Chapter 4
Playing With Numbers

Exercise 4.1

Question 1.

Fill in the blanks:

(i) A number having exactly two factors is called a …….

(ii) A number having more than two factors is called a…….

(iii) 1 is neither ……..nor …….

(iv) The smallest prime number is…….

(v) The smallest odd prime number is…….

(vi) The smallest composite number is……..

(vii)The smallest odd composite number is…….

(viii) All prime numbers (except 2) are……

Solution:

(i) False

(ii) True

(iii) True

(iv) True

(v) False

(vi) False

(vii) False

(viii) False

(ix) True

(x) False

(xi) True

Question 3.

Write all the factors of the following natural numbers:

(i) 68

(ii) 27

(iii) 210

Solution:

(i) 68

The factors of 68 are : 1, 2, 4, 17, 34, 68

(ii) 27

The factors of 27 are : 1, 3, 9, 27

(iii) 210

The factors of 210 are:

1, 2,3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210

Question 4.

Write first six multiples of the following natural numbers:

(i) 3

(ii) 5

(iii) 12

Solution:

(i) 3
The firsts six multiple of 3 are
3, 6, 9, 12, 15, 18

(ii) 5
The first six multiples of 3 are
3, 6 ,9, 12, 15, 18

(iii) 12
The first six multiples of 12 are
12, 24, 36, 48, 60, 72

Question 5.

Match the items in column 1 with the items in colums 2:

Column 1	Column 2
(i) 15	(a) Multiple of 8
(ii) 36	(b) Factor of 30
(iii) 16	(c) Multiple of 70
(iv) 20	(d) Factor of 50
(v) 25	(e) Multiple of 9
(vi) 210	(f) Factor of 20

Question 6.

Find the common factors of :

(i) 20 and 28

(ii) 35 and 50

(iii) 56 and 120

Solution:

(i) 20 and 28

The factors of 20 are:

1, 2, 4, 5, 10, 20 The factors of 28 are:

1, 2, 4, 7, 14, 28

The common factors of 20 and 28 are: 1, 2, 4

(ii) 35 and 20

The factors of 35 are:

1, 5, 7, 35

The factors of 20 are:

1, 2, 4, 5, 10, 20

The common factors of 35 and 20 are 1, 5

(iii) 56 and 120

The factors of 56 are:

1, 2, 3, 4, 5 ,6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

The common factors of 56 and 120 are 1, 2, 4, 8

Question 7.

Find the common factors of:

(i) 4, 8, 12

(ii) 10, 30 and 45

Solution:

(i) The factors of 4 are :

1, 2, 4

The factors of 8 are :

1, 2, 4, 8

The factors of 12 are :

1, 2, 3, 4, 6, 12

The common factors of 4, 8, 12 are 1, 2, 4

(ii) 10, 30 and 45.

The factor of 10 are:

1, 2, 5, 10

The factor of 30 are:

1, 2, 3, 5, 10, 15, 30

The factor of 45 are:

1, 3, 5, 9, 15, 45

The common factors of 10, 30, 45 are 1, 5

Question 8.

Write all natural numbers less than 100 which are common multiples of 3 and 4.

Solution:

Multiples of 3 are : 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87,

90, 93, 96, 99, 102, 105, 108,……

Multiple of 4 are : 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, 104, 108,…….

∴ Common multiples of 3 and 4 are : 12, 24, 36, 48, 60, 72, 84, 96, 108, …….

All the numbers less than 100 which are common multiples of 3 and 4 are 12, 24, 36, 48, 60, 72, 84 and 96.

Question 9.

(i) Write the odd numbers between 36 and 53.

(ii) Write the even numbers between 232 and 251.

Solution:

(i) The odd numbers between 36 and 53 are:

37, 39, 41, 43, 45, 47, 49, 51.

(ii) The even numbers sbetween 232 and 251 are :

234, 236, 238, 240, 242, 244, 246, 248, 250.

Question 10.

(i) Write four consecutive odd numbers succeesing 79.

(ii) Write three consecutive even numbers preceding 124.

Solution:

(i) Four consecutive odd numbers succeeding 79 are : 81, 83, 85, 87.

(ii) Three consecutive even numbers preceding 124 are: 118, 120, 122.

Question 11.

What is greatest prime number between 1 and 15?

Solution:

The greatest prime number between 1 and 15 is 13.

Question 12.

Which of the following numbers are prime ?

(i) 29

(ii) 57

(iii) 43

(iv) 61

Solution:

(i) 29

We have, $29 = 1 \times 29$

= 29 has exactly two factors 1 and 29 itself.

$\therefore$ 29 is a prime number.

(ii) 57

We have, $57 = 1 \times 57 = 3 \times 19 = 57$

$\therefore$ Factors of 57 are 1, 3, 19 and 57

$\Rightarrow$ 57 has more than two factors

$\therefore$ 57 is not a prime.

(iii) 43

We have, $43 = 1 \times 43$

$\Rightarrow$ 43 has exactly two factors 1 and 43 itself.

$\therefore$ 43 is a prime number.

(iv) 61

We have, $61 = 1 \times 61$

$\Rightarrow$ 61 has exactly two factors 1 and 61 itself.

$\therefore$ 61 is a prime number.

Question 13.

Which of the following pairs of numbers are co-prime?

(i) 12 and 35

(ii) 15 and 37

(iii) 27 and 32

(iv) 17 and 85

(v) 515 and 516

(vi) 215 and 415

Solution:

(i) 12 and 35

The factors of 12 are 1, 2, 3, 4, 6, 12

The factors of 35 are 1, 5, 7, 35

Since, the common factor of 12 and 35 is 1

∴ They are co-prime.

(ii) 15 and 37

The factors of 15 are 1, 3, 5, 15

The factors off 37 are 1, 37

The common factor of 15 and 37 is 1

∴ They are co-prime.

(iii) 27 and 32

The factors of 27 are 1, 3, 9, 27

The factors of 32 are 1, 2, 4, 8, 16, 32

Since, the common factor of 27 and 32 is 1 They are co-prime.

(iv) 17 and 85

The factors of 17 are 1, 17

The factors of 85 are 1, 5, 17, 85

∴ They are not co-prime because they have more than 1 common factor.

(v) 515 and 516

The factors of 515 are 1, 5, 103, 515

The factors of 516 are 1, 2, 3, 4, 6, 12, 43, 86, 129, 172, 258, 516

Since, the common factor of 515 and 516 are 1 and 5

∴ So, they are not co-prime.

(vi) 215 and 415

The factors of 215 are 1, 5, 43, 215

The factors of 415 are 1, 5, 83, 415

Since, the common factor of 215 and 415 are 1 and 5

∴ So, they are not co-prime.

Question 14.

Express each of the following numbers as the sum of two odd primes:

(i) 24

(ii) 36

(iii) 84

(iv) 98

Solution:

(i) 24

$\Rightarrow 24 = 5 + 19$

(ii) 36

$\Rightarrow \ 36 = 7 + 29$

(iii) 84

$\Rightarrow \ 84 = 17 + 67$

(iv) 98

$\Rightarrow \ 98 = 19 + 79$

Question 15.

Express each of the following numbers as the sum of twin-primes:

(i) 24

(ii) 36

(iii) 84

(iv) 120

Solution:

(i) 24

$\Rightarrow$ 24 = 11 + 13

(ii) 36

$\Rightarrow$ 36 = 17 + 19

(iii) 84

$\Rightarrow$ 84 = 17 + 19

(iii) 84

$\Rightarrow$ 84 = 41 + 43

(iv) 120

$\Rightarrow$ 120 = = 59 + 61

Question 16.

Express each of the following numbers as the sum of three odd primes:

(i) 21

(ii) 35

(iii) 49

(iv) 63

Solution:

(i) 21

$\Rightarrow 21 = 3 + 7 + 11$

(ii) 35

$\Rightarrow 35 = 5 + 11 + 19$

(iii) 49

$\Rightarrow 49 = 7 + 11 + 31$

(iv) 63

$\Rightarrow 63 = 7 + 13 + 43$

Exercise 4.2

Question 1.

