation of the line segment AB is $y = x$. If A and B lie on the same side of the line mirror $2x - y = 1$, the image of AB has the equation

(a) $x + y = 2$ (b) $8x + y = 9$

(c) $7x - y = 6$ (d) none of these

22. A is a point on either of two lines $y + \sqrt{3}|x| = 2$ at a distance of $\frac{4}{\sqrt{3}}$ units from their point of intersection. The co-ordinates of the foot of perpendicular from A on the bisector of the angle between them are

(a) $\left(-\frac{2}{\sqrt{3}}, 2\right)$ (b) (0, 0)

(c) $\left(\frac{2}{\sqrt{3}}, 2\right)$ (d) (0, 4)

23. On the portion of the straight line, $x + 2y = 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co-ordinates :

(a) (2, 3) (b) (3, 2)

(c) (3, 3) (d) none

24. The equations of three lines, AB, CD and EF are, $(b - c) x + (c - a) y + (a - b) = 0$, $(c - a) x + (a - b) y + (b - c) = 0$ and $(a - b) x + (b - c) y + (c - a) = 0$. Which one of the following inferences is correct.

(a) the lines are parallel to each other

(b) AB and BC are perpendicular to EF

(c) all the lines are coincident

(d) the lines are concurrent

25. The base BC of a ΔABC is bisected at the point (p, q) and the equation to the side AB and AC are $px + qy = 1$ and $qx + py = 1$. The equation of the median through A is :

(a) $(p - 2q)x + (q - 2p)y + 1 = 0$

(b) $(p + q)(x + y) - 2 = 0$

(c) $(2pq - 1)(px + qy - 1) = (p^2 + q^2 - 1)(qx + py - 1)$

(d) none

26. Given the family of lines, $a(3x+4y+6)+b(x+y+2)=0$. The line of the family situated at the greatest distance from the point P (2, 3) has equation :

(a) $4x+3y+8=0$ (b) $5x+3y+10=0$
(c) $15x+8y+30=0$ (d) none

27. Let $P=(1, 1)$ and $Q=(3, 2)$. The point R on the x-axis such that PR + RQ is the minimum is

(a) $\left(\frac{5}{3},0\right)$ (b) $\left(\frac{1}{3},0\right)$
(c) (3, 0) (d) none of these

28. The co-ordinates of a point P on the line $2x-y+5=0$ such that $|PA-PB|$ is maximum where A is (4, –2) and B is (2, –4) will be :

(a) (11, 27) (b) (–11, –17)
(c) (–11, 17) (d) (0, 5)

29. A right angle triangle ABC having 'C' a right angle has AC = a and BC = b units. The points A and B slide along the cartesian axes (A on x – axis and B on y – axis). Then the locus of 'C' is :

(a) $by\pm ax=0$ (b) $ay\pm bx=0$
(c) $x^2+y^2=0$ (d) $xy=ab$

30. A line is drawn through the point A (p, q) in the direction θ to meet the line, $ax+by+c=0$ in B, then $|AB|=$

(a) $\left|\frac{ap+bq+c}{a\cos\theta+b\sin\theta}\right|$ (b) $\left|\frac{aq+bp+c}{a\cos\theta+b\sin\theta}\right|$

(c) $\left|\frac{ap+bq+c}{a\sin\theta+b\cos\theta}\right|$ (d) $\left|\frac{aq+bp+c}{a\sin\theta+b\cos\theta}\right|$

31. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line $a^2x+aby+1=0\ \forall\ a\in R, b>0$ are :

(a) $b\in(2, 4)$ (c) $b\in(0, 2)$
(c) $b\in[0, 2]$ (d) $b\in(2, \infty)$

32. The acute angle between two straight lines passing through the point M(–6, –8) and the points in which the line segment $2x+y+10=0$ enclosed between the co-ordinate axes is divided in the ratio 1 : 2 : 2 in the direction from the point of its intersection with the x–axis to the point of Intersection with the y –axis is :

(a) π/3 (b) π/4
(c) π/6 (d) π/12

33. The equations of the sides of a square whose each side is of length 4 units and centre is (1, 1). Given that one pair of sides is parallel to $3x-4y=0$.

