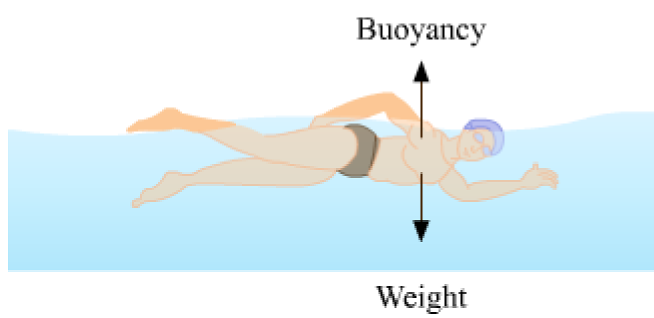
Mechanical Properties Of Fluids

Archimedes' Principle: An Overview

'Eureka! Eureka!' Screaming thus, Archimedes came out of his bathtub and ran straight to his king. A popular legend related to the discovery of the principle of buoyancy ends in this manner.

What is this principle of buoyancy? And why is it so important? If you have wondered about this phenomenon, then the following questions must have arisen in your mind.

- What has buoyancy to do with the floatation of bodies in liquids?
- Why does a piece of cork rise back to the surface of water even after you force it harder into water?
- Why does a piece of nail made of steel sink but a ship made of the same material float in water?
- Why do you feel lighter while swimming in a pool?
- How can Archimedes' discovery be used in determining the purity of a substance?



Let us go through this lesson to get the answers to all the above questions.

Buoyancy

When an object is immersed partially or fully in a liquid, it experiences an upward force. This **upward force** is known as **buoyant force** and the phenomenon is called **buoyancy**.

When an object is immersed in a liquid, its weight seems to be less than its actual weight. The buoyant force exerted by the liquid is responsible for this phenomenon.

Cause of buoyant force

When a body is partially or fully immersed in a liquid, the displaced **fluid has the tendency to regain its original position due to gravity**. An upward force—called the buoyant force—is, thus, exerted on the body by the displaced fluid.

In equilibrium, the buoyant force is balanced by the weight of the immersed body or the force of gravity acting on it.

The magnitude of the buoyant force acting on the immersed body depends upon two factors.

- Volume of the immersed body
- Density of the liquid

The density of a substance, with respect to the density of a liquid, determines whether the substance will sink or float in the liquid. An iron nail sinks in water because the density of iron is greater than that of water. On the other hand, a cork floats in water as the density of cork is less than that of water.

Density is expressed in terms of the volume of a substance. Hence, volume plays a major role in deciding whether a substance will sink or float. Such a relation was given by Archimedes.

Know Your Scientist

Archimedes (287−212 BC) was a Greek mathematician and physicist. According to a legend, he discovered the principle of buoyancy (Archimedes' principle) while taking a bath. It is said that he was so excited with his discovery that he ran naked in the street shouting '*Eureka*'.

Apart from this principle, Archimedes made some very important contributions to the fields of mechanics and geometry. He is considered one of the three greatest mathematicians of all time.

Archimedes' Principle

Archimedes' principle states that **when a body is immersed wholly or partially in a liquid, it experiences an upward buoyant force of magnitude equal to the weight of the liquid displaced by it**.

Buoyant force on an immersed body = Weight of the displaced liquid

Weight of the displaced liquid = Mass of the displaced liquid × Acceleration due to gravity

= Density of the liquid × Volume of the displaced liquid × Acceleration due to gravity

Volume of the displaced liquid = Volume of the immersed body

So,

Weight of the displaced liquid = Volume of the immersed body × Density of the liquid × Acceleration due to gravity

Hence, we can write the magnitude of the upthrust on a body immersed in a liquid as follows:

Buoyant force on an immersed body = Volume of the immersed body × Density of the liquid

× Acceleration due to gravity

The buoyant force on an immersed body depends on the density of the liquid in which the body is immersed. So, this force is different in different liquids for the same body.

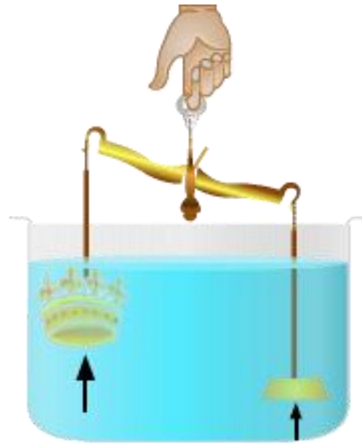
Application of Archimedes' Principle

Archimedes' principle can be used for determining the purity of substances such as gold.

