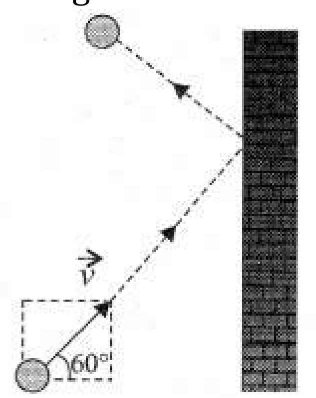
Chapter – 9

Kinetic Theory of Gases

Multiple Choice Questions

Question 1.
A particle of mass m is moving with speed u in a direction which makes 60° with respect to x axis. It undergoes elastic collision with the wall. What is the change in momentum in x and y direction?



(a) $\Delta p_x = -mu, \Delta p_y = 0$

(b) $\Delta p_x = -2mu, \Delta p_y = 0$

(c) $\Delta p_x = 0, \Delta p_y = mu$

(d) $\Delta p_x = mu, \Delta p_y = 0$

Answer:
$$\Delta p_x = -2mu, \Delta p_y = 0$$

Solution:
The change in momentum of the molecule in x direction

Δp_x = Final momentum – Initial momentum ,

= After collision – Before collision

= – mu – mu = – 2mu

The change in momentum of the molecule in Y direction Δp_y = 0

Question 2.
A sample of ideal gas is at equilibrium. Which of the following quantity is zero?
(a) rms speed
(b) average speed
(c) average velocity
(d) most probable speed

Answer:
(c) average velocity

Question 3.
An ideal gas is maintained at constant pressure. If the temperature of an ideal gas increases from 100K to 1000K then the rms speed of the gas molecules
(a) increases by 5 times
(b) increases by √10 times
(c) remains same
(d) increases by 7 times

Answer:
(b) increases by √10 times

Solution:

$$v_{rms} \propto \sqrt{T}$$

$$\frac{v_{rms_2}}{v_{rms_1}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1000}{100}} = \frac{\sqrt{10}}{\sqrt{1}}$$

Question 4.
Two identically sized rooms A and B are. connected by an open door. If the room A is air conditioned such that its temperature is 4° lesser than room B, which room has more air in it?
(a) Room A
(b) RoomB
(c) Both room has same air
(d) Cannot be determined

Answer:
(a) Room A

Question 5.
The average translational kinetic energy of gas molecules depends on ……
(a) number of moles and T
(b) only on T
(c) P and T
(d) P only

Answer:
(a) number of moles and T

Question 6.
If the internal energy of an ideal gas U and volume V are doubled then the pressure ……
(a) doubles
(b) remains same
(c) halves
(d) quadruples

Answer:
(b) remains same

Solution:

The internal energy of the gas $U = \frac{3}{2}PV$

U and V are doubled, So $2U = \frac{3}{2}P(2V)$

$$U = \frac{3}{2}PV$$

Question 7.
The ratio $\gamma = \frac{C_P}{C_V}$ for a gas mixture consisting of 8 g of helium and 16 g of oxygen is …… [Physics Olympiad – 2005]
(a) 23/15
(b) 15/23
(c) 27/17

(d) 17/27

Answer:
(c) 27/17

Solution:

$$\text{Number of moles of helium } (n) = \frac{8}{4} = 2$$

$$\text{Number of moles of Oxygen } (n') = \frac{16}{32} = \frac{1}{2}$$

In helium atom, $C_V = \frac{3}{2}R$

Oxygen atom, $C'_V = \frac{5}{2}R$

$$C_V\,(\text{mixture}) = \frac{nC_V + n'C'_V}{n+n'} = \frac{\left(2\times\frac{3}{2}R\right)+\left(\frac{1}{2}\times\frac{5}{2}R\right)}{\left(2+\frac{1}{2}\right)} = \frac{17}{10}R$$

$$\gamma\,(\text{mixture}) = \frac{C_P}{C_V} = 1 + \frac{R}{C_V(\text{mix})} = 1 + \frac{R}{\left(\frac{17}{10}R\right)}$$

$$\gamma\,(\text{mixture}) = \frac{27}{17}$$

Question 8.
A container has one mole of monoatomic ideal gas. Each molecule has /degrees of freedom.

What is the ratio of $\gamma = \frac{C_P}{C_V} = ?$

(a) f (b) $\frac{f}{2}$ (c) $\frac{f}{f+2}$ (d) $\frac{f+2}{f}$

Answer:

(d) $\frac{f+2}{f}$

Solution:

Energy associated with 1 mole of gas, $U = \frac{f}{2}RT$

$$C_V = \frac{f}{2}R$$

$$C_P = \left(1 + \frac{f}{2}\right)R$$

$$\gamma = \frac{C_P}{C_V} + \frac{\left(1+\frac{f}{2}\right)}{\frac{f}{2}} = \frac{\left(\frac{2+f}{2}\right)}{\left(\frac{f}{2}\right)}\ ; \gamma = \frac{f+2}{f}$$

Question 9.
If the temperature and pressure of a gas is doubled the mean free path of the gas molecules
(a) remains same
(b) doubled
(c) tripled
(d) quadruples

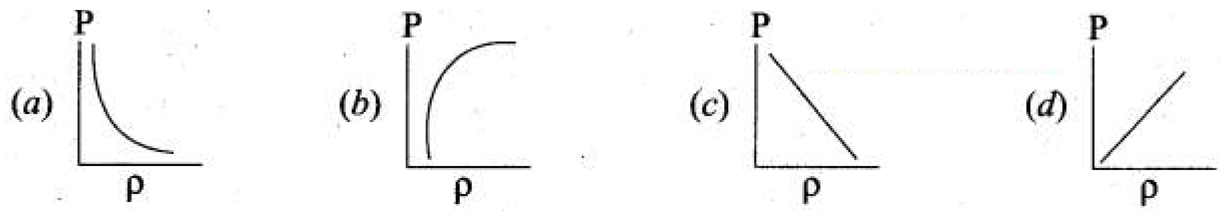
Answer:
(a) remains same

Solution:
Mean free path of the molecule

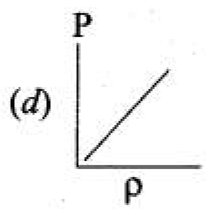
$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$$

If T and P are doubled, $\lambda = \frac{k(2T)}{\sqrt{2}\pi d^2 (2P)} = \frac{kT}{\sqrt{2}\pi d^2 P}$

Question 10.
Which of the following shows the correct relationship between the pressure and density of an ideal gas at constant temperature?



