QUICK RECAP

Electric potential : Electric potential at a point is defined as amount of work done in bringing a unit positive charge from infinity to that point. It is denoted by symbol V.

$$V = \frac{W}{q}$$

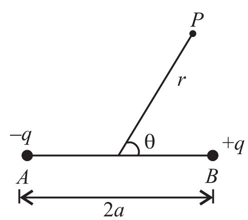
- Electric potential is a scalar quantity. The SI unit of potential is volt and its dimensional formula is $[ML^2T^{-3}A^{-1}]$.
- Electric potential at a point distant r from a point charge q is

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

- Electric potential due to group of charges : The electric potential at a point due to a group of charges is equal to the algebraic sum of the electric potentials due to individual charges at that point.

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}+\ldots\frac{q_n}{r_n}\right) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

- Electric potential at any point due to an electric dipole



The electric potential at point P due to an electric dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

– At axial point : When the point P lies on the axial line of dipole *i.e.*, $\theta = 0°$.

$$V = \frac{p}{4\pi\varepsilon_0 r^2}.$$

– At equatorial point : When the point P lies on the equatorial line of the dipole, *i.e.*, $\theta = 90° \quad \therefore \quad V = 0$.

- Electric potential due to a uniformly charged spherical shell of uniform surface charge density σ and radius R at a distance r from the centre the shell is given as follows :

– At a point outside the shell *i.e.*, $r > R$

$$V = \frac{\sigma R^2}{\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

– At a point on the shell *i.e.*, $r = R$

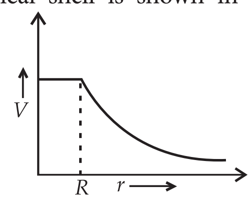
$$V = \frac{\sigma R}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

– At a point inside the shell *i.e.*, $r < R$

$$V = \frac{\sigma R}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Here, $q = 4\pi R^2\sigma$

The variation of V with r for a uniformly charged thin spherical shell is shown in the figure.



- Electric potential due to a non-conducting solid sphere of uniform volume charge density ρ and radius R at distant r from the sphere is given as follows :

– At a point outside the sphere *i.e.*, $r > R$

$$V = \frac{\rho R^3}{3\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

– At a point on the sphere *i.e.*, $r = R$

$$V = \frac{\rho R^2}{3\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

– At a point inside the sphere *i.e.*, $r < R$

$$V = \frac{\rho}{3\varepsilon_0}\frac{(3R^2 - r^2)}{2} = \frac{1}{4\pi\varepsilon_0}\frac{q(3R^2 - r^2)}{2R^3}$$

Here $q = \frac{4}{3}\pi R^3\rho$

Equipotential surface : A surface on which the electric potential is constant is known as equipotential surface.

- **Properties of an equipotential surface :**

– Electric field lines are always perpendicular to an equipotential surface.

– Work done in moving an electric charge from one point to another on an equipotential surface is zero.

– Two equipotential surfaces can never intersect one another.

Relationship between $\vec{E}$ and $\vec{V}$

$$\vec{E} = -\vec{\nabla} V$$

where $\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$

–ve sign shows that the direction of $\vec{E}$ is the direction of decreasing potential.

Electric potential energy

- Electric potential energy of a system of two point charges

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

- Electric potential energy of a system of n point charges

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}; j > k$$

- The SI unit of electric potential energy is joule.

Conductors : Those substances which can easily allow electricity to pass through them are known as conductors. They have a large number of free charge carriers that are free to move inside the material. *e.g.*, metals, human beings, earth etc.

- Basic electrostatics properties of a conductor are as follows :
  – Inside a conductor, electric field is zero.
  – At the surface of a charged conductor, electric field must be normal to the surface at every point.
  – The interior of a conductor can have no excess charge in the static situation.
  – Electric potential is constant throughtout the volume of the conductor and has the same value (as inside) on its surface.
  – Electric field at the surface of a charged conductor, $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

  where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.
- **Electrostatic shielding :** It is the phenomenon of protecting a certain region of space from external electric field.

Polar and non-polar molecule

- Polar molecule : A polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). A polar molecule has a permanent dipole moment *e.g.*, water (H_2O) and HCl.
- Non-polar molecule : A non-polar molecule is one in which the centres of positive and negative charges coincide. A non polar molecule has no permanent dipole moment. *e.g.*, oxygen (O_2) and hydrogen (H_2).

Capacitance : Capacitance (C) of a capacitor is the ratio of charge (Q) given and the potential (V) to which it is raised. *i.e.*, $C = Q/V$.

- The SI unit of capacitance is farad (F).
  – 1 millifarad (mF) = 10^{-3} farad
  – 1 microfarad (μF) = 10^{-6} farad
  – 1 picofarad (pF) = 10^{-12} farad.
- The dimensional formula of capacitance is $[M^{-1}L^{-2}T^4A^2]$.
- Capacitance of a spherical conductor of radius R is $C = 4\pi\varepsilon_0 R$

  Taking earth to be a conducting sphere of radius 6400 km, its capacity will be

$$C = 4\pi\varepsilon_0 R = \frac{6.4\times10^6}{9\times10^9} = 711\ \mu\text{F}$$

Capacitor : A condenser or a capacitor is a device that stores electric charge. It consists of two conductors separated by an insulator or dielectric. The two conductors carry equal and opposite charges $\pm Q$.

- Capacitance of an air filled parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

where A is area of each plate and d is separation between the two plates.

- Capacitance of an air filled spherical capacitor

$$C = 4\pi\varepsilon_0\frac{ab}{b-a}$$

where a and b are the inner and outer radii.

- Capacitance of an air filled cylindrical capacitor

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

where a and b are the inner and outer radii and L is the length.

- Capacitance of a parallel plate capacitor with a dielectric slab of dielectric constant K, completely filled between the plates of the capacitor, is given by

$$C = \frac{K\varepsilon_0 A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

- When a dielectric slab of thickness t and dielectric constant K is introduced between the plates, then the capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

- When a metallic conductor of thickness t is introduced between the plates, then capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t}$$

Combination of capacitors in series and parallel

- Capacitors in series : For n capacitors connected in series the equivalent capacitance C_S is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

- Capacitors in parallel : For n capacitors connected in parallel, the equivalent capacitance C_P is given by

$$C_P = C_1 + C_2 + ... + C_n$$

- When capacitors are connected in series, the charge through each capacitor is same. When capacitors are connected in parallel, the potential difference across each capacitor is same.
- When two capacitors charged to different potentials are connected by a conducting wire, charge flows from the one at higher potential to the other at lower potential till their potentials become equal. The equal potential is called common potential (V), where

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

It should be clearly understood that in sharing charges, there is absolutely no loss of charge. Some energy is, however, lost in the process in the form of heat etc. which is given by

$$U_1 - U_2 = \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)}.$$

Energy stored in a capacitor : Work done in charging a capacitor gets stored in the capacitor in the form of its electric potential energy and it is given by

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

- Energy density : The energy stored per unit volume in the electric field between the plates is known as energy density (u). It is given by

$$u = \frac{1}{2}\varepsilon_0 E^2$$

- When a dielectric slab of dielectric constant K is introduced between the plates of a charged parallel plate capacitor and the charging battery remains connected, then
  - Potential difference between the plates remains constant *i.e.*, $V = V_0$
  - Capacitance C increases *i.e.*, $C = KC_0$
  - Charge on a capacitor increases *i.e.*, $Q = KQ_0$
  - Electric field between the plates remains unchanged *i.e.*, $E = E_0$
  - Energy stored in a capacitor increases *i.e.*, $U = KU_0$
- When a dielectric slab of dielectric constant K is introduced in between the plates of a charged parallel plate capacitor and the charging battery is disconnected, then
  - Charge remains unchanged *i.e.*, $Q = Q_0$
  - Capacitance increases *i.e.*, $C = KC_0$
  - Potential difference between the plates decreases *i.e.*, $V = \frac{V_0}{K}$
  - Electric field between the plates decreases *i.e.*, $E = \frac{E_0}{K}$
  - Energy stored in the capacitor decreases *i.e.*, $U = \frac{U_0}{K}$

where Q_0, C_0, V_0, E_0 and U_0 represents the charge, capacitance, potential difference, electric field and energy stored in the capacitor of a charged air filled parallel plate capacitor.

Previous Years' CBSE Board Questions

2.2 Electrostatic Potential

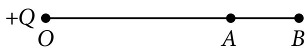
VSA (1 mark)

1. The physical quantity having SI unit $N\,C^{-1}\,m$ is ______. *(2020)*

2.3 Potential due to a Point Charge

VSA (1 mark)

2. A point charge $+Q$ is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero?



(Delhi 2016, Foreign 2016, Delhi 2011)

SA (2 marks)

3. Draw a plot showing the variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge Q. *(Delhi 2012)*

2.4 Potential due to an Electric Dipole

LA I (3 marks)

4. Derive the expression for the electric potential due to an electric dipole at a point on its axial line. *(2/3, Delhi 2017)*

LA II (5 marks)

5. Obtain the expression for the potential due to an electric dipole of dipole moment p at a point 'x' on the axial line. *(2/5, AI 2013C)*

2.5 Potential due to a System of Charges

SA (2 marks)

6. N small conducting liquid droplets, each of radius r, are charged to a potential V each. These droplets coalesce to form a single large drop without any charge leakage. Find the potential of the large drop. *(2020)*

7. Two point charges q and $-2q$ are kept 'd' distance apart. Find the location of point relative to charge 'q' at which potential due to this system of charges is zero. *(AI 2014C)*

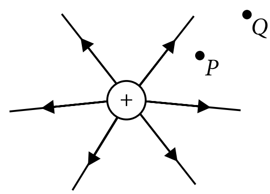
2.6 Equipotential Surfaces

VSA (1 mark)

8. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor? *(AI 2015C)*

9. "For any charge configuration, equipotential surface through a point is normal to the electric field." Justify. *(Delhi 2014)*

10. Figure shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? Give reason. *(Foreign 2014)*



11. What is the geometrical shape of equipotential surfaces due to a single isolated charge? *(Delhi 2013, AI 2010C)*

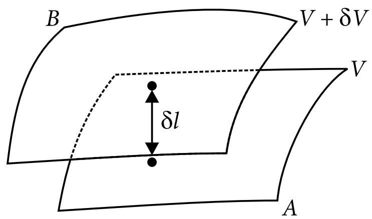
12. Two charges 2 μC and –2 μC are placed at points A and B, 5 cm apart. Depict an equipotential surface of the system. *(Delhi 2013C)*

13. What is the amount of work done in moving a point charge around a circular arc of radius r at the centre of which another point charge is located ? *(AI 2013C)*

SA (2 marks)

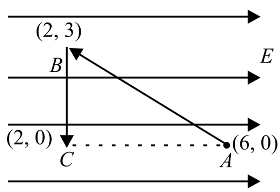
14. Two closely spaced equipotential surfaces A and B with potentials V and $V + \delta V$, (where δV is the change in V), are kept δl distance apart as shown in the figure. Deduce the relation between the electric field and the potential gradient between them. Write the two important conclusions concerning the

relation between the electric field and electric potentials.



