

# MAGNETIC EFFECT OF CURRENT - II

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## Lorentz Magnetic Force:

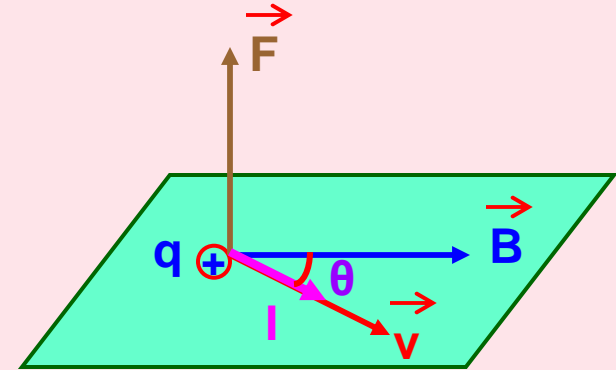
A current carrying conductor placed in a magnetic field experiences a force which means that a moving charge in a magnetic field experiences force.

$$\vec{F}_m = q (\vec{v} \times \vec{B})$$

or

$$\vec{F}_m = (q v B \sin \theta) \hat{n}$$

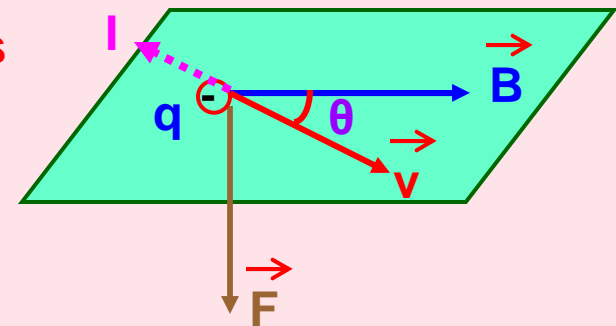
where  $\theta$  is the angle between  $\vec{v}$  and  $\vec{B}$



### Special Cases:

- i) If the charge is at rest, i.e.  $v = 0$ , then  $F_m = 0$ .  
So, a stationary charge in a magnetic field does not experience any force.
- ii) If  $\theta = 0^\circ$  or  $180^\circ$  i.e. if the charge moves parallel or anti-parallel to the direction of the magnetic field, then  $F_m = 0$ .
- iii) If  $\theta = 90^\circ$  i.e. if the charge moves perpendicular to the magnetic field, then the force is maximum.

$$F_{m(\max)} = q v B$$

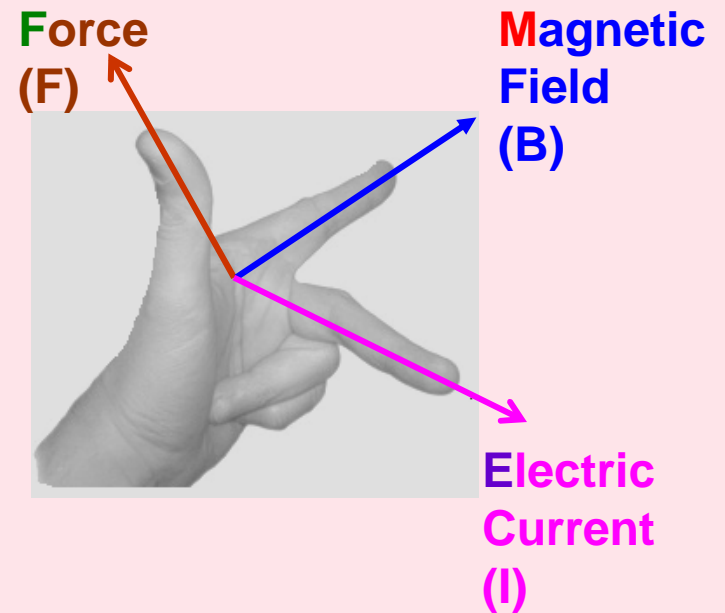


## Fleming's Left Hand Rule:

If the central finger, fore finger and thumb of left hand are stretched mutually perpendicular to each other and the central finger points to current, fore finger points to magnetic field, then thumb points in the direction of motion (force) on the current carrying conductor.