Which of the following numbers are divisible by 5 or by 10:

(i) 3725

(ii) 48970

(iii) 56823

(iv) 760035

(v) 7893217

(vi) 4500010

Solution:

(i) 3725 : divisible by 5 as last digit is 5.

(ii) 48970 : divisible by 5 and 10 both as last digit is 0.

(iii) 56283 : not divisible by 5 and neither by 10 as last digit is 3.

(iv) 760035 : divisible by 5 as last digit is 5.

(v) 7893217 : not divisible by 5 and neither 10 as last digit is 7.

(vi) 4500010 : divisible by both 5 and 10 as last digit is 0.

Question 2.

Which of the following numbers are divisible by 2, 4 or 8:

(i) 54014

(ii) 723840

(iii) 6531088

(iv) 75689604

(v) 78689604

(vi) 5321048

Solution:

(i) 54014

The last digit is 4, hence it is divisible by 2 but not by 4 and 8.

(ii) 723840

This number is divisible by 8, hence it should get divided by all its factors i.e. 2 and 4 (using property 1).

So, 723840 is divisible by 2, 4 and 8.

(iii) 6531088

This number is divided by 8.

So, by using property 1, it should also get divided by all its factorise 2 and 4.

Hence, 6531088 is divisible by 2, 4 and 8.

(iv) 75689604

This number is divisible by 4 and not by 8. By using property 1, if it is divisible by 4, then it should also get divisible by its factors also i.e. 2.

(v) 786235

Since, the last digit of the number is 5, which is even. Hence, it is not divisible by 2, 4 and 8.

(vi) 5321048

This number is divisible by 8.

So, by using property 1, if it is divisible by all its factors i.e. 2 and 4.

Question 3.

Which of the following numbers are divisible by 3 or 9:

(i) 7341

(ii) 59031

(iii) 12345678

(iv) 560319

(v) 720634

(vi) 37211509

Solution:

A number is divisible by 3 if the sum of its digit is divisible by 3 or 9.

(i) 7341 = 7 + 3 + 4 + 1 = 15 : divisible by 3.

(ii) 59031 = 5+ 9 + 0 + 3 + 1 = 18; divisible by 3, 9.

(iii) 12345678 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36; divisible by 3,9.

(iv) 560319 = 5 + 6 + 0 + 3 + 1 + 9 = 24; divisible by 3.

(v) 720634 = 7 + 2 + 0 + 6 + 3 + 4 = 22; not divisible by 3,9.

(vi) 3721509 = 3 + 7 + 2 + 1 + 5 + 0 + 9 = 27; divisible by 3,9.

Question 4:

Examine the following numbers for divisibilty by 11:

(i) 10428

(ii) 70169803

(iii) 7136985

Solution:

(i) 10428 = 1 + 4 + 8 = 13 and 0 + 2 = 2

Their differences = 13- 2 = 11, divisible by 11

(ii) 70169803 = 7 + 1 + 9 + 0 = 17 and 0 + 6 + 8 + 3 = 17

Their differences = 17 -17 = 0, divisible by 11

(iii) 7136985 = 7 + 3 + 9 + 5 = 24 and 1 + 6 + 8 = 15

Their differences = 24 -15 = 9 not divisible by 11

Question 5.

(i) 93573

(ii) 217944

(iii) 5034126

(iv) 901352

(v) 639210

(vi) 1790184

Solution:

A number is divisible by 6 if it is divisible by 2 as well as by 3.

(i) 93573 : not divisible by 6 because, it is not divisible by 2.

(ii) 217944: divisible by 6, as it is divisible by both 2 and 3.

The last digit of 217944 is 4, which is divisible by 2.

∴ The number is divisible by 2.

Now, the sum of the digits 217944 = 2 + 1 + 7 + 9 + 4 + 4 = 27

27 is divisible by 3.

Hence, given number is divisible by $2 \times 3 = 6$

(iii) 5034126: divisible by 6, as it is divisible by both 2 and 3.

The last digit of 5034126 is 6, which is divisible by 2.

Now, sum of 5034126 = 5 + 0 + 3 + 4 + 1 + 2 + 6 = 21

21 is divisible by 3.

Hence, given number 5034126 is divisible by 6.

(iv) 901352

The last digit 901352 is 2, which is divisible by 2

∴ The given number is divisible by 2

Now, the sum of the digits of 901352 is

9 + 0 + 1 + 3 + 5 + 2 = 20

20 is not divisible by 3

The given number 901352 is not divisible by 6

So we can say 93573 is not divisible by 6

(v) 639210

The last digit of 639120 is 0, which is divisible by 2,

∴ The given number is divisible by 2

Now, the sum of the digits of 639120 is

6 + 3 + 9 + 1 + 2 + 0 =21

21 is divisible by 3

The given number 639120 is divisible by 6

(vi) 1790184

The last digit of 1790184 is 4, which is divisible by 2,

∴ The given number is divisible by 2

Now, the sum of the digits of 1790184 is

1 + 7 + 9 + 0 + 1 + 8 + 4 = 30

30 is divisible by 3

∴ The given number 1790184 is divisible by 6

Question 6.

In each of the following replace '*' by a digit so that the number formed is divisible by 9 :

(i) 4710*82

(ii) 70*356722

Solution:

(i) 4710*82

The given number = 4710*81 Sum of its given digits

= 4 + 7 + 1 + 0 + 8 + 2 = 22

The number next to 22 which is divisible by 9 is 27.

∴ Required smallest number = 27 – 22 = 5

(ii) 70*356722

The given number = 70*356722 Sum of its given digits

= 7 + 0 + 3 + 5 + 6 + 7 + 2 + 2 = 32

The number next to 32 which is divisible by 9 is 36.

∴ Required smallest number = 36 – 32 = 4

Question 7.

In each of the following replace '*' by (i) the smallest digit (ii) the greatest digit so that the number formed is divisible by 3:

(a) 4*672

(b) 4756*2

Solution:

(a) 4*672

(i) Smallest digit

Sum of the given digits = 4 + 6 + 7 + 2 = 19

∵ 19 is not divisible by 3

∴ Smallest digit (non-zero) is = 2

(ii) Greatest digit

The greatest digit is 8

i.e. 19 + 8 = 27 which is divisible by 3

(b) 4756*2

(i) Smallest digit

Sum of the given digits = 4 + 7 + 5 + 6 = 24

∵ 24 is divisible by 3

∴ Smallest digit is 0.

(ii) Greatest digit

The greatest digit is 9

i.e. 24 + 9 = 33 which is divisible by 3.

Question 8.

In each of the following replace '*" by a digit so that the number formed is divisible by 11 :

(i) 8*9484

(ii) 9*53762

Solution :

(i) 8*9484

Sum of the given digits (at odd places) from the right

= 4 + 4 + required digit

= 8 + required digit

Sum of the given digits (at even places) from the right = 8 + 9 + 8 = 25
Difference of sums = 25 – (8 + required digit) = 17 – required digit

11 is the number smaller than 17, who gets divided by 11

∴ For the above difference to be divisible by 11 required digit = 6

Hence the required number is 869784.

(ii) 9 *53762

Sum of the given digits (at odd places) from the right

= 2 + 7 + 5 + 9 = 23

Sum of the given digits(at even places) from the right

= 6 + 3 + required number = 9

Differences of suns = 23 – (9 + required number)

= 14- required number for the above differences to be divisible by 11 required digit = 3

= 14 – 3 = 11

11 is divisible by 11

Hence, the required number is 9353762

Question 9

In each of the following replace '*' by (i) the smallest digit 00 the greatest digit so that the number formed is divisible by 6 :

(a) 2*4706

(b) 5825*34

Solution:

(a) 2*4706

If the number is divisible by 6 then the number should also get divisible by 2 and 3.

⇒ The last digit of 2*4706 is 6, so it is divisible by 2.

⇒ The sum of 2*4706

$\Rightarrow 2 + 4 + 7 + 0 + 6 = 19$

(i) Smallest required number to be added in 19 is 2.