Answer:



Question 11.
A sample of gas consists of μ_1 moles of monoatomic molecules, μ_2 moles of diatomic molecules and μ_3 moles of linear triatomic molecules. The gas is kept at high temperature. What is the total number of degrees of freedom?
(a) $[3\mu_1 + 7(\mu_2 + \mu_3)]\ N_A$
(b) $[3\mu_1 + 7\mu_2 + 6\mu_3]\ N_A$
(c) $[7\mu_1 + 3(\mu_2 + \mu_3)]\ N_A$
(d) $[3\mu_1 + 6(\mu_2 + \mu_3)]\ N_A$

Answer:
(a) $[3\mu_1 + 7(\mu_2 + \mu_3)]\ N_A$

Question 12.
If S_P and S_V denote the specific heats of nitrogen gas per unit mass at constant pressure and constant volume respectively, then …….. [JEE 2007]
(a) $S_P - S_V = 28\ R$
(b) $S_P - S_V = R/28$
(c) $S_P - S_V = R/14$
(d) $S_P - S_V = R$

Answer:
(b) $S_P - S_V = R/28$

Question 13.
Which of the following gases will have least rms speed at a given temperature?

(a) Hydrogen
(b) Nitrogen
(c) Oxygen
(d) Carbon dioxide

Answer:
(d) Carbon dioxide

Question 14.
For a given gas molecule at a fixed temperature, the area under the Maxwell-Boltzmann distribution curve is equal to

(a) $\frac{PV}{kT}$ (b) $\frac{kT}{PV}$ (c) $\frac{P}{NkT}$ (d) PV

Answer:
(a) PV/kT

Question 15.
The following graph represents the pressure versus number density for ideal gas at two different temperatures T_1 and T_2. The graph implies ....
(a) $T_1 = T_2$
(b) $T_1 > T_2$
(c) $T_1 < T_2$
(d) Cannot be determined

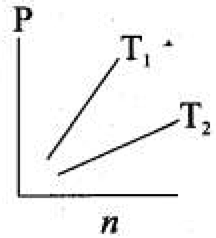
Answer:
(b) $T_1 > T_2$

Short Answer Questions

Question 1.
What is the microscopic origin of pressure?

Answer:
According to the kinetic theory of a gases, the pressure exerted by the molecules depends on
(i) Number density $n = N/V$
(ii) Mass of the molecule

(iii) Mean square speed



The pressure is given by $P = \frac{1}{3} nm\overline{v}^2$ (or) $P = \frac{1}{3}\frac{N}{V} m\overline{v}^2$

Question 2.
What is the microscopic origin of temperature?

Answer:
The average K.E per molecule $\overline{\mathrm{KE}} = \epsilon = \frac{3}{2} k\mathrm{T}$

The equation implies that the temperature of a gas is a measure of the average translational K.E. per molecule of the gas.

Question 3.
Why moon has no atmosphere?

Answer:
Moon has no atmosphere. The escape speed of gases on the surface of Moon is much less than the root mean square speeds of gases due to low gravity. Due to this all the gases escape from the surface of the Moon.

Question 4.
Write the expression for rms speed, average speed and most probable speed of a gas molecule.

Answer:

Root mean square speed : (v_{rms})

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$$

Average Speed: ($\overline{v}$)

$$\overline{v} = \frac{v_1 + v_2 + v_3 + ... v_n}{N}$$

$$\overline{v} = \sqrt{\frac{8\,RT}{\pi M}} = \sqrt{\frac{8\,kT}{\pi m}} = 1.60\sqrt{\frac{kT}{m}}$$

Most probable speed: (v_{mp})

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$$

Question 5.
What is the relation between the average kinetic energy and pressure?

Answer:

The internal energy of the gas U = $\frac{3}{2}NkT$

The above equation can also be written as U = $\frac{3}{2}PV$

since PV = NkT

$$P = \frac{2}{3}\frac{U}{V} = \frac{2}{3}u \qquad ...(1)$$

From the equation (1), we can state that the pressure of the gas is equal to two thirds of internal energy per unit volume or internal energy density $\left(u = \frac{U}{V}\right)$

Writing pressure in terms of mean kinetic energy density

$$P = P = \frac{1}{3}nm\overline{v^2} = \frac{1}{3}\rho\overline{v^2} \qquad ..(2)$$

where ρ = nm = mass density (Note n is number density)
Multiply and divide R.H. S of equation (2) by 2, we get

$$P = \frac{2}{3}\left(\frac{\rho}{2}\overline{v^2}\right) \quad ...(3)$$

From the equation (3), pressure is equal to 2/3 of mean kinetic energy per unit volume.

Question 6.
Define the term degrees of freedom.

Answer:
The minimum number of independent coordinates needed to specify the position and configuration of a thermo-dynamical system in space is called the degree of freedom of the system.

Question 7.
State the law of equipartition of energy.

Answer:
According to kinetic theory, the average kinetic energy of system of molecules in thermal equilibrium at temperature T is uniformly distributed to all degrees of freedom (x or y or z directions of motion) so that each degree of freedom will get 1/2 kT of energy. This is called law of equipartition of energy.

Question 8.
Define mean free path and write down its expression.

Answer:
The average distance travelled by the molecule between collisions is called mean free path (λ).

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$$

Question 9.
Deduce Chailes' law based on kinetic theory.

Answer:
Charles' law: From PV = 2/3 U. For a fixed pressure, the volume of the gas is proportional to internal energy of the gas or average kinetic energy of the gas

and the average kinetic energy is directly proportional to absolute temperature. It implies that

$$V \propto T \text{ or } \frac{V}{T} = \text{constant}$$

This is Charles' law.

