CHAPTER

5 COORDINATE GEOMETRY

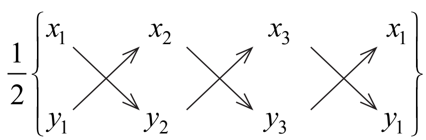
I. AREA OF A TRIANGLE AND QUADRILATERAL :

Key Points

- ✓ The area of Δ ABC is the absolute value of the expression

$$= \frac{1}{2}\{(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

- ✓ The vertices A(x_1, y_1), B (x_2, y_2) and C (x_3, y_3) of ΔABC are said to be “taken in order” if A, B, C are taken in counter-clock wise direction.
- ✓ The following pictorial representation helps us to write the above formula very easily.



Area of ΔABC = $\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}$ sq. units.

- ✓ Three distinct points A(x_1, y_1), B (x_2, y_2) and C (x_3, y_3) will be collinear if and only if area of ΔABC = 0 .
- ✓ Area of the quadrilateral ABCD = $\frac{1}{2}\{(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3\}$ sq. units.

Example 5.1

Find the area of the triangle whose vertices are (-3,5) , (5,6) and (5,-2).

Solution :

Plot the points in a rough diagram and take them in counter-clockwise order.

Let the vertices be A(–3, 5) , B(5, –2), C(5,6)

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ (x_1, y_1) & (x_2, y_2) & (x_3, y_3) \end{array}$$

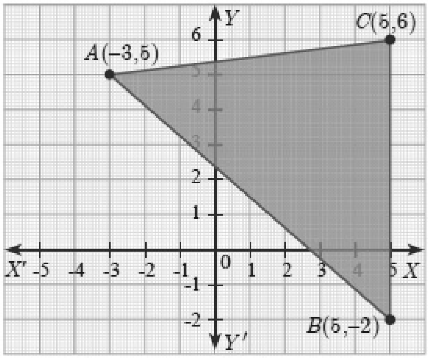
The area ΔABC is

$$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}$$

$$= \frac{1}{2}\{(6 + 30 + 25) - (25 - 10 - 18)\}$$

$$= \frac{1}{2}\{61 + 3\}$$

$$= \frac{1}{2}(64) = 32 \text{ sq. units}$$



Example 5.2

Show that the points P(–1.5, 3) , Q(6, –2) , R(–3, 4) are collinear.

Solution :

The points are P(– 1.5, 3) , Q(6, –2) , R(–3, 4)

Area of ΔPQR

$$= \frac{1}{2}\left\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right\}$$

$$= \frac{1}{2}\{(3 + 24 - 9) - (18 + 6 - 6)\}$$

$$= \frac{1}{2}\{18 - 18\} = 0$$

Therefore, the given points are collinear.

Example 5.3

If the area of the triangle formed by the vertices A(–1, 2) , B(k , –2) and C(7, 4) (taken in order) is 22 sq. units, find the value of *k*.

Solution :

The vertices are A(–1, 2) , B(k , –2) and C(7, 4)

Area of triangle ABC is 22 sq.units

$$\frac{1}{2}\left\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right\} = 22$$

$$\frac{1}{2}\{(2 + 4k + 14) - (2k - 14 - 4)\} = 22$$

$$2k + 34 = 44$$

$$\text{gives } 2k = 10 \text{ so } k = 5$$

Example 5.4

If the points P(–1, – 4), Q (b, c) and R(5, –1) are collinear and if 2b + c = 4 , then find the values of b and c.

Solution :

Since the three points P(-1,-4), Q(b,c) and R(5, –1) are collinear,

Area of triangle PQR = 0

$$\frac{1}{2}\left\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right\} = 0$$

$$\frac{1}{2}\{(-c - b - 20) - (-4b + 5c + 1)\} = 0$$

$$-c - b - 20 + 4b - 5c - 1 = 0$$

b – 2c = 7 — (1)

Also, 2b + c = 4 — (2)

(from given information)

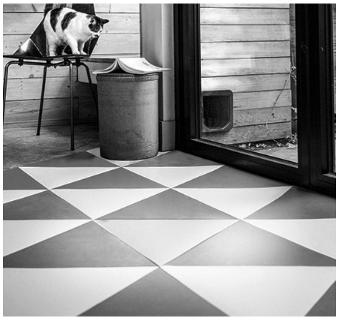
Solving (1) and (2) we get b = 3 , c = −2

Example 5.5

The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at (– 3, 2) , (–1, –1) and (1, 2) . If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

Solution :

Vertices of one triangular tile are at (– 3, 2) , (–1, –1) and (1, 2)



Area of this tile

$$=\frac{1}{2}\left\{(3-2+2)-(-2-1-6)\right\} \text{ sq. units}$$

$$=\frac{1}{2}(12)=6 \text{ sq. units}$$

Since the floor is covered by 110 triangle shaped identical tiles,

Area of floor = 110×6 = 660 sq.units

Example 5.6

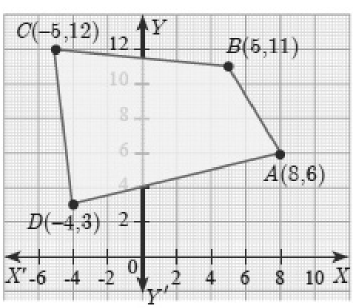
Find the area of the quadrilateral formed by the points (8, 6), (5, 11), (–5,12) and (– 4, 3) .

Solution :

Before determining the area of quadrilateral, plot the vertices in a graph.

Let the vertices be A(8,6), B(5,11), C(–5, 12) and D(– 4, 3)

Therefore, area of the quadrilateral ABCD



$$=\frac{1}{2}\begin{Bmatrix}(x_1y_2+x_2y_3+x_3y_4+x_4y_1)\\-(x_2y_1+x_3y_2+x_4y_3+x_1y_4)\end{Bmatrix}$$

$$=\frac{1}{2}\left\{(88+60-15-24)-(30-55-48+24)\right\}$$

$$=\frac{1}{2}\{109+49\}$$

$$=\frac{1}{2}\{158\}=79 \text{ sq. units}$$

Example 5.7

The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹1300 per square feet. What will be the total cost for making the parking lot?

Solution :

The parking lot is a quadrilateral whose vertices are at A(2,2), B(5,5) , C(4,9) and D(1,7)

Therefore, Area of parking lot

$$=\frac{1}{2}\{(10+45+28+2)-(10+20+9+14)\}$$

$$=\frac{1}{2}\{85-53\}$$

Use : $\frac{1}{2}\begin{Bmatrix} 2 & 5 & 4 & 1 & 2 \\ 2 & 5 & 9 & 7 & 2 \end{Bmatrix}$

$$=\frac{1}{2}(32)=16 \text{ sq. units.}$$

So, area of parking lot = 16 sq. feets

Construction rate per square feet= ₹1300

Therefore, total cost for constructing the parking lot = 16 × 1300 = ₹20800

EXERCISE 5.1

1. Find the area of the triangle formed by the points

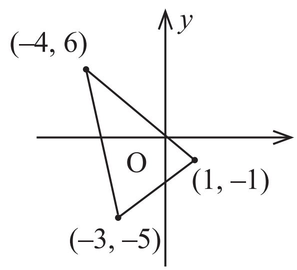
(i) (1, –1), (–4, 6) and (–3, –5)

(ii) (–10, – 4), (–8, –1) and (–3, –5)

Solution:

i) Given points are (1, –1), (–4, 6), (–3, –5)

Taking A, B, C in anti clockwise direction.



Let A (1, –1), B (–4, 6), C (–3, –5)

$\therefore$ Area of triangle ABC

$$= \frac{1}{2}\begin{Bmatrix} 1 & -4 & -3 & 3 \\ -1 & 6 & -5 & 1 \end{Bmatrix}$$

$$= \frac{1}{2}\{(6 + 20 + 3) - (4 - 18 - 5)]$$

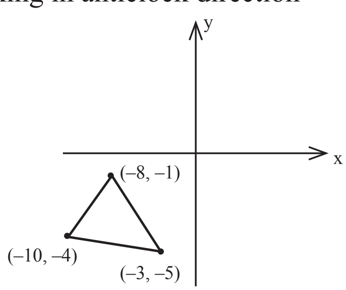
$$= \frac{1}{2}[29 + 19]$$

$$= \frac{1}{2}(48)$$

= 24 sq. units

(ii) Given points are (–10, –4), (–8, –1), (–3, –3)

Taking in anticlock direction



Let A (–10, –4), B (–3, –5), C (–8, –1)

$\therefore$ Area of triangle ABC

$$= \frac{1}{2}\begin{Bmatrix} -10 & -3 & -8 & -10 \\ -4 & -5 & -1 & -4 \end{Bmatrix}$$

$$= \frac{1}{2}\left[(50+3+32)-(12+40+10)\right]$$

$$= \frac{1}{2}[85-62]$$

$$= \frac{23}{2}$$

$= 11.5$ sq.units

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2. Determine whether the sets of points are collinear ?

(i) $\left\{-\frac{1}{2}, 3\right\}$, (–5, 6) and (–8, 8)

(ii) (*a, b* + *c*), (*b, c* + *a*) and (*c, a* + *b*)

Solution :

Area of triangle formed by 3 points

$$= \frac{1}{2}\begin{bmatrix} -1/2 & -5 & -8 & -1/2 \\ 3 & 6 & 8 & 3 \end{bmatrix}$$

$$= \frac{1}{2}[(-3-40-24)-(-15-48-4)]$$

$$= \frac{1}{2}[-67-(67)]$$

$$= \frac{1}{2}(0) = 0$$

$\therefore$ The 3 points are collinear.

ii) Given points are A (*a, b* + *c*) , B (*b, c* + *a*), C (*c, a* + *b*)

Areaof tirangle by 3 points

$$= \frac{1}{2}\begin{bmatrix} a & b & c & a \\ b+c & c+a & a+b & b+c \end{bmatrix}$$

$$=\frac{1}{2}[(ac+a^2+ab+b^2+bc+c^2)$$
$$-(b^2+bc+c^2+ac+a^2+ab)]$$
$$=\frac{1}{2}[(a^2+b^2+c^2+ab+bc+ca)$$
$$-(a^2+b^2+c^2+ab+bc+ca)]$$
$$=\frac{1}{2}[0]$$

$\therefore$ The 3 points are collinear.

3. Vertices of given triangles are taken in order and their areas are provided below. In each case, find the value of 'p'.

S.No.	Vertices	Area (sq. units)
(i)	(0, 0), (p, 8), (6, 2)	20
(ii)	(p, p), (5, 6), (5, –2)	32

Solution :

Area of triangle formed by 3 points

$$=\frac{1}{2}\begin{bmatrix}0 & p & 6 & 0\\ 0 & 8 & 2 & 0\end{bmatrix}$$

$\Rightarrow \quad [0 + 2p + 0] – [0 + 48 + 0] \quad = 40$

$\Rightarrow \quad 2p – 48 = 40$

$\Rightarrow \quad 2p = 88$

$\therefore\ p = 44$

ii) Given vertices are (p, p), (5, 6), (5, –2), Area = 32 sq. units.