Suppose we have a gold crown and need to determine if it is pure gold or not. We also have a block of pure gold as reference. The block and the crown have the same mass (as shown in the figure). Using Archimedes' principle, we can compare the densities of the crown and the block.

If the crown is less dense than the block, then it will displace more water —owing to its greater volume. Consequently, the crown will experience a greater buoyant force than the block (as shown in the figure). This will indicate that the gold used in making the crown is not pure, but has some other metal or alloy mixed in it.

In air weight of the crown is equal to the weight of the piece of pure gold.



In water apparent weight of the crown is less than the apparent weight of pure gold.

Solved Examples

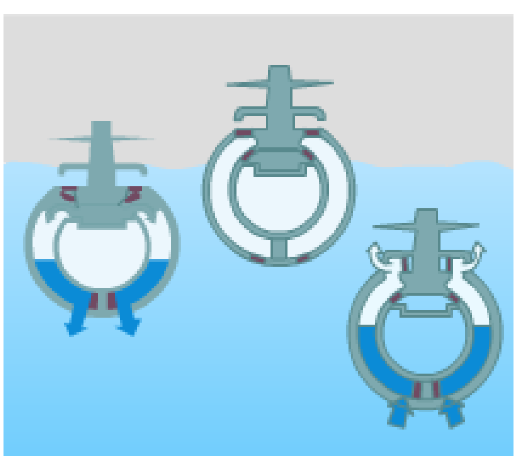
Easy

Example 1:

Do you know how submarines are made to float or sink as desired?

Solution:

A submarine has large tanks onboard which control how deep it sinks or how high it rises. These tanks are called ballast tanks. To sink the submarine, the tanks are filled with water. The greater the amount of water in the tanks, the deeper does the submarine sink.



To raise the submarine, water is released from the tanks and compressed air (kept onboard in flasks) is let into them. The greater the amount of compressed air in the tanks, the higher does the submarine rise.

Example 2:

When an iron block is dipped in water, it displaces 10 kg of water. Calculate the amount of buoyant force (in Newton) acting on the iron block. (Take g = 9.8 m/s^2)

Solution:

According to Archimedes' principle, the buoyant force on the iron block is equal to the weight of the water displaced by it.

It is given that:

Mass of the water displaced = 10 kg

Acceleration due to gravity = 9.8 m/s^2

∴ Weight of the water displaced = Mass of the water displaced × Acceleration due to gravity

= 10 × 9.8 = 98 N

Hence, the buoyant force acting on the iron block is 98 N.

Medium

Example 3:

How do the densities of an object and a liquid affect the sinking or floating of the object in the liquid?

Solution:

Suppose an object of density ρ and volume V is immersed completely in a liquid of density σ.

Then,

Apparent weight of the object = $W - w = V\rho g - V\sigma g$

Where, W = Weight of the object

w = Weight of the water displaced by the immersed part of the object

Case I: If $W > w$ ($\rho > \sigma$), then $W - w$ is positive.

In this case, the object will sink.

Case II: If $W < w$ ($\rho < \sigma$), then $W - w$ is negative.

In this case, the object will float.

Case III: If $W = w$ ($\rho = \sigma$), then $W - w = 0$.

In this case, the object will rest anywhere within the liquid.

Hard

Example 4:

An object weighs 300 N in air and 150 N in water. Find its relative density.

Solution:

Let us take:

Volume of the object = V

Density of the object = ρ

Density of water = σ_w

Acceleration due to gravity = g

It is given that the weight of the object in air is 300 N.

We know that:

Weight of the object = $V\rho g$

So,

$V\rho g$ = 300 N (Neglecting the buoyancy of air)

$$\rho = \frac{300}{Vg} \quad \ldots \text{(i)}$$

It is also given that the apparent weight of the object in water is 150 N.

We know that:

Apparent weight of the object = Weight of the object − Weight of the water displaced by it

So,

$$V\rho g - v\sigma_{\mathrm{w}} g = 150$$
$$\Rightarrow 300 - v\sigma_{\mathrm{w}} g = 150$$
$$\Rightarrow \sigma_{\mathrm{w}} = \frac{300-150}{Vg} \qquad \ldots . \text{(ii)}$$

Now, the relative density of the object can be calculated as follows:

$$\text{Relative density of the object} = \frac{\text{density of the object}}{\text{density of water}} = \frac{\rho}{\sigma \mathrm{w}}$$
$$= \frac{\frac{300}{Vg}}{\frac{300-150}{Vg}}$$
$$= 2$$

Example 5:

An object of density *ρ* floats in kerosene of density 0.7×10^3 kg/m^3 up to a certain mark. If the same object is placed in water of density 1×10^3 kg/m^3, will it sink more or less in water?