As 19 + 2 = 21(i.e. 21 is divisible by 3)

(ii) Greatest required number to be added in 19 is 8

As 19 + 8 = 27 (i.e. 27 is divisible by 3)

(b) 5825*34

If the number is divisible by 6, then it should get divisible by 2 and 3.

⇒ The last number is 4, so it is divisible by 2

⇒ The sum of 5825*34

⇒ $5 + 8 + 2 + 5 + 3 + 4 = 27$

(i) The smallest number to be added in 27 is 0

27 + 0 = 27 (27 is 9 i.e. 27 + 9 = 36

36 is divided by 3

Question 10.

Which of the following numbers are prime:

(i) 101

(ii) 251

(iii) 323

(iv) 397

Solution:

(i) 101

We have, 101 = 1 × 101

⇒ 101 has exactly two factors 1 and 101 itself.

∴ 101 is a prime number.

(ii) 251

We have, 251 = 1 × 251

⇒ 251 has exactly two factors 1 and 251 itself.

∴ 251 is a prime number.

(iii) 323

We have, 323 = 1 × 323 = 17 × 19

∴ Factors of 323 are 1, 17, 19, 323

⇒ 323 has more than two factors.

323 is not a prime number.

(iv) 397

We have, 397 = 1 × 397

⇒ 397 has exactly two factors 1 and 397 itself.

397 is a prime number.

Question 11.

Determin if 372645 is divisible by 45.

Solution:

To determine if 25110 is divisible by 45, we test it for divisible by 5 and 9 both. Divisibility of 372645 by 5

∴ Number in the unit's place of 372645 = 5

∴ 372645 is divisible by 5

Divisibility of 372645 by 9

Sum of the digits of the number 372645 = 3 + 7 + 2 + 6 + 4 + 5 = 27

∵ 27 is divisible by 9

∴ 372645 is divisible by 9

As 372645 is divisible by 5 and 9 both and d5 and 9 are co-prime numbers, so 372645 is divisible by $5 \times 9 = 45$

Question 12.

A number is divisible by 12. By what other numbers will that number be divisible ?

Solution:

The number divisible by 12. should also get divisible by all its factors.

⇒ So, the numbers by which the given number is divisible are : 1, 2, 3, 4 ,6.

Question 13.

A number is divisible by both 3 and 8. By which other numbers will that number be always divisible ?

Solution:

Let a natural number, say n, be divisible by both 3 and 8.

As 3 and 8 are co-prime numbers using property n is divisible by 3×8.

i.e. 24.

Thus, the given number is always divisible by 24.

⇒ So the given number should get divided by all thefactors of 24.

Hence, the other number by which the given number is always divisible are:

1, 2, 4, 6, 12, 24.

Question 14:

State whether the following statements are true (T) or false (F) :

(i) If a number is divisible by 4, it must be divisible by 8.

(ii) If a number is divisible by 3, it must be divisible by 9.

(iii) If a number is divisible by 9, it must be divisible by 3.

(iv) If a number is divisible by 9 and 10 both, it must be divisible by 90.

(v) If a number divides two numbers separately, then it must divide their sum.

(vi) If a number divides the sum of two numbers, then it must divide the two numbers separately.

(vii) If a number is divisible by 3 and 8 both, it must be divisible by 12.

(viii) If a number is divisible by 6 and 15 both, it must be divisible by 90.

Solution:

(i) False

(ii) False

(iii) True

(iv) True

(v) False

(vi) True

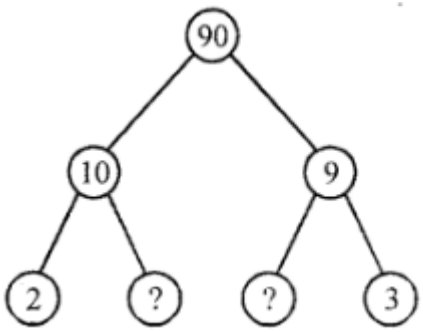
(vii) False

Exercise 4.3

Question.1 **Here are two different factors tress of the number 90. factor trees of the numer**

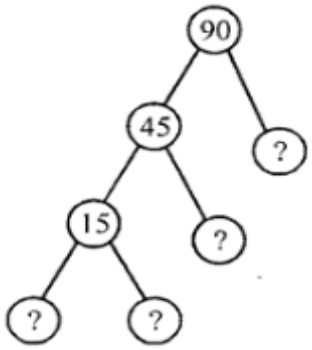
Solution:

(i)



Factor trees = $90 = 2 \times 5 \times 3 \times 3$

(ii)



Factor trees = $5 \times 3 \times 3 \times 2$

Question 2.

Find the prime factorisation of the following numbers :

(i) 72

(ii) 172

(iii) 450

(iv) 980

(v) 8712

(vi) 13500

Solution:

(i) 72

2	**72**
2	**36**
2	**18**
3	**9**
3	**3**
	1

$\therefore \quad 72 = 2 \times 2 \times 2 \times 3 \times 3$

(ii) 172

2	**172**
2	**86**
43	**43**
	1

$\therefore$ $172 = 2 \times 2 \times 43$

(iii) 450

2	**450**
3	**225**
3	**75**
5	**25**
5	**5**
	1

$\therefore$ $450 = 2 \times 3 \times 3 \times 5 \times 5$

(iv) 980

2	**980**
2	**490**
5	**245**
7	**49**
7	**7**
	1

$\therefore \quad 980 = 2 \times 2 \times 5 \times 7 \times 7$

(v) 8712

2	**8712**
2	**4356**
2	**2178**
3	**1089**
3	**363**
11	**121**
11	**11**
	1

$\therefore 8712 = 2 \times 2 \times 2 \times 3 \times 3 \times 11 \times 11$

(vi) 13500

2	**13500**
2	**6750**
3	**3375**
3	**1125**
3	**375**
5	**125**
5	**25**
5	**5**
	1

$\therefore 13500 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

Question 3.

Write the smallest and the greatest 3-digit numbers and express them as the product of prime.

Solution:

Smallest 3 digit number =100

2	100
2	50
5	25
5	5
	1

$\therefore 100 = 2 \times 2 \times 5 \times 5$

Greatest 3 digit number =999

3	**999**
3	**333**
3	**111**
37	**37**
	1

$\therefore 999 = 3 \times 3 \times 3 \times 37$

Question 4.

Write the smallest five digit number and express it in the form of its prime factors.

Solution:

The smallest 5-digit number is 10000

2	10000
2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

$\therefore 10000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

Question 5.

1 am the smallest number, having four different prime factors. Can you find me ?

Solution:

The smallest four different prime numbers are 2, 3, 5 and 7.

$\therefore$ The smallest number, having four different prime factor is $2 \times 3 \times 5 \times 7 = 210$.

Exercise 4.4

Question 1.

Find the H.C.F of the given numbers by prime factorisation method:

(i) 28, 36

(ii) 54, 72,90

(iii) 105, 140, 175

Solution:

(i) Prime factorisation of the given numbers are:

$28 = 2 \times 2 \times 7$

$36 = 2 \times 2 \times 3 \times 3$

Notice that 2 occurs as a common prime factor atleast 2 times in both given numbers.

$\therefore$ H.C.F. $= 2 \times 2 = 4$

(ii) Prime factorisation of the given numbers are:

$54 = 2 \times 3 \times 3 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$90 = 2 \times 3 \times 3 \times 5$

Notice that 2 occurs as a common prime factor atleast one times and 3 atleat two times in given numbers.

$\therefore$ H.C.F. $= 2 \times 3 \times 3 = 18$

(iii) Prime factorisation of given numbers are:

$105 = 3 \times 5 \times 7$

$140 = 2 \times 2 \times 5 \times 7$

$175 = 5 \times 5 \times 7$

Notice that 5 occurs as a common prime factor atleast one time and 7 one time in given numbers.