Area of triangle formed by 3 points

$$=\frac{1}{2}\begin{bmatrix}p & 5 & 5 & p\\ p & 6 & -2 & p\end{bmatrix}=32$$

$\Rightarrow (6p – 10 + 5p) – (5p + 30 – 2p) = 64$

$\Rightarrow (11p – 10) – (3p + 30) \quad = 64$

$\Rightarrow \quad 8p = 104$

$p = \frac{104}{8}$

$p = 13$

4. In each of the following, find the value of '*a*' for which the given points are collinear. (i) (2, 3), (4, *a*) and (6, –3) (ii) (*a*, 2 – 2*a*), (–a+1, 2*a*) and (– 4 – *a*, 6 – 2*a*)

Solution :

i) Given, 3 points (2, 3), (4, a), (6, –3) are collinear.

$$\text{ie, } \frac{1}{2}\begin{bmatrix}2 & 4 & 6 & 2\\ 3 & a & -3 & 3\end{bmatrix}=0$$

$\Rightarrow (2a – 12 + 18) – (12 + 6a – 6) = 0$

$\Rightarrow \quad (2a + 6) – (6a + 6) = 0$

$\Rightarrow \quad – 4a = 0$

$\Rightarrow \quad a = 0$

ii) Given 2 points (a, 2 – 2a), (– a + 1, 2a), (– 4 – a , 6 – 2a) are collinear.

$\therefore$ Area of triangle formed by 3 points is 0.

$$\text{ie, } \frac{1}{2}\begin{bmatrix}a & -a+1 & -4-a & a\\ 2-2a & 2a & 6-2a & 2-2a\end{bmatrix}=0$$

$$\Rightarrow [2a^2+(-a+1)(6-2a)+(-4-a)(2-2a)]$$
$$-[(2-2a)(-a+1)+2a(-4-a)+a(6-2a)]=0$$
$$\Rightarrow (2a^2+2a^2-8a+6+2a^2+6a-8)$$
$$-[2a^2-4a+2-2a^2-8a+6a-2a^2]=0$$
$$\Rightarrow (6a^2-2a-2)-(-2a^2-6a+2)=0$$
$$\Rightarrow 8a^2+4a-4=0$$
$$\Rightarrow 2a^2+a-1=0$$
$$\Rightarrow a=-1,\ \frac{1}{2}$$

5. **Find the area of the quadrilateral whose vertices are at**

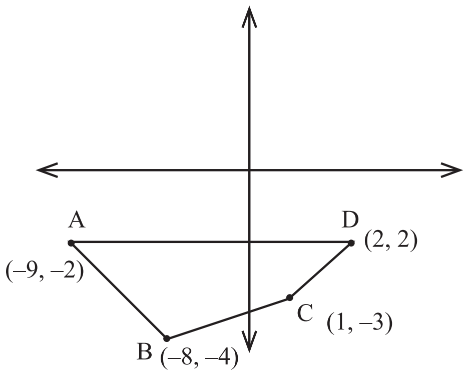
(i) (–9, –2), (–8, –4), (2, 2) and (1, –3) (ii) (–9, 0), (–8, 6), (–1, –2) and (–6, –3)

Solution :

(i) Given vertices of quadrilateral are (–9, –2), (–8, –4), (2, 2), (1, –3)

First we plot the points in the plane

Let A (–9, –2), B (–8, –4), C (1, –3), D (2, 2) $\therefore$ Area of qadrilateral



$$= \frac{1}{2}\begin{bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{bmatrix}$$

$$= \frac{1}{2}\left[(36+24+2-4)-(16-4-6-18)\right]$$

$$= \frac{1}{2}[58-(-12)]$$

$$= \frac{1}{2}[70]$$

$= 35$ sq. units

ii) Given vertices of quadrilateral are (–9, 0), (–8, 6), (–1, –2), (–6, –3)

First, we plot the points in the plane.

Let A (–8, 6), B (–9, 0), C (–6, –3), D (–1, –2)

Area of quadrilateral

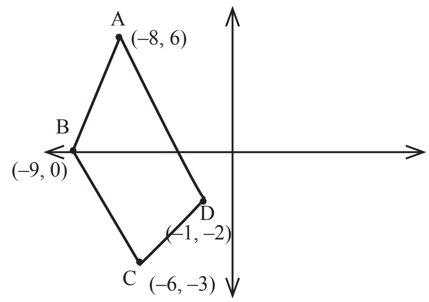
$$= \frac{1}{2}\begin{bmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{bmatrix}$$

$$= \frac{1}{2}[(0+27+12-6)-(-54+0+3+16)]$$

$$= \frac{1}{2}[33-(-35)]$$

$$= \frac{1}{2}[68]$$

$= 34$ sq. units



6. **Find the value of k, if the area of a quadrilateral is 28 sq.units, whose vertices are (–4, –2), (–3, k), (3, –2) and (2, 3)**

Solution :

$$= \frac{1}{2}\begin{bmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{bmatrix} = 28$$

$\Rightarrow$ (–4k + 6 + 9 – 4) – (6 + 3k – 4 – 12) = 56

$\Rightarrow$ (11 – 4k) – (3k – 10) = 56

$\Rightarrow$ 21 – 7k = 56

$\therefore$ 7k = – 35

k = – 5

7. **If the points A(-3, 9) , B (*a, b*) and C(4,-5) are collinear and if *a* + *b* = 1 , then find *a* and *b*.**

Solution :

Given, A (– 3, 9), B (*a, b*), C (4, –5) are collinear and *a* + *b* = 1. — (1)

Area of triangle formed by 3 points = 0.

ie) $\frac{1}{2}\begin{bmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{bmatrix} = 0$

$\Rightarrow$ (– 3b – 5a + 36) – (9a + 4b + 15) = 0

$\Rightarrow$ – 5a – 3b + 36 – 9a – 4b – 15 = 0

$\Rightarrow$ – 14a – 7b + 21 = 0

$\Rightarrow$ 2a + b – 3 = 0

$\Rightarrow$ 2a + 1 – a – 3 = 0 (from (1))

$\therefore \Rightarrow$ $a = 2$ b = – 1

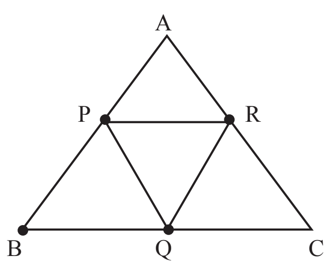
8. Let P(11,7) , Q(13.5, 4) and R(9.5, 4) be the mid- points of the sides AB, BC and AC respectively of ΔABC . Find the coordinates of the vertices A, B and C. Hence find the area of ΔABC and compare this with area of ΔPQR .

Solution :

In ΔABC, given that P, Q, R are the mid points of AB, BC, CA respectively.

P (11, 7), Q (13.5, 4), R (9.5, 4)

Let A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) be the vertices.



$\therefore \frac{x_1 + x_2}{2} = 11, \ \frac{y_1 + y_2}{2} = 7$

$\Rightarrow x_1 + x_2 = 22, \qquad y_1 + y_2 = 14$ .....(1)

$\frac{x_2 + x_3}{2} = \frac{27}{2}, \ \frac{y_2 + y_3}{2} = 4$

$\Rightarrow x_2 + x_3 = 27, \qquad y_2 + y_3 = 8$ .....(2)

$\frac{x_1 + x_3}{2} = \frac{19}{2}, \ \frac{y_1 + y_3}{2} = 4$

$\Rightarrow x_1 + x_3 = 19, \qquad y_1 + y_3 = 8$ .....(3)

$\therefore$ Adding (1), (2) and (3)

$2(x_1 + x_2 + x_3) = 68, \qquad 2(y_1 + y_2 + y_3) = 30$

$x_1 + x_2 + x_3 = 34, \qquad y_1 + y_2 + y_3 = 15$

$22 + x_3 = 34, \qquad 14 + y_3 = 15$

$x_3 = 12 \qquad y_3 = 1$

$\therefore$ C (12 , 1)

Also, (2) $\Rightarrow$ $x_2 + 12 = 27, \ y_2 + 1 = 8$

$x_2 = 15 \qquad y_2 = 7$

$\therefore$ B is (15, 7)

(1) $\Rightarrow$ $x_1 + 15 = 22, \ y_1 + 7 = 14$

$x_1 = 7 \qquad y_1 = 7$

$\therefore$ A is (7, 7)

Area of ΔABC

$= \frac{1}{2}\begin{bmatrix} 7 & 15 & 12 & 7 \\ 7 & 7 & 1 & 7 \end{bmatrix}$

$= \frac{1}{2}[(49 + 15 + 84) - (105 + 84 + 7)]$

$= \frac{1}{2}[148 - 196]$

$= \frac{1}{2}(-48)$

$= 24$ ($\because$ Area can't be –ve)

Area of ΔPQR

$$= \frac{1}{2}\begin{bmatrix} 11 & \frac{27}{2} & \frac{19}{2} & 11 \\ 7 & 4 & 4 & 7 \end{bmatrix}$$

$$= \frac{1}{2}[(44+54+66.5)-(94.5+38+44)]$$

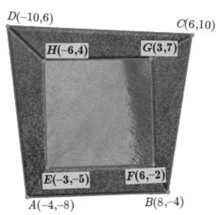
$$= \frac{1}{2}[164.5-176.5]$$

$$= \frac{1}{2}[-12]$$

$= 6$ $\quad$ ($\because$ Area can't be –ve)

$\therefore$ Area of $\Delta ABC = 4$ (Area of ΔPQR).

9. In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.



Solution :

Required area of the patio = Area of portion ABCD – Area of portion EFGH

$$= \frac{1}{2}\begin{bmatrix} -4 & 8 & 6 & -10 & -4 \\ -8 & -4 & 10 & 6 & -8 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{bmatrix}$$

$$= \frac{1}{2}[(16+80+36+80)-(-64-24-100-24)]$$

$$- \frac{1}{2}[(6+42+12+30)-(-30-6-42-12)]$$

$$= \frac{1}{2}[212-(-212)] - \frac{1}{2}[90-(-90)]$$

$$= \frac{1}{2}[424] - \frac{1}{2}[180]$$

10. A triangular shaped glass with vertices at A(-5,-4) , B(1,6) and C(7,-4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

Solution :

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{bmatrix} -5 & 1 & 7 & -5 \\ -4 & 6 & -4 & -4 \end{bmatrix}$$

$$= \frac{1}{2}[(-30-4-28)-(-4+42+20)]$$

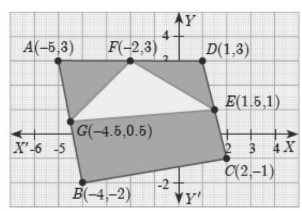
$$= \frac{1}{2}[-62-(58)]$$

$$= \frac{1}{2}[-120]$$

= 60 sq. units (Area can't be –ve).