Solution:

Let us take:

Volume of the object = *V*

Height of the cross-section of the object = *h*

Area of the cross-section of the object = *A*

Height of the object when immersed in kerosene = *h*'

Height of the object when immersed in water = *h*''

Acceleration due to gravity = *g*

It is given that:

Density of the object = ρ

Density of kerosene, $\rho_k = 0.7 \times 10^3$ kg/m^3

Density of water, $\rho_w = 1 \times 10^3$ kg/m^3

According to Archimedes' principle:

Weight of the object = Weight of the kerosene displaced by the object

= Weight of the water displaced by the object

$$\Rightarrow V\rho g = V'\rho_k g = V''\rho_w g$$
$$\Rightarrow (hA)\rho g = (h'A)\rho_k g = (h''A)\rho_w g$$
$$\Rightarrow (h'A)\rho_k g = (h''A)\rho_w g$$
$$\Rightarrow (h'A)\rho_k = (h''A)\rho_w$$
$$\Rightarrow \frac{h'}{h''} = \frac{\rho_w}{\rho_k} = \frac{1\times10^3}{0.7\times10^3} = 1.43$$
$$\text{So, } h' = 1.43h''$$

Therefore, the object will sink less in water.

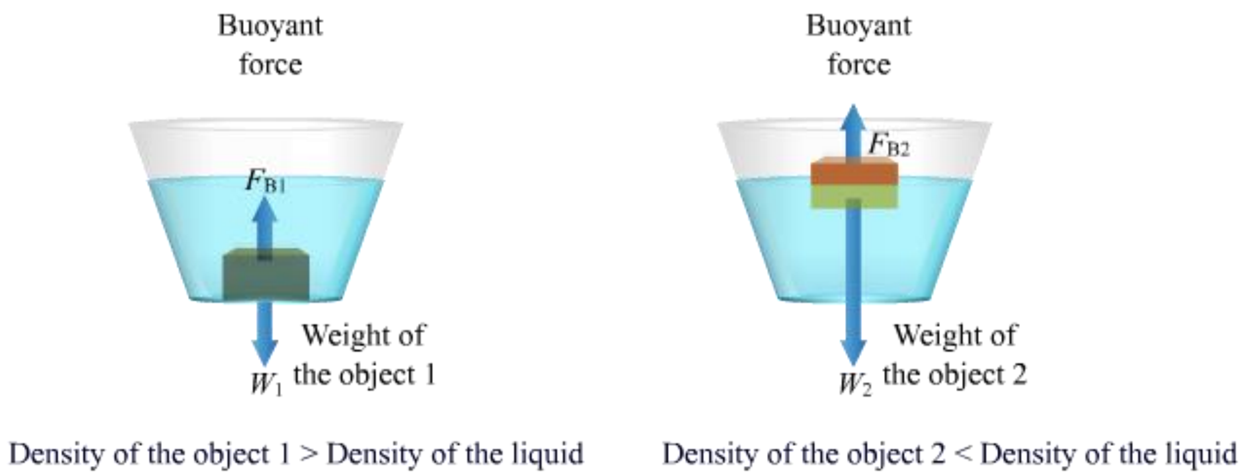
Sinking or Floating

Two forces act on an object placed in a liquid:

- Weight (*W*) of the object, which acts downwards
- Buoyant force or upthrust (*W'*) exerted by the liquid, which acts upwards

For any object immersed in a liquid:

- If the density of the object is less than the density of the liquid then it will float. The object will be immersed to an extent until the weight of the volume of liquid displaced is equal to the weight of the object
- If the density of the object is more than the density of the liquid then the object will sink as its weight is more than the weight of the liquid displaced.
- If the density of the object is equal to the density of the liquid then it will neither float nor sink in the liquid. It will remain in equilibrium within the liquid wherever it is placed.



Law of Flotation

According to the law of flotation, an object will float in a liquid if its weight is equal to or less than the weight of the liquid displaced by it.

The floating object may be partially or fully submerged in the liquid. Liquid is displaced by the submerged portion of the object.

We can tell whether an object will float or sink in a liquid by comparing its density (or average density) to that of the liquid.

For any object immersed in a liquid:

- If the average density of the object is less than that of the liquid, then the object will float in the liquid.
- If the density of the object is equal to that of the liquid, then the object will float in the liquid but no part of it will be above the surface of the liquid.
- If the density of the object is greater than that of the liquid, then the object will sink in the liquid.