### TIP:

Remember the phrase 'e m f' to represent electric current, magnetic field and force in anticlockwise direction of the fingers of left hand.



## Force on a moving charge in uniform Electric and Magnetic Fields:

When a charge  $q$  moves with velocity  $\vec{v}$  in region in which both electric field  $\vec{E}$  and magnetic field  $\vec{B}$  exist, then the Lorentz force is

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) \quad \text{or} \quad \vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

## Force on a current-carrying conductor in a uniform Magnetic Field:

Force experienced by each electron in the conductor is

$$\vec{f} = -e (\vec{v}_d \times \vec{B})$$

If  $n$  be the number density of electrons,  $A$  be the area of cross section of the conductor, then no. of electrons in the element  $dl$  is  $n A dl$ .

Force experienced by the electrons in  $dl$  is

$$\begin{aligned} d\vec{F} &= n A dl [-e (\vec{v}_d \times \vec{B})] = -n e A v_d (d\vec{l} \times \vec{B}) \\ &= I (d\vec{l} \times \vec{B}) \end{aligned}$$

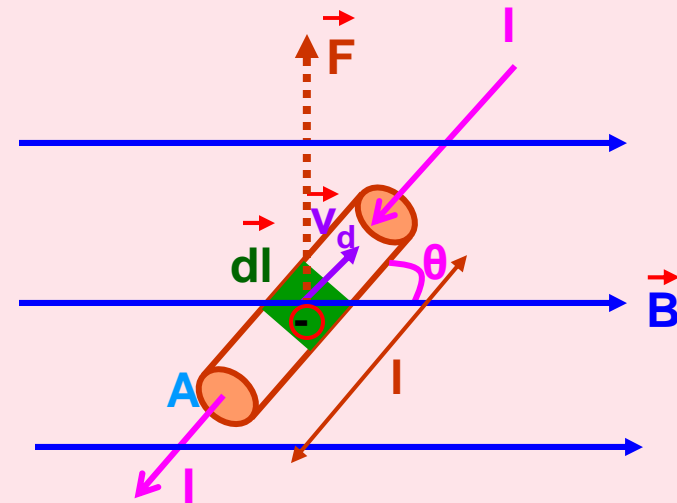
$$\vec{F} = \int d\vec{F} = \int I (d\vec{l} \times \vec{B})$$

where  $I = neAv_d$  and -ve sign represents that the direction of  $dl$  is opposite to that of  $v_d$

$$\vec{F} = I (\vec{l} \times \vec{B})$$

or

$$F = I l B \sin \theta$$



## Forces between two parallel infinitely long current-carrying conductors:

Magnetic Field on RS due to current in PQ is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \quad (\text{in magnitude})$$

Force acting on RS due to current  $I_2$  through it is

$$F_{21} = \frac{\mu_0 I_1}{2\pi r} I_2 l \sin 90^\circ \quad \text{or} \quad F_{21} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

$B_1$  acts perpendicular and into the plane of the diagram by Right Hand Thumb Rule. So, the angle between  $I$  and  $B_1$  is  $90^\circ$ .  $l$  is length of the conductor.

Magnetic Field on PQ due to current in RS is

$$B_2 = \frac{\mu_0 I_2}{2\pi r} \quad (\text{in magnitude})$$

Force acting on PQ due to current  $I_1$  through it is

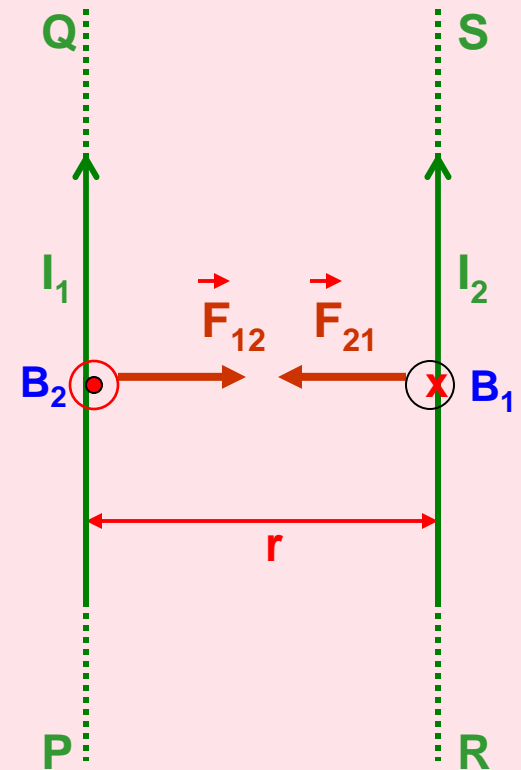
$$F_{12} = \frac{\mu_0 I_2}{2\pi r} I_1 l \sin 90^\circ \quad \text{or} \quad F_{12} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