$\therefore$ No. of paint cans needed = $\frac{60}{6} = 10$

11. In the figure, find the area of (i) triangle AGF (ii) triangle FED (iii) quadrilateral BCEG.



Solution :

i) Area of ΔAGF

$$= \frac{1}{2}\begin{bmatrix} -5 & -9/2 & -2 & -5 \\ 3 & 1/2 & 3 & 3 \end{bmatrix}$$

$$= \frac{1}{2}[(-2.5-13.5-6)-(-13.5-1-15)]$$

$$= \frac{1}{2}[(-22)-(-29.5)]$$

$$= \frac{1}{2}[7.5]$$

= 3.75 sq. units

ii) Area of ΔFED

$$= \frac{1}{2}\begin{bmatrix} -2 & \frac{3}{2} & 1 & -2 \\ 3 & 1 & 3 & 3 \end{bmatrix}$$

$$= \frac{1}{2}[(-2+4.5+3)-(-4.5+1-6)]$$

$$= \frac{1}{2}[5.5-(-0.5)]$$

$$= \frac{1}{2}[6]$$

$= 3$ sq. units

iii) Area of quadrilateral BCEG

$$= \frac{1}{2}\begin{bmatrix} -4 & 2 & \frac{3}{2} & \frac{-9}{2} & -4 \\ -2 & -1 & 1 & \frac{1}{2} & -2 \end{bmatrix}$$

$$= \frac{1}{2}[(4+2+0.75+9)-(-4-1.5-4.5-2)]$$

$$= \frac{1}{2}[15.75+12]$$

$$= \frac{27.75}{2}$$

$= 13.875$

$\simeq 13.88$ sq. units

II. INCLINATION AND SLOPE OF STRAIGHT LINE

Key Points

- ✓ The inclination of a line or the angle of inclination of a line is the angle which a straight line makes with the positive direction of X axis measured in the counter-clockwise direction to the part of the line above the X axis.
- ✓ The inclination of X axis and every line parallel to X axis is $0°$.
- ✓ The inclination of Y axis and every line parallel to Y axis is 90°.
- ✓ If θ is the angle of inclination of a non-vertical straight line, then tan θ is called the slope or gradient of the line and is denoted by *m*.
- ✓ The slope of the straight line is m = tan θ, $0 \le 180°$, $\theta \neq 90°$.
- ✓ The slope of the line through (x_1, y_1) and (x_2, y_2) with $x_1 \neq y_1$ is $\dfrac{y_2 - y_1}{x_2 - x_1}$.
- ✓ If $\theta = 0^o$, the line is parallel to the positive direction of X axis.
- ✓ If $0 < \theta < 90^o$, the line has positive slope.
- ✓ If $90^o < \theta < 180^o$, the line has negative slope.
- ✓ If $\theta = 180^o$, the line is parallel to the negative direction of X axis.
- ✓ If $\theta = 90^o$, the slope is undefined.
- ✓ Non vertical lines are parallel if and only if their slopes are equal.
- ✓ Two non-vertical lines with slopes m_1 and m_2 are perpendicular if and only if $m_1m_2 = -1$.

Example 5.8

(i) What is the slope of a line whose inclination is 30^o ?

(ii) What is the inclination of a line whose slope is $\sqrt{3}$?

Solution :

(i) Here $\theta = 30^o$

Slope $m = \tan\theta$

Therefore, slope $m = \tan 30^o = \dfrac{1}{\sqrt{3}}$

(ii) Given $m = \sqrt{3}$, let θ be the inclination of the line

$$\tan\theta = \sqrt{3}$$

We get, $\theta = 60^o$

Example 5.9

Find the slope of a line joining the given points

(i) $(-6, 1)$ and $(-3, 2)$ (ii) $\left(-\frac{1}{3}, \frac{1}{2}\right)$ and $\left(\frac{2}{7}, \frac{3}{7}\right)$ (iii) $(14, 10)$ and $(14, -6)$

Solution :

(i) $(-6, 1)$ and $(-3, 2)$

$$\text{The slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{-3+6} = \frac{1}{3}.$$

(ii) $\left(-\frac{1}{3}, \frac{1}{2}\right)$ and $\left(\frac{2}{7}, \frac{3}{7}\right)$

$$\text{The slope} = \frac{\frac{3}{7} - \frac{1}{2}}{\frac{2}{7} + \frac{1}{3}} = \frac{\frac{6-7}{14}}{\frac{6+7}{21}}$$

$$= -\frac{1}{14} \times \frac{21}{13} = -\frac{3}{26}.$$

(iii) $(14, 10)$ and $(14, -6)$

$$\text{The slope} = \frac{-6-10}{14-14} = \frac{-16}{0}.$$

The slope is undefined.

Example 5.10

The line r passes through the points $(-2, 2)$ and $(5, 8)$ and the line s passes through the points $(-8, 7)$ and $(-2, 0)$. Is the line r perpendicular to s ?

Solution :

The slope of line r is $m_1 = \dfrac{8-2}{5+2} = \dfrac{6}{7}$

The slope of line s is $m_2 = \dfrac{0-7}{-2+8} = \dfrac{-7}{6}$

The product of slopes $= \dfrac{6}{7} \times \dfrac{-7}{6} = -1$

That is, $m_1 m_2 = -1$

Therefore, the line r is perpendicular to line s.

Example 5.11

The line p passes through the points $(3, -2)$, $(12, 4)$ and the line q passes through the points $(6, -2)$ and $(12, 2)$. Is p parallel to q ?

Solution :

The slope of line p is $m_1 = \dfrac{4+2}{12-3} = \dfrac{6}{9} = \dfrac{2}{3}$

The slope of line q is $m_2 = \dfrac{2+2}{12-6} = \dfrac{4}{6} = \dfrac{2}{3}$

Thus, slope of line p = slope of line q.

Therefore, line p is parallel to the line q.

Example 5.12

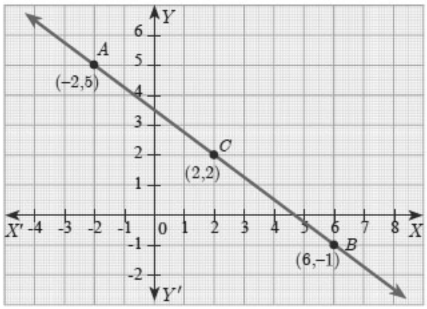
Show that the points $(-2, 5)$, $(6, -1)$ and $(2, 2)$ are collinear.

Solution :

The vertices are A $(-2,5)$, B $(6, -1)$ and C $(2, 2)$.

Slope of AB $= \dfrac{-1-5}{6+2} = \dfrac{-6}{8} = \dfrac{-3}{4}$

Slope of BC $= \dfrac{2+1}{2-6} = \dfrac{3}{-4} = \dfrac{-3}{4}$



We get, Slope of AB = Slope of BC.

Therefore, the points A, B, C all lie in a same straight line.

Hence the points A, B and C are collinear.

Example 5.13

Let A (1, –2), B (6, –2), C(5, 1) and D (2, 1) be four points.

(i) Find the slope of the line segments (a) AB (b) CD

(ii) Find the slope of the line segments (a) BC (b) AD

(iii) What can you deduce from your answer?

Solution :

(i) (a) Slope of AB = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2+2}{6-1} = 0$

(b) Slope of CD = $\dfrac{1-1}{2-5} = \dfrac{0}{-3} = 0$

(ii) (a) Slope of BC = $\dfrac{1+2}{5-6} = \dfrac{3}{-1} = -3$

(b) Slope of AD = $\dfrac{1+2}{2-1} = \dfrac{3}{1} = 3$

(iii) The slope of AB and CD are equal so AB, CD are parallel.

Similarly the lines AD and BC are not parallel, since their slopes are not equal.

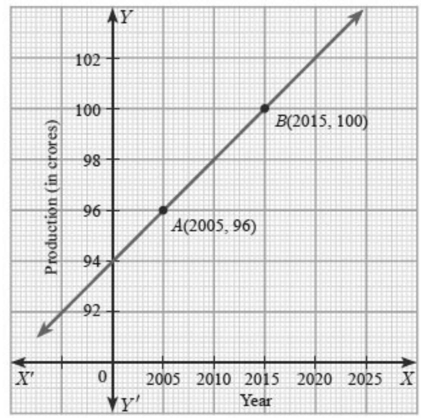
So, we can deduce that the quadrilateral ABCD is a trapezium.

Example 5.14

Consider the graph representing growth of population (in crores). Find the slope of the line AB and hence estimate the population in the year 2030?

Solution :

The points A(2005,96) and B(2015,100) are on the line AB.



$$\text{Slope of AB} = \frac{100-96}{2015-2005} = \frac{4}{10} = \frac{2}{5}$$

Let the growth of population in 2030 be k crores.

Assuming that the point C(2030,k) is on AB,

we have, slope of AC = slope of AB

$$\frac{k-96}{2030-2005} = \frac{2}{5} \text{ gives } \frac{k-96}{25} = \frac{2}{5}$$

$$k - 96 = 10$$

$$k = 106$$

Hence the estimated population in 2030 = 106 Crores.

Example 5.15

Without using Pythagoras theorem, show that the points (1,-4) , (2, –3) and (4, –7) form a right angled triangle.

Solution :

Let the given points be A(1, – 4) , B(2, –3) and C(4, –7) .

$$\text{The slope of AB} = \frac{-3+4}{2-1} = \frac{1}{1} = 1$$

$$\text{The slope of BC} = \frac{-7+3}{4-2} = \frac{-4}{2} = -2$$

$$\text{The slope of AC} = \frac{-7+4}{4-1} = \frac{-3}{3} = -1$$

Slope of AB slope of AC = (1)(−1) = −1

AB is perpendicular to AC. $\angle A = 90°$

Therefore, ΔABC is a right angled triangle.

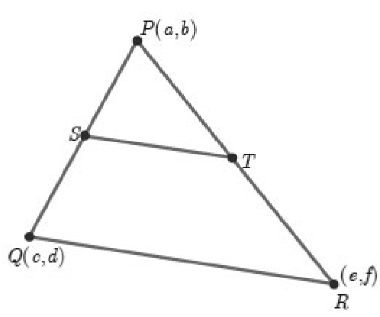
Example 5.16

Prove analytically that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and is equal to half of its length.

Solution :

Let P (*a, b*) Q (*c, d*) and R (e, f) be the vertices of a triangle.

Let S be the mid-point of PQ and T be the mid-point of PR



$$\text{Therefore } S = \left(\frac{a+c}{2}, \frac{b+d}{2}\right)$$

$$\text{and T} = \left(\frac{a+e}{2}, \frac{b+f}{2}\right)$$

$$\text{Now, slope of ST} = \frac{\frac{b+f}{2} - \frac{b+d}{2}}{\frac{a+e}{2} - \frac{a+c}{2}} = \frac{f-d}{e-c}$$

$$\text{And slope of QR} = \frac{f-d}{e-c}$$

Therefore, ST is parallel to QR. (since, their slopes are equal)

Also

$$\text{ST} = \sqrt{\left(\frac{a+e}{2} - \frac{a+c}{2}\right)^2 + \left(\frac{b+f}{2} - \frac{b+d}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(e-c)^2 + (f-d)^2}$$

$$\text{ST} = \frac{1}{2}QR$$

Thus ST is parallel to QR and half of it.

EXERCISE 5.2

1. **What is the slope of a line whose inclination with positive direction of *x*-axis is**
   (i) 90° **(ii) 0°**

Solution :

i) $\theta = 90^o$ m = tan 90^o = undefined.

ii) $\theta = 0^o$ m = tan0^o = 0

2. **What is the inlination of a line whose slope is (i) 0** **(ii) 1**

Solution :

i) m = 0 $\Rightarrow$ tan θ = 0 $\therefore \theta = 0^o$

ii) m = 1 $\Rightarrow$ tan θ = 1 $\therefore \theta = 45^o$

3. Find the slope of a line joining the points
(i) (5, $\sqrt{5}$) with the origin
(ii) (sin θ, – cos θ) and (– sin θ, cos θ)

Solution :

i) Slope of the line joining (5, $\sqrt{5}$), (0, 0)

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - \sqrt{5}}{0 - 5} = \frac{1}{\sqrt{5}}$$

$$\therefore \text{Slope} = \frac{1}{\sqrt{5}}$$

ii) Slope of line joining (sin θ, – cos θ) (– sin θ, cos θ)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\cos\theta + \cos\theta}{-\sin\theta - \sin\theta}$$

$$= \frac{2\cos\theta}{-2\sin\theta}$$

$$= -\cot\theta$$

4. What is the slope of a line perpendicular to the line joining A (5, 1) and P where P is the mid-point of the segment joining (4, 2) and (–6, 4).