Density and Pressure - Pascal's Law

- Volume of solid, liquid or gas depends on the stress or pressure on it.
- Solids and liquids have lower compressibility compared to gases.

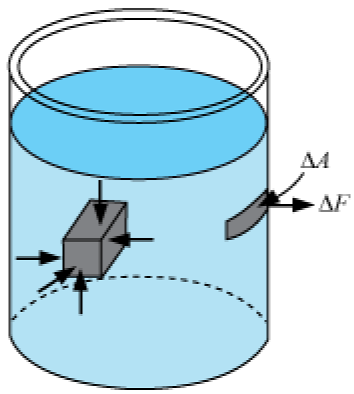
Pressure

- Average pressure (P_{av}) is normal force (F) acting per unit area (A).

- $P_{av} = \frac{F}{A}$ (Scalar quantity)

- For very small area:

  Limiting pressure, $P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$



- Force exerted by a liquid on the submerged object is normal at all points on the surface.

- Pressure is a scalar quantity.

- Its SI unit is Nm^{-2} (pascal).

Density (*ρ*)

- Mass (*m*) per unit volume (*v*)

- p = mvρ = mv(scalar quantity)

- Its SI unit is kg m^{-3}.

- A liquid is largely incompressible and therefore, its density is nearly constant at all pressures.

- The density of water at 4°C is 1×10^3 kg m^{-3}.

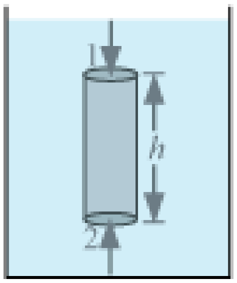
- The relative density of a substance is the ratio of its density to the density of water at 4°C.

Pascal's Law

- It states that the pressure in a fluid at rest is the same at all points if they are at the same height.
- The pressure exerted in all directions in a fluid at rest is the same.

Variation of Pressure with Depth and Hydraulic Machine

Effect of gravity on fluid pressure:



Consider a liquid of density ρ in a vessel.

Pressure at point 1 = P_1

Pressure at point 2 = P_2

Mass of the fluid inside the cylinder = m

Area of the base of the cylinder = A

Height of the cylinder = h

Density of the fluid = ρ
Force on face 1 = $F_1 = P_1A + mg$
Force on face 2 = $F_2 = P_2A$
Since the vessel is in equilibrium,
$F_1 = F_2$

$$(P_2 - P_1) = \frac{mg}{A}$$

$$\because m = \rho V$$

(Volume of the cylinder = $V = hA$)

$m = \rho hA$

$\therefore P_2 - P_1 = \rho gh$

In the absence of gravity, $P_2 = P_1$ and the pressure at every point inside the liquid in equilibrium would be same. In presence of gravity, P_2 is not equal to P_1. The pressure at all points at the same horizontal level is same but increases with depth due to gravity.

When point 1 is exposed to the atmosphere:

P_1 = Atmospheric pressure (P_a)

$P_2 = P$ (absolute pressure)

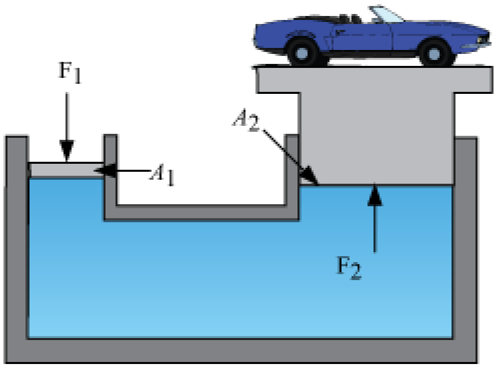
$$\boxed{\therefore P = P_a + \rho g h}$$

- Gauge pressure = $P - P_a = \rho g h$
- Pressure is same at all points at the same depth.
- The liquid pressure at a point is independent of the quantity of liquid but depends upon the depth of the point below the liquid surface. This is known as hydrostatic paradox.
- The atmospheric pressure at any point is equal to the weight of a column of air of unit cross-sectional area, extending from that point to the top of the earth's atmosphere.
- Atmospheric pressure at sea level is 1.013×10^5 P_a (1 atm).
- Two devices for measuring pressure are mercury barometer and open-tube manometer.

Hydraulic Machines

- These are based on Pascal's Law for Transmission of Fluid Pressure, which states that external pressure applied on any part of a fluid contained in a vessel is transmitted undiminished and equally in all directions.
- Hydraulic lift, hydraulic brakes and hydraulic press are some examples of hydraulic machines.