(a) $3x-4y+11=0$, $3x-4y-9=0$, $4x+3y+3=0$, $4x+3y-17=0$

(b) $3x-4y-15=0$, $3x-4y+5=0$, $4x+3y+3=0$, $4x+3y-17=0$

(c) $3x-4y+11=0$, $3x-4y-9=0$, $4x+3y+2=0$, $4x+3y-18=0$

(d) none

34. If $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ are the values of n for which $\sum_{r=0}^{n-1}x^{2r}$ is divisible by $\sum_{r=0}^{n-1}x^{r}$, then the triangle having vertices (α_1, β_1), (α_2, β_2) and (α_3, β_3) cannot be.

(a) an isosceles triangle
(b) a right angled isosceles triangle
(c) a right angled triangle
(d) an equilateral triangle

35. Consider the straight line $ax+by=c$, where $a, b, c\in R^+$ this line meets the coordinate axes at A and B respectively. If the area of the ΔOAB, O being origin, does not depend upon a, b and c, then

(a) a, b, c are in AP (b) a, c, b are in GP
(c) a, b, c are in HP (d) none of these

36. ABC is an equilateral triangle such that the vertices B and C lie on two parallel lines at a distance 6. If A lies between the parallel lines at a distance 4 from one of them, then the length of a side of the equilateral triangle is

(a) 8 (b) $\sqrt{\frac{88}{3}}$

(c) $\frac{4\sqrt{7}}{\sqrt{3}}$ (d) none of these

37. All the points lying on or inside the triangle formed by the points (1, 3), (5, 6) and (–1, 2) satisfy

(a) $3x+2y\geq 0$ (b) $2x+y+1\geq 0$
(c) $2x+3y-12\geq 0$ (d) $-2x+11\geq 0$

38. Given two straight lines $x - y - 7 = 0$ and $x - y + 3 = 0$. Equation of a line which divides the distance between them in the ratio 3 : 2 can be and || to them.

(a) $x - y - 1 = 0$ (b) $x - y - 3 = 0$

(c) $y = x$ (d) $x - y + 1 = 0$

39. A light beam emanating from the point A(3, 10) reflects from the straight line $2x + y - 6 = 0$ and then passes through the point B(4, 3). The equation of the reflected beam is :

(a) $3x - y + 1 = 0$ (b) $x + 3y - 13 = 0$

(c) $3x + y - 15 = 0$ (d) $x - 3y + 5 = 0$

40. If $\frac{x}{c} + \frac{y}{d} = 1$ is a line through the intersection of $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ and the lengths of the perpendiculars drawn from the origin to these lines are equal in lengths then :

(a) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2} + \frac{1}{d^2}$ (b) $\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{c^2} - \frac{1}{d^2}$

(c) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c} + \frac{1}{d}$ (d) none

41. Equation of a straight line passing through the point (4, 5) and equally inclined to the lines, $3x = 4y + 7$ and $5y = 12x + 6$ is

(a) $9x - 7y = 1$ (b) $9x + 7y = 71$

(c) $7x + 9y = 73$ (d) $7x - 9y + 17 = 0$

42. Let $u \equiv ax + by + a\sqrt[3]{b} = 0,\ v \equiv bx - ay + b\sqrt[3]{a} = 0, a\, b \in R$ be two straight lines. The equation of the bisectors of the angle formed by $k_1u - k_2v = 0$ and $k_1u + k_2v = 0$ for non zero real k_1 and k_2 are :

(a) $u = 0$ (b) $k_2u + k_1v = 0$

(c) $k_2u - k_1v = 0$ (d) $v = 0$

43. If the equation, $2x^2 + k\,xy - 3y^2 - x - 4y - 1 = 0$ represents a pair of lines then the value of k can be :

(a) 1 (b) 5

(c) –1 (d) –5

Assertion and Reason

(A) If both assertion and reason are correct and reason is the correct explanation of assertion.

(B) If both assertion and reason are true but reason is not the correct explanation of assertion.

(C) If assertion is true but reason is false.

(D) If assertion is false but reason is true.

44. **Assertion :** The lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

Reason : The area of the triangle formed by three concurrent lines must be zero.

(a) A (b) B

(c) C (d) D

45. **Assertion :** Each point on the line $y - x + 12 = 0$ is equidistant from the lines $4y + 3x - 12 = 0$, $3y + 4x - 24 = 0$.

Reason : The locus of a point which is equidistant from two given lines is the angular bisector of the two lines.

(a) A (b) B

(c) C (d) D

46. **Assertion :** If $(a_1x + b_1y + c_1) + (a_2x + b_2y + c_2) + (a_3x + b_3y + c_3) = 0$, then lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$ cannot be parallel.

Reason : If sum of three straight lines equations is identically zero then they are either concurrent or parallel.