Question 10.
Deduce Boyle's law based on kinetic theory.

Answer:

Boyle's law: From PV = $\frac{2}{3}U$

But the internal energy of an ideal gas is equal to N times the average kinetic energy (ε) of each molecule. U = Nε. For a fixed temperature, the average translational kinetic energy ε will remain constant. It implies that

$$PV = \frac{2}{3} N\epsilon$$

Thus PV = constant Therefore, pressure of a given gas is inversely proportional to its volume provided the temperature remains constant. This is Boyle's law.

Question 11.
Deduce Avogadro's law based on kinetic theory.

Answer:
Avogadro's law: This law states that at constant temperature and pressure, equal volumes of all gases contain the same number of molecules. For two different gases at the same temperature and pressure, according to kinetic theory of gases,

From $$P = \frac{1}{3}\frac{N}{V} m\overline{v^2}$$

$$P = \frac{1}{3}\frac{N_1}{V} m_1 \overline{v_1^2} = \frac{1}{3}\frac{N_2}{V} m_2 \overline{v_2^2} \quad \text{...(1)}$$

At the same temperature, average kinetic energy per molecule is the same for two gases.

$$\frac{1}{2}m_1\overline{v_1^2} = \frac{1}{2}m_2\overline{v_2^2} \quad ...(2)$$

Dividing the equation (1) by (2) we get $N_1 = N_2$
This is Avogadro's law. It is sometimes referred to as Avogadro's hypothesis or Avogadro's Principle.

Question 12.
List the factors affecting the mean free path.

Answer:

1. Mean free path increases with increasing temperature. As the temperature increases, the average speed of each molecule will increase. It is the reason why the smell of hot sizzling food reaches several metre away than smell of cold food.

2. Mean free path increases with decreasing pressure of the gas and diameter of the gas molecules.

Question 13.
What is the reason for Brownian motion?

Answer:
According to kinetic theory, any particle suspended in a liquid or gas is continuously bombarded from all the directions so that the mean free path is almost negligible. This leads to the motion of the particles in a random and zig-zag manner.

Long Answer Questions

Question 1.
Write down the postulates of kinetic theory of gases.

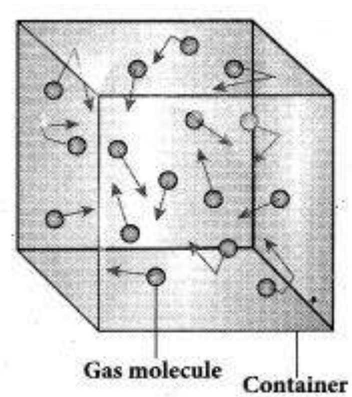
Answer:

1. All the molecules of a gas are identical, elastic spheres.
2. The molecules of different gases are different.

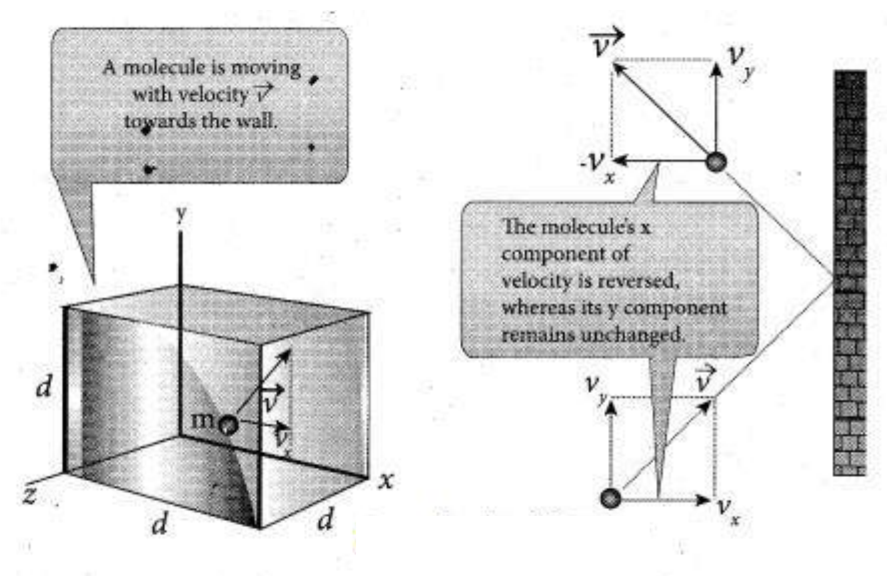
3. The number of molecules in a gas is very large and the average separation between them is larger than size of the gas molecules.
4. The molecules of a gas are in a state of continuous random motion.
5. The molecules collide with one another and also with the walls of the container.
6. These collisions are perfectly elastic so that there is no loss of kinetic energy during collisions.
7. Between two successive collisions, a molecule moves with uniform velocity.
8. The molecules do not exert any force of attraction or repulsion on each other except during collision. The molecules do not possess any potential energy and the energy is wholly kinetic.
9. The collisions are instantaneous. The time spent by a molecule in each collision is very small compared to the time elapsed between two consecutive collisions.
10. These molecules obey Newton's laws of motion even though they move randomly.

Question 2.
Derive the expression of pressure exerted by the gas on the walls of the container.

Answer:
Expression for pressure exerted by a gas: Consider a monoatomic gas of N molecules each having a mass m inside a cubical container of side l.
The molecules of the gas are in random motion. They collide with each other and also with the walls of the container. As the collisions are elastic in nature, there is no loss of energy, but a change in momentum occurs.



Container of gas molecules



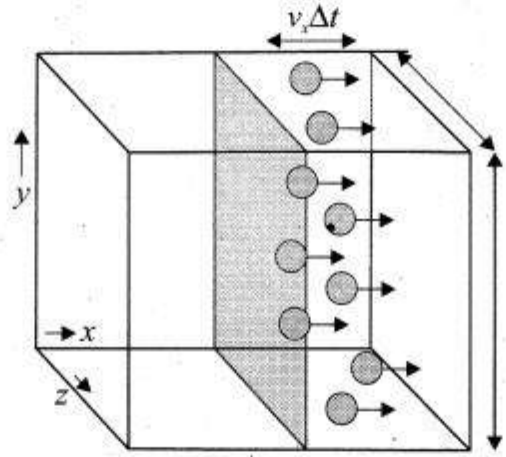
Collision of a molecule with the wall

The molecules of the gas exert pressure on the walls of the container due to collision on it. During each collision, the molecules impart certain momentum to the wall. Due to transfer of momentum, the walls experience a continuous force. The force experienced per unit area of the walls of the container determines the pressure exerted by the gas. It is essential to determine the total momentum transferred by the molecules in a short interval of time.

