7. Integration

Objective Type Questions

Q.1. Choose the correct answer –

(1) $\left[\sqrt{x}+\frac{1}{\sqrt{x}}\right]$ **is counter derivative of–**

(a) $\frac{1}{3}x^{1/3}+2x^{1/2}+c$

(b) $\frac{2}{3}x^{2/3}+\frac{1}{2}x^{2}+c$

(c) $\frac{2}{3}x^{3/2}+2x^{1/2}+c$

(d) $\frac{3}{2}x^{3/2}+\frac{1}{2}x^{1/2}+c$

(2) **If** $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$ **in which** $f(2)=0$ **then –**

(a) $x^4+\frac{1}{x^3}-\frac{129}{8}$ (b) $x^3+\frac{1}{x^4}-\frac{129}{8}$

(c) $x^4+\frac{1}{x^{43}}-\frac{129}{8}$ (d) $x^3+\frac{1}{x^4}-\frac{129}{8}$

(3) **Value of** $\int x^2+ex^{-1}dx$ **is –**

(a) $\frac{1}{3}e^{x3}+c$ (b) $\frac{1}{3}e^{x2}+c$

(c) $\frac{1}{2}e^{x3}+c$ (d) $\frac{1}{2}e^{x2}+c$

(4) $\int e^x\sec x(1+\tan x)\,dx$ **is equal to–**

(a) $e^x\cos x+c$ (b) $e^x\sec x+c$

(c) $e^x\sin x+c$ (d) $e^x\tan x+c$

(5) $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ is equal to –

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

(6) $\int_0^{2/3} \frac{dx}{4+9x^2}$ is equal to –

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{24}$ (d) $\frac{\pi}{4}$

Ans. (1) (a) (2) (a) (3) (a) (4) (b) (5) (d) (6) (c).

Q.2. Fill in the blanks–

(1) If f(x) = $\int_0^x t\sin at$ then $f^1(x)$ is ....

(2) The value of Integral $\int_{1/3}^{1} \frac{(x-x^2)^{1/3}}{x^4}dx$ ....

(3) The value of $\int_{-\pi/2}^{\pi/2} (x^3+\cos x+\tan^5 x+1)\cdot$ dx is ....

(4) The value of $\int_0^{\frac{\pi}{2}}\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ log...dx is ....

(5) $\int\frac{dx}{e^x+e^{-x}}dx$ is equal to ....

Ans. (1) xsinx (2) 4 (3) π (4) 0 (5) $\tan(e^x)+c$.

Q.3. Answer in one word/sentence –

(1) If f(a + b – x) = f(x) then $\int_a^b xf(x)dx$ is equal to –

(2) $\int\frac{dx}{x^2+2x+2}$ is equal to –

(3) $\int\frac{dx}{\sqrt{9x-4x^2}}$ is equal to –

Ans. (1) $\frac{a+b}{2}\int_a^b f(x)dx$ (2) $\tan^{-1}(x+1)+c$ (3) $\frac{1}{2}$

Q.4. (A) Match the pairs–

(1) $\int\frac{10x^9+10^x\log 10dx}{x^{10}+10}$	(a) $\tan x-\cot t+C$
(2) $\int\frac{dx}{\sin^2x\cos^2x}$	(b) $\log(10^x+x^{10})+C$
(3) $\int\frac{\sin^2x-\cos^2x}{\sin^2x\cos^2x}dx$	(c) $\tan(xe^x)+C$
(4) $\int\frac{e^x(1+x)}{\cos^2(e^x x)}dx$	(d) $\tan x+\cot x+C$
(5) $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	(e) 1
(6) $\int_0^1 ex\,dx$	(f) $\frac{\pi}{4}$

Ans. (1) (b) (2) (a) (3) (d) (4) (c) (5) (f) (6) (e).

Q.4. (B) Match the Pair –

(1) $\int\frac{dx}{\sqrt{x^2-a^2}}$	(a) $\frac{1}{a}\tan^{-1}\frac{x}{a}+c$
(2) $\int\frac{dx}{\sqrt{a^2-x^2}}$	(b) $\frac{1}{2a}\log\left[\frac{a+x}{a-x}\right]+c$
(3) $\int\frac{dx}{x^2+a^2}$	(c) $\sin^{-1}\frac{x}{a}+c$
(4) $\int\frac{dx}{x^2+a^2}$	(d) $\log\left\|x+\sqrt{x^2-a^2}\right\|+c$
(5) $\int\sqrt{x^2+a^2}$	(e) $\frac{1}{2}x\sqrt{a^2-a^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$
(6) $\int\sqrt{a^2-x^2}$	(f) $\frac{1}{2}x\sqrt{x^2}+a^2+\log\left\|x+\sqrt{x^2-a^2}\right\|+C$

Ans. (1) (d) (2) (c) (3) (a) (4) (b) (5) (f) (6) (e).