Hydraulic Lift



- In the illustration:

  Area of the smaller piston = A_1

  Force exerted on it = F_1

  Pressure transmitted, $P = \frac{F_1}{A_1}$

  Area of the larger piston = A_2

  Upward force on the piston = $P \times A_2$

  Force supported by the large piston, $F_2 = PA_2$

  $F_2 = F_1 \frac{A_2}{A_1}$

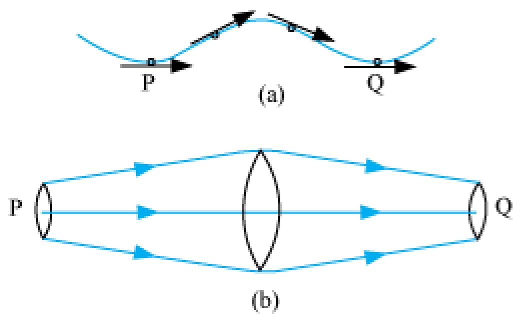
  Since A_2 is greater than A_1, force F_2 on the larger piston will also be much larger than the force F_1 applied on the smaller piston.

- Mechanical advantage of the device is $\frac{A_2}{A_1}$.

Streamline Flow and Bernoulli's Principle

- Streamline − Path taken by a fluid particle under a steady flow

  The tangent to streamline at any point gives the direction of the flow of liquid at that point.



- Two streamlines do not cross each other.
- Flow rate remains constant throughout the pipe of flow.

  Av = Constant

  Where,

  A − Area of cross-section

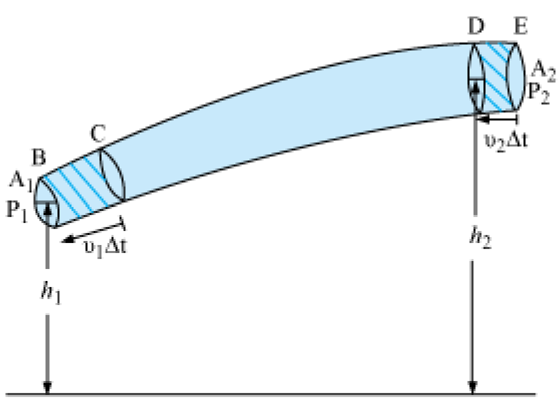
  v − Velocity of flow

  At two points of streamline,

  $A_1v_1 = A_2v_2$ [This is called equation of continuity]

- Steady flow is possible only at low speed.
- Turbulent flow − unsteady motion of the elements of the liquid along a zig-zag path
- Critical velocity − limiting value of velocity beyond which the liquid stops flowing as streamline and becomes turbulent

Bernoulli's Principle



- The sum of pressure energy, kinetic energy, and potential energy per unit mass is always constant for the streamline flow of a non-viscous and incompressible fluid.

  Consider,

  A_1 − Area of cross-section at B

  P_1 − Pressure at A_1

  h_1 − Height of B

  h_2 − Height of D

  A_2 − Area of cross-section at E

  P_2− Pressure at A_2

  v_1 − Speed at B

  v_2 − Speed at D

  BC = $v_1\Delta t$

  DE = $v_2\Delta t$

  Δt − Time interval

  Work done (BC), $W_1 = P_1A_1 (v_1\Delta t) = P_1\Delta V$

  Work done (DE), $W_2 = P_2A_2 (v_2\Delta t) = P_2\Delta V$

  Work done on fluid = $W_1 - W_2$

  $= (P_1 - P_2)\,\Delta V$

  Change of gravitational potential energy, $\Delta U = \rho g\Delta V\,(h_2 - h_1)$

  Change in kinetic energy, $\Delta k = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2)$

  According to work-energy theorem,

$$(P_1 - P_2)\Delta V = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2) + \rho g\Delta V(h_2 - h_1)$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

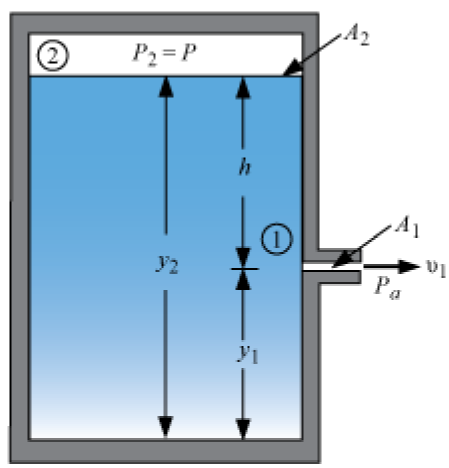
General expression for Bernoulli's equation,

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{ constant}}$$

- Bernoulli's equation does not hold for non-steady flows.