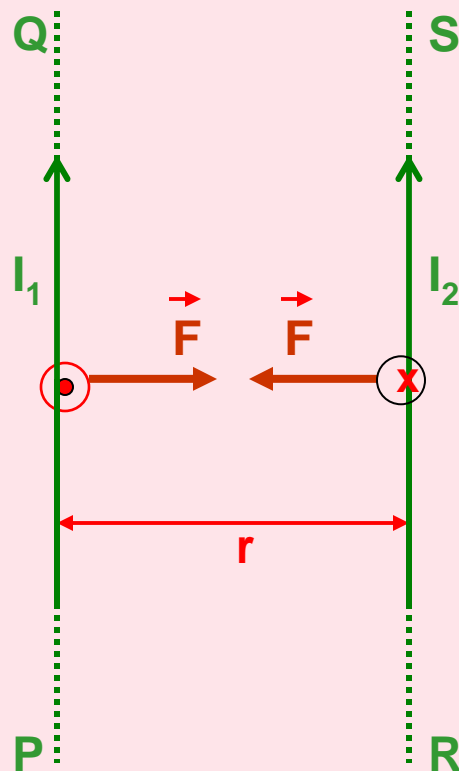
(The angle between  $I$  and  $B_2$  is  $90^\circ$  and  $B_2$  is emerging out)

$$F_{12} = F_{21} = F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

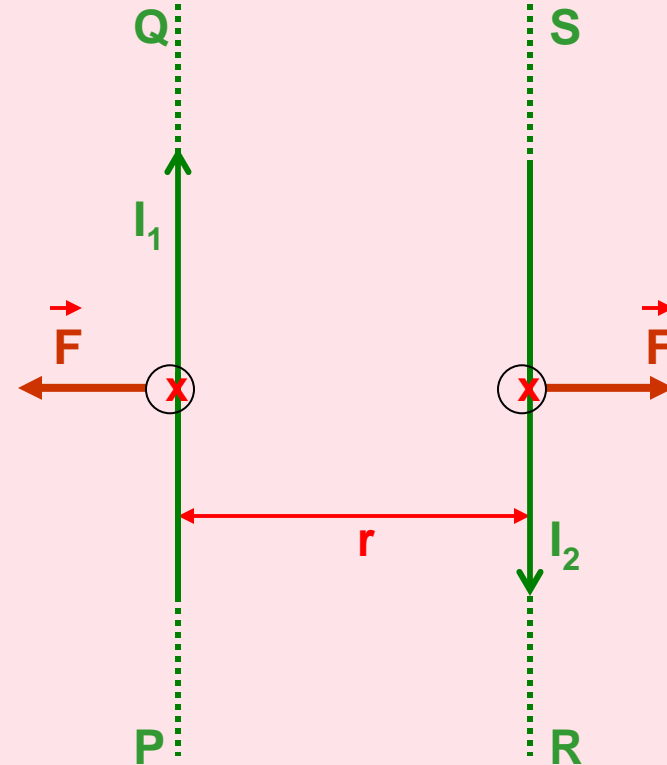
Force per unit length of the conductor is

$$F / l = \frac{\mu_0 I_1 I_2}{2\pi r} \quad \text{N / m}$$





By Fleming's Left Hand Rule,  
the conductors experience  
**force towards each other** and  
hence **attract** each other.



By Fleming's Left Hand Rule,  
the conductors experience  
**force away from each other**  
and hence **repel** each other.