(Delhi 2014C)

15. A test charge 'q' is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure. (i) Calculate the potential difference between A and C. (ii) At which point (of the two) is the electric potential more and why? *(AI 2012)*



16. Two uniformly large parallel thin plates having densities $+\sigma$ and $-\sigma$ are kept in the X-Z plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge $-q$ remains stationary between the plates, what is the magnitude and direction of this field? *(Delhi 2011)*

17. (a) Draw equipotential surfaces due to point $Q > 0$.
(b) Are these surfaces equidistant from each other? If no, explain why? *(Delhi 2011C)*

18. Can two equipotential surfaces intersect each other? Give reasons. *(Delhi 2011C)*

19. Two point charges 2 μC and –2μC are placed at points A and B, 6 cm apart.
(i) Draw equpotential surfaces of the system.
(ii) Why do the equipotential surfaces get closer to each other near the point charges? *(AI 2011C)*

LA I (3 marks)

20. Draw the equipotential surface due to an electric dipole. (*1/3, Delhi 2019*)

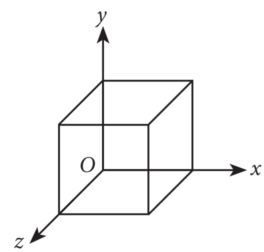
21. Depict the equipotential surfaces due to an electric dipole. (*2/3, Delhi 2017*)

22. Define an equipotential surface. Draw equipotential surfaces:
(i) in the case of a single point charge and
(ii) in a constant electric field in Z-direction. Why the equipotential surface about a single charge are not equidistant ?
(iii) Can electric field exist tangential to an equipotential surface? Give reason. *(AI 2016)*

LA II (5 marks)

23. Draw equipotential surfaces due to an isolated point charge ($-q$) and depict the electric field lines. *(1/5, 2020)*

24. A cube of side 20 cm is kept in a region as shown in the figure. An electric field $\vec{E}$ exists in the region such that the potential at a point is given by $V = 10x + 5$, where V is in volt and x is in m.



Find the
(i) electric field $\vec{E}$, and
(ii) total electric flux through the cube. *(3/5, 2020)*

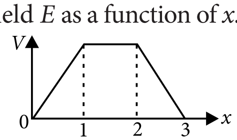
25. Write two important characteristics of equipotential surfaces. *(2/5, 2020)*

26. The magnitude of electric field (in N C^{-1}) in a region varies with the distance r (in m) as

$E = 10\,r + 5$

By how much does the electric potential increase in moving from point at $r = 1$ m to a point at $r = 10$ m. *(2/5, 2020)*

27. The electric potential as a function of distance 'x' is shown in the figure. Draw a graph of the electric field E as a function of x.



(*1/5, AI 2019*)

28. Is the electrostatic potential necessarily zero at a point where the electric field is zero? Give an example to support your answer. (*2/5, AI 2019*)

29. Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero.

(2/5, AI 2013)

30. Write two properties of equipotential surfaces. Depict equipotential surfaces due to an isolated point charge. Why do the equipotential surfaces get closer as the distance between the equipotential surface and the source charge decreases? *(AI 2012C)*

2.7 Potential Energy of a System of Charges

SA (2 marks)

31. Calculate the amount of work done to dissociate a system of three charges 1μC, 1μC and –4μC placed on the vertices of an equilateral triangle of side 10 cm. *(AI 2013C)*

32. Two charges $-q$ and $+q$ are located at point $A(0, 0, -a)$ and $B(0, 0, +a)$ respectively. How much work is done in moving a test charge from point $P(7, 0, 0)$ to $Q(-3, 0, 0)$?

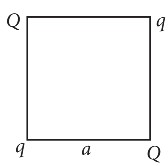
(1/2, Delhi 2011C)

LA I (3 marks)

33. (a) Two point charges $+Q_1$ and $-Q_2$ are placed r distance apart. Obtain the expression for the amount of work done to place a third charge Q_3 at the midpoint of the line joining the two charges.

(b) At what distance from charge $+Q_1$ on the line joining the two charges (in terms of Q_1, Q_2 and r) will this work done be zero. *(2020)*

34. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure.

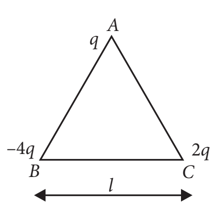


Find the

(a) resultant electric force on a charge Q, and

(b) potential energy of this system. *(2018)*

35. (a) Three point charges q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q.



(b) Find out the amount of the work done to separate the charges at infinite distance.

(2018)

LA II (5 marks)

36. Three point charges +1 μC, –1 μC and +2 μC are initially infinite distance apart. Calculate the work done in assembling these charges at the vertices of an equilateral triangle of side 10 cm. *(2/5, 2020)*

2.8 Potential Energy in an External Field

LA II (5 marks)

37. Find the expression for the potential energy of a system of two point charges q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$, respectively in an external electric field $\vec{E}$. *(2/5, 2020)*

38. Two point charges q_1 and q_2 are kept r distance apart in a uniform external electric field $\vec{E}$. Find the amount of work done in assembling this system of charges. *(2/5, 2020)*

39. Derive an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium. *(3/5, AI 2019)*

40. An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r, in front of the charged plane sheet. *(3/5, AI 2017)*

2.9 Electrostatics of Conductors

VSA (1 mark)

41. Why is the potential inside a hollow spherical charged conductor constant and has the same value as on its surface? *(Foreign 2012)*

42. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the centre of the sphere? *(AI 2011)*

2.10 Dielectrics and Polarisation

VSA (1 mark)

43. Distinguish between a dielectric and a conductor? *(Delhi 2012C)*

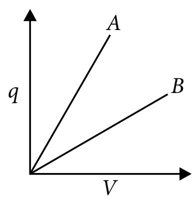
LA II (5 marks)

44. Explain, using suitable diagrams, the difference in the behaviour of a (i) conductor and (ii) dielectric in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility. *(Delhi 2012C)*

2.11 Capacitors and Capacitance

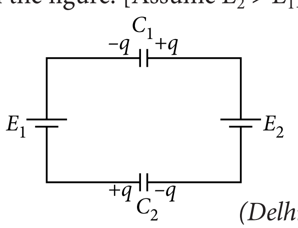
VSA (1 mark)

45. The given graph shows variation of charge 'q' versus potential difference 'V' for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than that of C_1. Which line (A or B) corresponds to C_1 and why? *(AI 2014C)*



SA (2 marks)

46. Determine the potential difference across the plates of the capacitor 'C_1' of the network shown in the figure. [Assume $E_2 > E_1$]



(Delhi 2012C)

2.12 The Parallel Plate Capacitor

SA (2 marks)

47. What is the area of the plates of 2 F parallel plate capacitor having separation between the plates is 0.5 cm? *(AI 2011)*

LA II (5 marks)

48. When a parallel plate capacitor is connected across a *dc* battery, explain briefly how the capacitor gets charged. (2/5, *AI 2019*)

49. If two similar large plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, find the expressions for

(a) field at points between the two plates and on outer side of the plates. Specify the direction of the field in each case.

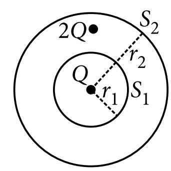
(b) the potential difference between the plates.

(c) the capacitance of the capacitor so formed. *(3/5, AI 2016)*

2.13 Effect of Dielectric on Capacitance

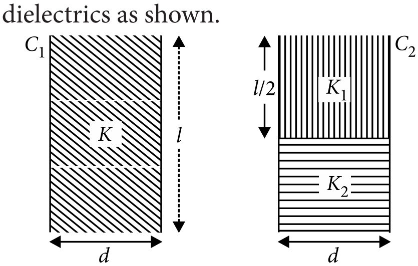
SA (2 marks)

50. A sphere S_1 of radius r_1 encloses a net charge Q. If there is another concentric sphere S_2 of radius $r_2 (r_2 > r_1)$ enclosing charge $2Q$, find the ratio of the electric flux through S_1 and S_2. How will the electric flux through sphere S_1 change if a medium of dielectric constant 5 is introduced in the space inside S_1 in place of air? *(AI 2014C)*



51. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. *(AI 2013)*

52. Two identical parallel plate (air) capacitor C_1 and C_2 have capacitances C each. The area between their plates in now filled with dielectrics as shown.



If the two capacitors still have equal capacitance, obtained the relation between dielectric constants K, K_1 and K_2.

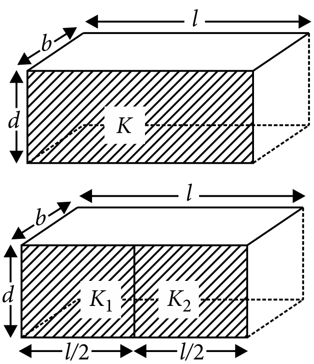
(Foreign 2011)

LA I (3 marks)

53. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}\,m^2$ and the separation between the plates is 3 mm.
    (i) Calculate the capacitance of the capacitor.
    (ii) If this capacitor is connected to 100V supply, what would be the charge on each plate?
    (iii) How would charge on the plates be affected, if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected?
    (Foreign 2014)

LA II (5 marks)

54. Two identical capacitors of plate dimensions $l \times b$ and plate separation d have dielectric slabs filled in between the space of the plates as shown in the figures.