(a) A (b) B

(c) C (d) D

47. **Assertion :** The four straight lines given by $12x^2 + 7xy - 12y^2 = 0$ and $12x^2 + 7xy - 12y^2 - x + 7y - 1 = 0$ are the sides of a square.

Reason : The lines represented by general equation of second degree $ax^2 + 2\,hxy + by^2 + 2\,gx + 2fy + c = 0$ are perpendicular if $a + b = 0$.

(a) A (b) B

(c) C (d) D

48. **Assertion :** The diagonals of parallelogram formed by the lines

$ax + by + c = 0, a'x + b'y + c' = 0,$

$ax + by + c' = 0, a'x + b'y + c' = 0$

will be perpendicular if $aa' + bb' = 0$

Reason : The diagonals of rhombus are always perpendicular.

(a) A (b) B

(c) C (d) D

Using the following passage, solve Q.49 to Q.51

Passage

Let A (0, β), B (–2, 0) and C (1, 1) be the vertices of a triangle then

49. Angle A of the triangle ABC will be obtuse if β lies in

(a) (–1, 2) (b) $\left(2, \frac{5}{2}\right)$

(c) $\left(-1, \frac{2}{3}\right) \cup \left(\frac{2}{3}, 2\right)$ (d) None of these

50. If I_1 is the interval of values of β for which A is obtuse and I_2 be the interval of values of β for which A is largest angle of ΔABC, then

(a) $I_1 = I_2$ (b) I_1 is a subset of I_2

(c) I_2 is a subset of I_1 (d) None of these

51. All the values of β for which angle A of the triangle ABC is largest lie in interval.

(a) (–2, 1) (b) $\left(-2, \frac{2}{3}\right) \cup \left(\frac{2}{3}, 1\right)$

(c) $\left(-2, \frac{2}{3}\right) \cup \left(\frac{2}{3}, \sqrt{6}\right)$ (d) None of these

Using the following passage, solve Q.52 to Q.54

Passage

Let OX and OY be two fixed lines inclined at a constant angle α. A variable line cuts OX at P and OY at Q. From P and Q perpendiculars PM and PN are drawn to OY and OX respectively.

52. Let the axes be chosen as OX and OY. If OP = a, OQ = b, then equation of PQ must be

(a) $\frac{x}{a\cos\alpha} + \frac{y}{b\sin\alpha} = 1$

(b) $\frac{x}{a\sin\alpha} + \frac{y}{b\cos\alpha} = 1$

(c) $\frac{x}{a} + \frac{y}{b} = 1$

(d) None of these

53. The equation of MN must be

(a) $\frac{x}{a\cos\alpha} + \frac{y}{b\sin\alpha} = 1$

(b) $\frac{x}{b\cos\alpha} + \frac{y}{a\cos\alpha} = 1$

(c) $\frac{x}{a\sin\alpha} + \frac{y}{b\sin\alpha} = 1$

(d) None of these

54. If AB passes through a fixed point (h, k) then MN passes through a fixed point

(a) (k cos α, h sin α) (b) (k cos α, h cos α)

(c) (k sin α, h cos α) (d) None of these.

Match the column

55. Match the values of k for which origin and (k, 3) lie

	Column–I		**Column–II**
(A)	in the same angle formed by lines $x-2y+3=0$ and $2x+y+5=0$	(P)	$(-4, 3)$
(B)	opposite angles formed by lines $x-2y+3=0$ and $2x+y+5=0$	(Q)	$(-\infty, -4)$
(C)	adjacent angles formed by lines $x-2y+3=0$ and $2x+y+5=0$	(R)	$(3, \infty)$

56. A line cuts x–axis at A and y–axis at B such that AB = l.

Match the following loci :

	Column - I		**Column - II**
(A)	Circumcentre of triangle ABC	(P)	$x^2+y^2=l^2/9$
(B)	Orthocentre of triangle ABC	(Q)	$x^2+y^2=l^2/4$
(C)	Incentre of the triangle ABC	(R)	$x^2+y^2=0$
(D)	Centroid of the triangle ABC	(S)	$y=x$

Subjective

57. For $0 \le \theta < 2\pi$, if the point $(2\cos\theta, 2\sin\theta)$ lies in the angle between the lines $y = |x-2|$ in which origin lies, then θ lies in the interval of length $k\pi$, then k must be