## Definition of Ampere:

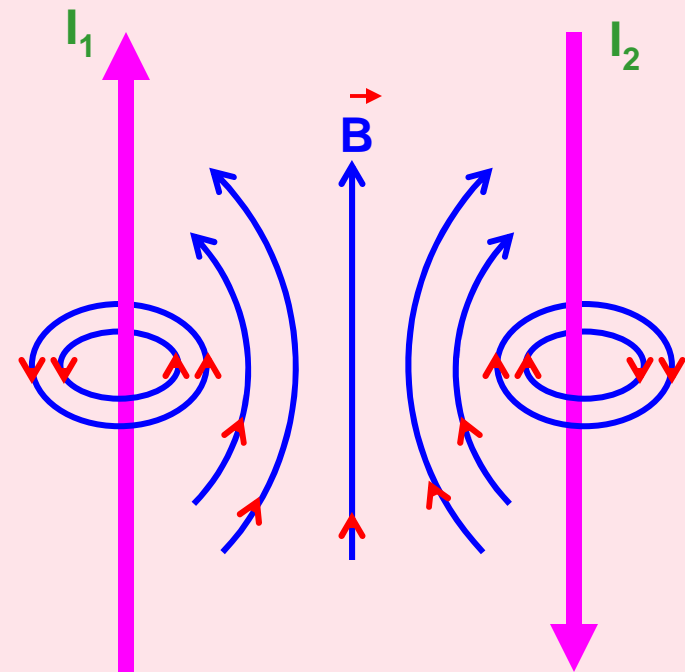
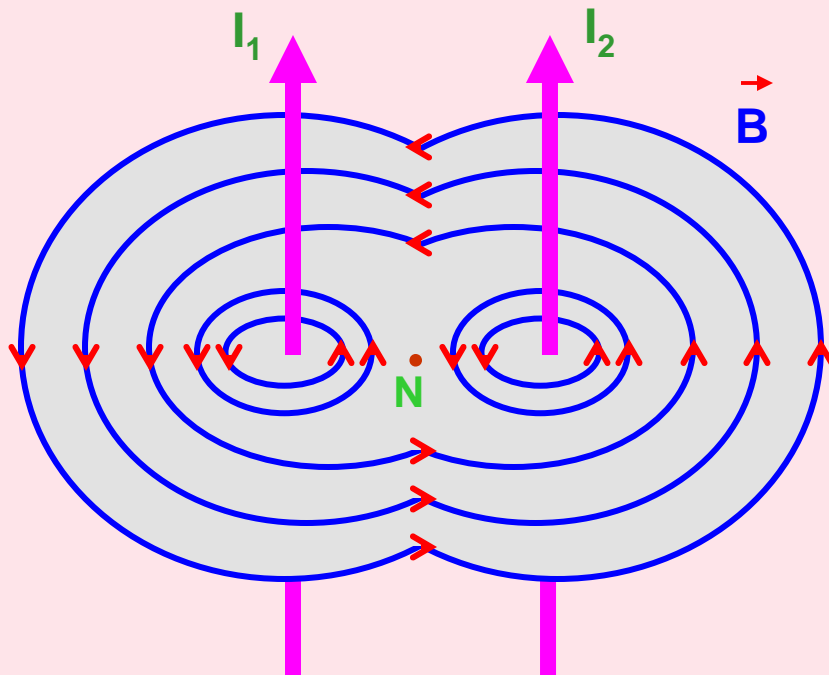
Force per unit length of the conductor is

$$F / l = \frac{\mu_0 I_1 I_2}{2\pi r} \quad \text{N / m}$$

When  $I_1 = I_2 = 1$  Ampere and  $r = 1$  m, then  $F = 2 \times 10^{-7}$  N/m.

One **ampere** is that current which, if passed in each of two parallel conductors of infinite length and placed 1 m apart in vacuum causes each conductor to experience a force of  $2 \times 10^{-7}$  Newton per metre of length of the conductor.