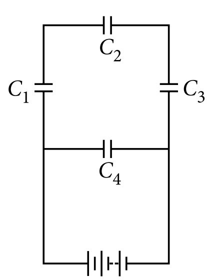


Obtain the relation between the dielectric constants K, K_1 and K_2. *(AI 2013C)*

2.14 Combination of Capacitors

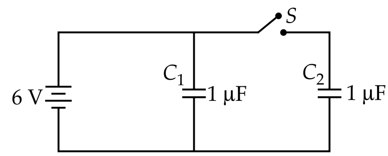
SA (2 marks)

55. A network of four capacitors, each of capacitance 15 μF, is connected across a battery of 100 V, as shown in the figure. Find the net capacitance and the charge on the capacitor C_4.



(AI 2012C)

56. 1 mF capacitance connected to a battery of 6 V. Initially switch S is closed. After sometime S is left open and dielectric slabs of dielectric constant $K = 3$ are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted?



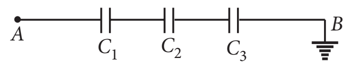
(Delhi 2011)

2.15 Energy Stored in a Capacitor

SA (2 marks)

57. Obtain the expression for the energy stored in a capacitor connected across a dc battery. Hence define energy density of the capacitor. *(2020)*

58. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given potential at A is 90 V, $C_1 = 20$ μF, $C_2 = 30$ μF and $C_3 = 15$ μF.



(AI 2015)

59. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. *(AI 2014)*

60. A parallel plate capacitor, each of plate area A and separation 'd' between the two plates, is charged with charges $+Q$ and $-Q$ on the two plates. Deduce the expression for the energy stored in capacitor. *(Foreign 2013)*

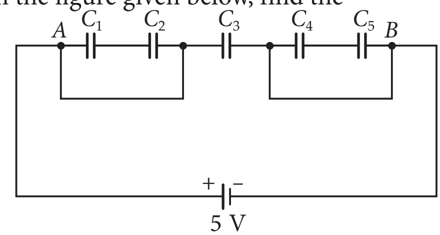
61. Deduce the expression for the electrostatic energy stored in a capacitor of capacitance 'C' and having charge 'Q'.

    How will the (i) energy stored and (ii) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant 'K'? *(AI 2012)*

62. Net capacitance of three identical capacitors in series is 1 μF. What will be their net capacitance if connected in parallel?
Find the ratio of energy stored in the two configurations if they are both connected to the same source. *(AI 2011)*

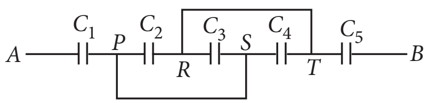
LA I (3 marks)

63. In the figure given below, find the



(a) equivalent capacitance of the network between points A and B.
Given : $C_1 = C_5 = 4$ μF, $C_2 = C_3 = C_4 = 2$ μF.
(b) maximum charge supplied by the battery, and
(c) total energy stored in the network.

64. (i) Find the equivalent capacitance between A and B in the combination given below. Each capacitor is of 2 μF capacitance.



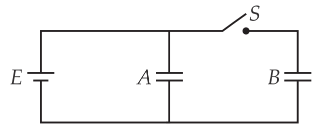
(ii) If a dc source of 7 V is connected across AB, how much charge is drawn from the source and what is the energy stored in the network? *(Delhi 2017)*

65. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor. *(Delhi 2017)*

66. Two identical capacitors of 12 pF each are connected in series across a battery of 50 V. How much electrostatic energy is stored in the combination? If these were connected in parallel across the same battery, how much energy will be stored in the combination now?

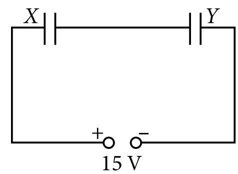
Also find the charge drawn from the battery in each case. *(Delhi 2017)*

67. Two identical parallel plate capacitors A and B are connected to a battery of V volt with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



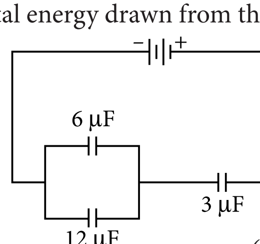
(AI 2017)

68. Two parallel plate capacitors X and Y have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric of $\varepsilon_r = 4$.



(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.
(ii) Calculate the potential difference between the plates of X and Y.
(iii) Estimate the ratio of electrostatic energy stored in X and Y. *(Delhi 2016)*

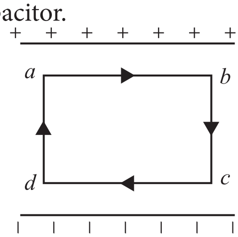
69. In the following arrangement of capacitors, the energy stored in the 6 μF capacitor is E. Find the value of the following
(i) Energy stored in 12 μF capacitor
(ii) Energy stored in 3 μF capacitor
(iii) Total energy drawn from the battery



(Foreign 2016)

70. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2. Also calculate the charge on each capacitor in parallel combination. *(Delhi 2015)*

71. (a) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.



(b) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop *abcda*. *(Delhi 2014)*

72. A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is 360 μC. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes 120 μC.
Calculate:
(i) The potential V and the unknown capacitance C.
(ii) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V? *(Delhi 2013)*

73. A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another uncharged capacitor of 100 pF. Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor. *(Foreign 2012)*

74. A parallel plate capacitor is charged by a battery. After sometime the battery is disconnected and a dielectric slab with its thickness equal to the plate separation is inserted between the plates. How will (i) the capacitance of the capacitor, (ii) potential difference between the plates and (iii) the energy stored in the capacitor be affected?
Justify your answer in each case.
(Delhi 2011C)

LA II (5 marks)

75. (a) Describe briefly the process of transferring the charge between the two plates of a parallel plate capacitor when connected to a battery. Derive an expression for the energy stored in a capacitor.
(b) A parallel plate capacitor is charged by a battery to a potential difference V. It is disconnected from battery and then connected to another uncharged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor.
(*Delhi 2019*)

76. A parallel plate capacitor of capacitance 'C' is charged to 'V' volt by a battery. After some time the battery is disconnected and the distance between the plates is doubled. Now a slab of dielectric constant $1 < K < 2$ is introduced to fill the space between the plates. How will the following be affected?
(i) The electric field between the plates of the capacitor?
(ii) The energy stored in the capacitor.
Justify your answer in each case.
(*2/5, AI 2019*)

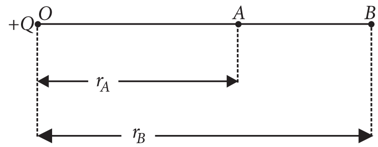
77. Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2 so that the energy stored in the two cases becomes the same. *(3/5, AI 2016)*

78. (a) Derive the expression for the energy stored in a parallel plate capacitor. Hence obtain the expression for the energy density of the electric field.
(b) A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than that stored initially in the single capacitor. *(AI 2015)*

Detailed Solutions

1. The physical quantity having SI unit N C^{-1} m is electrostatic potential.

2.



Potential difference due to a point charge Q at a distance r is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$\therefore$ From the given figure

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_A}; \quad V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_B}$$

$$\therefore\ V_A - V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_A} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_B} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

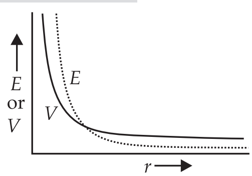
$$r_B > r_A \Rightarrow \frac{1}{r_B} < \frac{1}{r_A} \Rightarrow \left(\frac{1}{r_A} - \frac{1}{r_B}\right) > 0$$

Hence, $(V_A - V_B) > 0$

i.e., potential difference $(V_A - V_B)$ is positive.

3. Electric field due to a point charge,

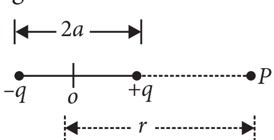
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}, \quad E \propto \frac{1}{r^2}$$



Potential due to a point charge, $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$; $V \propto \frac{1}{r}$

The variation of electric field E with distance r and also the variation of potential v with r as shown in the figure.

4.



Let P be an axial point at distance r from the centre of the dipole. Electric potential at point P will be

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{r+a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r-a}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right] = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a}{r^2-a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2-a^2} \qquad [\because p = q\,(2a)]$$

For a far away point, $r >> a$

$$\therefore\ V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} \text{ or } V \propto \frac{1}{r^2}$$

Thus, due to a dipole potential at a point is $V \propto \frac{1}{r^2}$.

5. *Refer to answer 4.*

6. Let q be the charge on each droplet.

Then $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ ...(i)

Volume of big drop = $N \times$ volume of small drop

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3,$$

where R is the radius of the big drop.

$\Rightarrow\ R = N^{1/3}\,r$ ...(ii)

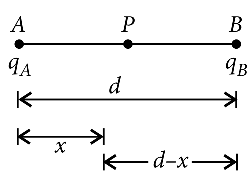
and $Q = Nq$, where Q is the charge of bigger drop

$\therefore$ Potential of larger drop,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Nq}{N^{1/3}r}$$

$$= \frac{N}{N^{1/3}}V = N^{2/3}V$$

7. $q_A = q$ and $q_B = -2q$



$$V_{PA} = \frac{kq_A}{x}$$

$$V_{PB} = \frac{kq_B}{(d-x)}$$

$$V_{PA} + V_{PB} = 0$$

$$\frac{kq}{x} = \frac{2kq}{(d-x)};\ d - x = 2x$$

$$3x = d;\ x = \frac{d}{3}$$

8. If the field were not normal to the equipotential surface, it would have a non zero component along the surface. So to move a test charge against this component, a work would have to be done. But there is no potential difference between any two points on an equipotential surface and consequently no work is required to move a test charge on the surface. Hence, the electric field must be normal to the equipotential surface at every point.