A molecule of mass m moving with a velocity $\vec{v}$ having components (v_x, v_y, v_z) hits the right-side wall. Since we have assumed that the collision is elastic, the particle rebounds with same speed and its x-component is reversed. This is shown in the figure. The components of velocity of the molecule after collision are (- v_x, v_y, v_z).

The x-component of momentum of the molecule before collision = mv_x
The x-component of momentum of the molecule after collision = – mv_x
The change in momentum of the molecule in x direction = Final momentum – initial momentum = – mv_x – mv_x = – $2mv_x$
According to law of conservation of linear momentum, the change in momentum of the wall = $2mv_x$
The number of molecules hitting the right side wall in a small interval of time At.
The molecules within the distance of $v_x\Delta t$ from the right side wall and moving towards' the right will hit the wall in the time interval Δt. The number of molecules that will hit the right-side wall in a time interval At is equal to the product of volume ($Av_x\Delta t$) and number density of the molecules (n). Here A is

area of the wall and n is number of molecules per unit volume N/V.
We have assumed that the number density is the same throughout the cube.



Number of molecules hitting the wall

Not all the n molecules will move to the right, therefore on an average only half of the n molecules move to the right and the other half moves towards left side.
The number of molecules that hit the right side wall in a time interval Δt

$$= \frac{n}{2} A v_x \Delta t \qquad ...(1)$$

In the same interval of time At, the total momentum transferred by the molecules

$$\Delta P = \frac{n}{2} A v_x \Delta t \times 2mv_x = A v_x^2\ mn\Delta t \qquad ...(2)$$

From Newton's second law, the change in momentum in a small interval of time gives rise to force.

The force exerted by the molecules on the wall (in magnitude)

$$F = \frac{\Delta P}{\Delta t} = nmAv_x^2 \qquad ...(3)$$

Pressure, P = force divided by the area of the wall

$$P = \frac{F}{A} = nmv_x^2 \qquad ...(4)$$

Since all the molecules are moving completely in random manner, they do not have same speed. So we can replace the term v_x^2 by the average $\overline{v_x^2}$ in equation (4).

$$P = nm\overline{v_x^2} \quad ...(5)$$

Since the gas is assumed to move in random direction, it has no preferred direction of motion (the effect of gravity on the molecules is neglected). It implies that the molecule has same average speed in all the three direction. So, $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$. The mean square speed is written as

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3\overline{v_x^2}$$

$$\overline{v_x^2} = \frac{1}{3}\overline{v^2}$$

Using this in equation (5), we get

$$P = \frac{1}{3}nm\overline{v^2} \text{ or } P = \frac{1}{3}\frac{N}{V}m\overline{v^2} \quad ...(6)$$

as $\left[n = \frac{N}{V}\right]$

Question 3.

Explain in detail the kinetic interpretation of temperature.

Answer:

To understand the microscopic origin of temperature in the same way, from pressure exerted by a gas,

$$P = \frac{1}{3}\frac{N}{V}m\overline{v^2}$$

$$PV = \frac{1}{3}Nm\overline{v^2} \quad ...(1)$$

Comparing the equation (1) with ideal gas equation PV = NkT,

$$NkT = \frac{1}{3}Nm\overline{v^2}$$

$$kT = \frac{1}{3}m\overline{v^2} \quad ...(2)$$

Multiply the above equation by 3/2 on both sides,

$$\frac{3}{2}kT = \frac{1}{2}m\overline{v^2} \quad ...(3)$$

R.H.S. of the equation (3) is called average kinetic energy of a single molecule $(\overline{KE})$.
The average kinetic energy per molecule

$$\overline{KE} = \epsilon = \frac{3}{2}kT \qquad ...(4)$$

Equation (3) implies that the temperature, of a gas is a measure of the average translational kinetic energy per molecule of the gas.
Equation (4) is a very important result from kinetic theory of gas. We can infer the following from this equation.

(i) The average kinetic energy of the molecule is directly proportional to absolute temperature of the gas. The equation (3) gives the connection between the macroscopic world (temperature) to microscopic world (motion of molecules).

(ii) The average kinetic energy of each molecule depends only on temperature of the gas hot on mass of the molecule. In other words, if the temperature of an ideal gas is measured using thermometer, the average kinetic energy of each molecule can be calculated without seeing the molecule through naked eye.

By multiplying the total number of gas molecules with average kinetic energy of each molecule, the internal energy of the gas is obtained.

Internal energy of ideal gas $U = N\left(\frac{1}{2}m\overline{v^2}\right)$

By using equation (3), $U = \frac{3}{2}NkT \qquad ...(5)$

Here, we understand that the internal energy of an ideal gas depends only on absolute temperature and is independent of pressure and volume.