Solution :

Given P is the midpoint of (4, 2), (–6, 4)

$$\Rightarrow P = \left(\frac{4-6}{2}, \frac{2+4}{2}\right)$$

$$= (-1, 3)$$

∴ Slope of the line joining A (5, 1), P (–1, 3)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{-1-5}$$

$$= \frac{2}{-6}$$

$$= \frac{-1}{3}$$

∴ Slope of the line perpendicular to the line joining A and P is $\frac{-1}{m} = 3$

5. Show that the given points are collinear: (– 3, – 4), (7, 2) and (12, 5).

Solution :

Given points are A (–3, –4), B (7, 2), C (12, 5)

$$\text{Slope of AB} = \frac{2+4}{7+3}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

$$\text{Slope of BC} = \frac{5-2}{12-7}$$

$$= \frac{3}{5}$$

∴ Slope of AB = Slope of BC

∴ AB and BC are parallel.

But B is the common point.

∴ A, B, C are collinear.

6. If the three points (3, –1), (*a*, 3) and (1, –3) are collinear, find the value of *a*.

Solution :

Given points A (3, –1), B (a, 3), C (1, –3) are collinear.

∴ Slope of AB = Slope of BC

$$\Rightarrow \frac{4}{a-3} = \frac{-6}{1-a}$$

$$\Rightarrow 4 - 4a = -6a + 18$$

$$\Rightarrow 2a = 14$$

$$a = 7$$

7. The line through the points (–2, *a*) and (9, 3) has slope $-\frac{1}{2}$. Find the value of *a*.

Solution :

Slope of the line joining (–2, *a*), (9, 3) = $-\frac{1}{2}$

$$\Rightarrow \frac{3-a}{9+2} = \frac{-1}{2}$$
$$\Rightarrow \frac{3-a}{11} = \frac{-1}{2}$$
$$\Rightarrow 6-2a = -11$$
$$\Rightarrow 2a = 17$$
$$\therefore a = \frac{17}{2}$$

8. The line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (*x*, 24). FInd the value of *x*.

Solution :

Slope of line joining (–2, 6), (4, 8)

$$m_1 = \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3}$$

Slope of line joining (8, 12), (*x*, 24)

$$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$$

Since two lines are perpendicular,

$$\Rightarrow \frac{1}{3} \times \frac{12}{x-8} = -1$$
$$\Rightarrow \frac{4}{x-8} = -1$$
$$\Rightarrow -x+8 = 4$$
$$\Rightarrow x = 4$$

9. Show that the given points form a right angled triangle and check whether they satisfies pythagoras theorem

(i) A (1, – 4), B (2, –3) and C (4, – 7)

(ii) L (0, 5), M (9, 12) and N (3, 14)

Solution :

i) Given A (1, – 4), B (2, – 3), C (4, – 7)

$$\text{Slope of AB} = \frac{-3+4}{2-1} = \frac{1}{1} = 1$$
$$\text{Slope of BC} = \frac{-7+3}{4-2} = \frac{-4}{2} = -2$$
$$\text{Slope of CA} = \frac{-7+4}{4-1} = \frac{-3}{3} = -1$$

$\therefore$ Slope of AB × Slope of CA = 1 × – 1 = – 1

$\therefore$ AB is perpendicular to AC.

$\therefore \angle A = 90^o$

$\therefore$ ΔABC is a right angled triangle.

ii) Given L (0, 5), M (9, 12), N (3, 14)

$$\text{Slope of LM} = \frac{12-5}{9-0} = \frac{7}{9}$$
$$\text{Slope of MN} = \frac{14-12}{3-9} = \frac{2}{-6} = \frac{-1}{3}$$
$$\text{Slope of LN} = \frac{14-5}{3-0} = \frac{9}{3} = 3$$

$\therefore$ Slope of MN × Slope of LN

$$= \frac{-1}{3} \times 3$$
$$= -1$$

$\therefore$ MN is perpendicular to LN.

$\therefore \angle N = 90^o$

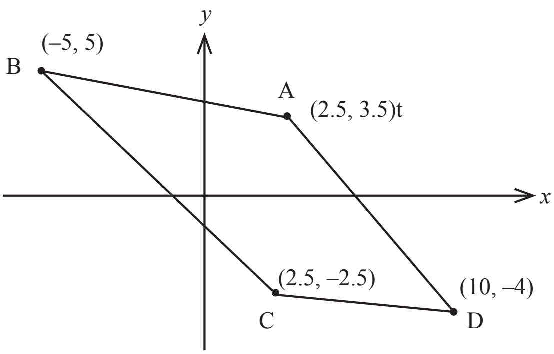
$\therefore$ ΔLMN is a right angled Δ.

10. Show that the given points form a parallelogram :

A (2.5, 3.5), B (10, – 4), C (2.5, – 2.5) and D (– 5, 5)

Solution :

Plot the points and taking in anticlockwise direction.



Let A (2.5, 3.5), B (– 5, 5), C (2.5, –2.5), D (10, –4)

$$\text{Slope of AB} = \frac{5-3.5}{-5-2.5} = \frac{1.5}{-7.5} = \frac{-1}{5}$$

$$\text{Slope of CD} = \frac{-4+2.5}{10-2.5} = \frac{-1.5}{7.5} = \frac{-1}{5}$$

$\therefore$ Slope of AB = Slope of CD.

$\therefore$ AB and CD are parallel.

Also,

$$\text{Slope of AD} = \frac{-4-3.5}{10-2.5} = \frac{-7.5}{7.5} = -1$$

$$\text{Slope of BC} = \frac{-2.5-5}{2.5+5} = \frac{-7.5}{7.5} = -1$$

$\therefore$ Slope of AD = Slope of BC.

$\therefore$ AD and BC are parallel.

$\therefore$ ABCD is a parallelogram.

11. If the points A (2, 2), B (–2, –3), C (1, –3) and D (*x, y*) form a parallelogram then find the value of *x* and *y*.

Solution :

Given A (2, 2), B (–2, –3), C (1, –3), D (*x, y*) form a parallelogram.

$\therefore$ Slope of AB = Slope of CD.

$$\Rightarrow \frac{-3-2}{-2-2} = \frac{y+3}{x-1}$$

$$\Rightarrow \frac{-5}{-4} = \frac{y+3}{x-1}$$

$$\Rightarrow \frac{5}{4} = \frac{y+3}{x-1}$$

$\Rightarrow 5x - 5 = 4y + 12$

$\Rightarrow 5x - 4y = 17$ — (1)

Also

Slope of AD = Slope of BC

$$\Rightarrow \frac{y-2}{x-2} = \frac{-3+3}{1+2}$$

$$\Rightarrow \frac{y-2}{x-2} = 0$$

$\Rightarrow y - 2 = 0$

$\Rightarrow \quad y = 2$

Sub. in (1)

$5x - 8 = 17$

$\Rightarrow 5x = 25$

$\therefore x = 5$

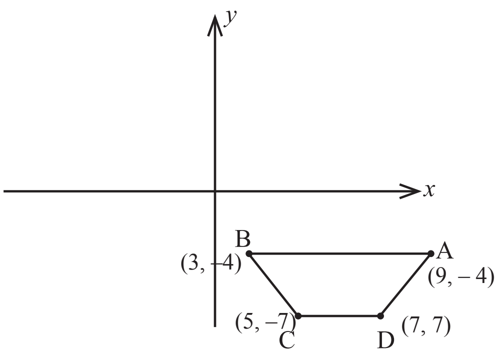
$\therefore x = 5, y = 2$

12. Let A (3, – 4), B (9, – 4), C (5, –7) and D (7, –7). Show that ABCD is a trapezium.

Solution :

Given points are (3, – 4), (9, –4), (5, –7), (7, –7)

Plotting the given points in a plane and taking in anti clockwise direction.



Let A (9, –4), B (3, –4), C (5, –7) D (7, –7)

Slope of AB = $\dfrac{-4+4}{3-9} = 0$

Slope of CD = $\dfrac{-7+7}{7-5} = 0$

$\therefore$ Slope of AB = Slope of CD.

$\therefore$ AB and CD are parallel.

Slope of AD = $\dfrac{-7+4}{7-9} = \dfrac{-3}{-2} = \dfrac{3}{2}$

Slope of BC = $\dfrac{-7+4}{5-3} = \dfrac{-3}{2}$

$\therefore$ Slope of AD $\neq$ Slope of BC

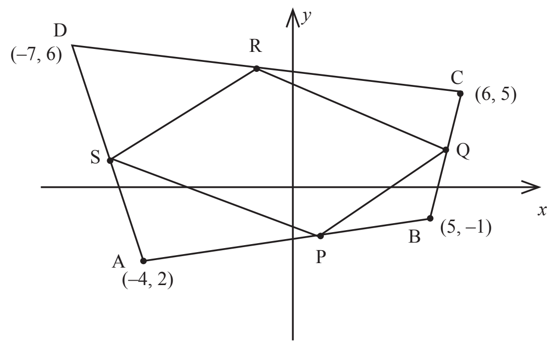
$\therefore$ One pair of opposite sides is equal.

$\therefore$ ABCD is a trapezium.

13. A quadrilateral has vertices at A (-4,-2) B(5, -1) , C(6, 5) and D(–7, 6). Show that the mid-points of its sides form a parallelogram.

Solution :

Given points are (– 4, – 2), (5, –1), (6, 5), (– 7, 6) which forms a quadrilateral.



Mid point of AB = $\left(\dfrac{-4+5}{2}, \dfrac{-2-1}{2}\right) = \left(\dfrac{1}{2}, \dfrac{-3}{2}\right)$ P

Mid point of BC = $\left(\dfrac{5+6}{2}, \dfrac{-1+5}{2}\right) = \left(\dfrac{11}{2}, 2\right)$ Q

Midpoint of CD = $\left(\dfrac{-7+6}{2}, \dfrac{6+5}{2}\right) = \left(\dfrac{-1}{2}, \dfrac{11}{2}\right)$ R

Midpoint of AD = $\left(\dfrac{-7-4}{2}, \dfrac{6-2}{2}\right) = \left(\dfrac{-11}{2}, 2\right)$ S

To prove : PQRS is a parallelogram.

$$\text{Slope of PQ} = \frac{2+\frac{3}{2}}{\frac{11}{2}-\frac{1}{2}}$$

$$= \frac{7/2}{10/2}$$

$$= \frac{7}{10}$$

$$\text{Slope of SR} = \frac{2-\frac{11}{2}}{\frac{-11}{2}+\frac{1}{2}}$$

$$= \frac{-7/2}{-10/2}$$

$$= \frac{7}{10}$$

$\therefore$ Slope of PQ = Slope of SR

$\therefore$ PQ and SR are parallel.

Also,

$$\text{Slope of PS} = \frac{2+\frac{3}{2}}{\frac{-11}{2}-\frac{1}{2}}$$

$$= \frac{7/2}{-12/2}$$

$$= \frac{-7}{12}$$

$$\text{Slope of QR} = \frac{11/2-2}{-1/2-11/2}$$

$$= \frac{7/2}{-12/2}$$

$$= \frac{-7}{12}$$

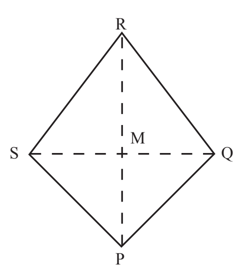
$\therefore$ Slope of PS = Slope of QR

$\therefore$ PS and QR are parallel.

$\therefore$ PQRS is a parallelogram.

14. PQRS is a rhombus. Its diagonals PR and QS intersect at the point M and satisfy QS = 2PR. If the coordinates of S and M are (1, 1) and (2, –1) respectively, find the coordinates of P.

Solution :



Given, In rhombus PQRS, diagonals PR and QS meet at M such that QS = 2PR.