9. *Refer to answer 8.*

10. Work done = q (Potential at Q – Potential at P), where q is the small positive charge.

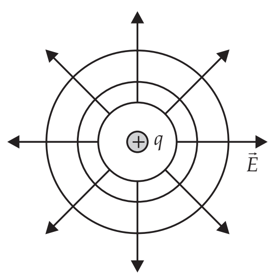
The electric potential at a point distance r due to the field created by a positive charge Q is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

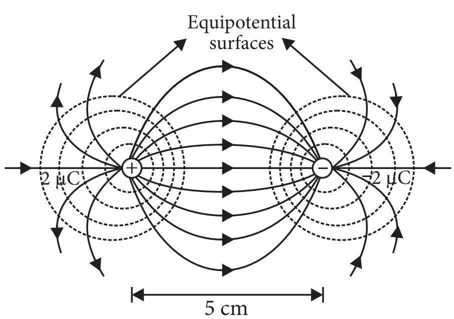
$\because \quad r_P < r_Q \;\therefore\; V_P > V_Q$

Hence, work done will be negative.

11. For an isolated charge the equipotential surfaces are concentric spherical shells and the separation between consecutive equipotential surfaces increases in the weaker electric field.



12.



13. Work done in carrying a charge on equipotential surface is always zero.

14. Electric field as gradient of potential consider a point charge $+q$ placed at point O. Suppose that V and $V+\delta V$ are electrostatic potential at points A and B, where distance from the charge $+q$ are r and $r - \delta r$ respectively.

$$(V+\delta V) = V + \frac{\delta W}{q_0}$$

$$\delta V = \frac{\delta W}{q_0} \qquad \text{...(i)}$$

If $\vec{E}$ is electric field at point P due to charge $+q$ placed at point O, then the test charge q_0 experiences a force equal to $q_0\vec{E}$ and the external force required to move the test charge against the electric field $\vec{E}$ is given by

$$\vec{F} = -q_0\vec{E}$$

Therefore, work done to move the test charge through an infinitesimally small displacement $\overrightarrow{PQ} = \overrightarrow{\delta l}$ is given by

$$\Delta W = \vec{F}\cdot\overrightarrow{\delta l} = (-q\vec{E})\cdot\overrightarrow{\delta l} = -q_0 E\delta l\cos 180° = q_0 E\delta l$$

As the distance r decreases in the direction of δl, then $\delta W = -q_0 E\,\delta r$

$$\frac{\delta W}{q_0} = -E\,\delta r \qquad \text{...(ii)}$$

From equations (i) and (ii), we get

$$\delta V = -E\,\delta r;\; E = -\frac{\delta V}{\delta r}$$

Therefore, electric field at a point is equal to the negative gradient of the electrostatic potential at that point.

Important conclusions :

(i) No work is done in moving a test charge over an equipotential surface.

(ii) The electric field is always at right angles to the equipotential surface.

(iii) The equipotential surfaces tell the direction of the electric field.

15. (i) In the relation

$$\boxed{E = \frac{-dV}{dr}} \Rightarrow E = -\left[\frac{V_C - V_A}{(2-6)}\right]$$

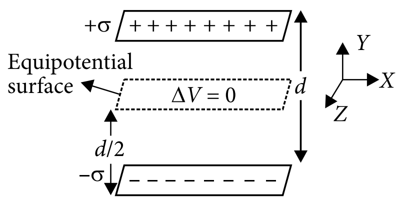
$$V_C - V_A = 4E$$

(ii) As $V_C - V_A = 4E$ is positive

$$\therefore V_C > V_A$$

Potential is greater at point C than point A, as potential decreases along the direction of electric field.

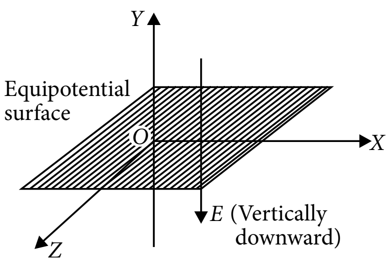
16.



The equipotential surface is at a distance $d/2$ from either plate in XZ-plane. $-q$ charge experiences a force in a direction opposite to the direction of electric field.

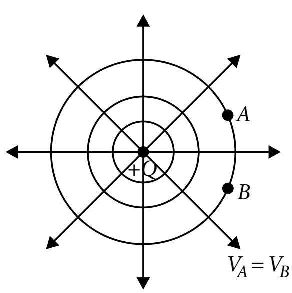
$\therefore$ $-q$ charge balances when $qE = mg$

$$E = \frac{mg}{q}$$



The direction of electric field along vertically downward direction. The XZ-plane is so chosen that the direction of electric field due to two plates is along vertically downward direction, otherwise weight (mg) of charge particle could not be balanced.

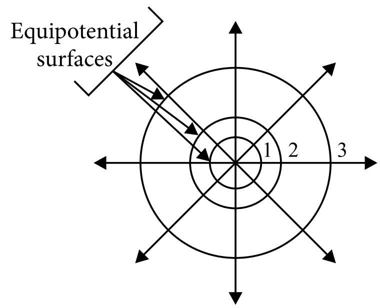
17. (a)



(b) These surfaces are not equidistant from each other because electric field at a point, distance r from point charge, is given by $E = +\dfrac{Q}{4\pi\varepsilon_0 r^2}$

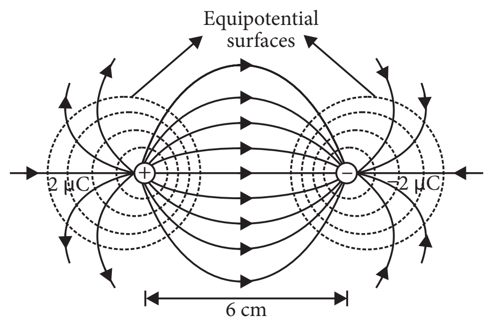
As electric field $E \propto \dfrac{1}{r^2}$, the field is non uniform.

So, distance between adjacent equipotential surfaces goes on increasing as shown in figure.



18. No, if two equipotential surfaces intersect then at the point of intersection, there will be two directions of electric field intensity which is not possible.

19. (i) Equipotential surface

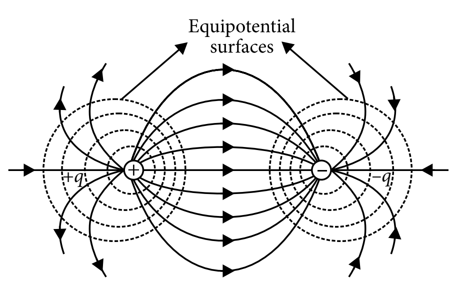


(ii) Equipotential surfaces get closer to each other near the point charges as strong electric field is produced there.

$\therefore$ $E = -\dfrac{\Delta V}{\Delta r} \Rightarrow E \propto -\dfrac{1}{\Delta r}$

For given equipotential surfaces, small Δr represents strong electric field and vice versa.

20.

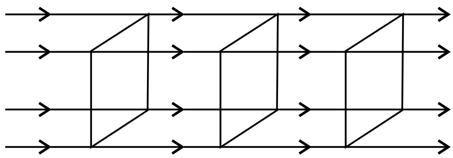


21. *Refer to answer 20.*

22. Equipotential surface is the surface with a constant value of potential at all points on the surface.

(i) *Refer to answer 17 (a).*

(ii) Equipotential surfaces in a constant electric field in Z-direction.



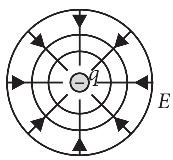
For constant electric field

Equipotential surfaces about a single charge are not equidistant because electric potential, $V \propto \frac{1}{r}$.

(iii) Electric field tangential to an equipotential surface cannot exist.

If the field lines are tangential, work will be done in moving a charge on the surface whereas on equipotential surface $W_{AB} = q_0(V_B - V_A) = 0$

23. For an isolated charge the equipotential surfaces are concentric spherical shells and the separation between consecutive equipotential surfaces increases in the weaker electric field.

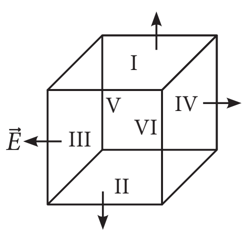


24. (i) Now electric field

$$\vec{E} = \frac{\partial V}{\partial r} = \frac{-dV}{dx} = \frac{-d}{dx}(10x+5) = -10\hat{i}$$

(ii) Now the total electric flux through the cube,

$\phi = \int E.ds$



$$\phi = \int_{\text{I}} E.ds + \int_{\text{II}} E.ds + \int_{\text{III}} E.ds + \int_{\text{IV}} E.ds + \int_{\text{V}} E.ds + \int_{\text{VI}} E.ds$$

$= 0 + 0 + (+10)(20\times10^{-2})^2 + (-10)(20\times10^{-2})^2 + 0 + 0$

$= 0$

25. (a) Properties of equipotential surface are:

(i) Work done in moving a test charge over an equipotential surface is zero.

(ii) Electric field is always directed normal to equipotential surface.

26. Given $E = 10r + 5$

Now the electric potential, $V = -\int E.dr$

$$= -\int_1^{10} (10r+5)dr = -\left[\frac{10r^2}{2} + 5r\right]_1^{10}$$

$$= -1\left[5r^2 + 5r\right]_1^{10}$$

$$= -[(5\times100+50)-(5+5)] = -540\text{ V}$$

27. Electric field $E = -\dfrac{dV}{dx}$ ...(i)

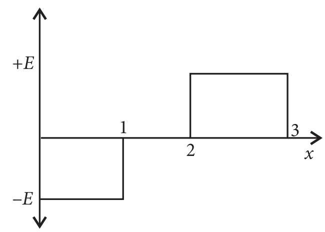
For x = 0 to 1, $V = kx$

x = 1 to 2, $V = k$

x = 2 to 3, $V = -kx$

where k is some constant

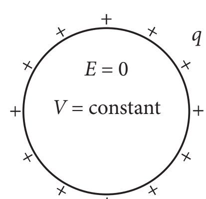
So, using (i) the variation of electric field is shown in figure.



28. The electric field $E = \dfrac{-dV}{dr}$

So, even for a constant electric potential electric field can be zero.

For example, for a hollow shell, the field inside is zero, whereas potential is non zero and constant.



29. *Refer to answer 20.*

Potential is zero on the points located on the line passing through the centre of dipole and perpendicular to the dipole axis.