Q.4. (C) Match the Pair –

(1) $\int_{-2}^{1}\frac{\|x\|}{x}dx$	(a) 1
(2) $\int_0^\infty e^{-x}dx$	(b) –1
(3) $\int\sqrt{x^2-a^2}\,dx$	(c) log : $\left\|x+\sqrt{x^2-a^2}\right\|+$
(4) $\int\frac{dx}{\sqrt{x^2+a^2}}$	(d) $\frac{1}{2a}\log\left\|\frac{x-a}{x-a}\right\|+c$
(5) $\int\frac{dx}{x^2-a^2}$	(e) $\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left\|x+\sqrt{x^2-a^2}\right\|+c$
(6) $\int\tan x$	(f) log sec x

Ans. (1) (b) (2) (a) (3) (e) (4) (c) (5) (d) (6) (f).

Q.4. (D) Match the Pair–

(1) $\int \frac{\cos 2x}{(\sin + \cos x)^2} dx$	(a) $\log\left\|\frac{(x+2)}{x-1}\right\| + c$
(2) $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$	(b) $\log\|\sin x + \cos\| + c$
(3) $\int \sqrt{1+x^2}\ dx$	(c) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7} + 9\log\left\|x+4+\sqrt{x^2-8x+7}\right\| + C$
(4) $\int \sqrt{x^2-8x+7}\ dx$	(d) $\log\|x\| - \frac{1}{2}\log(x^2+1)$
(5) $\int \frac{xdx}{(x-1)(x-2)}$	(e) 0
(6) $\int \frac{dx}{x(x^2+1)}$	(f) $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left\|x+\sqrt{1+x^2}+C\right\|$

Ans. (1) (b) (2) (e) (3) (f) (4) (c) (5) (a) (6) (d).

Q.4. (E) Match the Pair –

(1) $\int \log x\, dx$	(a) $\frac{\pi}{12}$
(2) $\int \cot x\, dx$	(b) 0
(3) $\int_0^{\pi} \cos x\, dx$	(c) $\tan(e^x)$
(4) $\int_0^{\frac{\pi}{2}} \log \sin x\, dx$	(d) $x \log - x$
(5) $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$	(e) $-\frac{\pi}{2}\log 2$
(6) $\int \frac{dx}{e^x+e^{-x}}$	(f) $\log \sin x$

Ans. (1) (d) (2) (f) (3) (b) (4) (e) (5) (a) (6) (c).

Very Short Answer Type Questions

Q.5. Fine the value of $\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos x^5}$

Solution : $\int_0^{\pi/2} \frac{\cos^2 x}{\sin^5 x + \cos^5 x} dx$

let $I = \int_0^{\pi/2} \frac{\cos^2 x}{\sin^5 x + \cos^5 x} dx$

$$I = \int_0^{\pi/2} \frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right) + \cos^5\left(\frac{\pi}{2}-x\right)} dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\pi/2} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} dx \quad ...(2)$$

By adding eqn. (1) and (2)

$$2I = \int_0^{\pi/2} \frac{\cos^5 x + \sin^5 x}{\cos^5 x + \sin^5 x} dx$$

$$= \int_0^{\pi/2} 1 dx$$

$$= [x]_0^{\pi/2} = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4} \quad \textbf{Ans.}$$

Q.6. Find the value of: $\int_0^{\pi/2} \log \sin x dx$

Solution : $I = \int_0^{\pi/2} \log \sin x\, dx \quad ...(1)$

$$= \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2}-x\right) dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\pi/2} \log \cos x dx \quad ...(2)$$