Also, given S (1, 1), M (2, –1)

Let Q be (x, y)

Midpoint of SQ = M ($\because$ rhombus)

$$\left(\frac{x+1}{2}, \frac{y+1}{2}\right) = (2, -1)$$

$\Rightarrow x + 1 = 4 \qquad y + 1 = -2$

$\Rightarrow x = 3, \qquad y = -3$

$\therefore$ Q is (3, – 3)

Since QS = 2 PR

$QS^2 = 4 \cdot PR^2$

$(3-1)^2 + (-3-1)^2 = 4 \cdot PR^2$

$\therefore PR^2 = 5$

$\therefore PR = \sqrt{5}$

$\Rightarrow PM = \frac{\sqrt{5}}{2}, \qquad M(2, -1)$

Let P be (l, m)

III. EQUATIONS OF STRAIGHT LINES:

Key Points

- ✓ Equation of OY(Y axis) is $x = 0$.
- ✓ Equation of OX (X axis) is y = 0.
- ✓ Equation of a straight line parallel to X axis is $y = b$.
- ✓ If b > 0 , then the line y=b lies above the X axis, If b < 0 , then the line y=b lies below the X axis, If b = 0 , then the line y=b is the X axis itself.
- ✓ Equation of a Straight line parallel to the Y axis is $x = c$.

  If c > 0 , then the line x=c lies right to the side of the Y axis

  If c < 0 , then the line x=c lies left to the side of the Y axis

  If c = 0 , then the line x=c is the Y axis itself.
- ✓ Slope-Intercept Form A line with slope m and y intercept c can be expressed through the equation $y = mx + c$.
- ✓ Point-Slope form $y - y_1 = m\,(x - x_1)$.
- ✓ $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$ is the equation of the line in two-point form.
- ✓ Intercept Form $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

Example 5.17

Find the equation of a straight line passing through (5,7) and is (i) parallel to X axis (ii) parallel to Y axis.

Solution :

(i) The equation of any straight line parallel to X axis is y=b.

Since it passes through (5,7), b = 7 .

Therefore, the required equation of the line is y=7.

(ii) The equation of any straight line parallel toY axis is x=c

Since it passes through (5,7), c = 5

Therefore, the required equation of the line is x = 5.

Example 5.18

Find the equation of a straight line whose (i) Slope is 5 and y intercept is – 9 (ii) Inclination is 45° and y intercept is 11

Solution :

(i) Given, Slope = 5, y intercept, c =–9

Therefore, equation of a straight line is

$y = mx + c$

y = 5x −9 gives 5x −y −9 = 0

(ii) Given, $\theta = 45°$, y intercept, $c = 11$

Slope m = tan θ = tan 45° = 1

Therefore, equation of a straight line is of the form $y = mx + c$

Hence we get, y = x +11 gives x −y +11 = 0.

Example 5.19

Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$.

Solution :

Equation of the given straight line is

$8x - 7y + 6 = 0$

$7y = 8x + 6$ (bringing it to the form

$y = mx + c$)

Slope $m = \frac{8}{7}$ and y intercept $c = \frac{6}{7}$

Example 5.20

The graph relates temperatures y (in Fahrenheit degree) to temperatures x (in Celsius degree) (a) Find the slope and y intercept (b) Write an equation of the line (c) What is the mean temperature of the earth in Fahrenheit degree if its mean temperature is 25° Celsius?

Solution :

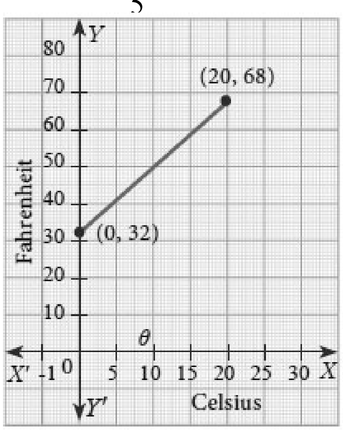
(a) From the figure, slope

$$= \frac{\text{change in } y \text{ coordinate}}{\text{change in } x \text{ coordinate}}$$

$$= \frac{68-32}{20-0} = \frac{36}{20} = \frac{9}{5} = 1.8$$

The line crosses the Y axis at (0, 32)

So the slope is $\frac{9}{5}$ and y intercept is 32.



(b) Use the slope and y intercept to write an equation

The equation is $y = \frac{9}{5}x + 32$.

(c) In Celsius, the mean temperature of the earth is 25°. To find the mean temperature in Fahrenheit, we find the value of y when $x = 25$.

$$y = \frac{9}{5}x + 32$$

$$y = \frac{9}{5}(25) + 32$$

$$y = 77$$

Therefore, the mean temperature of the earth is 77° F.

Example 5.21

Find the equation of a line passing through the point (3, – 4) and having slope $\frac{-5}{7}$.

Solution :

Given, $(x, y) = (3, -4)$ and $m = \frac{-5}{7}$

The equation of the point-slope form of the straight line is $y - y_1 = m(x - x_1)$

we write it as $y + 4 = -\frac{5}{7}(x - 3)$

gives us $5x + 7y + 13 = 0$.

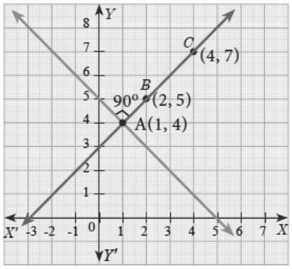
Example 5.22

Find the equation of a line passing through the point A(1, 4) and perpendicular to the line joining points (2,5) and (4,7) .

Solution :

Let the given points be A(1, 4) , B(2,5) and C(4,7) .

Slope of line BC = $\frac{7-5}{4-2} = \frac{2}{2} = 1$.



Let m be the slope of the required line.

Since the required line is perpendicular to BC,

$m \times 1 = -1$

$m = -1$

The required line also pass through the point A(1,4).

The equation of the required straight line is

$y - y_1 = m\,(x - x_1)$

$y - 4 = -1(x - 1)$

$y - 4 = -x + 1$

we get, $x + y - 5 = 0$.

Example 5.23

Find the equation of a straight line passing through (5, –3) and (7, – 4).

Solution :

The equation of a straight line passing through the two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Substituting the points we get,

$$\frac{y+3}{-4+3} = \frac{x-5}{7-5}$$

gives $2y + 6 = -x + 5$

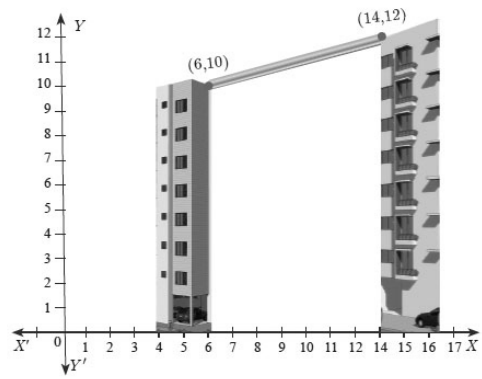
Therefore, $x + 2y + 1 = 0$.

Example 5.24

Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from (6, 10) to (14, 12), find the equation of the rod joining the buildings ?

Solution :

Let A(6,10) , B(14,12) be the points denoting the terrace of the buildings.



The equation of the rod is the equation of the straight line passing through A(6,10) and B(14,12)

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \text{ gives } \frac{y-10}{12-10} = \frac{x-6}{11-6}$$

$$\frac{y-10}{2} = \frac{x-6}{8}$$

Therefore, $x - 4y + 34 = 0$.

Hence, equation of the rod is $x - 4y + 34 = 0$.

Example 5.25

Find the equation of a line which passes through (5,7) and makes intercepts on the axes equal in magnitude but opposite in sign.

Solution :

Let the x intercept be ‘a’ and y intercept be ‘–a’.

The equation of the line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

gives $\frac{x}{a}+\frac{y}{-a}=1$ (Here $b=-a$)

Therefore, $x-y=a$ ....(1)

Since (1) passes through (5,7)

Therefore, $5-7=a$ gives $a=-2$

Thus the required equation of the straight line is $x-y=-2$; or $x-y+2=0$.

Example 5.26

Find the intercepts made by the line $4x-9y+36=0$ on the coordinate axes.

Solution :

Equation of the given line is $4x-9y+36=0$.

we write it as $4x-9y=-36$

(bringing it to the normal form)

Dividing by -36 we get, $\frac{x}{-9}+\frac{y}{4}=1$ ....(1)

Comparing (1) with intercept form, we get x intercept $a=-9$; y intercept $b=4$

Example 5.27

A mobile phone is put to use when the battery power is 100%. The percent of battery power 'y' (in decimal) remaining after using the mobile phone for x hours is assumed as $y=-0.25x+1$

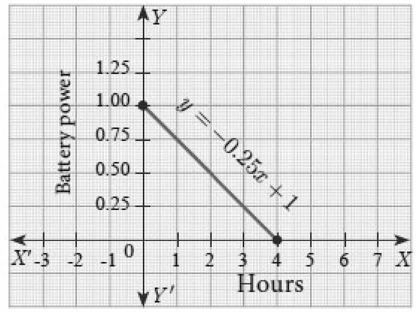
(i) Draw a graph of the equation.

(ii) Find the number of hours elapsed if the battery power is 40%.

(iii) How much time does it take so that the battery has no power?

Solution :

(i)



(ii) To find the time when the battery power is 40%, we have to take y = 0.40

$0.40=-0.25x+1$ gives $0.25x=0.60$

we get, $x=\frac{0.60}{0.25}=2.4$ hours.

(iii) If the battery power is 0 then y = 0

Therefore, $0=-0.25x+1$ gives $-0.25x=1$ hence $x=4$ hours.

Thus, after 4 hours, the battery of the mobile phone will have no power.

Example 5.28

A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through (–3, 8). Find its equation.

Solution :

If a and b are the intercepts then $a+b=7$ or $b=7-a$

By intercept form $\frac{x}{a}+\frac{y}{b}=1$ ...(1)

We have $\frac{x}{a}+\frac{y}{7-a}=1$

As this line pass through the point (–3, 8), we have

$$\frac{-3}{a}+\frac{8}{7-a}=1$$

gives – 3 (7 – a) + 8a = a (7 – a).

– 21 + 3a + 8a = 7a – a^2

So, a^2 + 4a – 21 = 0.

Solving this equation (a – 3) (a + 7) = 0

a = 3 or a = – 7

Solving this equation (a – 3) (a + 7) = 0

a = 3 or a = – 7

Since a is positive, we have a = 3 and

b = 7 – a = 7 – 3 = 4.

Hence $\frac{x}{3}+\frac{y}{4}=1$

Therefore, 4x + 3y – 12 = 0 is the required equation.

Example 5.29

A circular garden is bounded by East Avenue and Cross Road. Cross Road intersects North Street at D and East Avenue at E. AD is tangential to the circular garden at A(3, 10). Using the figure.

(a) Find the equation of

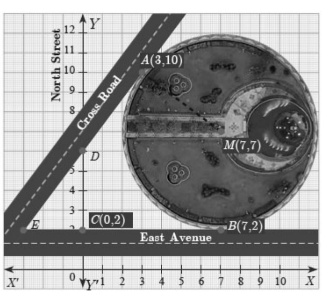
(i) East Avenue.

(ii) North Street

(iii) Cross Road

(b) Where does the Cross Road intersect the

(i) East Avenue ? (ii) North Street ?



Solution :

(a) (i) East Avenue is the straight line joining C(0, 2) and B(7, 2). Thus the equation of East Avenue is obtained by using two-point form which is

$$\frac{y-2}{2-2}=\frac{x-0}{7-0}$$

$$\frac{y-2}{0}=\frac{x}{7} \text{ gives } y=2$$

(ii) Since the point D lie vertically above C(0, 2). The x coordinate of D is 0.

Since any point on North Street has x coordinate value 0.

The equation of North Street is x = 0

(iii) To find equation of Cross Road.