30. *Refer to answer 25(a)*

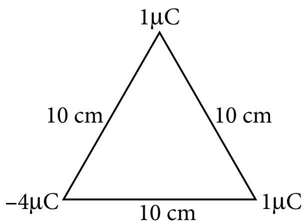
Refer to answer 11.

$$\because \quad E = -\frac{dV}{dr}, \quad i.e., \; dr = -\frac{dV}{E}$$

for given dV, $dr \propto \dfrac{1}{E}$

Hence, dr is small, then E is large. Hence, for small dr, equipotential surfaces are crowded.

31. $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times10^{-6}(-4\times10^{-6})}{0.10} + \frac{1\times10^{-6}(1\times10^{-6})}{0.10} + \frac{-4\times10^{-6}(1\times10^{-6})}{0.10}\right]$



$U = \frac{1}{4\pi\varepsilon_0}\times10^{-12}[-4\times10+10-4\times10]$

$= -9\times10^{9}\times10^{-12}\times70$

$= -0.630$ J.

Work done to dissociate the system of charges

$W = -V = 0.630$ J

32. Potential at $P(7, 0, 0)$ is

$$V_1 = \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(7-0)^2+0+(-a-0)^2}} + \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(7-0)^2+0+(a-0)^2}}$$

$$= \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{49+a^2}} + \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{49+a^2}} = 0$$

Potential at $Q(-3, 0, 0)$ is

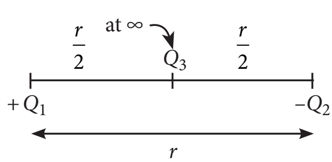
$$V_2 = \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(-3-0)^2+(-a)^2}} + \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(-3-0)^2+(-a)^2}}$$

$$= \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{9+a^2}} + \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{9+a^2}} = 0$$

$\therefore$ Work done $= q(V_2 - V_1) = q(0 - 0) = 0$

Hence, $W = 0$.

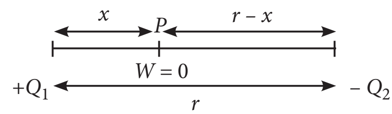
33. (a) The work done to bring the charge Q_3 from infinity to $\frac{r}{2}$,



$$W = U = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_3}{r/2} - \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{r/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2Q_3}{r}[Q_1 - Q_2]$$

(b) Consider a point P at a distance x from Q_1 where work done is zero. Then



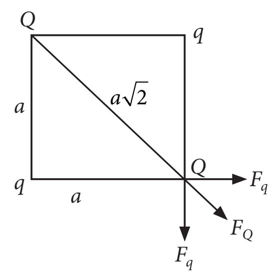
$\therefore$ Potential at P due to Q_1 = potential at P due to Q_2

$$\frac{kQ_1}{x} = \frac{kQ_2}{(r-x)} \Rightarrow (r-x)Q_1 = xQ_2$$

$$rQ_1 - xQ_1 = xQ_2 \Rightarrow rQ_1 = x(Q_1+Q_2)$$

$$\Rightarrow \quad x = \frac{rQ_1}{Q_1+Q_2}$$

34. (a) Force on charge Q due to charge q.



$$F_q = \frac{1}{4\pi\varepsilon_0}\times\frac{qQ}{a^2}$$

Force on charge Q due to another charge Q,

$$F_Q = \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{(a\sqrt{2})^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}$$

Net force on charge Q is

$$F_{\text{net}} = F_Q + \sqrt{F_q^2 + F_q^2} = F_Q + F_q\sqrt{2}$$

$$= \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{2a^2} + \frac{1}{4\pi\varepsilon_0}\times\frac{qQ}{a^2}\sqrt{2}$$

$$= \frac{Q}{4\pi\varepsilon_0 a^2}\left[\frac{Q}{2} + \sqrt{2}\,q\right] \text{ along diagonal}$$

(b) Potential energy of the given system,

$U = U_{qQ} + U_{Qq} + U_{qQ} + U_{Qq} + U_{qq} + U_{QQ}$

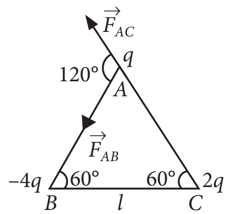
$= 4U_{qQ} + U_{qq} + U_{QQ}$

$$= \frac{4qQ}{4\pi\varepsilon_0 a} + \frac{q^2}{4\pi\varepsilon_0(\sqrt{2}a)} + \frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}a)}$$

$$= \frac{1}{4\pi\varepsilon_0 a}\left[4qQ + \frac{q^2}{\sqrt{2}a} + \frac{Q^2}{\sqrt{2}a}\right]$$

35. (a) $F_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q(4q)}{l^2}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{l^2}$$



$$F_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q(2q)}{l^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{l^2}$$

Angle between forces $\vec{F}_{AB}$ and $\vec{F}_{AC}$ is 120°.
Magnitude of resultant force,

$$F = \sqrt{F_{AB}^2 + F_{AC}^2 + 2F_{AB}F_{AC}\cos 120°}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{l^2}\right)\sqrt{(4)^2 + (2)^2 + 2\times 4\times 2\times\left(\frac{-1}{2}\right)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\sqrt{16+4-8} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}(2\sqrt{3})$$

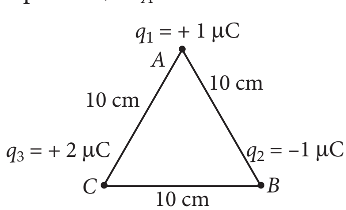
(b) Required work done = Change in potential energy of the system = $U_f - U_i$

$= 0 - (U_{AB} + U_{BC} + U_{CA})$

$$= \frac{-1}{4\pi\varepsilon_0 l}[q(-4q) + (-4q)(2q) + (q)(2q)]$$

$$= \frac{-1}{4\pi\varepsilon_0 l}[-4q^2 - 8q^2 + 2q^2] = \frac{10q^2}{4\pi\varepsilon_0 l}$$

36. The work done is bringing a charge q_1 from infinity to point A, $W_A = 0$.



Work done in bringing charge q_2 to point B from infinity

$$W_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{-1\times 10^{-12}}{10\times 10^{-2}}$$

Work done in bringing a point charge q_3 to point C from infinity,

$$W_C = \frac{1}{4\pi\varepsilon_0}\frac{1\times 2\times 10^{-12}}{10\times 10^{-2}} + \frac{1}{4\pi\varepsilon_0}\frac{2\times(-1)\times 10^{-12}}{10\times 10^{-2}}$$

$\therefore$ Total work done, $W = W_A + W_B + W_C$

$$\frac{1}{4\pi\epsilon_0}\left[0 + \frac{-1\times 10^{-12}}{10\times 10^{-2}} + \frac{2\times 10^{-12}}{10\times 10^{-2}} + \frac{(-2)\times 10^{-12}}{10\times 10^{-2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{-1\times 10^{-12}}{10\times 10^{-2}}$$

$$= 9\times 10^9 \times (-0.1)\times 10^{-10} = -0.9\times 10^{-1} = -0.09\ \text{J}$$

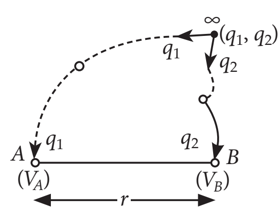
37. The work done in bringing charge q_1 in the external electric field at a distance $\vec{r}_1 = q_1\ V(r_1)$ work done in bringing charge q_2 in the external electric field at a distance $\vec{r}_2 = q_2\ V(r_2)$
The work done in moving q_2 against the force of q_1

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

where r_{12} is the distance between q_1 and q_2.
$\therefore$ Potential energy of the system

$$q_1 V(r_1) + q_2 V(r_2) + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

38. Potential energy of a system of two charges in an external field



$W_1 = q_1\ (V_A - 0)$
$= q_1 V_A$
where V_A is the potential at point A due the external field
Now work done in bringing charge q_2 to point B

$$W_2 = q_2 V_B + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

where V_B is the potential due to the external field at point B.
Total work done in assembling the configuration of two charges in an electric field is

$$W = W_1 + W_2$$

$$\therefore\ W = q_1 V_A + q_2 V_B + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

39. Since net force on electric dipole in uniform electric field is zero, so no work is done in moving the electric dipole in uniform electric field, however some work is done in rotating the dipole against the torque acting on it. So, small work done in rotating the dipole by an angle $d\theta$ in uniform electric field E is

$$dW = \tau\, d\theta = pE \sin\theta\, d\theta$$

Hence, net work done in rotating the dipole from angle θ_i to θ_f in uniform electric field is

$$W = \int_{\theta_i}^{\theta_f} pE \sin\theta\, d\theta = pE\,[-\cos\theta]_{\theta_i}^{\theta_f}$$

or $\quad W = pE\,[-\cos\theta_f + \cos\theta_i] = pE\,[\cos\theta_i - \cos\theta_f]$

If initially, the dipole is placed at an angle $\theta_i = 90°$ to the direction of electric field, and is then rotated to the angle $\theta_f = \theta$, then net work done is

$$W = pE\,[\cos 90° - \cos\theta]$$

or $\quad W = -\,pE\cos\theta$

This gives the work done in rotating the dipole through an angle θ in uniform electric field, which gets stored in it in the form of potential energy *i.e.*,

$$U = -\,pE\cos\theta$$

This gives potential energy stored in electric dipole of moment p when placed in uniform electric field at an angle θ with its direction.

(i) When $\theta = 0°$, then $U_{min} = -pE$

So, potential energy of an electric dipole is minimum, when it is placed with its dipole moment p parallel to the direction of electric field E and so it is called its most stable equilibrium position.

(ii) When $\theta = 180°$, then $U_{max} = +\,pE$

So, potential energy of an electric dipole is maximum, when it is placed with its dipole moment p anti parallel to the direction of electric field E and so it is called its most unstable equilibrium position.