Question 4.
Describe the total degrees of freedom for monoatomie molecule, diatomic molecule and triatomic molecule,
Monoatomie molecule: A monoatomie molecule by virtue of its nature has only three translational degrees of freedom.
Therefore f = 3

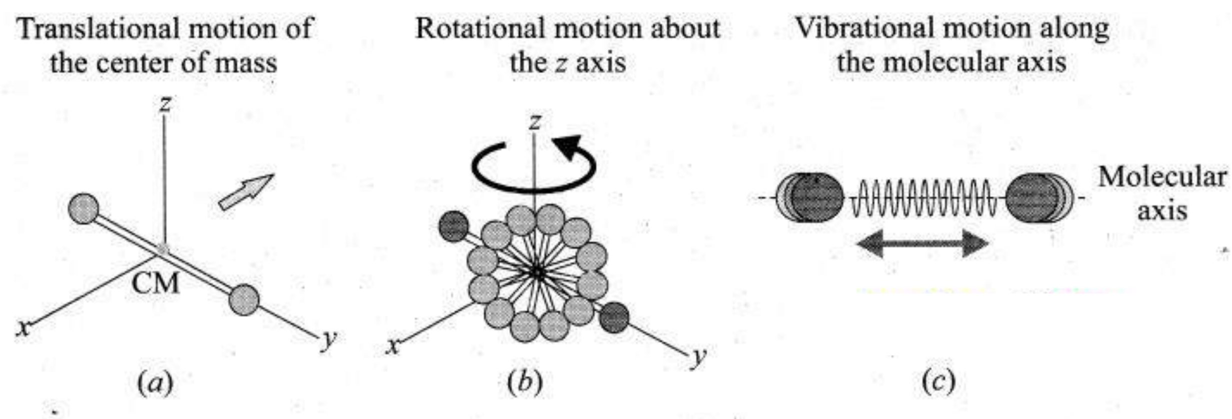
Example: Helium, Neon, Argon

Diatomic molecule: There are two cases.

(i) At Normal temperature: A molecule of a diatomic gas consists of two atoms bound to each other by a force of attraction. Physically the molecule can be regarded as a system of two point masses fixed at the ends of a massless elastic spring. The center of mass lies in the center of the diatomic molecule. So, the motion of the center of mass requires three translational degrees of freedom (figure a). In addition, the diatomic molecule can rotate about three mutually perpendicular axes (figure b). But the moment of inertia about its own axis of rotation is negligible. Therefore, it has only two rotational degrees of freedom (one rotation is about Z axis and another rotation is about Y axis). Therefore totally there are five degrees of freedom.
f = 5

2. At High Temperature: At a very high temperature such as 5000 K, the diatomic molecules possess additional two degrees of freedom due to vibrational motion [one due to kinetic energy of vibration and the other is due to potential energy] (figure c). So totally there are seven degrees of freedom.
f = 7

Examples: Hydrogen, Nitrogen, Oxygen.



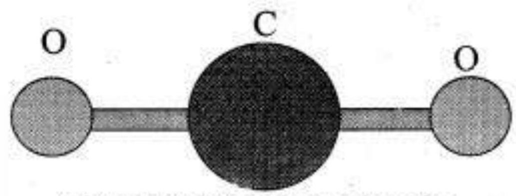
Degree of freedom of diatomic molecule

Triatomic molecules: There are two cases.

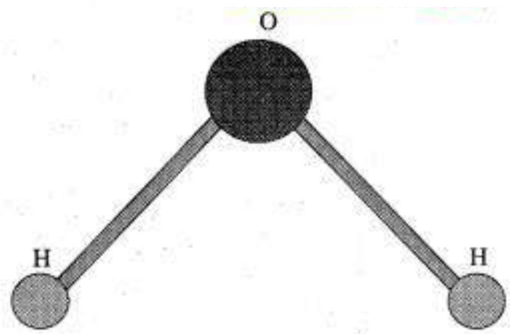
Linear triatomic molecule:
In this type, two atoms lie on either side of the central atom. Linear triatomic molecule has three translational degrees of freedom. It has two rotational degrees of freedom because it is similar to diatomic molecule except there is

an additional atom at the center. At normal temperature, linear triatomic molecule will have five degrees of freedom. At high temperature it has two additional vibrational degrees of freedom. So a linear triatomic molecule has seven degrees of freedom.

Example: Carbon dioxide.



A linear triatomic molecule



A non-linear triatomic molecule

Non-linear triatomic molecule: In this case, the three atoms lie at the vertices of a triangle.

It has three translational degrees of freedom and three rotational degrees of freedom about three mutually orthogonal axes. The total degrees of freedom. $f = 6$
Example: Water, Sulphurdioxide.

Question 5.
Derive the ratio of two specific heat capacities of monoatomic, diatomic and triatomic molecules.

Answer:
Application of law of equipartition energy in specific heat of a gas, Meyer's relation $C_P - C_V = R$ connects the two specific heats for one mole of an ideal gas.

Equipartition law of energy is used to calculate the value of $C_P - C_V$ and the ratio between them $\gamma = \frac{C_P}{C_V}$. Here γ is called adiabatic exponent.

(*i*) **Monoatomic molecule:** Average kinetic energy of a molecule = $\left[\frac{3}{2}kT\right]$

Total energy of a mole of gas $= \frac{3}{2}kT \times N_A = \frac{3}{2}RT$

For one mole, the molar specific heat at constant volume

$$C_V = \frac{dU}{dT} = \frac{d}{dT}\left[\frac{3}{2}RT\right] ; C_V = \left[\frac{3}{2}R\right]$$

$$C_P = C_V + R = \frac{3}{2}R + R = \frac{5}{2}R$$

The ratio of specific heats, $\gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3} = 1.67$

(*ii*) **Diatomic molecule:** Average kinetic energy of a diatomic molecule at low temperature $= \frac{5}{2}kT$

Total energy of one mole of gas $= \frac{5}{2}kT \times N_A = \frac{5}{2}RT$

(Here, the total energy is purely kinetic)

For one mole Specific heat at constant volume

$$C_V = \frac{dU}{dT} = \left[\frac{5}{2}RT\right] = \frac{5}{2}R$$

But, $\quad C_P = C_V + R = \frac{5}{2}R + R = \frac{7}{2}R$

$$\therefore \quad \gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5} = 1.40$$

Energy of a diatomic molecule at high temperature is equal to $\frac{7}{2}RT$

$$C_V = \frac{dU}{dT} = \frac{d}{dT}\left[\frac{7}{2}RT\right] = \frac{7}{2}R$$

$\therefore C_P = C_V + R = \frac{7}{2}R + R \; ; \quad C_P = \frac{9}{2}R$

Note that the C_V and C_P are higher for diatomic molecules than the mono atomic molecules. It implies that to increase the temperature of diatomic gas molecules by 1°C it require more heat energy than monoatomic molecules.

$$\therefore \qquad \gamma = \frac{C_P}{C_V} = \frac{\frac{9}{2}R}{\frac{7}{2}R} = \frac{9}{7} = 1.28$$