Torricelli's Law

- Speed of efflux (fluid outflow) is same as the speed of a free falling body.



- According to equation of continuity,

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

Since $A_2 >> A_1$,

$v_2 = 0$ (fluid at rest)

Applying Bernoulli's theorem,

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$P_1 = P_a$ (atmospheric pressure)

$y_2 - y_1 = h$ (shown in the figure)

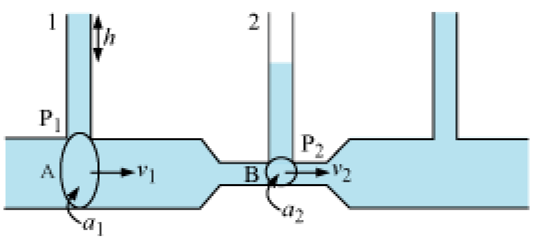
$$P_a + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

$$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}$$

when $P >> P_a$

$$\boxed{\therefore v_1 = \sqrt{2gh}}$$

VENTURIMETER



- It is a gauge used to measure the rate of flow of fluid when fluid is steady.

Consider:

a_1 − Cross-sectional area at A

a_2 − Cross-sectional area at B

v_1 − Velocity at A

v_2 − Velocity at B

P_1 − Pressure at A

P_2 − Pressure at B

h − Height difference of columns 1 and 2 ρ − Density of liquid contained in the venturimeter

Speed at the constriction, $v_2 = \frac{a_1}{a_2} v_1$ [Using equation of continuity]

According to Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_1^2 \left(\frac{a_1}{a_2}\right)^2$$

$$P_1 - P_2 = \rho g h = \frac{1}{2}\rho v_1^2 \left[\left(\frac{a_1}{a_2}\right)^2 - 1\right]$$

$$v_1 = \sqrt{2gh}\left[\left(\frac{a_1}{a_2}\right)^2 - 1\right]^{-1/2}$$

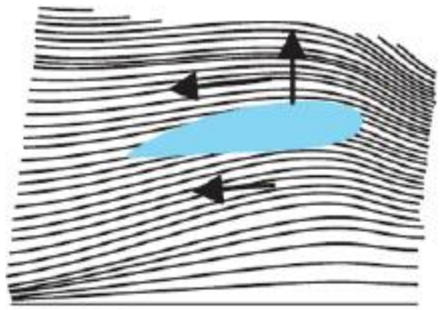
- This principle is used in spray gun.

Blood Flow and Heart Attack

- Accumulation of plaque constricts the artery.
- To drive blood through it, the activity of heart increases.
- Speed of blood in the region increases, lowering the inside pressure of artery.
- Artery may collapse due to high external pressure.
- Heart exerts further pressure and opens the artery to force the blood through it.
- Blood rushes through the opening and the internal pressure of artery drops.
- This leads to repeat collapse and results in heart attack.

Dynamic Lift

Aerofoil or lift on aircraft wing:



- Provides upward dynamic lift when aerofoil moves horizontal
- Wings of aeroplane look similar to an aerofoil.
- Aerofoil moving against the wind causes the streamline to crowd more above the wing than below it.
- Therefore, the speed on top is more than it is below it.
- Upward force resulting in a dynamic lift of wings balances the weight of the plane.

Viscosity and Reynolds Number

- It is the resistance of the fluid motion. This force exists when there is relative motion between the layers of liquid.
- Laminar − For any layer of liquid, its upper layer pulls it forward while lower layer pulls it backward. This results in force between the layers. This type of flow is known as laminar.

- Co-efficient of viscosity, $\eta = \frac{\text{Shearing stress}}{\text{Strain rate}}$

Shearing stress = $\frac{F}{A} = \frac{\text{Force}}{\text{Area}}$

Strain rate = $\frac{\Delta x}{l \Delta t} = \frac{v}{l}$

Where Δx → Change in length

Δt → Time internal

l → Original length

$\therefore \eta = \frac{F / A}{v / l} = \frac{Fl}{vA}$

- Unit of viscosity is poiseiulle (P1) or Nsm^{-2} or Pa s.

- Thin liquids are less viscous than thick liquids.

- Viscosity of liquids decreases with temperature while it increases in case of gases.

Newton's Law of Viscosity

It states that the shear stress (τ) on fluid layers is directly proportional to the rate of shear strain i.e.

$$\tau = \eta \frac{dv}{dl}$$

Poiseuille's Formula

According to Poiseuille, the volume (*V*) of liquid flowing per second through a capillary tube is

- directly proportional to the pressure difference (*P*) across the two ends of the tube
- directly proportional to the fourth power of radius (*r*) of the tube
- inversely proportional to the length (*l*) of the tube
- inversely proportional to the coefficient of viscocity (ηη) of the liquid

Thus,

$$V \propto \frac{Pr^4}{\eta l}$$

or

$$V = \frac{\pi P r^4}{8 \eta l}$$

Stokes' Law

- An object moving through a fluid drags the liquid in contact. This force between the layers of the fluid makes the body experience a retarding force.