58. The co-ordinates of A, B, C are (6, 3), (–3, 5), (4, –2) respectively. For any point P (x, y) if the ratio of the areas of the triangles ΔPBC and ΔABC is $\left|\frac{x+y-2}{\lambda}\right|$. Then the numerical quantity λ must be equal to

59. Two sides of a rhombus lying in the first quadrant are given by $3x-4y=0$ and $12x-5y=0$. The length of the longer diagonal is 12. If the equations of other two sides are $3x-4y=-\frac{180}{\sqrt{k}}$ and $12x-5y=\frac{468}{\sqrt{k}}$ then the numerical quantity k should be

60. The base of a triangle passes through a fixed point (1, 1) and its sides are bisected at right angle by the lines $y^2-8xy-9x^2=0$. If the locus of its vertex is a circle of radius $\sqrt{\frac{\lambda}{32}}$. Then the numerical quantity λ must be equal.

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Question I [Only one correct option]

1. The points (–a, –b), (0, 0), (a, b) and are **(1979)**

(a) collinear

(b) vertices of a rectangle

(c) vertices of a parallelogram

(d) None of the above

2. Given the four lines with the equations

$x + 2y - 3 = 0, 3x + 4y - 7 = 0,$

$2x + 3y - 4 = 0, 4x + 5y - 6 = 0$ then : **(1980)**

(a) they are all concurrent

(b) they are the sides of a quadrilateral

(c) only three lines are concurrent

(d) none of the above

3. The point (4, 1) undergoes the following three transformations successively.

(I) Reflection about the line y = x

(II) Transformation through a distance 2 unit along the positive direction of x-axis.

(III) Rotation through an angle $\frac{\pi}{4}$ about the origin in the counter clockwise direction.

Then, the final position of the point is given by the coordinates. **(1980)**

(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $\left(-\sqrt{2}, 7\sqrt{2}\right)$

(c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $\left(\sqrt{2}, 7\sqrt{2}\right)$

4. The straight lines $x + y = 0, 3x + y - 4 = 0, x + 3y - 4 = 0$ form a triangle which is : **(1983)**

(a) isosceles (b) equilateral

(c) right angled (d) none of these

5. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are vertices of : **(1986)**

(a) an obtuse angled triangle

(b) an acute angled triangle

(c) a right angled triangled

(d) none of these

6. If P = (1, 0), Q = (–1, 0) and R = (2, 0) are three given points, then locus of the points satisfying the relation $SQ^2 + SR^2 = 2SP^2$, is : **(1988)**

(a) a straight line parallel to x-axis

(b) a circle passing through the origin

(c) a circle with the centre at the origin

(d) a straight line parallel to y-axis

7. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then **(1990)**

(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

(c) $a^2 + p^2 = b^2 + q^2$ (d) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

8. If the sum of the distance of a point from two perpendicular lines in a plane is 1, then its locus is : **(1992)**

(a) square (b) circle

(c) straight line (d) two intersecting line

9. The orthocentre of the triangle formed by the line xy = 0 and x + y = 1, is **(1995)**

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{3}\right)$

(c) (0, 0) (d) $\left(\frac{1}{4}, \frac{1}{4}\right)$

10. The graph of the function $\cos x \cos(x + 2) - \cos^2(x + 1)$ is **(1997)**

(a) a straight line passing through $(0, -\sin^2 1)$ with slope 2

(b) a straight line passing through (0, 0)

(c) a parabola with vertex $(1, -\sin^2 1)$

(d) a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to the x-axis.

11. The diagonals of a parallelogram PQRS are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then PQRS must be : **(1998)**

(a) rectangle (b) square
(c) cyclic quadrilateral (d) rhombus

12. If P (1, 2), Q (4, 6), R (5, 7) and S (a, b) are the vertices of a parallelogram PQRS,then: **(1998)**

(a) $a = 2, b = 4$ (b) $a = 3, b = 4$
(c) $a = 2, b = 3$ (d) $a = 3, b = 5$

13. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/(are) always rational points(s) **(1998)**

(a) centroid (b) incentre
(c) circumcentre (d) othocentre

(A rational point is a point both of whose coordinates are rational numbers)

14. Let $A_0, A_1, A_2, A_3, A_4, A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments $A_0 A_1$, $A_0 A_2$ and $A_0 A_4$ is : **(1998)**

(a) $\frac{3}{4}$ (b) $3\sqrt{3}$
(c) 3 (d) $\frac{3\sqrt{3}}{2}$

15. Let PQR be a right angled isosceles triangle, right angled at P (2, 1). If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is **(1999)**