By adding eqn. 1 and 2

$$\therefore \quad 2I = \int_0^{\pi/2} \log \sin x dx + \int_0^{\pi/2} \log \cos x\, dx$$

$$= \int_0^{\pi/2} (\log \sin x + \log \cos x) dx$$

$$= \int_0^{\pi/2} (\log \sin x \cos x) dx$$

$$= \int_0^{\pi/2} \log\left(\frac{2 \sin x \cos x}{2}\right) dx$$

$$= \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right) dx$$

$$= \int_0^{\pi/2} \log \sin 2x\, dx - \int_0^{\pi/2} \log 2 dx$$

$$= \int_0^{\pi/2} \log\sin 2x\,dx - \log 2[x]_0^{\pi/2}$$

$\therefore \quad 2I = I_1 = \frac{\pi}{2}\log 2$

Where $I_1 = \int_0^{\pi/2} \log\sin 2x\,dx$

Let $2x = 1 \Rightarrow 2dx = dt \Rightarrow dx = \frac{dt}{2}$

If $x = 0$ then $t = 0$ and

if $x = \frac{\pi}{2}$ then $t = \pi$

$$I_1 = \frac{1}{2}\int_0^{\pi} \log\sin t\,dt$$

$$= \frac{1}{2} \times 2 \int_0^{\pi/2} \log\sin t\,dt$$

$$= \int_0^{\pi/2} \log\sin t\,dt$$

$$= \int_0^{\pi/2} \log\sin x\,dx = I$$

$$\left[\because \int_a^b f(x)dx = f(t)dr\right]$$

$$2I = I_1 - \frac{\pi}{2}\log 2$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = -\frac{\pi}{2}\log 2 \text{ or } I = \frac{\pi}{2}\log 2\frac{1}{2}$$

Q.7. Find the value of: $\int_1^2 |x^2 - x|\,dx$ –

Solution :

Let $f(x) = |x^3 - x|$

then $f(x) = \begin{cases} x^3 - x, x \in (-1,0) \cup (1,2) \\ -(x^3 - x), x \in (0,1) \end{cases}$

Now, $\int_{-1}^2 |x^3 - x|\,dx$

$$\int_{-1}^2 |x^3 - x|dx + \int_0^2 |x^3 - x|dx + \int_1^2 |x^3 - x|dx$$

$$\left[\because \int_a^b f(x0dx = \int_a^c f(x)dx + \int_c^b f(x)dx\right]$$

$$= \int_0^1 (x^3 - x)dx + \int_0^1 (-(x^3 - x))dx + \int_0^2 (x^3 - x)dx$$

$$= \int_{-1}^0 (x^3 - x)dx - \int_0^1 (x^3 - x)dx + \int_1^2 (x^3 - x)x$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^0 - \left[\frac{x^4}{4} - \frac{x^2}{2}\right] + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]^2$$

$$= -\left[\frac{1}{4} - \frac{1}{2}\right] - \left[\frac{1}{4} - \frac{1}{2}\right] + \left[\frac{16}{4} - \frac{4}{4} - \left\{\frac{1}{4} - \frac{1}{2}\right\}\right]$$

$$= \frac{3}{4} + 4 - 2 = \frac{11}{4}$$

Hence $\int_{-1}^2 |x^3 - x|\,dx = \frac{11}{4}$

Q.8. Find the value of $\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx$ –

Solution. Let $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}x = dt$

If $x = 0$ then $t = 1$ and if $x = 1$ then $t = \pi/4$

$$\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx = \int_0^{\pi/4} t\,dt = \left[\frac{t^2}{2}\right]_0^{\pi/4}$$

$$\Rightarrow \left(\frac{\pi}{4}\right)^2 \cdot \frac{1}{2} - 0^2 \cdot \frac{1}{2}$$

$$\Rightarrow \frac{\pi^2}{32}$$ **Ans.**

Q.9. Find the value of $\int_{-1}^1 (x+1)dx$

Solution :

Let $I = \int_{-1}^1 (x+1)dx = \left[\frac{x^2}{2} + x\right]$

$$= \left[\frac{I^2}{2} + 1\right] - \left[\frac{(-1)^2}{2} - 1\right]$$

$$I = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$$ **Ans.**

Q.10. Find the following :

(i) $\int \cos^2 x\,dx$ **(ii)** $\int \sin^3 x\,dx$

(iii) $\int \sin^3 x \cos^{2x} dx$ **(iv)** $\int \cot x\,dx$

(v) $\int \frac{\sin(\tan^{-1}x)}{1+x^2}$ **(vi)** $\int \frac{1}{x^2 - 16}dx$

(vii) $\int \frac{1}{(x+1)(x+2)}dx$ **(viii)** $\int \frac{1}{x(x^n + 1)}dx$

(ix) $\int x \cos dx$

Solution : (i) let $\int \cos^2 x dx = A$

as $\cos^2 x = \left(\frac{1+\cos 2x}{2}\right)$

$$A = \int\left(\frac{1+\cos 2x}{2}\right)dx$$

$$= \frac{1}{2}\int 1 dx + \frac{1}{2}\int \cos 2x \; dx$$

$$= \frac{x}{2} + \frac{\sin 2x}{4} + c$$

Solution : (ii)