$\therefore$ H.C.F $= 5 \times 7 = 35$

Question :2

Find the H.C.F. of the given numbers by division method:

(i) 198, 429

(ii) 20, 64, 104

(iii) 120, 144, 204

Solution:

(i)

$$
\begin{array}{r}
198\overline{)\ 429\ (\ 2} \\
396 \\
\hline
33\overline{)\ 198\ (\ 5} \\
165 \\
\hline
33\overline{)\ 33\ (\ 1} \\
33 \\
\hline
0
\end{array}
$$

Last remainder = 0, stop here

$\therefore$ H.C.F. = 33 (Last divisor)

(ii) Let us find H.C.F. of 20 and 64

$$
\begin{array}{r}
20\overline{)\ 64\ (\ 3} \\
60 \\
\hline
4\overline{)\ 20\ (\ 5} \\
20 \\
\hline
0
\end{array}
$$

$\therefore$ H.C.F. of 20 and 64 = 4

Now, find H.C.F. 4 and 104

$$
\begin{array}{r}
3\overline{)\ 104\ (\ 26} \\
8 \\
\hline
24 \\
24 \\
\hline
0
\end{array}
$$

∴ H.C.F. of 20, 64, 104 = 4

(iii) 120, 144, 204

First, find the H.C.F. of 120 and 144

```
120 ) 204 ( 1
      120
      ---------
       84 ) 120 ( 1
             84
             ---------
             36 ) 84 ( 2
                  72
                  --------
                  12 ) 36 ( 3
                       36
                       -------
                        ×
```

H.C.F. of 120 and 204 is 12

Now, find the H.C.F. of 12 and 144

```
12 ) 144 (12
     12
     ----
      24
      24
     ----
      ×
     ----
```

∴ H.C.F. of 120, 144 and 204 = 12.

Question 3.

Fill in the blanks:

(i) HCF of two consecutive natural numbers is…….

(ii) HCF of two consecutive odd numbers is …….

(iii) HCF of two consecutive even numbers is……..

Solution:

(i) HCF of two consecutive natural numbers is 1.

(ii) HCF of two consecutive odd numbers is 1.

(iii) HCF of two consecutive even numbers is 2.

Question 4.

Find the greatest number which can divide 257 and 329 so as to leave a remainder 5 in each case.

Solution:

When 257 is divided by the required, 5 is left as a remainder.

So 257 – 5 = 252 i.e. 252 is exactly divisible by that number.

Similarly 329 – 5 = 324 is exactly divisible by that number.

∴ 252 and 324 are both divisible by that number. Thus, the required number is the H.C.F. of 252 and 324.

$$
\begin{array}{r|l|l}
252 & 324 & 1 \\
 & 252 & \\
\hline
72 & 252 & 3 \\
 & 216 & \\
\hline
36 & 72 & 2 \\
 & 72 & \\
\hline
 & 00 &
\end{array}
$$

Hence the required number = 36

Question 5.

Find the largest number that will divide 623, 729 and 841 leaving remainder 3, 9 and 1 respectively.

Solution:

Numbers are 623, 729 and 841 and remainders are 3, 9, 1 respectively

$\therefore$ Numbers will be $623 - 3 = 620$

$729 - 9 = 720$

$841 - 1 = 840$

Now, let us find the H.C.F. of 620, 720 and 840.

$$
\begin{array}{r|l}
20 & 840 \,(\, 42 \\
 & \underline{80} \\
 & 40 \\
 & \underline{40} \\
 & \times
\end{array}
$$

The H.C.F. of 620, 720 and 840 is 20

∴ The required largest number = 20

Question 6.

Meenu purchases two bags of rice of weights 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the rice exact number of times.

Solution:

Weights of two bags = 75kg, 60kg

3	75
5	25
5	5
	1

$\Rightarrow\ 75 = 3 \times 5 \times 5$

$\Rightarrow\ 69 = 3 \times 23$

∴ H.C.F. of 75 and 69 = 3

⇒ The maximum value of weigh which can measure the weight of the rice exact number of times is 3kg.

Question 7.

Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a container that can measure the diesel of three containers exact number of times.

Solution:

Capacity of 3 tankers = 403/, 434/, 465/

∴ Maximum capacity of a container, that can measure the diesel of three tankers.

403) 434 (1
403
31) 403 (13
403
×
31) 465 (15
31
155
155
×

= HCF of 403, 434, 465 = 31

∴ Required measure = 31 litres.

Exercise 4.5

Question 1.

Find the L.C.M. of the given numbers by prime factorisation method:

(i) 28, 98

(ii) 36, 40, 126

(iii) 108, 135, 162

(iv) 24, 28, 196.

Solution:

(i) Prime factorisation of the given numbers are :

$28 = 2 \times 2 \times 7$

$98 = 2 \times 7 \times 7$

Here 2 and 7 occurs as a prime factor maximum 2 times

$\therefore$ L.C.M. $= 2 \times 2 \times 7 \times 7 = 196$

(ii) Prime factorisation of the given numbers are:

$36 = 2 \times 2 \times 3 \times 3$

$40 = 2 \times 2 \times 2 \times 5$

$126 = 2 \times 3 \times 3 \times 7$

Notice that 2 occurs as a prime factor maximum 3 times, 3 two times, 5 one times and 7 one times.

$\therefore$ L.C.M. $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

(iii) Prime factorisation of given numbers are

$108 = 2 \times 2 \times 3 \times 3 \times 3$

$135 = 3 \times 3 \times 3 \times 5$

$162 = 2 \times 3 \times 3 \times 3 \times 3$

= Notice that 2 occurs as a prime factor.

maximum 2 times, 3, four time and 5, one time

$\therefore$ L.C.M. $= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 = 1620$

(iv) Prime factorisation of the given numbers are

$24 = 2 \times 2 \times 2 \times 3$

$28 = 2 \times 2 \times 7$

$196 = 2 \times 2 \times 7 \times 7$

Notice that 2 occurs as a prime factor maximum 3 times, 3 one times, 7 two times

$\therefore$ L.C. M. $= 2 \times 2 \times 2 \times 3 \times 7 \times 7 = 1176$

Question 2.

Find the L.C.M. of the given numbers by division method:

(i) 480, 672

(ii) 6, 8, 45

(iii) 24, 40, 84

(iv) 20, 36, 63, 67

Solution:

(i) 480, 672

2	480, 672
2	240, 336
2	120, 168
2	60, 84
2	30, 42
3	15, 21
5	5, 7
7	1, 7
	1,1

$\therefore$ L.C. M. $= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 3360$

(ii) 6, 8, 45

2	6, 8, 45
2	3, 4, 45
2	3, 2, 45
3	3, 1, 45
3	1, 1, 15
5	1, 1, 5
	1, 1, 1

$\therefore$ LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

(iii) 24, 40, 84

2	24, 40, 84
2	12, 20, 42
2	6, 10, 21
3	3, 5, 21
5	1, 5, 7
7	1, 1, 7
	1, 1, 1

$\therefore$ LCM = $2 \times 2 \times 2 \times 3 \times 5 \times 7 = 3360$

(iv) 20, 36, 63, 97

2	20, 36, 63, 77
2	10, 18, 63, 77
3	5, 9, 63, 77
3	5, 3, 21, 77
5	5, 1, 7, 77
7	1, 1, 7, 77
11	1, 1, 1, 11
	1, 1, 1, 1

$\therefore$ LCM = $2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 11 = 13860$

Question 3.

Find the least number which when increased by 15 is exactly divisible by 15, 35 and 48.

Solution:

First, we find the least number which is exactly divisible by the numbers 15,35 and 48. For this, we find L.C.M. of 15, 35 and 48.

3	15, 35, 48
5	5, 35, 16
7	1, 7, 16
2	1, 1, 16
2	1, 1, 8
2	1, 1, 4
2	1, 1, 2
	1, 1, 1

∴ L.C.M. = 3 × 5×7 × 2× 2 × 2 × 2 = 1680

According to given condition, the required number will be 15 less than 1680.

∴ The required least number = 1680 – 15 = 1665

Question 4.