40. Let P be a point at distance r from the sheet.

$$W = q \cdot (V_P - V_\infty) \qquad ...(i)$$

Now, $V_P - V_\infty$

$$= -\int_\infty^r \vec{E}\cdot d\vec{r} = -\int_\infty^r E dr = -\int_\infty^r \left(\frac{\sigma}{2\varepsilon_0}\right)\cdot dr$$

(Field from an infinitely large plane sheet of charge q is uniform and is given by $\frac{\sigma}{2\varepsilon_0}$).

$$-\frac{\sigma}{2\varepsilon_0}\int_\infty^r dr = -\frac{\sigma}{2\varepsilon_0}\cdot[r]_\infty^r$$

$$-\frac{\sigma}{2\varepsilon_0}(r-\infty) = \infty \text{ or, } V_P - V_\infty = \infty$$

From eq. (i) $W = \infty$

41. Electric field intensity is zero inside the hollow spherical charge conductor. So, no work is done in moving a test charge inside the conductor and on its surface. Therefore, there is no potential difference between any two points inside or on the surface of the conductor.

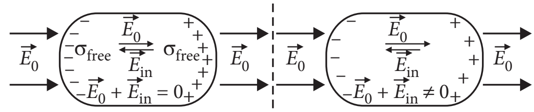
42. Potential inside the charged sphere is constant and equal to potential on the surface of the conductor. Therefore, potential at the centre of the sphere is 10 V.

43. Dielectrics are non-conductors and do not have free electrons at all. While conductors has free electrons which makes it able to pass the electricity through it.

44. (i) When a conductor is placed in an external electric field, the free charges present inside the conductor redistribute themselves in such a manner that the electric field due to induced charges opposes the external field within the conductor. This happens until a static situation is achieved *i.e.*, when the two fields cancel each other and the net electrostatic field in the conductor becomes zero.

(ii) Dielectrics are non-conducting substances *i.e.*, they have no charge carriers. Thus, in a dielectric, free movement of charges is not possible. It turns out that the external field induces dipole moment by reorienting molecules of the dielectric. The collective effect of all the molecular dipole moments is the net charge on the surface of the dielectric which produce a field that opposes the external field, unlike a conductor in an external electric field. However, the opposing field so induced does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of the dielectric.

The effect of electric field on a conductor and a dielectric is shown in the figure.



Conductor

Dielectric

The dipole moment per unit volume is called polarisation and is denoted by P. For linear isotropic dielectrics, $P = \chi E$

where χ is electric susceptibility of the dielectric medium.

45. The plate area of C_2 is greater than that of C_1. Since capacitance of a capacitor is directly proportional to the area of the plates,

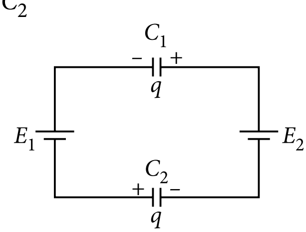
$\therefore \quad C_2 > C_1$

Now, $C = \dfrac{q}{V}$

Therefore, slope of a line ($=q/V$) is directly proportional to the capacitance of the capacitor, it represents. Since the slope of line A is more than that of B, line A represents C_2 and the line B represents C_1.

46. $\dfrac{-q}{C_1} - E_1 - \dfrac{q}{C_2} + E_2 = 0$

or $\quad \dfrac{q}{C_1} + \dfrac{q}{C_2} = E_2 - E_1$



Now, $V_1 = \dfrac{q}{C_1}, V_2 = \dfrac{q}{C_2}$

47. Here $C = 2$ F

$d = 0.5$ cm $= 0.5 \times 10^{-2}$ m

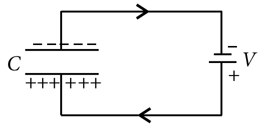
$\varepsilon_0 = 8.854 \times 10^{-12}\, C^2 N^{-1} m^{-2}$

$\therefore \quad C = \dfrac{\varepsilon_0 A}{d}$

$$A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 0.5 \times 10^{-2}}{8.854 \times 10^{-12}}$$

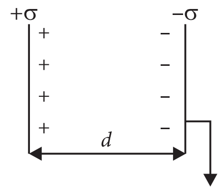
$A = 1.13 \times 10^9$ m^2

48. Consider a parallel plate capacitor is connected across a battery as shown in figure.



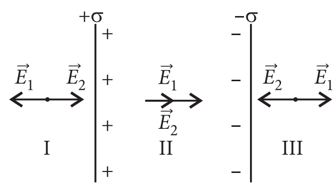
Then the electric current will flow through the circuit. As the charges reach the plate, the insulating gap does not allow the charges to move further; hence, positive charges get deposited on one side of the plate and negative charges get deposited on the other side of the plate. As the voltage begins to develop, the electric charges begins to resist the deposition of further charges. Thus the current flowing through the circuit gradually becomes less and then zero till the voltage of the capacitor is exactly equal but opposite to the voltage of the battery. This is how capacitor gets charged.

49. Capacitor is based on the principle of electrostatic induction. The capacitance of an insulated conductor increases significantly by bringing an uncharged earthed conductor near to it. This combination forms parallel plate capacitor.



(a) Magnitude of electric field intensities

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$



(i) In region I (outside)

$$E_I = E_2 - E_1 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

(ii) In region II (inside)

$$E_{II} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

(iii) In region III (outside)

$$E_{III} = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the region II *i.e.*, in the space between the plates, resultant electric field $\vec{E}_{II}$ is directed normal to plates, from positive to negative charge plate.

(b) The potential difference between the plates is

$$V = E_{II}.d = \frac{\sigma}{\varepsilon_0} d \quad \text{or} \quad V = \frac{Q}{A\varepsilon_0} d$$

(c) Capacitance of the capacitor so formed is

$$C = \frac{Q}{V} = \frac{Q}{Qd / A\varepsilon_0} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

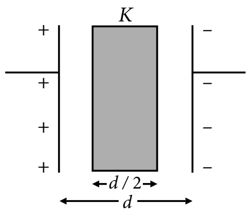
50. (i) $\phi_1 = \frac{Q}{\varepsilon_0}, \ \phi_2 = \frac{3Q}{\varepsilon_0}$

$$\frac{\phi_1}{\phi_2} = \frac{1}{3}$$

(ii) If a medium of dielectric constant 5 is filled in the space inside S_1, the flux inside S_1

$$\phi_1' = \frac{Q}{5\varepsilon_0} = \frac{\phi_1}{5}$$

51.



Capacitance of a capacitor partially filled with a dielectric

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{2\varepsilon_0 AK}{d(K+1)}$$

52. Let $A \rightarrow$ area of each plate and C_1 and C_2 are capacitance of each slab.

Let initially $C_1 = C = \frac{\varepsilon_0 A}{d} = C_2$

After inserting respective dielectric slabs:

$C_1' = KC$ ...(i)

and $C_2' = K_1 \frac{\varepsilon_0 (A/2)}{d} + K_2 \frac{\varepsilon_0 (A/2)}{d}$

$= \frac{\varepsilon_0 A}{2d}(K_1 + K_2); \ C_2' = \frac{C}{2}(K_1 + K_2)$ ...(ii)

From (i) and (ii)

$C_1' = C_2'$

$$KC = \frac{C}{2}(K_1 + K_2)$$

$$K = \frac{1}{2}(K_1 + K_2)$$

53. (i) Capacitance $C = \frac{\varepsilon_0 A}{d}$

$$= \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-4}} = 17.7 \times 10^{-11} \text{ F}$$

(ii) Charge $Q = CV = 17.7 \times 10^{-11} \times 100$

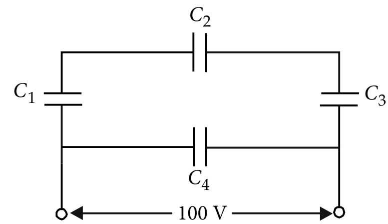
$= 17.7 \times 10^{-9}$ C

(iii) $C' = KC$

$\therefore \quad Q' = KQ = 10.62 \times 10^{-8}$ C

54. *Refer to answer 52.*

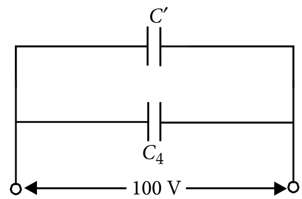
55. (a)



Here C_1, C_2 and C_3 are in series, hence their equivalent capacitance is C' given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C' = \frac{15}{3} \mu\text{F}$$



$C' = 5$ μF

The circuit can be redrawn as shown, in the figure.

Since C' and C_4 are in parallel

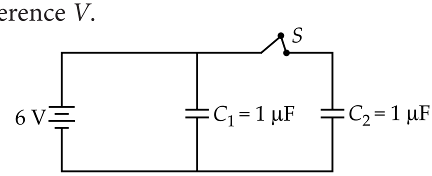
$\therefore \quad C_{net} = C' + C_4 = 5$ μF + 15 μF = 20 μF

(b) Since C' and C_4 are in parallel, potential difference across both of them is 100 V.

$\therefore \quad$ Charge across C_4 is $Q_4 = C_4 \times 100$ C

$= 15 \times 10^{-6} \times 100$ C = 1.5 mC

56. When the switch *S* is closed, the two capacitors in parallel will be charged by the same potential difference *V*.



So, charge on capacitor C_1

$q_1 = C_1 V$

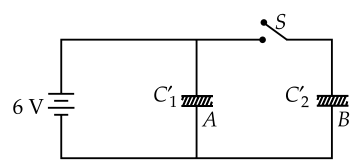
$q_1 = 1 \times 6 = 6$ μC

and charge on capacitor C_2

$q_2 = C_2 V = 1 \times 6 = 6$ μC

$\therefore \quad q = q_1 + q_2 = 6 + 6 = 12$ μC.