(a) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$
(b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
(c) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
(d) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

16. Let PS be the median of the triangle with vertices P (2, 2), Q (6, –1) and R (7, 3). The equation of the line passing through (1, –1) and parallel to PS is : **(2000)**

(a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$
(c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$

17. The incentre of the triangle with vertices $(1, \sqrt{3})$, (0, 0) and (2, 0) is : **(2000)**

(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$
(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

18. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ & $y = mx + 1$ is also an integer, is : **(2001)**

(a) 2 (b) 0
(c) 4 (d) 1

19. Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals : **(2001)**

(a) $\frac{|m+n|}{(m-n)^2}$ (b) $\frac{2}{|m+n|}$
(c) $\frac{1}{|m+n|}$ (d) $\frac{1}{|m-n|}$

20. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. Then, the equations of the bisector of the angle PQR is : **(2002)**

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$

21. Let $0 < \alpha < \frac{\pi}{2}$ be a fixed angle. If $P = (\cos\theta, \sin\theta)$ and $Q = \{\cos(\alpha - \theta), \sin(\alpha - \theta)\}$, then Q is obtained from p by **(2002)**

(a) clockwise rotation around origin through an angle α
(b) anticlockwise rotation around origin through an angles α
(c) reflection in the line through origin with slope $\tan\alpha$
(d) reflection in the line through origin with slope $\tan\frac{\alpha}{2}$

22. A straight line through the origin O meets the parallel lines $4x+2y=9$ and $2x+y+6=0$ at points P and Q respectively. Then the point O divides the segment PQ in the ratio : **(2003)**

(a) 1 : 2 (b) 3 : 4

(c) 2 : 1 (d) 4 : 3

23. Area of triangle formed by the lines $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is **(2004)**

(a) 2 sq. unit (b) 4 sq. unit

(c) 6 sq. unit (d) 8 sq. unit

24. Let O (0, 0), P (3, 4), Q (6, 0) be the vertices of a triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are : **(2007)**

(a) $\left(\frac{4}{3}, 3\right)$ (b) $\left(3, \frac{2}{3}\right)$

(c) $\left(3, \frac{4}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{2}{3}\right)$

25. Let a and b be non-zero and real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents **(2008)**

(a) Four straight lines, when c = 0 and a, b are of the same sign.

(b) Two straight lines and a circle, when a = b and c is of sign opposite to that of a.

(c) Two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(d) A circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

26. The locus of the orthocentre of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0$,
$(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$ where $p \neq q$, is **(2009)**

(a) a hyperbola (b) a parabola

(c) an ellipse (d) a straight line

27. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x+y=1$. If L also intersects the x-axis, then the equation of L is **(2011)**

(a) $y+\sqrt{3}x+2-3\sqrt{3}=0$ (b) $y-\sqrt{3}x+2+3\sqrt{3}=0$

(c) $\sqrt{3}y-x+3+2\sqrt{3}=0$ (d) $\sqrt{3}y+x-3+2\sqrt{3}=0$

28. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then, **(2013)**

(a) $a + b - c > 0$ (b) $a - b + c < 0$

(c) $a - b + c > 0$ (d) $a + b - c < 0$

29. In a triangle PQR, let $\angle PQR=30^o$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE? **(2018)**

(a) $\angle QPR=45^o$

(b) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QPR=120^o$

(c) The radius of the incircle of the triangle PQR is $10\sqrt{3}-15$

(d) The area of the circumcircle of the triangle PQR is 100π.

30. Let T be the line passing through the points P(-2, 7) and Q(2, -5). Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1, 1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2.
Then, which of the following statement(s) is (are) TRUE? **(2018)**

(a) The point (-2, 7) lies in E_1

(b) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does NOT lie in E_2

(c) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2

(d) The point $\left(0, \frac{3}{2}\right)$ does NOT lies in E_1

Assertion and Reason

For the following questions choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.

(a) Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

31. Line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement I : The ratio PR : RQ equal $2\sqrt{2}:\sqrt{5}$.

Because

Statement II : In any triangle, bisector of an angle divides the triangle into two similar triangles. **(2007)**

(a) a (b) b

(c) c (d) d

Match the Columns

Match the conditions/expressions in Column I with statement in Column II.