Find the least number which when divided by 6, 15 and 18 leaves remainder 5 in each case.

Solution:

LCM of 6, 15 and 18

2	6, 15, 18
3	3, 15, 9
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

$= 2 \times 3 \times 3 \times 5 = 90$

Hence, the required number is 90 + 5 i.e. 95 48

Question 5.

Find the least number which when divided by 24, 36, 45 and 54 leaves a remainder of 3 in each case.

Solution:

24, 36, 45 and 54

2	24, 36, 45, 54
2	12, 18, 45, 27
2	6, 9, 45, 27
3	3, 9, 45, 27
3	1, 3, 15, 9
3	1, 1, 5, 3
5	1, 1, 5, 1
	1, 1, 1, 1

∴ L.C.M. = $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 1080$

According to given condition, the required number will be 3 more than 1080.

∴ The required number = 1080 + 3 = 1083

Question 6.
Find the greatest 3-digit number which is exactly divisible by 8, 20 and 24.
Solution:

First, we find the LCM of 8, 20 and 24

2	8, 20, 24
2	4, 10, 12
2	2, 5, 6
3	1, 5, 3
5	1, 1, 1
	1, 1, 1

∴ LCM of given numbers = 2 × 2 × 2 × 3 × 5 = 120

Greatest number of 3 digit is 999

We divide 999 by 120 and find the remainder.

$$\begin{array}{r} 8 \\ 120\overline{)\,999} \\ -\,960 \\ \hline 39 \end{array}$$

According to given condition, we need a greatest 3-digit number which is exactly divisible by 120.

∴ The required number = 999 – 39 = 960

Question 7.

Find the smallest 4-digit number which is exactly divisible by 32, 36 and 48.

Solution:

First, we find the LCM of 32, 36 and 48

2	32, 36, 48
2	16, 18, 24
2	8, 9, 12
2	4, 9, 6
2	2, 9, 3
3	1, 9, 3
3	1, 3, 1
	1, 1, 1

∴ LCM of given number

$= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$

Smallest number of 4-digit = 1000

We divide 1000 by 288 and find the remainder

$$288 \overline{)\,1000\,}(3$$
$$\underline{-\,864}$$
$$\underline{136}$$

According to given condition, we need a least number of 4-digit which is exactly divisible by 288.

∴ The required number = 1000 + (288 – 136) = 1152

Question 8.

Find the greatest 4-digit number which is exactly divisible by each of 8, 12 and 20.

Solution:

first, we find the LCM of 8, 12 and 20

2	8, 12, 20
2	4, 6, 10
2	2, 3, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

∴ LCM of given numbers = $2 \times 2 \times 2 \times 3 \times 5$ =120

According to given condtion, we need a greatest number of 4-digit which is exactly divisible by 120.

Greatest number of 4-digit = 9999

We divide 9999 by 120 and find the remainder.

$$\begin{array}{r|l}
120 \,) \; 9999 \; (\; 83 & \\
-960 & \\
\hline
399 & \\
-\;360 & \\
\hline
39 & \\
\hline
\end{array}$$

∴ The required number = 9999 – 39 = 9960

Question 9.

Find the least number of give digits which is exactly divisible by 32, 36 and 45.

Solution:

First we, find the LCM of 32, 36 and 45

2	32, 36, 45
2	16, 18, 45
2	8, 9, 45
2	4, 9, 45
3	2, 9, 45
3	1, 3, 15
5	1, 1, 5
	1, 1, 1

∴ LCM of given numbers

$= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 1140$

Smallest 5-digit number = 10000

We divided 10000 by 1440 and find the remainder

1440) 10000 (
8640
―――
1360

According to given condition,

We need a least 5-digit number which is exactly divisible by 1440

The required number

$= 10000 + 1440 - 1360$

$= 10080$

Question 10.

Three boys step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm repectively. What is the minimum distance each should cover so that all can cover the same distance in complete steps?

Solution:

The L.C.M. of 63, 70 and 77

3	63
3	21
7	7
	1

2	70
5	35
7	7
	1

7	77
11	11
	1

$\Rightarrow$ $63 = 3 \times 3 \times 7$

$70 = 2 \times 5 \times 7$

$77 = 7 \times 11$

∴ L.C.M. = $3 \times 3 \times 2 \times 5 \times 7 \times 11 = 6930$

∴ The minimum distance each shall cover is 6930 cm i.e. 69m 30 cm

Question 11.

Traffic lights at three different road crossing change after 48 seconds, 72 seconds and 108 seconds respectively. At what time will they change together again if they change simultaneously at 7 A.M. ?

Solution:

LCM of 48, 72 and 108

2	48, 72, 108
2	24, 36, 54
2	12, 18, 27
2	6, 9, 27
3	3, 9, 27
3	1, 3, 9
3	1, 1, 3
	1, 1, 1

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 432$

432 seconds = 7 minutes 12 seconds past 7 A.M.

Question 12.

If the product of two numbers is 4032 and their HCF is 12, find their LCM.

Solution:

Product of two number = 4032

H C F = 12

L.C.M. = 4032 ÷ 12 = 336

```
12 ) 4032 ( 336
     36
     ----
      43
      36
     ----
       72
       72
     ----
        ×
     ----
```

Question 13.

The HCF and LCM of two numbers are 9 and 270 respectively. If one of the numbers is 45, find the other number.

Solution:

HCF × LCM = one number × 2^{nd} number

9 × 270 = 45 × 2^{nd} number

2430 = 45 × 2^{nd} number

$= \frac{\mathbf{2430}}{\mathbf{45}} = \frac{\mathbf{162}}{\mathbf{3}} = 54$

∴ 54 is other number.

Question 14.

Find the HCF of 180 and 336. Hence, find their LCM.

Solution:

Division method : HCF of 180 and 336

$$
\begin{array}{r}
180 \overline{)\,336\,(\,1} \\
\underline{180} \\
156 \overline{)\,180\,(\,1} \\
\underline{156} \\
24 \overline{)\,156\,(\,6} \\
\underline{144} \\
12 \overline{)\,24\,(\,2} \\
\underline{24} \\
\times
\end{array}
$$

∴ H.C.R. of 180 and 336 = 12

Products of numbers LCM of 180 and 336= $\frac{Products\ of\ numbers}{their\ H.C.F.}$

$= \frac{180 \times 336}{12} = 15 \times 336 = 5040$

Question 15.

Can two numbers have 15 as their HCF and 110 as their LCM ? Give reason to justify your answer.

Solution:

On dividing 110 by 15, we get

$$\begin{array}{r} 7 \\ 15\overline{)\,110} \\ \underline{105} \\ 5 \end{array}$$

7 as quotient and 5 as remainder

We find that the remainder $\neq 0$

So 110 is not exactly divisible by 15

$\therefore$ HCF and L.C.M. of two numbers cannot be 15 and 110 respectively.

As LCM of two numbers is always exactly divisible by their HCF.

Objective Type Questions

Mental Maths

Question 1.

Fill in the blanks:

(i) The only natural number which has exactly one factor is …….

(ii) The only prime number which is even is…….

(iii) The HCF of two co-prime numbers is…….

(iv) Two perfect numbers are…… and…….

(v) The only prime-triplet is 3, 5, 7.

Question 2.

State whether the following statements are true (T) or false (F):

(i) Every natural number has a finite number of factors.

(ii) Every natural number has an infinite number of its multiples.

(iii) There are infinitely many prime numbers.

(iv) If two numbers are separately divisible by a number, then their difference is also divisible by that number.

(v) LCM of two prime numbers equals their product.

(vi) LCM of two co-prime numbers equals their product.

Solution:

(i) Every natural number has a finite number of factors. **True**

(ii) Every natural number has an infinite number of its multiples. **True**

(iii) There are infinitely many prime numbers. **True**

(iv) If two numbers are separately divisible by a number, then their difference is also divisible by that number. **True**

(v) LCM of two prime numbers equals their product. **True**

(vi) LCM of two co-prime numbers equals their product. **True**

Question 3.

State whether the following statements are ture or false. If a statement is false, justify your answer.