## Representation of Field due to Parallel Currents:



# Torque experienced by a Current Loop (Rectangular) in a uniform Magnetic Field:

Let  $\theta$  be the angle between the plane of the loop and the direction of the magnetic field. The axis of the coil is perpendicular to the magnetic field.

$$\vec{F}_{SP} = I (\vec{b} \times \vec{B})$$

$$|F_{SP}| = I b B \sin \theta$$

$$\vec{F}_{QR} = I (\vec{b} \times \vec{B})$$

$$|F_{QR}| = I b B \sin \theta$$

Forces  $\vec{F}_{SP}$  and  $\vec{F}_{QR}$  are equal in magnitude but opposite in direction and they cancel out each other. Moreover they act along the same line of action (axis) and hence do not produce torque.

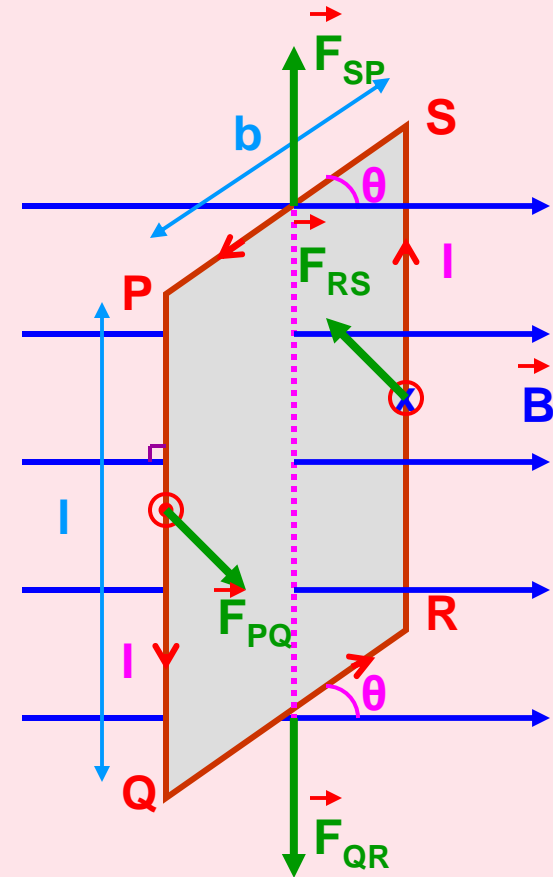
$$\vec{F}_{PQ} = I (\vec{l} \times \vec{B})$$

$$|F_{PQ}| = I l B \sin 90^\circ = I l B$$

$$\vec{F}_{RS} = I (\vec{l} \times \vec{B})$$

$$|F_{RS}| = I l B \sin 90^\circ = I l B$$

Forces  $\vec{F}_{PQ}$  and  $\vec{F}_{RS}$  being equal in magnitude but opposite in direction cancel out each other and do not produce any translational motion. But they act along different lines of action and hence produce torque about the axis of the coil.





**Torque experienced by the coil is**

$$\tau = F_{PQ} \times PN \quad (\text{in magnitude})$$

$$\tau = I l B (b \cos \theta)$$

$$\tau = I l b B \cos \theta$$

$$\tau = I A B \cos \theta \quad (A = lb)$$

$$\tau = N I A B \cos \theta \quad (\text{where } N \text{ is the no. of turns})$$

If  $\Phi$  is the angle between the **normal to the coil** and the **direction of the magnetic field**, then