Center of circular garden M is at (7, 7), A is (3, 10)

We first find slope of MA, which we call m_1

$$\text{Thus } m_1=\frac{10-7}{3-7}=\frac{-3}{4}$$

Since the Cross Road is perpendicular to MA, if m_2 is the slope of the Cross Road then,

$$m_1m_2=-1 \text{ gives } \frac{-3}{4}m_2=-1 \text{ so } m_2=\frac{4}{3}$$

Now, the cross road has slope $\frac{4}{3}$ and it passes through th epoint A (3, 10).

The equation of the Cross Road is

$$y-10=\frac{4}{3}(x-3).$$

3y – 30 = 4x – 12

Hence, 4x – 3y + 18 = 0

(b) (i) If D is (0, k) then D is a point on the Cross Road.

Therefore, substituting $x = 0$, $y = k$ in the equation of Cross Road,

we get, $0 - 3k + 18 = 0$

Value of $k = 6$

Therefore, D is (0, 6)

(ii) To find E, let E be (q,2)

Put y =2 in the equation of the Cross Road,

we get, $4q - 6 + 18 = 0$

$$4q = -12 \text{ gives } q = -3$$

Therefore, The point E is (-3,2)

Thus the Cross Road meets the North Street at D(0, 6) and East Avenue at E (-3,2) .

EXERCISE 5.3

1. Find the equation of a straight line passing through the mid-point of a line segment joining the points (1,-5), (4,2) and parallel to (i) X axis (ii) Y axis

Solution :

Mid point of the line joining the points (1, –5), (4, 2) is

$$= \left(\frac{1+4}{2}, \frac{-5+2}{2}\right)$$

$$= \left(\frac{5}{2}, \frac{-3}{2}\right)$$

i) Equation of straight line passing through $\left(\frac{5}{2}, \frac{-3}{2}\right)$ and

a) Parallel to x-axis is

$$y = {}^{-3}\!/_{2} \Rightarrow 2y + 3 = 0$$

b) Parallel to y-axis is

$$x = {}^{5}\!/_{2} \Rightarrow 2x - 5 = 0$$

2. The equation of a straight line is $2(x - y) + 5 = 0$. Find its slope, inclination and intercept on the Y axis.

Solution :

Given equation of a straight line is

$2(x - y) + 5 = 0$

$\Rightarrow 2x - 2y + 5 = 0$ — (1)

i) Slope of the line $= \dfrac{-\text{coefficient of } x}{\text{coefficient of } y}$

$$= \frac{-2}{-2}$$

$$= 1$$

ii) Slope of the line = 1

$\tan\theta = 1$

$\therefore \theta = 45^{o}$.

iii) Interecept on y-axis

Put $x = 0$ in (1)

$-2y + 5 = 0$

$\Rightarrow -2y = -5$

$\Rightarrow y = {}^{5}\!/_{2}$

$\therefore y$ - intercept $= {}^{5}\!/_{2}$

3. Find the equation of a line whose inclination is 30° and making an intercept -3 on the Y axis.

Solution :

Given $\theta = 30^{o} \Rightarrow m = \tan 30^{o} = \dfrac{1}{\sqrt{3}}$ and

y-intercept = – 3

The required equation of the line is

$y = mx + c$

$$\Rightarrow y = \frac{1}{\sqrt{3}}x - 3$$

$$\Rightarrow \sqrt{3}y = x - 3\sqrt{3}$$

$$\Rightarrow x - \sqrt{3}y - 3\sqrt{3} = 0$$

4. Find the slope and y intercept of $\sqrt{3}\,x + (1-\sqrt{3})y = 3$.

Solution :

Given line is $\sqrt{3}\,x + (1-\sqrt{3})\,y - 3 = 0$

$$\Rightarrow (1-\sqrt{3})y = -\sqrt{3}x + 3$$

$$\Rightarrow y = \frac{-\sqrt{3}}{1-\sqrt{3}}x + \frac{3}{1-\sqrt{3}}$$

This is of the form $y = mx + c$.

$$m = \frac{-\sqrt{3}}{1-\sqrt{3}} \qquad c = \frac{3}{1-\sqrt{3}}$$

$$= \frac{\sqrt{3}}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1} \qquad = \frac{3}{1-\sqrt{3}}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{3\sqrt{3}+3}{2} \qquad = \frac{3+3\sqrt{3}}{-2}$$

$$\therefore \text{ Slope} = \frac{3+\sqrt{3}}{2},\quad y-\text{intercept} = \frac{3+3\sqrt{3}}{-2}.$$

---

5. Find the value of 'a', if the line through (–2, 3) and (8, 5) is perpendicular to $y = ax + 2$.

Solution :

Slope of the line joining (– 2, 3), (8, 5).

$$= \frac{5-3}{8+2}$$

$$= \frac{2}{10}$$

$$m_1 = \frac{1}{5}$$

Slope of the line $y = ax + 2$ is $m_2 = a$.

Since the 2 lines are perpendicular.

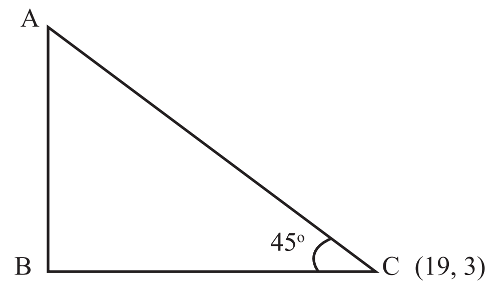
$$m_1 m_2 = -1$$

$$\Rightarrow \frac{1}{5}\times a = -1$$

$$\Rightarrow \quad a = -5$$

6. The hill in the form of a right triangle has its foot at (19, 3) . The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top.

Solution :



C – Foot of the hill.

$\therefore$ Slope of AC = m = tan 45° = 1

$\therefore$ Equation of AC whose slope 1 and passing through C (19, 3) is

$$y - y_1 = m\,(x - x_1)$$

$$\Rightarrow y - 3 = 1\,(x - 19)$$

$$\Rightarrow x - y - 16 = 0.$$

---

7. Find the equation of a line through the given pair of points

(i) $\left(2, \frac{2}{3}\right)$ and $\left(\frac{-1}{2}, -2\right)$

(ii) (2, 3) and (–7, –1)

Solution :

Given points are

$$\left(2, \frac{2}{3}\right), \left(\frac{-1}{2}, -2\right)$$

The equation of the line passing through 2 points

$$\frac{y - \frac{2}{3}}{-2 - \frac{2}{3}} = \frac{x-2}{\frac{-1}{2} - 2}$$

$$\Rightarrow \frac{3y-2}{-8} = \frac{x-2}{\frac{-5}{2}}$$

$$\Rightarrow \frac{3y-2}{-8} = \frac{2x-4}{-5}$$
$$\Rightarrow 15y - 10 = 16x - 32$$
$$\Rightarrow 16x - 15y - 22 = 0$$

ii) Given points are (2, 3), (–7, –1)

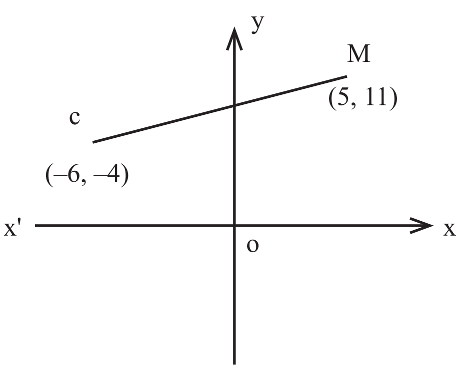
Equation through 2 points is

$$\frac{y-3}{-1-3} = \frac{x-2}{-7-2}$$
$$\Rightarrow \frac{y-3}{-4} = \frac{x-2}{-9}$$
$$\Rightarrow 9y - 27 = 4x - 8$$
$$\Rightarrow 4x - 9y + 19 = 0$$

---

8. A cat is located at the point(-6,-4) in xy plane. A bottle of milk is kept at (5,11). The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

Solution :



C (– 6, –4) is the position of cat.

M (5, 11) is the position of milk.

Equation of the path CM is

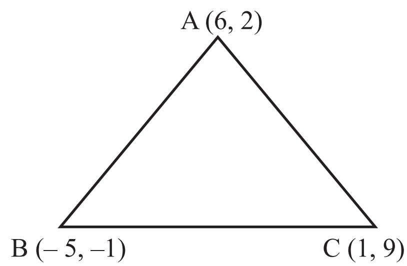
$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y+4}{11+4} = \frac{x+6}{5+6}$$
$$\Rightarrow \frac{y+4}{15} = \frac{x+6}{11}$$
$$\Rightarrow 15x + 90 = 11y + 44$$
$$\Rightarrow 15x - 11y + 46 - 0$$

---

9. Find the equation of the median and altitude of ΔABC through A where the vertices are A(6,2) , B(-5,-1) and C(1,9)

Solution :



i) Equation of the median through A.

mid point of BC = $\left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$

= D (– 2, 4)

Equation of AD is [A (6, 2), D (– 2, 4).

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$
$$\Rightarrow \frac{y-2}{4-2} = \frac{x-6}{-2-6}$$
$$\Rightarrow \frac{y-2}{2} = \frac{x-6}{-8}$$
$$= \frac{y-2}{1} = \frac{x-6}{-4}$$
$$\Rightarrow x - 6 = -4y + 8$$
$$\Rightarrow x + 4y - 14 = 0$$

---

ii) Equation of altitude through 'A'

Slope of BC = $\frac{9+1}{1+5} = \frac{10}{6} = \frac{5}{3}$

Since AD ⊥ BC, slope of AD = $\frac{-3}{5}$

and A is (6, 2).

∴ Equation of altitude AD is

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = \frac{-3}{5}(x - 6)$

$\Rightarrow 5y - 10 = -3x + 18$

$\Rightarrow 3x + 5y - 28 = 0$

10. Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point (–1,2).

Solution :

Given slope of the line is $\frac{-5}{4}$ and (–1, 2) is a point on the line.

∴ its equation is $y - y_1 = m\,(x - x_1)$

$\Rightarrow y - 2 = \frac{-5}{4}(x + 1)$

$\Rightarrow 4y - 8 = -5x - 5$

$\Rightarrow 5x + 4y - 3 = 0$

11. You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in x seconds is given by y = −0.1x +1.

(i) graph the equation.

(ii) find the total MB of the song.

(iii) after how many seconds will 75% of the song gets downloaded?

(iv) after how many seconds the song will be downloaded completely?

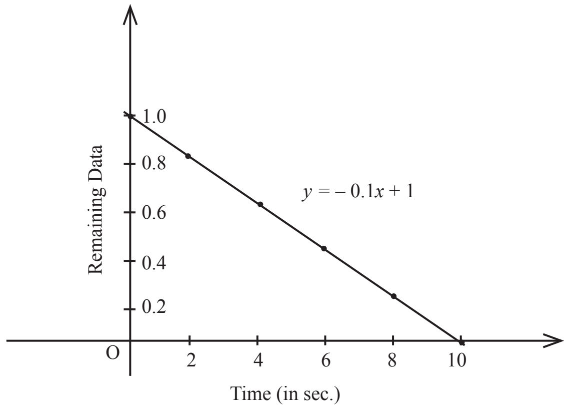
Solution :

Given $y = -0.1x + 1$.

where x - time (in sec.)

y - remaining data to be downloaded.

x	0	2	4	6	8	10
y	1	0.8	0.6	0.4	0.2	0



ii) $y = -0.1x + 1$

when $x = 0$, $y = 1$ (∵ x can't be –ve)

∴ Total MB of the song = 1 MB

iii) 75% of MB downloaded.

∴ 25% of MB to be downloaded

∴ Put y = 0.25 in (1)

$\Rightarrow 0.25 = -0.1x + 1$

$\Rightarrow 0.1x = 0.75$

$x = \frac{0.75}{0.10} = \frac{15}{2} = 7.5$

∴ Required time = 7.5 sec.

iv) Time when songs completely downloaded

Put $y = 0$ in (1)

ie, $0.1x = 1$

$x = 10$ sec.