(i) The sum of two prime numbers is always an even number.

(ii) The sum of two prime numbers is always a prime number.

(iii) The sum of two prime numbers can never be a prime number

(iv) No odd number can be written as the sum of two prime numbers.

(v) If two numbers are co-prime, then atleast one of them must be prime.

(vi) If a number is divisible by 18, it must be divisible by 3 and 6 both.

(vii) If a number is divisible by 2 and 4 both, it must be divisible by 8.

(viii) If a number is divisible by 3 and 6 both, it must be divisible by 18.

(ix) HCF of an even number and an odd number is always 1.

Solution:

(i) The sum of two prime numbers is always an even number.

False

Correct:

2 and 7 both are prime numbers nut their sum = 2 + 7 =9, which is an odd number.

(ii) The sum of two prime numbers is always a prime number. False

Correct:

3 and 5 both are prime numbers but their sum = 3 + 5 = 8, which is a composite number.

(iii) The sum of two prime numbers can never be a prime number

False

Correct:

2 and 5 both are prime numbers but their sum = 2 + 5 = 7, which is a prime number.

(iv) No odd number can be written as the sum of two prime numbers.

False

Correct:

13 is an odd number and 13 = 2 + 11, Which is the sum of two prime numbers.

(v) If two numbers are co-prime, then atleast one of them must be prime.

False

Correct:

8 and 15 are co - prime numbers but neither 8 is prime nor 15 is prime.

(vi) If a number is divisible by 18, it must be divisible by 3 and 6 both.

True

(vii) If a number is divisible by 2 and 4 both, it must be divisible by 8.

False

Correct:

20 is divisible by 2 and 4 both but 20 is not divisible by 8.

(viii) If a number is divisible by 3 and 6 both, it must be divisible by 18.

False

Correct: 12 is divisible by 3 and 6 both but 12 is not divisible by 18.

(ix) HCF of an even number and an odd number is always 1.

False

Correct: 6 is even and 9 is odd but HCF of 6 and 9 is 3.

Multiple Choice Questions

Choose the correct answer from the given four options (4 to 28) :

Question 4.

All factors of 6 are

(a) 1, 6

(b) 2, 3

(c) 1, 2, 3

(d) 1, 2, 3, 6

Solution :

The factors of 6 are 1, 2, 3, 6(d)

Question 5.

Which of the following is an odd composite number ?

(a) 7

(b) 9

(c) 11

(d) 12

Solution:

9, is an odd composite number.(c)

Question 6.

The number of even numbers between 68 and 90 is

(a) 10

(b) 11

(c) 12

(d) 31

Solution:

The even numbers between 68 and 90 is 70, 72, 74, 76, 78, 80, 82, 84, 86, 88 = 10 numbers (a)

Question 7.

Which of the following is a prime number ?

(a) 69

(b) 87

(c) 91

(d) 97

Solution:

Since, the factors of 97 are 1 and 97

97 is a prime number. (d)

Question 8.

Which of the following is a pair of twin-prime number ?

(a) 19, 21

(b) 43, 47

(c) 59, 61

(d) 73, 79

Solution:

59, 61

Pairs of prime numbers whose difference is 2 are called twin-prime numbers. (c)

Question 9.

The number of distinct prime factors of the largest 4-digit number is

(a) 2

(b) 3

(c) 5

(d) none of these

Solution:

Largest 4 digit number = 9999

3	9999
3	3333
10	101
	1

3 is prime factor.(b)

Question 10.

The number of distinct prime factors of the smallest 5-digit number is

(a) 2

(b) 4

(c) 6

(d) 8

Solution:

Smallest 5-digit number = 10000

2	10000
2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

Number of distinct prime factors of smallest 5-digit number=2(a)

Question 11.

The sum of the prime factors of 1729 is

(a) 13

(b) 19

(c) 32

(d) 39

Solution:

7	1729
13	247
19	19
	1

Prime factors of 1729 are 7, 13 and 19 Sum of prime factors

= 7 + 13 + 19

= 39(d)

Question 12.

Which of the following is a pair of co-prime numbers ?

(a) 8, 45

(b) 3,18

(c) 5, 35

(d) 6, 39

Solution:

8, 45

The factors of 8 are 1, 2, 4, 8 The factors of 15 are 1, 3, 5, 15 The common factors of 8 and 15 is 1 They are co-prime. (a)

Question 13.

Every natural number has an infinte number of

(a) prime factors

(b) factors

(c) multiples

(d) none of these

Solution:

Multiples

e.g. Multiples of 2 = 2, 4, 6, 8, 10, 12, 14, 16,…….(c)

Question 14.

Which of the following numbers is divisible by 4?

(a) 308594

(b) 506784

(c) 732106

(d) 9301538

Solution:

(b) 506784

Because the number formes by tens and ones digits is divisible by 4 i.e.

$84 \div 4 = 21$(b)

Question 15.

Which of the following numbers is divisible by 8?

(a) 503786

(b) 505268

(c) 305678

(d) 703568

Solution:

(d) 703568

Because the number formed by hundred, tens and ones digit is divisible by 8.

i.e. $568 - 8 = 71$(d)

Question 16.

Which of the following numbers is divisible by 3?

(a) 50762

(b) 42063

(c) 52871

(d) 37036

Solution:

(b) 42063

Because sum of its digits is = 4 + 2 +0 + 6 + 3 = 15

Which is divisible by 3(b)

Question 17.

Which of the following numbers is divisible by 9 ?

(a) 972063

(b) 730542

(c) 785423

(d) 5612844

Solution:

(a) 972063

Because sum of digits

= 9 + 7 + 2 + 0 + 6 + 3 = 27 which is divisible by 9(a)

Question 18.

Which of the following numbers is divisible by 6 ?

(a) 560324

(b) 650374

(c) 798653

(d) 750972

Solution:

(d) 750972

Because sum of its digit

= 7 + 5 + 0 + 9 + 7 + 2 = 30 Which is divisible by 3.

Hence it is divisible by 6.(d)

Question 19.

The digit by which '*' should be replaced in 54* 281 so that the number formed is divisible by 9 is

(a) 6

(b) 7

(c) 8

(d) 9

Solution:

For a number to be divisible by 9, sum of its digits should be divisible by 9.

Sum of given digits in 54*281

= 5 + 4 + 2 + 8 + 1 = 20.

If we add 7, it becomes 27, which is divisible by 9.

∴ * is to be replaced by 7.(b)

Question 20.

The digit by which should be replaced in 7254*98 so that the number formed is divisible by 22 is

(a) 0

(b) 1

(c) 2

(d) 6

Solution:

For a number to be divisible by 22, sum of its digits should be divisible by 2 and by 11.

Since, the last digit of 7254*98 is 8, which is divisible by 2.

Now,

Sum of the digits at odd places = 7 + 5 + 8 = 20

Sum of the digits at even places = 9 + 4 +12 = 15

∴ Their Difference = 20 – 15 = 5

Since, 5 is not divisible by 11, so to make a number divisible by 11 we must add 6.

∴ * is to be replaced by 6(d).

Question 21.

If a number is divisible by 5 and 6 both, then it may not be divisible by

(a) 10

(b) 15

(c) 30

(d) 60

Solution:

60 (d)

Question 22.

The number of common prime factors of 60, 75 and 105 is

(a) 2

(b) 3

(c) 4

(d) 5

Solution:

60, 75 and 105

2	60, 70, 105
2	30, 75, 105
2	15, 75, 105
5	5, 25, 35
5	1, 5, 7
5	1, 1, 7
	1, 1, 1

$= 2 \times 2 \times 3 \times 5 \times 5 \times 7 = 2(a)$

Question 23.

The H.C.F. of 144 and 198 is

(a) 6

(b) 9

(c) 12

(d) 18

Solution :

H.C.f. of 144 = $2 \times 2 \times 2 \times 2 \times 3 \times 3$

H.C.F. od 198 = $2 \times 3 \times 3 \times 11$

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	198
3	99
3	33
11	11
	1

$\therefore$ H.C.F. of 144 and 198

$\therefore = 2 \times 3 \times 3 = 18$(d)

Question 24.