$$\Phi + \theta = 90^\circ \quad \text{i.e.} \quad \theta = 90^\circ - \Phi$$

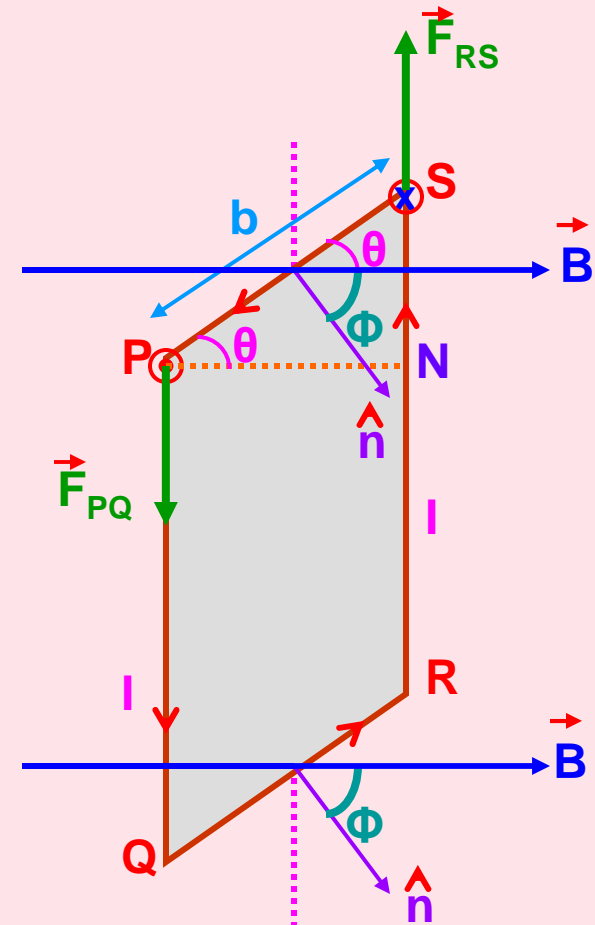
So,

$$\tau = I A B \cos (90^\circ - \Phi)$$

$$\tau = N I A B \sin \Phi$$

**NOTE:**

One must be very careful in using the formula in terms of **cos** or **sin** since it depends on the angle taken whether with the **plane of the coil** or the **normal of the coil**.



## Torque in Vector form:

$$\tau = N I A B \sin \Phi$$

$$\vec{\tau} = (N I A B \sin \Phi) \hat{n} \quad (\text{where } \hat{n} \text{ is unit vector normal to the plane of the loop})$$

$$\vec{\tau} = N I (\vec{A} \times \vec{B})$$

or

$$\vec{\tau} = N (\vec{M} \times \vec{B})$$

(since  $\vec{M} = I \vec{A}$  is the Magnetic Dipole Moment)

### Note:

- 1) The coil will rotate in the anticlockwise direction (from the top view, according to the figure) about the axis of the coil shown by the dotted line.
- 2) The torque acts in the upward direction along the dotted line (according to Maxwell's Screw Rule).
- 3) If  $\Phi = 0^\circ$ , then  $\tau = 0$ .
- 4) If  $\Phi = 90^\circ$ , then  $\tau$  is maximum. i.e.  $\tau_{\max} = N I A B$
- 5) Units:  $B$  in Tesla,  $I$  in Ampere,  $A$  in  $\text{m}^2$  and  $\tau$  in Nm.
- 6) The above formulae for torque can be used for any loop irrespective of its shape.