32. Consider the lines given by

$L_1 : x + 3y - 5 = 0$

$L_2 : 3x - ky - 1 = 0$

$L_3 : 5x + 2y - 12 = 0$

Column I	Column II
(A) L_1, L_2, L_3 are concurrent if	(p) $k = -9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k = -\frac{6}{5}$
(C) L_1, L_2, L_3 form a triangle, if	(r) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(s) $k = 5$

(2008)

Objective Questions II [One or more than one correct option]

33. Three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent, if **(1985)**

(a) $p + q + r = 0$ (b) $p^2 + q^2 + r^2 = pr + rq$

(c) $p^3 + q^3 + r^3 = 3pqr$ (d) None of these

34. All points lying inside the triangle formed by the points (1, 3), (5, 0) and (–1, 2) satisfy. **(1986)**

(a) $3x + 2y \geq 0$ (b) $2x + y - 13 \geq 0$

(c) $2x - 3y - 12 \leq 0$ (d) $-2x + y \geq 0$

35. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equation can represent L_1 ? **(1999)**

(a) $x + y = 0$ (b) $x - y = 0$

(c) $x + 7y = 0$ (d) $x - 7y = 0$

Integer Answer Type Questions

36. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \leq 6$ into two parts. If

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\},$$

then the number of point(s) is S lying inside the smaller part is ...... **(2011)**

37. For a point P in the plane, let d_1 (P) and d_2 (P) be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is **(2014)**

Fill in the Blanks.

38. $y = 10^x$ is the reflection of $y = \log_{10} x$ in the line whose equation is ..... **(1982)**

39. The set of lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$ is concurrent at the point ... **(1982)**

40. If a, b and c are in A.P., then the straight line $ax + by + c = 0$ will always pass through a fixed point whose coordinates are (.....). **(1984)**

41. The sides AB, BC and CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices is .... **(1984)**

42. The set of all real numbers a such that $a^2 + 2a$, $2a + 3$ and $a^2 + 3a + 8$ are the sides of a triangle is. **(1985)**

43. The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in quadrant number ..... **(1985)**

44. Let the algebraic sum of the perpendicular distance from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero; then the line passes through a fixed point whose coordinates are ... **(1991)**

45. The vertices of a triangle are A (–1, –7), B (5, 1) and C (1, 4). The equation of the bisector of the angle $\angle ABC$ is .... **(1993)**

True/False.

46. The straight line 5x + 4y = 0 passes through the point of intersection of the straight lines x + 2y – 10 = 0 and 2x + y + 5 = 0. **(1983)**

47. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$, then the two triangles with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) and (a_1, b_1), (a_2, b_2), (a_3, b_3) must be congruent. **(1985)**

48. The lines 2x + 3y + 19 = 0 and 9x + 6y – 17 = 0 cut the coordinate axes in concyclic points. **(1988)**

Analytical and Descriptive Questions.

49. One side of a rectangle lies along the line 4x+7y+5=0. Two of its vertices are (–3, 1) and (1, 1). Find the equations of the other three sides. **(1978)**

50. Two vertices of a triangle are (5, –1), and (–2, 3). If the orthocentre of the triangle is the origin, find the coordinates of the third vertex. **(1978)**

51. Find the equation of the line which bisects the obtuse angle between the lines x – 2y + 4 =0 and 4x – 3y + 2 =0. **(1978)**

52. A straight line L is perpendicular to the line 5x – y = 1. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L. **(1980)**

53. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line y = 2x + c. Find c and the remaining vertices. **(1981)**

54. The ends A, B of a straight line segement of constant length c slide upon the fixed rectangular axes OX, OY respectively. If the rectangle OAPB be completed, then show that the locus of the foot of the perpendicular drawn from P to AB is $x^{2/3} + y^{2/3} = c^{2/3}$ **(1983)**

55. The coordinates of A,B,C are (6, 3), (–3, 5), (4, –2) respectively and P is any point (x, y). Show that the ratio of the areas of the triangles ΔPBC & ΔABC is $\left|\frac{x+y-2}{7}\right|$. **(1983)**

56. The vertices of a triangle are

$[at_1t_2, a(t_1 + t_2)], [at_2t_3, a(t_2 + t_3)], [at_3t_1, a(t_3 + t_1)]$.