∴ Songs will be downloaded completely after 10 sec.

12. Find the equation of a line whose intercepts on the x and y axes are given below.

(i) 4, – 6 **(ii) – 5, $\frac{3}{4}$**

Solution :

i) Given x-intercept = 4 = a

y - intercept = – 6 = b

Equation of line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \frac{x}{4}+\frac{y}{-6}=1$$

$$\Rightarrow \frac{x}{4}-\frac{y}{6}=1$$

$$\Rightarrow \frac{3x-2y}{12}=1$$

$$\Rightarrow 3x-2y-12=0$$

ii) Given x-intercept = – 5 = a

y-intercept = $\frac{3}{4}=b$

Equation of line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \frac{x}{-5}+\frac{y}{3/4}=1$$

$$\Rightarrow \frac{x}{-5}-\frac{4y}{3}=1$$

$$\Rightarrow \frac{-3x+20y}{15}=1$$

$$\Rightarrow -3x+20y-15=0$$

$$\Rightarrow 3x-20y+15=0$$

13. Find the intercepts made by the following lines on the coordinate axes.

(i) 3x −2y − 6 = 0 (ii) 4x + 3y +12 = 0

Solution :

Method - 1 :

$$3x-2y=6$$

$$\Rightarrow \frac{3x}{6}-\frac{2y}{6}=1$$

$$\Rightarrow \frac{x}{2}+\frac{y}{-3}=1$$

$\therefore$ x-int = 2, y-int = – 3

Method - 2 :

3x – 2y = 6

Put y = 0 $\Rightarrow$ 3x = 6

$\Rightarrow x = 2$ (x-int)

Put x = 0 $\Rightarrow$ – 2y = 6

$\Rightarrow y = -3$ (y - int)

ii) Given line is 4x + 3y + 12 = 0

$$\Rightarrow 4x+3y=-12$$

$$\Rightarrow \frac{4x}{-12}+\frac{3y}{-12}=1$$

$$\Rightarrow \frac{x}{-3}+\frac{y}{-4}=1$$

$\therefore$ x-int = – 3, y-int = – 4.

14. Find the equation of a straight line

(i) passing through (1,-4) and has intercepts which are in the ratio 2:5

(ii) passing through (-8, 4) and making equal intercepts on the coordinate axes

Solution :

i) Required line is passing through (1, – 4) and has intercepts in the ratio 2 : 5.

Equation of line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \frac{x}{a}+\frac{y}{\frac{5a}{2}}=1$$

$$\Rightarrow \frac{x}{a}+\frac{2y}{5a}=1 \quad ...(1)$$

where $a : b = 2 : 5$

$$\Rightarrow \frac{a}{b}=\frac{2}{5}$$

$$\Rightarrow b=\frac{5a}{2}$$

Since (1) passes through (1, – 4)

$$\frac{1}{a}-\frac{8}{5a}=1$$

$$\Rightarrow \frac{5-8}{5a}=1$$

$$\Rightarrow \quad 5a=-3$$

$$a=\frac{-3}{5}$$

$$\therefore (1) \Rightarrow \frac{x}{-3/5}+\frac{2y}{-3}=1$$

$$\Rightarrow \frac{5x}{-3}+\frac{2y}{-3}=1$$

$$\Rightarrow 5x+2y=-3$$

$$\Rightarrow 5x+2y+3=0$$

ii) Required line is passing through (– 8, 4) and making equal intercepts on the axes.

Equation o fline in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \quad \text{where} \quad a=b$$

$$\Rightarrow \frac{x}{a}+\frac{y}{a}=1$$

$$\Rightarrow x+y=a \qquad ....(1)$$

Since (1) passes through (– 8, 4)

$-8 + 4 = a$

$a = -4$

$\therefore (1) \Rightarrow x + y = -4 \qquad \Rightarrow x + y + 4 = 0$

IV. GENERAL FORM OF A STRAIGHT LINE

Key Points

- ✓ The equation of all lines parallel to the line ax +by +c = 0 can be put in the form $ax + by + k = 0$ for different values of k.
- ✓ The equation of all lines perpendicular to the line $ax + by + c = 0$ can be written as $bx - ay + k = 0$ for different values of k.
- ✓ Slope of a straight line $m = \dfrac{-\text{coefficient of } x}{\text{coefficient of } y}$
- ✓ y intercept $= \dfrac{-\text{ constant term}}{\text{coefficient of } y}$

Example 5.30

Find the slope of the straight line $6x+8y + 7 = 0$.

Solution :

Given $6x + 8y + 7 = 0$

$$\text{slope } m=\frac{-\text{coefficient of } x}{\text{coefficient of } y}=-\frac{6}{8}=-\frac{3}{4}$$

Therefore, the slope of the straight line is $-\dfrac{3}{4}$.

Example 5.31

Find the slope of the line which is

(i) parallel to $3x - 7y = 11$

(ii) perpendicular to $2x - 3y + 8 = 0$.

Solution :

(i) Given straight line is $3x - 7y = 11$

gives $3x - 7y - 11 = 0$

Slope $m = \frac{-3}{-7} = \frac{3}{7}$

Since parallel lines have same slopes, slope of any line parallel to

$3x - 7y = 11$ is $\frac{3}{7}$.

(ii) Given striaght line is $2x - 3y + 8 = 0$

Slope $m = \frac{-2}{-3} = \frac{2}{3}$

Since product of slopes is −1 for perpendicular lines, slope of any line perpendicular to $2x - 3y + 8 = 0$ is $\frac{-1}{\frac{2}{3}} = \frac{-3}{2}$

Example 5.32

Show that the straight lines $2x + 3y - 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Solution :

Slope of the straight line $2x + 3y - 8 = 0$ is

$$m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$$

$$m_1 = \frac{-2}{3}$$

Slope of the straight line $4x + 6y + 18 = 0$ is

$$m_2 = \frac{-4}{6} = \frac{-2}{3}$$

Here, $m_1 = m_2$

That is, slopes are equal. Hence, the two straight lines are parallel.

Example 5.33

Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Solution :

Slope of the straight line $x - 2y + 3 = 0$ is

$$m_1 = \frac{-1}{-2} = \frac{1}{2}$$

Slope of the straight line $6x + 3y + 8 = 0$ is

$$m_2 = \frac{-6}{3} = -2$$

Now, $m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$

Hence, the two straight lines are perpendicular.

Example 5.34

Find the equation of a straight line which is parallel to the line $3x - 7y = 12$ and passing through the point (6, 4).

Solution :

Equation of the straight line, parallel to $3x - 7y - 12 = 0$ is $3x - 7y + k = 0$

Since it passes through the point (6,4)

$3(6) - 7(4) + k = 0$

$k = 28 - 18 = 10$

Therefore, equation of the required straight line is $3x - 7y + 10 = 0$.

Example 5.35

Find the equation of a straight line perpendicular to the line $y = \frac{4}{3}x - 7$ and passing through the point (7, – 1).

Solution :

The equation $y = \frac{4}{3}x - 7$ can be written as $4x - 3y - 21 = 0$.

Equation of a straight line perpendicular to $4x - 3y - 21 = 0$ is $3x + 4y + k = 0$

Since it is passes through the point (7, –1),

$21 - 4 + k = 0$ we get, $k = -17$

Therefore, equation of the required straight line is $3x + 4y - 17 = 0$.

Example 5.36

Find the equation of a straight line parallel to Y axis and passing through the point of intersection of the lines $4x + 5y = 13$ and $x - 8y + 9 = 0$.

Solution :

Given lines $4x + 5y - 13 = 0$ ...(1)

$x - 8y + 9 = 0$ ...(2)

$$\begin{array}{cccccccc} & x & & y & & 1 & & \\ 5 & & -13 & & 4 & & 5 \\ -8 & & 9 & & 1 & & -8 \end{array}$$

$$\frac{x}{45-104} = \frac{y}{-13-36} = \frac{1}{-32-5}$$

$$\frac{x}{-59} = \frac{y}{-49} = \frac{1}{-37}$$

$$x = \frac{59}{37}, y = \frac{49}{37}$$

Therefore, the point of intersection

$$(x, y) = \left(\frac{59}{37}, \frac{49}{37}\right)$$

The equation of line parallel to Y axis is x = c.

It passes through $(x, y) = \left(\frac{59}{37}, \frac{49}{37}\right)$.

Therefore, $c = \frac{59}{37}$

The equation of the line is $x = \frac{59}{37}$ gives $37x - 59 = 0$.

Example 5.37

The line joining the points A(0, 5) and B(4, 1) is a tangent to a circle whose centre C is at the point (4, 4) find

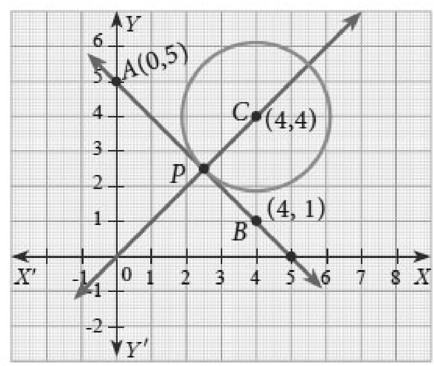
(i) the equation of the line AB.

(ii) the equation of the line through C which is perpendicular to the line AB.

(iii) the coordinates of the point of contact of tangent line AB with the circle.

Solution :

(i) Equation of line AB, A(0,5) and B(4,1)



$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{1 - 5} = \frac{x - 0}{4 - 0}$$

$4(y - 5) = -4x$ gives $y - 5 = -x$

$x + y - 5 = 0$

(ii) The equation of a line which is perpendicular to the line AB : $x + y - 5 = 0$ is $x - y + k = 0$

Since it is passing through the point (4,4), we have

$4 - 4 + k = 0$ gives $k = 0$

The equation of a line which is perpendicular to AB and through C is

$x - y = 0$ ...(2)

(iii) The coordinate of the point of contact P of the tangent line AB with the circle is

$x + y - 5 = 0$ and $x - y = 0$

Solving, we get $x = \frac{5}{2}$ and $y = \frac{5}{2}$

Therefore, the coordinate of the point of contact is P $\left(\frac{5}{2}, \frac{5}{2}\right)$.

EXERCISE 5.4

1. Find the slope of the following straight lines

(i) 5y −3 = 0 **(ii)** $7x - \frac{3}{17} = 0$

Solution :

i) Given line is $5y - 3 = 0$

$$\text{Slope} = \frac{\text{Co.eff. of } x}{\text{Co.eff. of } y}$$

$$= \frac{-0}{5}$$

$$= 0$$

ii) Given line is $7x - \frac{3}{17} = 0$

$$\text{Slope} = \frac{\text{Co.eff. of } x}{\text{Co.eff. of } y}$$

$$= \frac{-7}{0}$$

= undefined

2. Find the slope of the line which is

(i) parallel to y = 0.7*x* −11

(ii) perpendicular to the line *x* = −11.

Solution :

i) Given line is $y = 0.7x - 11$

whose slope is 0.7

∴ Slope of the line parallel to

$y = 0.7x - 11$ is also 0.7

ii) Given line is $x = -11$,

whose slope is 0

∴ Slope of the line perpendicular to

$x = -11$ is , $\frac{-1}{0}$

which is undefined.