The L.C.M. of 30 and 45 is

(a) 15

(b) 30

(c) 45

(d) 90

Solution:

L.C.M. of 30 and 45 is 90.

3	30, 45
5	10, 15
2	2, 3
3	1, 3
	1, 1

$\therefore$ L.C.M. $= 3 \times 5 \times 2 \times 3 = 90$(d)

Question 25.

The L.C.M. of 4 and 44 is

(a) 4

(b) 11

(c) 44

(d) 176

Solution:

LCM of 4 and 44 is 44

4	44
11	11
	1

∴ L.C.M. = 4 × 11 = 44(c)

Question 26.

The LCM of 7 and 13 is 1

(a) 1

(b) 7

(c) 13

(d) 91

Solution:

LCM of 7 and 13 is

L.C.M. = 13 × 7 = 91(d)

Question 27.

If H.C.F. of two numbers is 15 and their product is 1575, then their L.C.M. is

(a) 15

(b) 105

(c) 525

(d) 1575

Solution:

Product of number = 1575

H.C.F. = 15

We know,

L.C.M. = Product of numbers H.C.F.

$= \frac{1575}{15} = 105$(b)

Question 28.

If the LCM of two natural numbers is 180, then which of the following is not the HCf of the numbers ?

(a) 45

(b) 60

(c) 75

(d) 90

Solution:

L.C.M. of 2 natural numbers = 180

We know that,

L.C.M. of 2 numbers is always exactly divisible by their H.C.F.

∴ Taking (a) 45 as H.C.F.

$$\begin{array}{r} 45\overline{)\ 180\ (\ 4} \\ \underline{180} \\ 0 \end{array}$$

Here remainder = 0

∴ 45 is H.C.F.

Now, taking (b) 60 as H.C.F.

$$60\overline{)\ 180\ }(3$$
$$\underline{180}$$
$$0$$

Here remainder $= 0$

$\therefore$ 60 is also H.C.F.

Now, taking 75 as H.C.F.

$$75\overline{)\ 180\ }(2$$
$$\underline{-150}$$
$$30$$

Here remainder = 30

i.e. remainder $\neq 0$

Hence, 75 is not the H.C.F. of two natural numbers whose L.C.M. is 180

Hence, answer is (c).

Value Based Questions

Question 1.

To teach the value of gratitude and appreciation to the students, a school organised a ‘Card Making’ activity in which the students were asked to make “THANK YOU CARDS” for the people who helped them in some way.

Assorted cards were made with different titles. Their numbers are given below:

T cards for teachers = 120

F cards for friends = 540

S cards for servants = 90

P cards for parents = 240 and

G cards for grandparents = 150

(i) Find the HCF and LCM of all the different number of cards.

(ii) Find HCF and LCM of maximum and minimum number of cards.

(iii) Is the number of T-cards is a factor of number of P-cards ?

Solution:

(i) HCF of 120, 540, 90, 240, 150

$$
\begin{array}{r}
120 \overline{)\,540\,(\,4} \\
\underline{480} \\
60 \overline{)\,120\,(\,2} \\
\underline{120} \\
\times
\end{array}
$$

$$
\begin{array}{r}
60 \overline{)\,90\,(\,1} \\
\underline{60} \\
30 \overline{)\,240\,(\,8} \\
\underline{240} \\
\times
\end{array}
$$

$$30 \overline{) 150} (5$$
$$- 150$$
$$\times$$

∴ HCF = 30

Now, LCM of 120, 540, 90, 240 and 150 is

2	120, 540, 90, 240, 150
2	60, 270, 45, 120, 75
2	30, 135, 45, 60, 75
2	15, 135, 45, 30, 75
3	15, 135, 45, 15, 75
3	5, 45, 15, 5, 25
3	5, 15, 5, 5, 25
5	5, 5, 5, 5, 25
5	1, 1, 1, 1, 5
	1, 1, 1, 1, 1

∴ LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$
= 10800

(ii) Maximum number of cards = 540

Minimum number of cards = 90

∴ HCF is as follow :

$$90 \overline{)540(} \quad 6$$

$$\underline{-\ 540}$$

$$\underline{\times}$$

Hence, HCF of 90 and 540 is 90

LCM of 90 and 540 is as follow:

2	90, 540
2	45, 270
3	45, 135
3	15, 45
3	5, 15
5	5, 5
	1, 1

LCM = $2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$

(iii) Yes.

T cards = 120

P cards = 240

240 = 120 × 2

Higher Order Thinking Skills (HOTS)

Question 1.

Write 2-digit odd numbers whose sum of digits is 8.

Solution: 17, 71, 35, 53

Question 2.

Write all paird of 2-digit twin primes such that on changing the places of their digits, they still remain prime numbers.

Solution:

11, 13, 71, 73

Question 3.

They are just four natural numbes less than 100, which have exactly three factors. One of them is 25, what are the other three ? What can be said about these numbers ?

Solution:

Four natural numbers less than 100 which have three factors :

One of them is 25 = 1, 5, 25

Second is 49 = 1, 7, 49

Third is 9 = 1, 3, 9

Fourth is 4 = 1, 2, 4

Check Your Progress

Question 1.

Write all factors of:

(i) 88

(ii) 105

(iii) 96

Solution:

(i) 88 = { 1, 2, 4, 8, 11, 22, 44, 88}

(ii) 105 = { 1, 3, 5, 7, 15, 21, 35, 105}

(iii) 96 = { 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96}

Question 2.

Find the common multiples of 8 and 12.

Solution:

The multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104.

The multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108

The common multiples of 8 and 12 are 24, 48, 72, 96

Question 3.

Which of the following pairs of numbers are co-prime ?

(i) 25 and 105

(ii) 59 and 97

(iii) 161 and 192

Solution:

(i) 25 and 105

The factors of 25 are 1, 5, 25

The factors of 105 are 1, 3, 5, 7, 15, 21, 35, 105

The common factors of 25 and 105 are 1, 5

∴ They are not co-prime.

(ii) 59 and 97

The factors of 59 are 1, 59

The factors of 97 are 1, 97

The common factors of 59 and 97 is 1.

∴ They are co-prime.

(iii) 161 and 192

The factors of 161 are 1, 161

The factors of 192 are 1, 97

The common factors of 59 and 97 is 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192

The common factors of 161, 192 is 1

∴ They are co-prime.

Question 4.

Using divisibility tests, determine which of the following numbers are divisible by 4, 6, 8, 9 or 11:

(i) 197244

(ii) 613440

(iii) 4100448

Solution:

197244:

divisible by 4, 6, 9.

It is divisible by 4 as last two digits is divisible by 4.

It is divisible by 6 as last digit of given number is divisible 2 and their sum is also divisible by 3.

It is not divisible by 8 as the sum of last three digit 2 + 4 + 4 = 10, is not divisible by 8.

It is divisible by 9 as the sum of its digits 27 is divisible by 9.

It is not divisible by 11 as the difference of the sum of alternate number

1 + 7 + 4 = 12 and 9 + 2 + 4 = 15, (15 - 12)

= 3 is not divisible by 11.

∴ 197244 is divisible by 4, 6 and 9.

(ii) 613440 : divisible by 4, 6, 8, 9.

It is divisible by 4 as last two digit is divisible by 4.

It is divisible by 6 as the sum of all digits (6 + 1 + 3 + 4 +4 + 0)

= 18 is divisible by 3 and by 2 also as last digit is 0.

It is divisible by 8 as the sum of last three digits (4 + 4 + 0) = 8 is divisible 8.

It is also divisible by 9 as the sum of its digits 18 is divisible by 9.

It is not divisible by 11 as the difference of the sum of alternate number 6 + 3 +4 =13 and 1 + 4 +0 = 5, (13- 5)

= 8 is not divisible by 11.

613440 is divisible by 4, 6, 8 and 9.

(iii) 4100448: divisible by 4, 6, 8 ,11.