## Moving Coil or Suspended Coil or D' Arsonval Type Galvanometer:

Torque experienced by the coil is

$$\tau = N I A B \sin \Phi$$

Restoring torque in the coil is

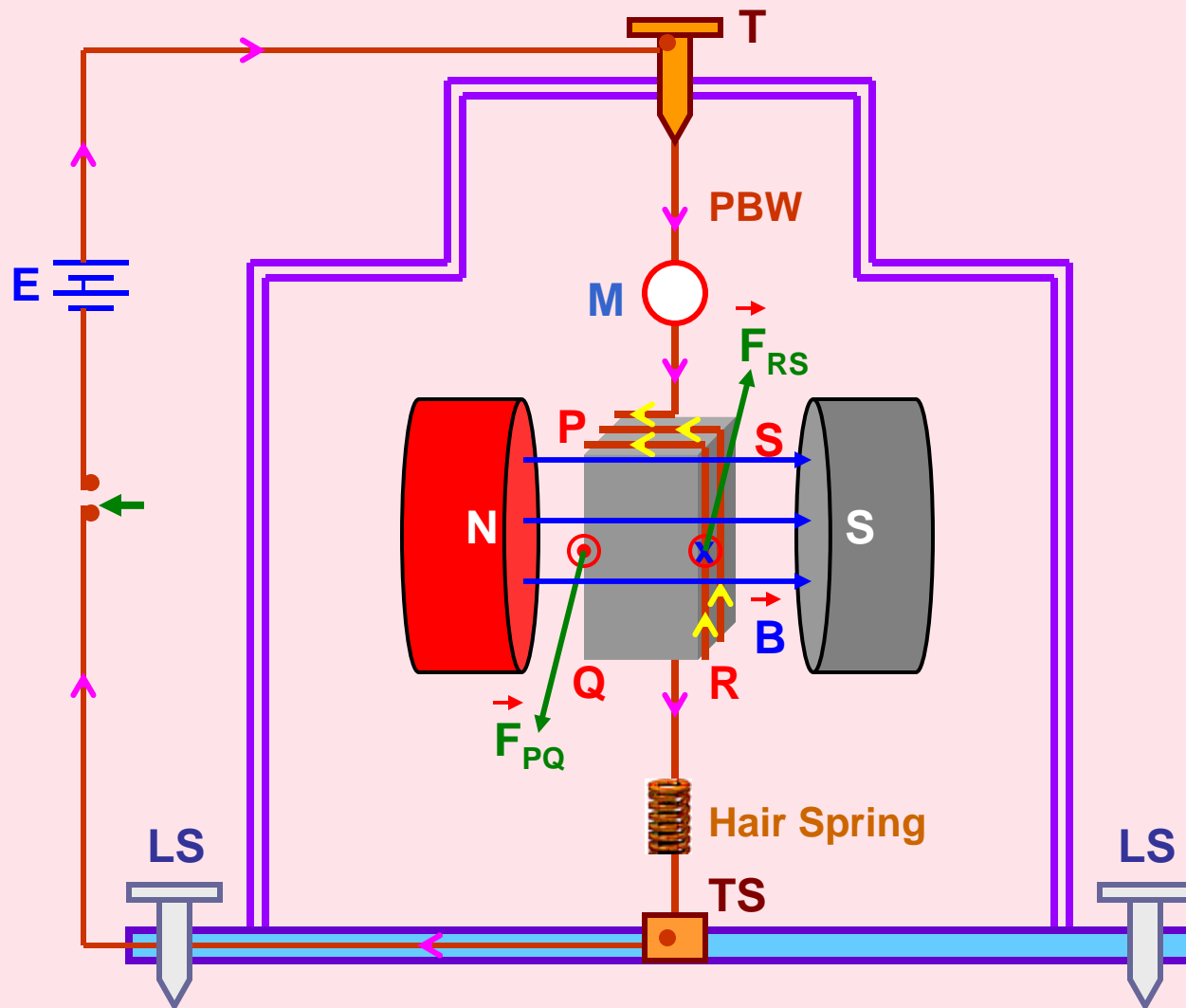
$$\tau = k \alpha \quad (\text{where } k \text{ is restoring torque per unit angular twist, } \alpha \text{ is the angular twist in the wire})$$

At equilibrium,

$$N I A B \sin \Phi = k \alpha$$

$$\therefore I = \frac{k}{N A B \sin \Phi} \alpha$$

The factor  $\sin \Phi$  can be eliminated by choosing Radial Magnetic Field.



T – Torsion Head, TS – Terminal screw, M – Mirror, N,S – Poles pieces of a magnet, LS – Levelling Screws, PQRS – Rectangular coil, PBW – Phosphor Bronze Wire

## Radial Magnetic Field:

The (top view PS of) plane of the coil PQRS lies along the magnetic lines of force in whichever position the coil comes to rest in equilibrium.

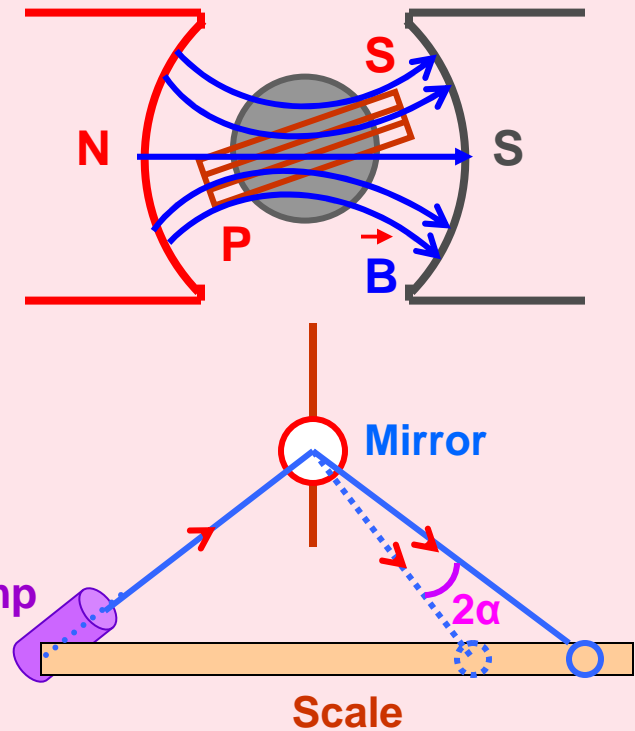
So, the angle between the plane of the coil and the magnetic field is  $0^\circ$ .