When switch *S* is opened and dielectric is introduced. Then



Capacity of both the capacitors becomes K times

i.e., $C_1' = C_2' = KC = 3 \times 1 = 3\ \mu F$

Capacitor A remains connected to battery

$\therefore \quad V_1' = V = 6$ V

$$q_1' = Kq_1 = 3 \times 6\ \mu C = 18\ \mu C$$

Capacitor B becomes isolated

$\therefore \quad q_2' = q_2$ or $C_2'V_2' = C_2V_2$ or $(KC)V_2' = CV$

or $\quad V_2' = \left(\dfrac{V}{K}\right) = \dfrac{6}{3} = 2$ V

57. Energy stored in a charged capacitor :

If q is the charge and V is the potential difference across a capacitor at any instant during its charging, then small work done is storing an additional small charge dq against the repulsion of charge q already stored on it is

$dW = V.dq = (q/C)dq$

So, the total amount of work done in storing the maximum charge Q on capacitor is

$$W = \int_0^Q \frac{q}{C} \cdot dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$$

which gets stored in the capacitor in the form of electrostatic energy. So the energy stored in capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

whereas the energy density *i.e.*, energy stored per unit volume in a charged parallel plate capacitor is given by

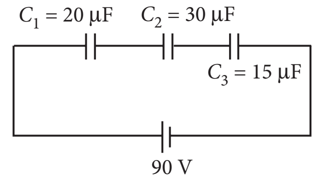
$$\text{Energy density} = \frac{\text{Total energy within plates}}{\text{Volume within plates}}$$

$$= \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\frac{\varepsilon_0 A}{d} \cdot E^2 d^2}{A \cdot d}$$

$$\boxed{\text{Energy density} = \frac{1}{2}\varepsilon_0 E^2}$$

58. The equivalent capacitance (C_{eq}) of the circuit is given by

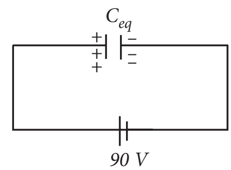
$$\frac{1}{C_{eq}} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15}$$



$$\frac{1}{C_{eq}} = \frac{3+2+4}{60}$$

$$C_{eq} = \frac{60}{9}\ \mu F$$

Charge on equivalent capacitor



$$Q = C_{eq}V = \frac{60}{9} \times 10^{-6} \times 90$$

$$Q = 600\ \mu C$$

Charge on each capacitor is same as they are in series.

Now, potential drop across C_2

$$V_2 = \frac{Q}{C_2} = \frac{600 \times 10^{-6}}{30 \times 10^{-6}} = 20 \text{ volt}$$

Energy, $U = \dfrac{1}{2}C_2V_2^2$

$$U = \frac{1}{2} \times 30 \times 10^{-6} \times (20)^2 = 6 \times 10^{-3} \text{ joule}$$

59. Energy stored in a capacitor

$$= \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

Capacitance of the (parallel) combination

$= C + C = 2C$

Here, total charge Q, remains the same

$\therefore \quad$ Initial energy (Single capacitor) $= \dfrac{1}{2}\dfrac{Q^2}{C}$

and final energy (Combined capacitor) $= \dfrac{1}{2}\dfrac{Q^2}{2C}$

$$\therefore \quad \frac{\text{Final energy}}{\text{Initial energy}} = \frac{1}{2}$$

60. *Refer to answer 57.*

61. *Refer to answer 57.*

When battery is disconnected

(i) Energy stored will be decreased or energy stored $= \dfrac{1}{K}$ times the initial energy.

(ii) Electric field would decrease

or $E' = \frac{E}{K}$

62. Net capacitance in series, $C_s = 1\ \mu F = 10^{-6}$ F

if $C_1 = C_2 = C_3 = C$

Let C be the capacitance of each of three capacitors and C_S and C_R be the capacitance of series and parallel combination respectively.

then, $\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$

$C_s = \frac{C}{3}\quad [C_s = 1\ \mu F]$

$\therefore\quad 1\ \mu F = \frac{C}{3};\ C = 3\ \mu F$

Also $C_P = C + C + C$

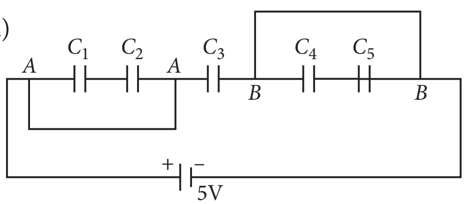
$= 3 + 3 + 3 = 9\ \mu F$

Energy stored in capacitor

$E = \frac{1}{2}CV^2$

$$\frac{E_S}{E_P} = \frac{\frac{1}{2}C_S V^2}{\frac{1}{2}C_P V^2} = \frac{C_S}{C_P} = \frac{1}{9}$$

63. (a)



So the equivalent capacitance between A and B is $C_3 = 2\ \mu F$

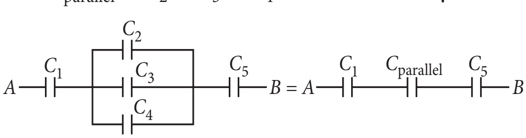
(b) Charge, $Q = CV = 2\ \mu F \times 5V = 10\ \mu C$

(c) Total energy stored

$= \frac{1}{2}CV^2 = \frac{1}{2} \times 2\mu F \times (5V)^2 = 25\ \mu J$

64. (i) In the circuit C_2, C_3 and C_4 are in parallel

$\therefore\quad C_{parallel} = C_2 + C_3 + C_4 = 2 + 2 + 2 = 6\ \mu F$



$\therefore$ Equivalent capacitance between A and B is

$$\frac{1}{C_{equivalent}} = \frac{1}{C_1} + \frac{1}{C_{parallel}} + \frac{1}{C_5}$$

$$= \frac{1}{2} + \frac{1}{6} + \frac{1}{2} = \frac{3+1+3}{6} = \frac{7}{6}$$

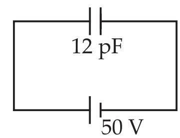
$\therefore\quad C_{equivalent} = \frac{6}{7} = 0.86\ \mu F$

(ii) $Q = C_{equivalent}V = 0.86 \times 7 = 6\ \mu C$.

Energy, $E = \frac{1}{2}QV = \frac{1}{2} \times 6 \times 7 = 21$ J

65. Electrostatic energy stored in the capacitor,

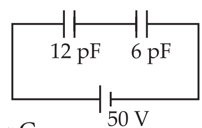
$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2$



(As C = 12 pF, V = 50 V)

$U = 1.5 \times 10^{-8}$ J

When 6 pF is connected in series with 12 pF, charge stored across each capacitor,



$$Q = \frac{C_1 C_2}{C_1 + C_2}V$$

$$= \frac{12 \times 6 \times 10^{-24}}{(12+6) \times 10^{-12}} \times 50 = 200\ pC$$

Now, potential difference across 12 pF is,

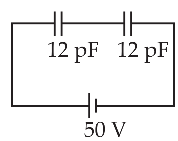
$$= \frac{Q}{C_1} = \frac{200 \times 10^{-12}}{12 \times 10^{-12}} = 16.67\ V$$

Potential difference across 6 pF is,

$$= \frac{Q}{C_2} = \frac{200 \times 10^{-12}}{6 \times 10^{-12}} = 33.33\ V$$

66. When two identical capacitors are in series,

Electrostatic energy,



$U = \frac{1}{2}C_s V^2$

As $C_s = \frac{C_1 C_2}{C_1 + C_2} = \frac{12 \times 12}{12 \times 12} = 6$ pF; $\quad V = 50$ V

$\therefore\ U_s = \frac{1}{2} \times 6 \times 10^{-12} \times (50)^2 = 7.5$ nJ

When two identical capacitors are in parallel then,

Stored energy, $U_p = \frac{1}{2}C_p V^2$

As $C_p = C_1 + C_2 = 12\ pF + 12\ pF = 24 \times 10^{-12}$ F

$\therefore\quad U_p = \frac{1}{2} \times 24 \times 10^{-12} \times (50)^2 = 30$ nJ

Charge drawn from the battery when two identical capacitor are in series,

$Q_s = C_s V = 6 \times 10^{-12} \times 50 = 300$ pC

Charge drawn from the battery when two capacitor are in parallel,

$Q_p = C_p V = 24 \times 10^{-12} \times 50 = 1200$ pC

67. Initially, when the switch is closed, both the capacitors A and B are in parallel and, therefore, the energy stored in the capacitors is

$$U_i = 2 \times \frac{1}{2} CV^2 = CV^2 \quad \text{...(i)}$$

When switch S is opened, B gets disconnected from the battery. The capacitor B is now isolated, and the charge on an isolated capacitor remains constant, often referred to as bound charge. On the other hand, A remains connected to the battery.

Hence, potential V remains constant on it.

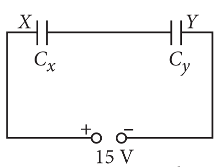
When the capacitors are filled with dielectric, their capacitance increases to KC. Therefore, energy stored in B changes to $Q^2/2KC$, where $Q = CV$ is the charge on B, which remains constant, and energy stored in A changes to $1/2\ KCV^2$, where V is the potential on A, which remains constant. Thus, the final total energy stored in the capacitors is

$$U_f = \frac{1}{2}\frac{(CV)^2}{KC} + \frac{1}{2}KCV^2 = \frac{1}{2}CV^2\left(K + \frac{1}{K}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we find

$$\frac{U_i}{U_f} = \frac{2K}{K^2+1}$$

68. Here, $C_x = \dfrac{\varepsilon_0 A}{d}$



$$C_y = \frac{\varepsilon_0 \varepsilon_r A}{d} = \varepsilon_r C_x = 4\,C_x$$

(i) C_x and C_y are in series, so equivalent capacitance is given by

$$C = \frac{C_x \times C_y}{C_x + C_y}$$

$$\Rightarrow \quad 4 = \frac{C_x \times 4\,C_x}{C_x + 4\,C_x} \qquad (\because\ C = 4\ \mu\text{F})$$

$$\Rightarrow \quad 4 = \frac{4\,C_x}{5} \quad \therefore\ C_x = 5\ \mu\text{F}$$

and $C_y = 4\,C_x = 20\ \mu$F

(ii) Charge on each capacitor, $Q = CV$

$Q = 4 \times 10^{-6} \times 15 = 60 \times 10^{-6}$ C

Potential difference between the plates of X,

$$V_x = \frac{Q}{C_x} = \frac{60\times10^{-6}}{5\times10^{-6}} = 12\ \text{V}$$

Potential difference between the plates of Y,

$V_y = V - V_x = 15 - 12 = 3$ V

(iii) Ratio of electrostatic energy stored,

$$\frac{U_x}{U_y} = \frac{\dfrac{Q^2}{2C_x}}{\dfrac{Q^2}{2C_y}} = \frac{C_y}{C_x} = \frac{4C_x}{C_x} = 4$$

69. (i) Given that energy of the 6 μF capacitor is E

Let V be the potential difference along the capacitor of capacitance 6 μF.