(iii) Triatomic molecule

(a) Linear molecule:

$$\text{Energy of one mole} = \frac{7}{2}kT \times N_A = \frac{7}{2}RT$$

$$C_V = \frac{dU}{dT} = \frac{d}{dT}\left[\frac{7}{2}RT\right]; C_V = \frac{7}{2}R$$

$$C_P = C_V + R = \frac{7}{2}R + R = \frac{9R}{2}$$

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{9}{2}R}{\frac{7}{2}R} = \frac{9}{7} = 1.28$$

(b) Non-linear molecule:

$$\text{Energy of a mole} = \frac{6}{2}kT \times N_A = \frac{6}{2}RT = 3RT$$

$$C_V = \frac{dU}{dT} = 3R$$

$$C_P = C_V + R = 3R + R = 4R$$

$$\therefore \qquad \gamma = \frac{C_P}{C_V} = \frac{4R}{3R} = \frac{4}{3} = 1.33$$

Note that according to kinetic theory model of gases the specific heat capacity at constant volume and constant pressure are independent of temperature. But in reality it is not sure.

The specific heat capacity varies with the temperature.

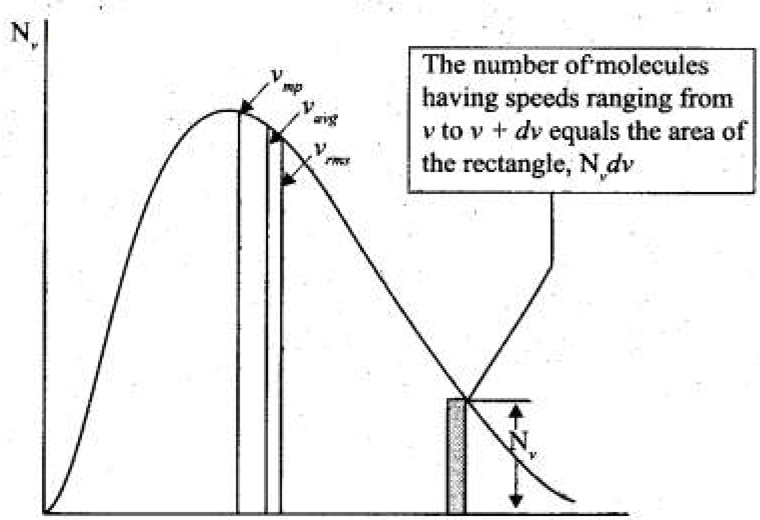
Question 6.
Explain in detail the Maxwell Boltzmann distribution function.

Answer: Maxwell-Boltzmann: In speed distribution function

Consider an atmosphere, the air molecules are moving in random directions. The speed of each molecule is not the same even though macroscopic parameters like temperature and pressure are fixed. Each molecule collides with every other molecule and they exchange their speed. In the previously we calculated the rms speed of each molecule and not the speed of each molecule which is rather difficult. In this scenario we can find the number of gas molecules that move with the speed of 5 ms^{-1} to 10 ms^{-1} or 10 ms^{-1} to 15 ms^{-1} etc. In general our interest is to find how many gas molecules have the range of speed from v to v + dv. This is given by Maxwell's speed distribution function.

$$N_v = 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}}$$

The above expression is graphically shown as follows:

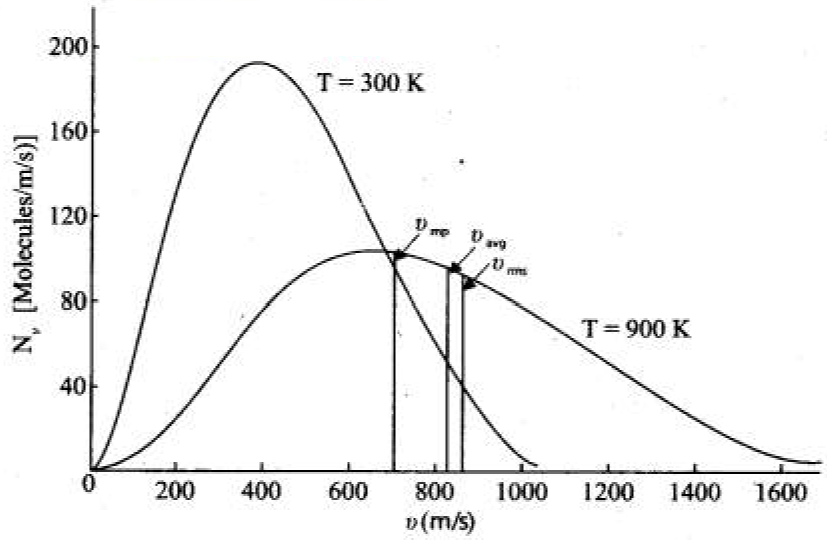


Maxwell's molecules speed distribution

From the above figure, it is clear that, for a given temperature the number of molecules having lower speed increases parabolically but decreases exponentially after reaching most probable speed. The ms speed, average speed and most probable speed are indicated in the figure. It can be seen that the rms speed is greatest among the three.

(i) The area under the graph will give the total number of gas molecules in the system

(ii) Figure 2 shows the speed distribution graph for two different temperatures. As temperature increases, the peak of the curve is shifted to the right. It implies that the average speed of each molecule will increase. But the area under each graph is same since it represents the total number of gas molecules.



Maxwell distribution graph for two different temperatures

Question 7.
Derive the expression for mean free path of the gas.

Answer:
Expression for mean free path
We know from postulates of kinetic theory that the molecules of a gas are in random motion and they collide with each other. Between two successive collisions, a molecule moves along a straight path with uniform velocity. This path is called mean free path. Consider a system of molecules each with diameter d. Let n be the number of molecules per unit volume. Assume that only one molecule is in motion and all others are at rest.

If a molecule moves with average speed v in a time t, the distance travelled is vt. In this time t, consider the molecule to move in an imaginary cylinder of volume $\pi d^2 vt$. It collides with any molecule whose center is within this cylinder. Therefore, the number of collisions is equal to the number of molecules in the volume of the imaginary cylinder. It is equal to $\pi d^2 vtn$. The total path length divided by the number of collisions in time t is the mean free path.