Find the orthocentre of the triangle. **(1983)**

57. Two equal sides of an isosceles triangle are given by the equations 7x – y + 3 = 0 and x + y – 3 = 0 and its third side passes through the point (1, –10). Determine the equation of the third side. **(1984)**

58. Two sides of a rhombus ABCD are parallel to the lines y = x + 2 and y = 7 x + 3. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on the y-axis, find possible coordinates of A. **(1985)**

59. One of the diameter of the circle circumscribing the rectangle ABCD 4y = x + 7. If A and B are the points (–3, 4) and (5, 4) respectively, then find the area of rectangle. **(1985)**

60. The equation of the perpendicular bisectors of the sides AB and AC of a triangle ABC are x – y + 5 = 0 and x + 2y = 0, respectively. If the point A is (1, –2), find the equation of the line BC. **(1986)**

61. Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point P and makes an angle θ with each other. Find the equation of a line L different from L_2 which passes through P and makes the same angle θ with L_1. **(1988)**

62. Let ABC be a triangle with AB = AC. If D is mid point of BC, the foot of the perpendicular drawn from D to AC is E and F the mid point of DE. Prove that AF is perpendicular to BE. **(1989)**

63. A line cuts the x–axis at A (7, 0) and the y–axis at B (0, –5), A variable line PQ is drawn perpendicular to AB cutting the x–axis in P and the y–axis in Q. If AQ and BP intersect at R, find the locus of R. **(1990)**

64. Straight lines 3x + 4y = 5 and 4x – 3y = 15 intersect at the point A. Points B and C are chosen on these two lines such that AB = AC. Determine the possible equations of the line BC passing through the point (1, 2) **(1990)**

65. Find the equation of the line passing through the point (2, 3) and making intercept of length 3 unit between the lines y + 2x = 2 and y + 2x = 5. **(1991)**

66. Show that all chords of curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin, pass through a fixed point. Find the coordinates of the point. **(1991)**

67. Determine all values of α for which the point (α, α^2) lies inside the triangles formed by the lines 2x + 3y – 1 = 0

x + 2y – 3 = 0, 5x – 6y – 1 = 0 **(1992)**

68. A line through A (–5, –4) meets the line $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at the points B, C and D respectively. If $(15/AB)^2 + (10/AC)^2 = (6/AD)^2$, find the equation of the line. **(1993)**

69. A rectangle PQRS has its sides PQ parallel to the line $y = mx$ and vertices P,Q and S on the lines $y = a$, $x = b$ and $x = -b$, respectively. Find the locus of the vertex R. **(1996)**

70. A variable straight line of slope 4 intersects the hyperbola $xy = 1$ at two points, Find the locus of the point which divides the line segment between these two points in the ratio 1 : 2. **(1997)**

71. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c and d denote the length of the sides of the quandrilateral, prove that $2 \le a^2 + b^2 + c^2 + d^2 \le 4$. **(1997)**

72. Using coordinate geometry, prove that the three altitudes of any triangle are concurrent. **(1998)**

73. For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the coordinate plane, a new distance d (P, Q) is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$.

Let O = (0, 0) and A = (3, 2). Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segement of finite length and an inifinite ray. Sketech this set in a labelled diagram. **(2000)**

74. A straight line L through the origin meets the lines $x + y = 1$ and $x + y = 3$ at P & Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to $2x - y = 5$ and $3x + y = 5$ respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R as L varies is a straight line. **(2002)**

75. A straight line L with negative slopes passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. Find the absolute minimum value of OP + OQ, as L varies, where O is the origin. **(2002)**

76. A line is drawn passing through P (h, k) and parallel to x–axis. If the area of triangle formed by this line and line $y = x$ and $x + y = 2$ is $4h^2$, then find the locus of P (h, k). **(2005)**

77. Let ABC and PQR by any two triangles in the same plane, assume that the perpendiculars from the points A,B,C to the sides QR, RP, PQ respectively are concurrent using vector methods or otherwise, prove that the perpendiculars form P, Q, R, to BC, CA, AB respectively are also concurrent. **(2005)**

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (c)	2. (d)	3. (c)	4. (b)	5. (b)	6. (a)	7. (d)	8. (a)	9. (a)	10. (b)
11. (d)	12. (a)	13. (c)	14. (b)	15. (a)	16. (b)	17. (b)	18. (c)	19. (c)	20. (b)
21. (a)	22. (c)	23. (b)	24. (d)	25. (b)	26. (d)	27. (b)	28. (a)	29. (c)	30. (a)
31. (b)	32. (b)	33. (a)	34. (c)	35. (b)	36. (c)	37. (d)	38. (b)	39. (b)	40. (a)
41. (c)	42. (c)	43. (b)	44. (b)	45. (b)	46. (c)	47. (b)	48. (b)	49. (c)	50. (c)
51. (a)	52. (d)	53. (d)	54. (c)	55. (c)	56. (a)	57. (a)	58. (d)	59. (a)	60. (d)
61. (c)	62. (b)	63. (b)	64. (a)	65. (a)	66. (a)	67. (c)	68. (c)	69. (b)	70. (c)
71. (d)	72. (c)	73. (b)	74. (c)	75. (b)					