Since $\frac{1}{2}CV^2 = E$

$$\therefore \quad \frac{1}{2}\times 6\times10^{-6}\times V^2 = E$$

$$\Rightarrow \quad V^2 = \frac{E}{3}\times10^6 \quad \text{...(i)}$$

Since potential is same for parallel connection, the potential through 12 μF capacitor is also V. Hence, energy of 12 μF capacitor is

$$E_{12} = \frac{1}{2}\times12\times10^{-6}\times V^2 = \frac{1}{2}\times12\times10^{-6}\times\frac{E}{3}\times10^6 = 2E$$

(ii) Since charge remains constant in series, the charge on 6 μF and 12 μF capacitors combined will be equal to the charge on 3 μF capacitor. Using the formula, $Q = CV$, we can write

$(6 + 12) \times 10^{-6} \times V = 3 \times 10^{-6} \times V'$

$V' = 6$ V

Using (i) and squaring both sides, we get

$V'^2 = 12E \times 10^6$

$$\therefore \quad E_3 = \frac{1}{2}\times3\times10^{-6}\times12E\times10^6 = 18E$$

(iii) Total energy drawn from battery is

$E_{total} = E + E_{12} + E_3 = E + 2E + 18E = 21E$

70. When two capacitors C_1 and C_2 are in parallel, Equivalent capacitance, $C_p = C_1 + C_2$

Energy stored, $U_p = \dfrac{1}{2} C_p V^2 = \dfrac{1}{2}(C_1 + C_2)V^2$

Here, $U_p = 0.25$ J, $V = 100$ V

$$C_1 + C_2 = \frac{2U_p}{V^2} = \frac{2\times0.25}{(100)^2}$$

$$\therefore \quad C_1 + C_2 = 5\times10^{-5} \quad \text{...(i)}$$

When C_1 and C_2 are connected in series

Equivalent capacitance, $C_s = \dfrac{C_1 C_2}{C_1 + C_2}$

Energy stored, $U_s = \dfrac{1}{2} C_s V^2 = \dfrac{1}{2}\left(\dfrac{C_1 C_2}{C_1 + C_2}\right) V^2$

Here, $U_s = 0.045$ J

$$\therefore \quad C_1 C_2 = \frac{2U_s(C_1 + C_2)}{V^2}$$

$$= \frac{2 \times 0.045 \times 5 \times 10^{-5}}{10^4} = 4.5 \times 10^{-10}$$

$$C_1 - C_2 = \sqrt{(C_1 + C_2)^2 - 4C_1C_2}$$

$$= \sqrt{(5 \times 10^{-5})^2 - 4 \times 4.5 \times 10^{-10}}$$

$$C_1 - C_2 = 2.64 \times 10^{-5} \quad \ldots\text{(ii)}$$

Solving eqn. (i) and (ii), we get

$C_1 = 38.2$ μF, $C_2 = 11.8$ μF

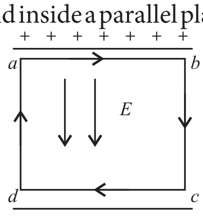
When capacitors are connected in parallel they have different amount of charge and given by

$Q_1 = C_1 V = 38.2 \times 10^{-6} \times 100 = 38.2 \times 10^{-4}$ C

$Q_2 = C_2 V = 11.8 \times 10^{-6} \times 100 = 11.8 \times 10^{-4}$ C.

71. (a) *Refer to answer 57.*

(b) Electric field inside a parallel plate capacitor $= E$



Here, electric field is conservative. Work done by the conservative force in closed loop is zero.

So, required work done = 0.

72. (i) Let the capacity of given capacitor is C and initial voltage $V_1 = V$

$Q_1 = 360\ \mu C$

$\therefore\ Q_1 = CV_1 \quad$ ..(i)

Changed potential, $V_2 = V - 120$

$Q_2 = 120\ \mu C$

$Q_2 = CV_2 \quad$ ...(ii)

Dividing equation (i) by (ii), we get $\dfrac{Q_1}{Q_2} = \dfrac{CV_1}{CV_2}$

$$\Rightarrow \frac{360}{120} = \frac{V}{V - 120}$$

$$\Rightarrow \quad V = 180$$

$$\therefore \quad C = \frac{Q_1}{V_1} = \frac{360 \times 10^{-6}}{180} = 2 \times 10^{-6}\,\text{F} = 2\,\mu\text{F}$$

(ii) If the voltage applied had increased by 120 V, then $V_3 = 180 + 120 = 300$ V. Hence, charge stored in the capacitor,

$Q_3 = CV_3 = 2 \times 10^{-6} \times 300 = 600$ μC

73. Initial energy of capacitor (U_i) = $\dfrac{1}{2} CV^2$

$$U_i = \frac{1}{2} \times 200 \times 10^{-12} \times (300)^2 = 9 \times 10^{-6}\ \text{J}$$

Charge on capacitor

$$Q = CV = 200 \times 10^{-12} \times 300 = 6 \times 10^{-8}\text{C}$$

When both capacitors are connected then let V be common potential difference across the two capacitros.

The charge would be shared between them.

Hence, $Q = q + q'$,

$q \rightarrow$ charge on capacitor (first)

$q' \rightarrow$ charge on capacitor (second)

$C = 200$ pF, $C' = 100$ pF

$$\frac{q}{200 \times 10^{-12}} = \frac{q'}{100 \times 10^{-12}} \quad \Rightarrow q = 2q'$$

Then $\quad Q = 2q' + q' = 3q'$

$$\Rightarrow \quad q' = \frac{Q}{3} = \frac{60\ \text{nC}}{3} = 20\ \text{nC}$$

and $\quad q = 2q' = 40$ nC

Hence, total final energy $U_f = \dfrac{q^2}{2C} + \dfrac{q'^2}{2C'}$

$$U_f = \frac{1}{2} \times \frac{(40 \times 10^{-9})^2}{200 \times 10^{-12}} + \frac{1}{2} \times \frac{(20 \times 10^{-9})^2}{100 \times 10^{-12}}$$

$U_f = 6 \times 10^{-6}$ J

Energy difference (ΔU) = $U_f - U_i$

$$= 6 \times 10^{-6} - 9 \times 10^{-6}\ \text{J} = -3 \times 10^{-6}\ \text{J}$$

$\Rightarrow\ \Delta U = 3 \times 10^{-6}$ J (in magnitude)

74. (i) On filling the dielectric of constant K in the space between the plates, capacitance of parallel plate capacitor becomes K times *i.e.*

$C = KC_0$

(ii) As the battery was disconnected, so the charge on the capacitor remains the same *i.e.*

$Q = Q_0$

So, the electric field in the space between the plates becomes

$$E = \frac{Q_0}{KA\varepsilon_0} \quad \text{or} \quad E = \frac{E_0}{K}$$

i.e. electric field becomes $\frac{1}{K}$ times.

(iii) Energy stored in capacitor becomes

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{KC} \text{ or } U = \frac{1}{K}U_0$$

i.e. becomes $\frac{1}{K}$ times

75. (a) *Refer to answer 48 and 57.*
(b) *Refer to answer 59.*

76. (i) The electric field between the plates is

$$E = \frac{V}{d}$$

The distance between plates is doubled, $d = 2d$

$$\therefore \quad E' = \frac{V'}{d'} = \left(\frac{V}{K}\right) \times \frac{1}{2d} = \frac{1}{2}\left(\frac{E}{K}\right)$$

Therefore, if the distance between the plates is double, the electric field will reduce to one half.

(ii) As the capacitance of the capacitor

$$C' = \frac{\varepsilon_0 KA}{d'} = \frac{\varepsilon_0 KA}{2d} = \frac{1}{2}C$$

Energy stored in the capacitor is $U = \frac{Q^2}{2C}$

New energy, $U' = \frac{Q^2}{2C'} = \frac{Q^2}{2(1/2)C} = 2\left(\frac{Q^2}{2C}\right) = 2U$

Therefore, when the distance between the plates is doubled, the capacitance reduces to half and the energy stored in the capacitor becomes double.

77. Given $\frac{C_1}{C_2} = \frac{1}{2}$ or $C_2 = 2C_1$

In parallel, $C_P = C_1 + C_2 = C_1 + 2C_1 = 3C_1$

In series, $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_1} + \frac{1}{2C_1} = \frac{2+1}{2C_1} = \frac{3}{2C_1}$

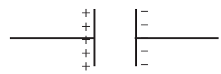
or $C_S = \frac{2}{3}C_1$

Given $U_S = U_P$

$$\frac{1}{2}C_S V_S^2 = \frac{1}{2}C_P V_P^2 \text{ or } \frac{2}{3}C_1 V_S^2 = 3C_1 V_P^2$$

$$\text{or } \frac{V_S^2}{V_P^2} = \frac{9}{2} \text{ or } \frac{V_S}{V_P} = \frac{3}{\sqrt{2}}$$

78. (a) *Refer to answer 57.*
(b) Let fully charge capacitor C has charge Q.

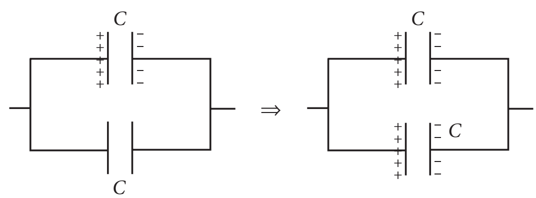


C, Q, V
$V = Q/C$

Energy stored in the capacitor

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

Now, the charged capacitor is connected to identical uncharged capacitor.



The two capacitor will have same potential.

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{Q+0}{2C} = \frac{Q}{2C}$$

Now, total energy

$$U' = \frac{1}{2}CV^2 + \frac{1}{2}CV^2$$

$$U' = \frac{1}{2}C\left(\frac{Q}{2C}\right)^2 + \frac{1}{2}C\left(\frac{Q}{2C}\right)^2 = \frac{Q^2}{4C}$$

So, $U > U'$

Energy lost as heat during charging the another capacitor.

$$U - U' = \frac{Q^2}{2C} - \frac{Q^2}{4C} = \frac{Q^2}{4C}$$