- Retarding force (*F*) depends on

- velocity of the object (*v*)

- viscosity of the fluid (*η*)

- radius of the sphere (a)

  $\therefore F = 6\pi\eta a v$

  This is known as Stokes' law.

  Terminal Velocity (v_t)

- When a spherical body falls through a viscous fluid, it experiences a viscous force. The magnitude of viscous force increases with the increase in velocity of the falling body under the action of its weight. As a result, the viscous force soon balances the driving force (weight of the body) and the body starts moving with a constant velocity known as its terminal velocity.

- Using Stokes' law,

  $$6\pi\eta a v_t = \left(\frac{4\pi}{3}\right)a^3\left(\rho - \sigma\right)g$$

  Where,

  ρ − Mass density of sphere

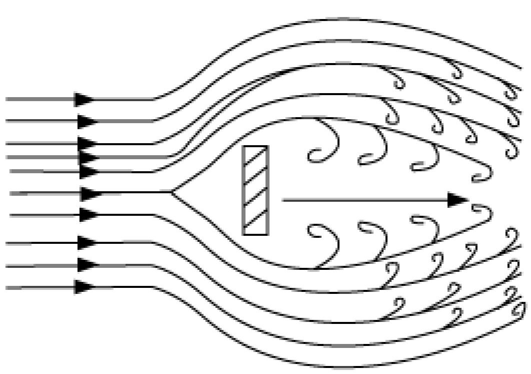
  σ − Mass density of fluid

  $$\text{Terminal velocity, } v_t = \frac{2a^2\left(\rho-\sigma\right)g}{9\eta}$$

  Reynolds Number

- At high flow rate, the liquid flow becomes turbulent. In a turbulent flow, the velocity of the fluids at any point in space varies rapidly and randomly with time.

- An obstacle placed in the path of a fast moving fluid causes turbulence.



- Reynolds (R_e) number implies if the flow would be turbulent or not.

  - $R_e = \frac{\rho v d}{\eta}$

  Where,

  ρ − Density of fluid

  d − Dimension of pipe

  v − Speed of fluid flow

  η − Viscosity of the fluid

- R_e is dimensionless.

  $R_e < 1000$ [Streamline or laminar flow]

  $2000 \geq R_e \geq 1000$ [Unsteady flow]

  $R_e > 2000$ [Turbulent flow]

- R_e is ratio of inertial force to viscous force.

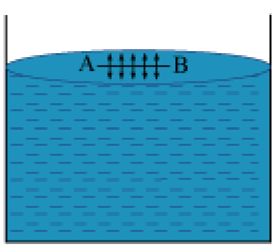
- Use − Turbulence promotes mixing; increases the transfer rate of mass, momentum, and energy.

Surface Tension and Surface Energy

Surface Tension

- Free surface of a liquid at rest behaves such as a stretched membrane, which tends to contract to possess minimum surface area.

- Surface tension is the force acting per unit length on either side of the imaginary line.

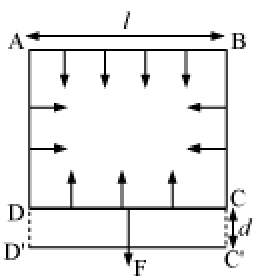


- SI Unit → Nm^{-1}

Surface Energy

- Surface tension decreases the surface area to minimum.
- To increase the surface area of liquid, work is done against the force of surface tension.
- The work done is stored in a form of potential energy in liquid surface film.
- Therefore, potential energy per unit area is called the surface energy of the surface film.

Relation between Surface Energy and Surface Tension



Consider, ABCD → Rectangular wire frame

DC → Movable end

- ABCD is dipped inside a soap solution. A film is formed which pulls the end DC inwards due to surface tension.
- Force due to surface tension, $F = (\sigma \times 2l)$

Where, σ → Surface tension of soap film

l → Length of the rectangular wire AB

d = A small distance between the position CD and $C'D'$

∴Work done, $W = \sigma \times 2l \times d$

$= \sigma \times 2ld$

Where, $2ld$ → Total increase in area on both side of soap film

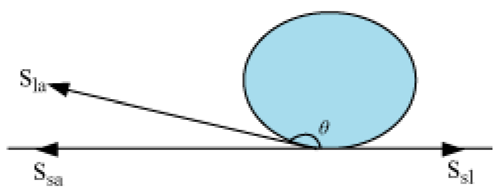
$$\therefore \text{Work done} = \sigma \times \text{Increase in area}$$

Results and Applications of Surface Tension

Angle of Contact

- The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is called the angle of contact.