$$\int \sin^3 x dx = \frac{1}{4}\int 4\sin^3 x dx$$

$$= \frac{1}{4}(3\sin x - \sin 3x)\, dx$$

$[\because \sin 3x = 3\sin x - 4\sin^3 x]$

$$= \frac{1}{4}\left[3\int \sin x dx - \int \sin 3x dx\right]$$

$$= \frac{1}{4}\left[3(-\cos x) - \left(-\frac{\cos 3x}{3}\right)\right]$$

$$= \frac{1}{4}\left[-3\cos x + \left(-\frac{\cos 3x}{3}\right)\right]$$

$$= \frac{1}{12}\cos 3x - \frac{3}{4}\cos x$$

Solution : (iii) $\int \sin^3 x . \cos^2 x \; dx$

$$= \int \sin x (\sin^2 x)\cos^2 x dx$$

$$= \int \sin x[1-\cos^2 x]\cos^2 x dx$$

Let $\cos x = t$

$-\sin x dx = dt$

$\sin x dt = -dt$

$$= -\int (1-t^2)t^2 dt$$

$$= -\int t^2 dt + \int t^4 dt$$

$$= \frac{t^3}{3} + \frac{t^5}{5} + C$$

$$= \frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$$ **Ans.**

Solution : (iv)

Let $I = \int \cot x dx$

$$= \int \frac{\cos x}{\sin x} dx$$

Let $\sin x = t \Rightarrow \cos x \; dx = dt$

$\therefore$ $I = \int \frac{dt}{t} = \log t + c = \log \sin x + c$

Hence $\int \cot x \; dx = \log(\sin x) + c = -\log(\operatorname{cosec} x) + c$

Solution : (v)

Let $\tan^{-1} x = t \Rightarrow \frac{dx}{1+x^2} = dt$

$$\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx = \int \sin t dt = -\cos t + c$$

$$= -\cos(\tan^{-1} x) + c$$ **Ans.**

Solution (vi)

$$\int \frac{dx}{x^2-16} = \int \frac{dx}{x^2-(4)^2}$$

$\because$ $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$

$\therefore$ $\int \frac{dx}{x^2-(4)^2} = \frac{1}{2\times 4}\log\left|\frac{x-4}{x+4}\right|$

$$= \frac{1}{8}\log\left|\frac{x-4}{x+4}\right| + c$$ **Ans.**

Solution (vii)

$$I = \int \frac{1}{(x+1)(x+2)} dx$$

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$\Rightarrow$ $1 = A(x+2) + B(x+1)$

$x = -2 \Rightarrow 1 = -B \Rightarrow B = -1$

$x = -1 \Rightarrow 1 = A \Rightarrow A = 1$

$$\int \frac{1}{(x+1)(x+2)} dx = \int \frac{dx}{x+1} + \left(\frac{-dx}{x+2}\right)$$

$$\Rightarrow \int \frac{dx}{x+1} - \int \frac{dx}{x+2} \Rightarrow \log(x+1) - \log(x+2)$$

$$\Rightarrow \log\left|\frac{x+1}{x+2}\right| + c$$ **Ans.**

Solution (viii)

Let $$I = \int \frac{dx}{x(x^n+1)}$$

$$= \int \frac{x^{n-1}dx}{x^n(x^n+1)}$$

on multiply and divide of x^{n-1}

Let $x^n = t \Rightarrow n\,x^{n-1}\,dx = dt \Rightarrow x^{n-1}\,dx = \frac{dt}{n}$

$$I = \frac{1}{n}\int \frac{1}{t(t+1)}dt$$

$$= \frac{1}{n}\int\left(\frac{1}{t} - \frac{1}{t+1}\right)dt$$

[Using fractional fractions]

$$= \frac{1}{n}[\log t - \log(t+1)]$$

$$= \frac{1}{n}\log\frac{t}{t+1}$$

$$I = \frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + c$$ **Ans.**

Solution (9) $\int x\cos x\,dx$

$$= x\int \cos x\,dx - \left[\frac{d}{dx}\times\int \cos x\,dx\right]dx$$

$$= x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x$$ **Ans.**

□