or the angle between the normal to the plane of the coil and the magnetic field is  $90^\circ$ .

$$\text{i.e. } \sin \Phi = \sin 90^\circ = 1$$

$$\therefore I = \frac{k}{NAB} \alpha \quad \text{or} \quad I = G \alpha \quad \text{where } G = \frac{k}{NAB}$$

is called Galvanometer constant



## Current Sensitivity of Galvanometer:

It is the deflection of galvanometer per unit current.

$$\frac{\alpha}{I} = \frac{NAB}{k}$$

## Voltage Sensitivity of Galvanometer:

It is the deflection of galvanometer per unit voltage.

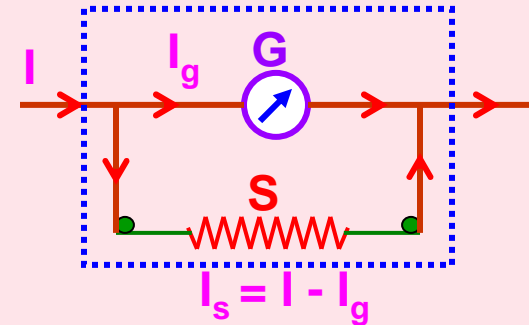
$$\frac{\alpha}{V} = \frac{NAB}{kR}$$

## Conversion of Galvanometer to Ammeter:

Galvanometer can be converted into ammeter by shunting it with a very small resistance.

Potential difference across the galvanometer and shunt resistance are equal.

$$\therefore (I - I_g) S = I_g G \quad \text{or} \quad S = \frac{I_g G}{I - I_g}$$

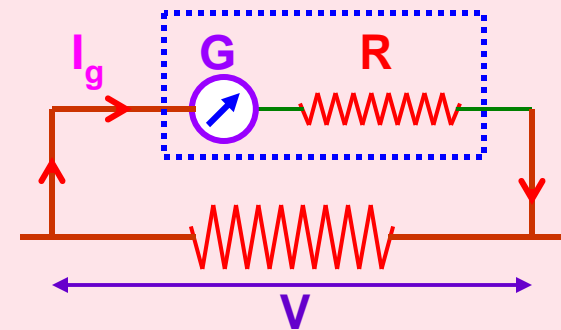


## Conversion of Galvanometer to Voltmeter:

Galvanometer can be converted into voltmeter by connecting it with a very high resistance.

Potential difference across the given load resistance is the sum of p.d across galvanometer and p.d. across the high resistance.

$$\therefore V = I_g (G + R) \quad \text{or} \quad R = \frac{V}{I_g} - G$$



## Difference between Ammeter and Voltmeter:

| S.No. | Ammeter  | Voltmeter  |
|-------|--|--|
| 1     | It is a low resistance instrument.                             | It is a high resistance instrument.                          |
| 2     | Resistance is $GS / (G + S)$                                   | Resistance is $G + R$  |
| 3     | Shunt Resistance is $(GI_g) / (I - I_g)$ and is very small.    | Series Resistance is $(V / I_g) - G$ and is very high.       |
| 4     | It is always connected in series.                              | It is always connected in parallel.                          |
| 5     | Resistance of an ideal ammeter is zero.                        | Resistance of an ideal voltmeter is infinity.                |
| 6     | Its resistance is less than that of the galvanometer.          | Its resistance is greater than that of the voltmeter.        |
| 7     | It is not possible to decrease the range of the given ammeter. | It is possible to decrease the range of the given voltmeter. |