Case I



(Water and oily surface interface)

Here,

S_{la} = Interfacial tension of the liquid–air interface

S_{sa} = Interfacial tension of the solid–air interface

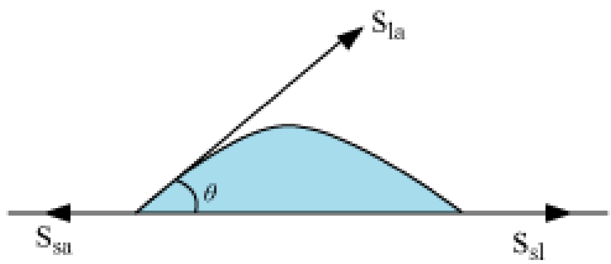
S_{ls} = Interfacial tension of the liquid–solid interface

- $S_{la} \cos\theta + S_{sl} = S_{sa}$

$$Cos\theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

- If $S_{sa} < S_{sl}$, then the angle of contact is obtuse and the molecules of the liquid are strongly attracted to themselves and weakly attracted to those of the solid.
- A lot of energy is used in creating the liquid–solid interface.

Case II

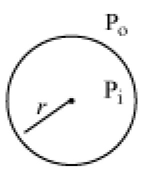


(Water and glass surface interface)

- If S_{sa} > S_{sl}, then the angle of contact is acute and the molecules of the liquid are strongly attached to those of the solid.
- Not enough energy is required to create the liquid–solid interface.

Drops and Bubbles

- Liquid drops are spherical, as they have least area and energy.



Spherical Drop

Here,

r = Radius of the drop

P_0= Pressure outside the bubble

P_i = Pressure inside the bubble

S = Surface tension of the bubble

- Surface energy = $4\pi r^2 S$

Let the radius increase by Δr.

Now,
Extra surface energy:

$[4\pi(r + \Delta r)^2 - 4\pi r^2]S$

$= 8\pi r\Delta r S$ ...(1)

- At equilibrium, the energy used is balanced by the energy gained.
- Energy gain is the pressure difference, i.e., $P_i - P_0$.

  ∴ Work done, $W = (P_i - P_0)4\pi r^2 \Delta r$ ...(2)

  From equations (1) and (2), we get:

  $$\boxed{P_i - P_0 = \frac{2S}{r}}$$

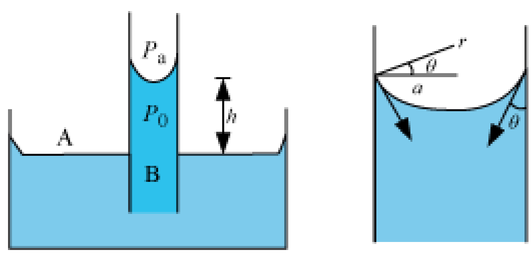
- Liquid−gas interface

- Convex side has higher pressure than concave side.

  For the bubble having two interfaces,

  $$P_i - P_0 = \frac{4S}{r}$$

  Capillary Rise



- Angle of contact between water and glass is acute.
- The surface of water in the capillary is concave.
- Pressure difference between two sides of the top surface: $P_a - P_0 = 2S/r$

  $$\therefore P_a - P_0 = \frac{2S}{a\sec\theta} = \frac{2S}{a}\cos\theta$$

- Consider points A and B. They should be at the same pressure.

  $P_0 + h\rho g = P_a$

  $$\therefore h\rho g = P_a - P_0 = \frac{2S\cos\theta}{a}$$

- Therefore, capillary rise is due to surface tension.

- Height of water rise, $h = \frac{2S}{\rho g a}$

Effect of Impurities on Surface Tension of Liquid

- When a liquid consists of soluble impurities, the surface tension of the liquid increases.
- When a sparingly soluble impurity like phenol is dissolved in water, the surface tension decreases.

Effect of Temperature on Surface Tension of Liquid

- The surface tension decreases in a liquid when the temperature increases. But in liquids like molten copper or molten cadmium, the surface tension increases with temperature.
- The surface tension of a liquid becomes zero at critical temperature.
- The surface tension of a liquid becomes zero at boiling point and maximum at freezing point.
- The surface tension of a liquid is dependent on temperature as $T = T_0(1 - \alpha\theta)$.