It is divisible by 4 as last two digit is divisible by 4.

It is divisible by 6 as the sum of all digits 4 + 1 + 0 + 0 + 4 + 4 + 8 = 21

is divisible by 3 and also last digit is divisible by 2.

It is divisible by 8 as the sum of last three digits 4 + 4 + 8 = 16 is divisible by 8.

It is not divisible by 9 as the sum of its digit 21 is not divisible by 9.

It is divisible by 11 as the difference of the sum of alternate nmber 4 + 0 + 4 + 8 =16 and 1+ 0 + 4 + 5, (16 – 6) = 11

Which is divisible by 11.

∴ 4100448 is divisible by 4, 6, 8, 11.

Question 5.

In 92*389, replace * by a digit so that the number formed is divisible by 11.

Solution:

The given number is 92*389

Here, * occur at odd place.

Sum of digits at odd place = 9 + 8 = 17(Except*)

Sum of digits at even place = 2 + 3 + 9 = 14

Their difference = 17 -14 = 3

If '*' is replaced by 8, then sum of digits at odd place = 9 + 8 + 8 =25

Their difference (Sum of digits at odd places – Sum of digits at evn places)

= 25 – 14 = 11

Which is divisible by 11

∴ '*' is to be replaced by the digit 8.

Question 6.

Find the prime factorisation of the following numbers:

(i) 168

(ii) 2304

Solution:

(i) 168

2	168
2	84
2	42
3	21
7	7
	1

$= 2 \times 2 \times 2 \times 3 \times 7$

(ii) 2304

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
2	9
3	3
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Question 7.

Find the G.C.D. of the given numbers by prime factorisation method:

(i) 24, 45

(ii) 180, 252, 324

Solution:

(i) 24, 45

$24 = 2 \times 2 \times 2 \times 3$

$45 = 3 \times 3 \times 5$

The greatest common factor is 3.
G.C.D = 3

2	24
2	12
2	6
3	3
	1

3	45
3	15
5	5
	1

(ii) 180, 252, 324

$180 = 2 \times 2 \times 3 \times 3 \times 5$

2	180
2	90
3	45
5	15
	1

$252 = 2 \times 2 \times 3 \times 7$

2	252
2	126
3	63
3	21
7	7
	1

$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

2	324
2	162
3	81
3	27
3	9
3	3
	1

G.C.D. $= 2 \times 2 \times 3 \times 3 = 36$

We notice that 2 and 3 both occurs as the common factor in the given numbers two time each.

Question 8.

Find the H.C.F. of the given numbers by division method.

(i) 54, 82

(ii) 84, 120, 156

Solution:

(i) 54, 82

H.C.F. = 2

54) 82 (1
54
28) 54 (1
28
26) 28 (1
26
2) 26 (13
26
×

(ii) 84, 120, 156

Solution:

84) 156 (1
84
72) 84 (1
72
12) 72 (6
72
×

12) 120 (10
12
×

Question 9.

Find the L.C.M. of the given numbers by prime factorisation method.

(i) 27, 90

(ii) 36, 48, 210

Solution:

(i) 27, 90

Solution:

(i) 27, 90

$= 2 \times 3 \times 3 \times 3 \times 5 = 270$

(ii) 36, 48, 210

$= 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 5 \times 7 = 5040$

2	36, 48, 210
2	18, 24, 105
3	9, 12, 105
3	3, 4, 35
2	1, 4, 35
2	1, 2, 35
5	1, 1, 35
7	1, 1, 7
	1, 1, 1

Question 10.

Find the L.C.M. of the given numbers by division method:

(i) 48, 60

(ii) 112, 168, 266

Solution:

(i) 48, 60

$= 2 \times 2 \times 3 \times 4 \times 5 = 240$

2	48, 60
2	24, 30
3	12, 15
4	4, 5
5	2, 5
5	1, 5
	1, 1

(ii) 112, 168, 266

$= 2 \times 2 \times 2 \times 7 \times 2 \times 3 \times 19 = 6384$

2	112, 168, 266
2	56, 84, 133
3	28, 42, 133
2	14, 21, 133
7	7, 21, 133
2	1, 3, 19
5	1, 1, 19
	1, 1, 1

Question 11.

Find the greatest numberwhich divides 2706, 7041 and 8250 leaving remainder 6,21 and 42 respectively.

Solution:

When 2706 is divided by the required number, 6 is left as a remainder.

So, 2706 – 6 = 2700 i.e. 2700 is exactly divisible by that number.

Similarly, 7041 – 21 = 7020 is exactly divisible by that number.

Similarly, also, 8250 – 42 - 8208 is exactly divisible by that number.

Therefore, 2700, 7020 and 8208 are divisible by that number.

Thus, the required number is the H.C.F. of 2700, 7020 and 8208.

First, we find H.C.F. of 2700 and 7020

$$
\begin{array}{r|l}
2700 & 7020 \;(\; 2 \\
 & 5400 \\
\hline
1620 & 2700 \;(\; 1 \\
 & 1620 \\
\hline
1080 & 1620 \;(\; 1 \\
 & 1080 \\
\hline
540 & 1080 \;(\; 2 \\
 & 1080 \\
\hline
 & 0
\end{array}
$$

Now find of H.C.F. 540 and 8208

$$
\begin{array}{r|l}
540 & 8208 \;(\; 15 \\
 & 8100 \\
\hline
108 & 540 \;(\; 5 \\
 & 540 \\
\hline
 & 0
\end{array}
$$

∴ The H.C.F. of 2700, 7020 and 8208 is 108.
Hence the required number is 108

Question 12.

Find the least number which on decreasing by 20 is exactly divisible by 18, 21, 28 and 30.

Solution:

First, we find the least number which is exactly divisible by the numbers 18, 21, 28 and 30. For this, we find the L.C.M. of 18, 21, 28 and 30.

2	18, 21, 28, 30
2	9, 21, 14, 15
3	9, 21, 7, 15
3	3, 7, 7, 5
5	1, 7, 7, 5
7	1,7, 7, 1
	1,1, 1, 1

$\therefore$ L.C.M. $= 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$

According to given, the required number will be 20 more than 1260.
The required number = 1260 + 20 = 1280

Question 13.
There are three heaps of rice weighing 120 kg, 144kg and 204 kg.
Find the maximum capacity of a bag so that the rice of each heap can be packed in exact number of bags.

Solution:
Weights of three heaps = 120kg, 144kg and 204 kg

```
120 ) 144 ( 1
      120
      ------
       24 ) 120 ( 5
            120
            ------
              ×
            ------
24  ) 204 ( 9
      192
      ------
       12 ) 24 ( 2
            24
            ------
              ×
            ------
```

∴ Maximum capacity of a bag, which exactly divides the heaps in exact number HCF of 120, 144, 204 = 12
∴ Required capacity of bag = 12kg

Question 14.

Three bells are ringing continuously at intervals of 30, 36 and 45 minutes respectively. At what time will they ring together again if they ring simultaneously at 8 a.m.

Solution:

L.C.M. = $2 \times 3 \times 3 \times 2 \times 5 = 180$

After 180 minute at 11:00 a.m.

2	30, 36, 45
3	15, 18, 45
3	5, 6, 15
2	5, 2, 5
5	5, 1, 5
	1, 1, 1

Question 15.

Two numbers are co-prime and their L.C.M. is 4940. If one of the numbers is 65, find the other number.

Solution:

One number = 65

and let the other number = x

We know that,

Two numbers are co-prime if their HCF is 1

Now, H.C.F. × L.C.M. of two numbers = Product of given two numbers

$= 1 \times 4940 = 65 \times x$

$\Rightarrow 4940 = 65 \times x$

$\Rightarrow 65 \times x = 4940$

$\Rightarrow x = 4940 \div 65 = 76$

$$\begin{array}{r|l|l} 65 & 4940 & 76 \\ & 455 & \\ \hline & \ \ 390 & \\ & \ \ 390 & \\ \hline \end{array}$$

$\therefore$ The other number is 76.