Mean free path, $\lambda = \dfrac{\text{Distance travelled}}{\text{Number of collisions}}$

$$\lambda = \frac{vt}{n\pi d^2 vt} = \frac{1}{n\pi d^2} \qquad ...(1)$$

Though we have assumed that only one molecule is moving at a time and other molecules are at rest, in actual practice all the molecules are in random motion. So the average relative speed of one molecule with respect to other molecules has to be taken into account. After some detailed calculations the correct expression for mean free path

$$\therefore \lambda = \frac{1}{\sqrt{2} n\pi d^2} \qquad ...(2)$$

The equation (2) implies that the mean free path is inversely proportional to number density. When the number density increases the molecular collisions increases so it decreases the distance travelled by the molecule before collisions.

Case 1: Rearranging the equation (2) using 'm' (mass of the molecule)

$$\therefore \lambda = \frac{m}{\sqrt{2}\pi d^2 mn}$$

But mn = mass per unit volume = ρ (density of the gas)

$$\therefore \lambda = \frac{m}{\sqrt{2}\pi d^2 \rho} \qquad ...(3)$$

Also we know that $PV = NkT$

$$P = \frac{N}{V}kT = nkT \Rightarrow n = \frac{P}{kT}$$

substituting $n = \dfrac{P}{kT}$ in equation (2), we get

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P} \qquad ...(4)$$

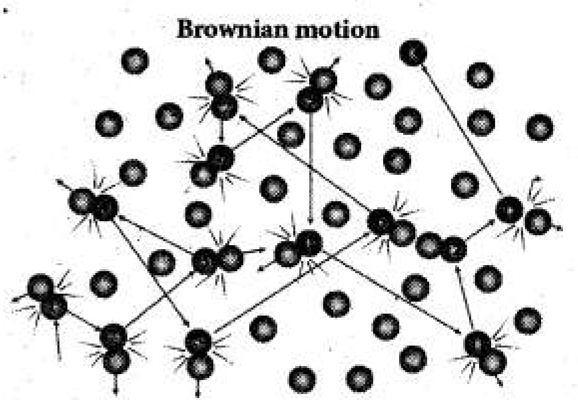
The equation (4) implies the following:

(i) Mean free path increases with increasing temperature. As the temperature increases, the average speed of each molecule will increase. It is the reason why the smell of hot sizzling food reaches several meter away than smell of cold food.

(ii) Mean free path increases with decreasing pressure of the gas and diameter of the gas molecules.

Question 8.
Describe the Brownian motion.

Answer:
Brownian motion is due to the bombardment of Brownian motion suspended particles by molecules of the surrounding fluid. But during 19th century people did not accept that every matter is made up of small atoms or molecules. In the year 1905, Einstein gave systematic theory of Brownian motion based on kinetic theory and he deduced the average size of molecules.



According to kinetic theory, any particle suspended – in a liquid or gas is continuously bombarded from all the directions so that the mean free path is almost negligible. This leads to the motion of the particles in a random and zig-zag manner. But when we put our hand in water it causes no random motion because the mass of our hand is so large that the momentum transferred by the molecular collision is not enough to move our hand.

Factors affecting Brownian Motion:

1. Brownian motion increases with increasing temperature.
2. Brownian motion decreases with bigger particle size, high viscosity and density of the liquid (or) gas.

Numerical Problems

Question 1.
A fresh air is composed of nitrogen N_2 (78%) and oxygen O_2 (21%). Find the rms speed of N_2 and O_2 at 20°C.

Answer:
Absolute temperature T = 20°C + 273 = 293K
Gas constant R = 8.32 J mol^{-1} K^{-1}

For, Nitrogen (N_2),
Molar mass (M) = 28 g per mol = 28×10^{-3} kg/mol

$$\therefore \quad v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.32 \times 293}{28 \times 10^{-3}}} = \sqrt{\frac{7313.28}{28 \times 10^{-3}}}$$

$$(v_{rms})_{N_2} = 511 \text{ ms}^{-1}$$

For, Oxygen (O_2),
Molar mass (M) = 32 g per mol = 32×10^{-3} kg/mol

$$\therefore \quad v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.32 \times 293}{32 \times 10^{-3}}} = \sqrt{\frac{7313.28}{32 \times 10^{-3}}}$$

$$(v_{rms})_{O_2} = 478 \text{ ms}^{-1}$$

Question 2.
If the rms speed of methane gas in the Jupiter's atmosphere is 471.8 m s^{-1}, show that the surface temperature of Jupiter is sub-zero.

Answer:
RMS speed of methane gas (v_{rms}) = 471.8 ms^{-1}
Molar mass of methane gas (M) = 16.04 g per mol

$$M = 16.04 \times 10^{-3} \text{ kg/mol}$$

Gas constant R = 8.31 J mol^{-1} K^{-1}

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$(v_{rms})^2 = \frac{3RT}{M}$$

$$T = \frac{(v_{rms})^2 \times M}{3R} = \frac{(471.8)^2 \times 16.04 \times 10^{-3}}{3 \times 8.31}$$

$$= \frac{3.57 \times 10^6 \times 10^{-3}}{24.93} = 0.143 \times 10^3$$

$$T = 143\ K - 273$$

$$T = -130°C$$

Question 3.
Calculate the temperature at which the rms velocity of a gas triples its value at S.T.P. (Standard temperature $T_1 = 273$ K)

Answer:
At STP temperature $T_1 = 273$ K

RMS velocity of a gas, $(v_{rms})_1 = v$

New RMS velocity of a gas, at temperature (T_2)

$$(v_{rms})_2 = 3v$$

New temperature $T_2 = ?$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{T_1}{T_2}}$$

$$T_2 = \left(\frac{(v_{rms})_2}{(v_{rms})_1}\right)^2 \times T_1 = \left(\frac{3v}{v}\right)^2 \times 273$$

$$T_2 = 9 \times 273 = 2457\ K$$

Question 4.
A gas is at temperature 80°C and pressure $5 \times 10^{-10} Nm^{-2}$. What is the number of molecules per m^3 if Boltzmann's constant is $1.38 \times 10^{-23} JK^{-1}$.