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (a)	2. (d)	3. (c)	4. (b)	5. (d)	6. (b)	7. (a)	8. (d)	9. (d)	10. (b)
11. (b)	12. (b)	13. (b)	14. (c)	15. (a)	16. (c)	17. (a)	18. (a)	19. (a)	20. (d)
21. (c)	22. (c)	23. (c)	24. (b)	25. (b)	26. (c)	27. (d)	28. (d)	29. (b)	30. (c)
31. (b)	32. (c)	33. (c)	34. (d)	35. (b)	36. (a)	37. (b)	38. (b)	39. (c)	40. (c)
41. (a)	42. (c)	43. (d)	44. (c)	45. (b)	46. (c)	47. (d)	48. (d)	49. (a)	50. (d)

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (c)	2. (a)	3. (c)	4. (a)	5. (c)	6. (b)	7. (c)	8. (c)	9. (c)	10. (c)
11. (a)	12. (a)	13. (a)	14. (d)	15. (a)	16. (b)	17. (c)	18. (b)	19. (a)	20. (a)
21. (c)	22. (b)	23. (c)	24. (d)	25. (c)	26. (a)	27. (a)	28. (b)	29. (a)	30. (a)
31. (b)	32. (b)	33. (a)	34. (d)	35. (b)	36. (c)	37. (a, b, d)	38. (a, b)	39. (b)	40. (a, c)
41. (a, c)	42. (a, d)	43. (a, d)	44. (a)	45. (a)	46. (d)	47. (b)	48. (d)	49. (c)	50. (b)
51. (c)	52. (c)	53. (b)	54. (b)	55. $A\to R, B\to Q, C\to P$			56. $A\to Q, B\to R, C\to S, D\to P$		
57. 0001	58. 0007	59. 0130	60. 0041						

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (a)	2. (c)	3. (c)	4. (a)	5. (d)	6. (d)	7. (b)	8. (a)	9. (c)	10. (d)
11. (d)	12. (c)	13. (a,c,d)	14. (c)	15. (b)	16. (d)	17. (d)	18. (a)	19. (d)	20. (c)
21. (d)	22. (b)	23. (a)	24. (c)	25. (b)	26. (d)	27. (b)	28. (a,c)	29. (b,c,d)	30. (b,d)
31. (c)	32. ($A\to s$; $B\to p,q$; $C\to r$; $D\to p, q, s$;)				33. (a,c)	34. (a,c)	35. (b,c)	36. 2	37 (6)

38. $y = x$ 39. $\left(\frac{3}{4}, \frac{1}{2}\right)$ 40. (1, –2) 41. 205 42. $a > 5$ 43. First 44. (1, 1) 45. $7y = x + 2$ 46. True

47. False 48. True 49. $7x - 4y + 25 = 0,\ 4x + 7y - 11 = 0,\ 7x - 4y - 3 = 0$ 50. (–4, –7)

51. $\left(-4+\sqrt{5}\right)x-\left(2\sqrt{5}+3\right)y+\left(4\sqrt{5}-2\right)=0$ 52. $x+5y=\pm 5\sqrt{2}$ 53. $c = -4$, (4, 4), (2, 0) 56. $[-a, a(t_1 + t_2 + t_3 + t_1t_2t_3)]$

57. $x - 3y - 31 = 0$ or $3x + y + 7 = 0$ 58. $\left(0, \frac{5}{2}\right), (0,0)$ 59. 32 sq. unit 60. $14x + 23y - 40 = 0$

61. $2(al + bm)(ax + by + c) - (a^2 + b^2)(lx + my + n) = 0$ 63. $x^2 + y^2 - 7x + 5y = 0$ 64. $x - 7y + 13 = 0$ and $7x + y - 9 = 0$

65. $x = 2$ and $3x + 4y = 18$ 66. (1, –2) 67. $-\frac{3}{2}<\alpha<-1\cup\frac{1}{2}<\alpha<1$ 68. $2x + 3y + 22 = 0$

69. $(m^2 - 1)x - my + b(m^2 + 1) + am = 0$ 70. $16x^2 + y^2 + 10xy = 2$ 76. 18 77. $y - 1 = 2x$ and $y - 1 = -2x$

Dream on !!