Answer:

Temperature of a gas (T) = 80°C + 273 = 353 K

Pressure of a gas (P) = 5×10^{-10} Nm^{-2}

Boltzmann's constant (K_B) = 1.38×10^{-23} J K^{-1}

$V = 1m^3$

$$\text{Number of molecules, } n = \frac{PV}{kT} = \frac{5 \times 10^{-10} \times 1}{1.38 \times 10^{-23} \times 353}$$

$$= \frac{5 \times 10^{-10}}{487.14 \times 10^{-23}} = 0.01026 \times 10^{13}$$

$n = 1.02 \times 10^{11}$

Question 5.
From kinetic theory of gases, show that Moon cannot have an atmosphere (Assume k = 1.38×10^{-23} J K^{-1} Temperature T = 0°C = 273 K).

Answer:
At absolute temperature T = 0°C = 273 K

Boltzmann's Constant $k_B = 1.38 \times 10^{-23}$ J K^{-1}

$m = 3.33 \times 10^{-27}$ kg (mass of hydrogen molecule)

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 273}{3.33 \times 10^{-27}}}$$

$$= \sqrt{\frac{1130.22 \times 10^{-23}}{3.33 \times 10^{-27}}} = 1842.11$$

$$v_{rms} = 1.84 \times 10^3 \text{ ms}^{-1}$$

Question 6.
If 10^{20} oxygen molecules per second strike 4 cm^2 of wall at an angle of 30° with the normal when moving at a speed of 2×10^3 ms^{-1}, find the pressure exerted on the wall.
(mass of 1 atom = 2.67×10^{-26} kg).

Answer:

$$\text{Mass of } O_2 \text{ atom} = 16 \times \text{mass of 1 atom}$$
$$= 16 \times 10^{20} \times 2.67 \times 10^{-26}; \; m = 42.72 \times 10^{-6} \text{ kg}$$
$$\text{Momentum of the } O_2 \text{ molecule (P)} = mv = 42.72 \times 10^{-6} \times 2 \times 10^3$$
$$P = 85.44 \times 10^{-3} \text{ kg ms}^{-1}$$

Momentum normal to the wall at angle 30°

$$= 85.44 \times 10^{-3} \times \cos 30° = 73.99104 \times 10^{-3} \text{ kg ms}^{-1}$$

$$\text{Pressure} = \frac{F}{A} = \frac{\text{Change in momentum}}{\text{Area}} = \frac{73.99104 \times 10^{-3}}{(4 \times 10^{-2})^2}$$

$$\frac{73.99104 \times 10^{-3} \times 10^4}{16} = 4.62444 \times 10$$
$$P = 46.2 \text{ Nm}^{-2}$$

Question 7.

During an adiabatic process, the pressure of a mixture of monoatomic and diatomic gases is found to be proportional to the cube of the temperature. Find the value of $\gamma = (C_p/C_V)$

Answer:

In adiabatic process, $T^{\gamma}P^{1-\gamma} = \text{constant}$

$$P \propto T^3$$
$$PT^{-3} = \text{constant} \quad \text{...(1)}$$
$$PT^{\frac{\gamma}{1-\gamma}} = \text{constant} \quad \text{...(2)}$$
$$PT^{\frac{\gamma}{1-\gamma}} = PT^{-3}$$

Comparing the powers, $\frac{\gamma}{1-\gamma} = -3 = -3 + 3\gamma$

$$2\gamma = 3 \; ; \gamma = \frac{3}{2}$$

Question 8.

Calculate the mean free path of air molecules at STP. The diameter of N_2 and O_2 is about 3×10^{-10} m.

Answer:

$P = 1\text{ atm} = 1.01 \times 10^5\text{ Pa}, k_B = 1.38 \times 10^{-23}\text{ JK}^{-1}, T = 273\text{ K}$

From ideal gas law, $n = \dfrac{P}{kT}$

$$n = \frac{1.01 \times 10^5}{1.38 \times 10^{-23} \times 273} = \frac{1.01 \times 10^5 \times 10^{23}}{376.74} = 2.68 \times 10^{-3} \times 10^{28}$$

$$n = 2.68 \times 10^{25}\text{ molecules/m}^3$$

Mean free path of the air molecule,

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{1}{1.414 \times 3.14 \times 2.68 \times 10^{25} \times (3 \times 10^{-10})^2}$$

$$= \frac{1}{1.0709 \times 10^{-18} \times 10^{25}} = 0.9338 \times 10^{-7}$$

$$\lambda = 9.3 \times 10^{-8}\text{ m}$$

Question 9.
A gas made of a mixture of 2 moles of oxygen and 4 moles of argon at temperature T. Calculate the energy of the gas in terms of RT. Neglect the vibrational modes.

Answer:
For two moles of diatomic nitrogen with no vibrational mode,

$$U_1 = 2 \times \frac{5}{2}RT = 5\text{ RT}$$

For four mole of monatomic argon,

$$U_2 = 4 \times \frac{3}{2}RT = 6\text{ RT}$$

Total energy of the gas, $U = U_1 + U_2 = 5\text{ RT} + 6\text{ RT}$
$U = 11\text{ RT}$

Question 10.
Estimate the total number of air molecules in a room of capacity 25 m^3 at a temperature of 27°C.

Answer:

$T = 27°C + 273 = 300\ K$, $k_B = 1.38 \times 10^{-23}\ JK^{-1}$, $V = 25\ m^3$

As Boltzmann's Constant, $k_B = \frac{R}{N} \Rightarrow R = k_B N$

Now, $PV = nRT = nk_B NT$

The number of molecules in the room,

$$nN = \frac{PV}{k_B T} = \frac{1.013 \times 10^5 \times 25}{1.38 \times 10^{-23} \times 300}$$

$$= \frac{25.325 \times 10^5}{414 \times 10^{-23}} = 0.06117 \times 10^{28}$$

$nN = 6.117 \times 10^{26}$ molecules