3. Check whether the given lines are parellel or perpendicular

(i) $\frac{x}{3} + \frac{y}{4} + \frac{1}{7} = 0$ **and** $\frac{2x}{3} + \frac{y}{2} + \frac{1}{10} = 0$

(ii) 5*x* + 23*y* + 14 = 0 and 23*x* – 5*y* + 9 = 0

Solution :

i) Given pair of lines

$$\frac{x}{3} + \frac{y}{4} + \frac{1}{7} = 0, \quad \frac{2x}{3} + \frac{y}{2} + \frac{1}{10} = 0$$

Their slope,

$$m_1 = \frac{-1/3}{1/4} \quad \& \quad m_2 = \frac{-2/3}{1/2}$$

$$= -4/3 \qquad = -4/3$$

$\therefore m_1 = m_2$

$\Rightarrow$ the 2 lines are parallel

ii) Given lines are

$5x + 23y + 14 = 0, \quad 23x - 5y + 9 = 0$

Their slope,

$$m_1 = \frac{-5}{23} \qquad m_2 = \frac{23}{5}$$

$\therefore m_1 \times m_2 = -1$

∴ the 2 lines are perpendicular.

4. If the straight lines 12y = −(p + 3)*x* +12,

$12x - 7y = 16$ are perpendicular then find 'p'.

Solution :

Given lines

$12y = -(p+3)x + 12$,

$12x - 7y = 16$ are perpendicular

$\Rightarrow (p+3)x + 12y = 12$

$$m_1 = \frac{-(p+3)}{12} \qquad m_2 = \frac{12}{7}$$

Since 2 lines are perpendicular,

$m_1 \times m_2 = -1$

$$\Rightarrow \quad \frac{-(p+3)}{12} \times \frac{12}{7} = -1$$

$$\Rightarrow \quad -(p+3) = -7$$

$$\Rightarrow \quad p+3 = 7$$

$$\Rightarrow \quad p = 4$$

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5. Find the equation of a straight line passing through the point P(-5,2) and parallel to the line joining the points Q(3,-2) and R(-5, 4) .

Solution :

The required line is passing through P $(-5, 2)$ and parallel to the line joining the points Q $(3, -2)$, R $(-5, 4)$

$$\text{Slope of QR} = \frac{4+2}{-5-3} = \frac{6}{-8} = \frac{-3}{4}$$

$\therefore$ Equn. of the line is

$y - y_1 = m(x - x_1)$

$$\Rightarrow \quad y - 2 = \frac{-3}{4}(x+5)$$

$$\Rightarrow \quad 4y - 8 = -3x - 15$$

$$\Rightarrow \quad 3x + 4y + 7 = 0$$

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6. Find the equation of a line passing through (6,–2) and perpendicular to the line joining the points (6,7) and (2,–3).

Solution :

The required line is passing through $(6, -2)$ and perpendicular to the line joining $(6, 7)$, $(2, -3)$

$\therefore$ Slope of the line joining $(6, 7)$, $(2, -3)$

$$= \frac{-3-7}{2-6} = \frac{-10}{-4} = \frac{5}{2}$$

$\therefore$ Slope of the line perpendicular to it is $\frac{-2}{5}$

$\therefore$ Equation of the required line is

$$y + 2 = \frac{-2}{5}(x - 4)$$

$$\Rightarrow \quad 5y + 10 = -2x + 12$$

$$\Rightarrow \quad 2x + 5y - 2 = 0$$

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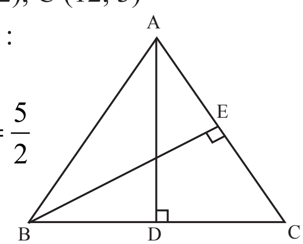
7. A(-3, 0) B(10,-2) and C(12, 3) are the vertices of ΔABC . Find the equation of the altitude through A and B.

Solution :

Given vertices of Δ are

A $(-3, 0)$, B $(10, -2)$, C $(12, 3)$

Equation of altitude AD :



$$\text{Slope of BC} = \frac{3+2}{12-10} = \frac{5}{2}$$

$\therefore$ Slope of AD is $= \frac{-2}{5}$ $(AD \perp BC)$

$\therefore$ Equation of AD is

$$y - 0 = \frac{-2}{5}(x+3)$$

$$5y = -2x - 6$$

$$\Rightarrow \quad 2x + 5y + 6 = 0$$

$\therefore$ Equation of altitude BE is

$$\text{Slope of } AC = \frac{3-0}{12+3} = \frac{3}{15} = \frac{1}{5}$$

$\therefore$ Slope of BE = -5 $\quad (\because BE \perp AC)$

$\therefore$ Equation of BE is

$$y + 2 = -5(x - 10)$$

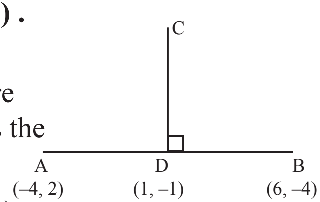
$$\Rightarrow \quad y + 2 = -5x + 50$$

$$\Rightarrow \quad 5x + y - 48 = 0$$

8. Find the equation of the perpendicular bisector of the line joining the points A(-4,2) and B(6,-4) .

Solution :

Given AB and CD are perpendicular & D is the midpoint of AB



$$\therefore D = \left(\frac{-4+6}{2}, \frac{2-4}{2}\right) = (1, -1)$$

$$\text{Slope of } AB = \frac{-4-2}{6+4} = \frac{-6}{10} = \frac{-3}{5}$$

$$\therefore \text{Slope of } CD = \frac{5}{3} \quad (\because CD \perp AB)$$

$\therefore$ Equation of perpendicular bisector CD is

$$y + 1 = \frac{5}{3}(x - 1)$$

$$\Rightarrow \quad 3y + 3 = 5x - 5$$

$$\Rightarrow \quad 5x - 3y - 8 = 0$$

9. Find the equation of a straight line through the intersection of lines 7x + 3y = 10 , 5x −4y = 1 and parallel to the line 13x + 5y +12 = 0.

Solution :

The required line is passing through the intersection of the lines

$7x + 3y = 10$ .... (1)

$5x - 4y = 1$ .... (2)

and parallel to the line $13x + 5y + 12 = 0$

Solving (1) & (2)

$$\begin{array}{lll} (1) \times 4 & \Rightarrow & 28x + 12y = 40 \\ (2) \times 3 & \Rightarrow & 15x - 12y = 3 \\ \hline & & 43x \quad = 43 \end{array}$$

$$x = 1$$

Sub x = 1 in (1)

$$7(1) + 3y = 10$$

$$\Rightarrow \quad 3y = 3$$

$$y = 1$$

$\therefore$ The required line is

$$13x + 5y + k = 0$$

Since it passes through (1, 1)

$$13 + 5 + k = 0$$

$$k = -18$$

$$\therefore 13x + 5y - 18 = 0$$

10. Find the equation of a straight line through the intersection of lines 5x −6y = 2 , 3x + 2y = 10 and perpendicular to the line 4x −7y +13 = 0.

Solution :

Given lines are

$5x - 6y = 2$ .... (1)

$3x + 2y = 10$ .... (2)

$$\begin{array}{lll} (1) & \Rightarrow & 5x - 6y = 2 \\ (2) \times 3 & \Rightarrow & 9x + 6y = 30 \\ \hline & & 14x \quad = 32 \end{array}$$

$$x = {}^{16}\!/_{7}$$

Sub in (2)

$$48/7 + 2y = 10 \Rightarrow 2y = 10 - 48/7$$

$$\Rightarrow 2y = 22/7$$

$$y = 11/7$$

The required line is perpendicular to

$4x - 7y + 13 = 0$

Equation of the required line is

$7x + 4y + k = 0$

$\therefore$ Since it passes through

$$\left(\frac{16}{7}, \frac{11}{7}\right)$$

$$\Rightarrow 7\left(\frac{16}{7}\right) + 4\left(\frac{11}{7}\right) + k = 0$$

$$\Rightarrow \quad 16 + \frac{44}{7} + k = 0$$

$$\Rightarrow \quad k = -16 - \frac{44}{7}$$

$$\Rightarrow \quad k = \frac{-156}{7}$$

$$\therefore \quad 7x + 4y - \frac{156}{7} = 0$$

$$\Rightarrow \quad 49x + 28y - 156 = 0$$

11. Find the equation of a straight line joining the point of intersection of 3x +y +2 = 0 and x −2y −4 = 0 to the point of intersection of 7x −3y = −12 and 2y = x + 3.

Solution :

$$\begin{aligned} (1) \times 2 \quad &\Rightarrow \quad 6x + 2y = -4 \\ (2) \quad &\Rightarrow \quad \underline{x - 2y = 4} \\ & \quad\quad 7x = 0 \end{aligned}$$

$x = 0$

$(1) \Rightarrow \quad y = -2$

$\therefore$ The point of int. of (1) & (2) is $(0, -2)$

Now, to find the point of int. of the lines

$7x - 3y = -12$ .... (3)

$x - 2y + 3 = 0$ .... (4)

$$\begin{aligned} (3) \quad &\Rightarrow \quad 7x - 3y = -12 \\ (4) \times 7 \quad &\Rightarrow \quad \underline{7x - 14y = -21} \\ & \quad\quad 11y = 9 \end{aligned}$$

$$y = \frac{9}{11}$$

Sub in (4)

$$x - \frac{18}{11} + 3 = 0$$

$$x = \frac{18}{11} - 3 = \frac{-15}{11}$$

The point of int. of (3) & (4) is

$$\left(\frac{-15}{11}, \frac{9}{11}\right)$$

The required equation of the line joining

$$(0, -2), \left(\frac{-15}{11}, \frac{9}{11}\right)$$

$$\frac{y+2}{\frac{9}{11}+2} = \frac{x-0}{\frac{-15}{11}}$$

$$\Rightarrow \frac{y+2}{31} = \frac{x}{-15}$$

$$\Rightarrow 31x = -15y - 30$$

$$\Rightarrow 31x + 15y + 30 = 0$$

12. Find the equation of a straight line through the point of intersection of the lines 8x + 3y = 18, 4x + 5y = 9 and bisecting the line segment joining the points (5,–4) and (–7,6).

Solution :

To find : The point of int. of

8x + 3y = 18 .... (1)

4x + 5y = 9 .... (2)

$$\begin{aligned} (1) &\Rightarrow 8x + 3y = 18 \\ (2)\times 2 &\Rightarrow 8x + 10y = 18 \\ & \quad -7y = 0 \\ & \quad y = 0 \end{aligned}$$

(2) ⇒ 4x = 9

$$\therefore x = \frac{9}{4}$$

∴ The point of int. of (1) & (2) in

$$\left(\frac{9}{4}, 0\right)$$

Mid point of the line joining

(5, −4), (−7, 6)

$$= \left(\frac{5-7}{2}, \frac{-4+6}{2}\right)$$

$$= (-1, 1)$$

Equation of the required line joining

$$\left(\frac{9}{5}, 0\right), (-1, 1)$$

$$\frac{y-0}{1} = \frac{x - \frac{9}{4}}{-1 - \frac{9}{4}}$$

$$y = \frac{4x-9}{-13}$$

$$\Rightarrow 4x + 13y - 9 = 0$$

EXERCISE 5.5

Multiple choice questions :

1. The area of triangle formed by the points (−5,0) , (0,−5) and (5,0) is

(1) 0 sq.units (2) 25 sq.units

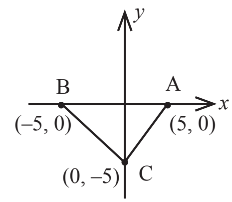
(3) 5 sq.units (4) none of these

Hint : **Ans : (2)**

$$\text{Area of ABC} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 10 \times 5$$

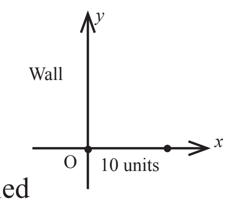
$$= 25 \text{ sq.units}$$



2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is

(1) x = 10 (2) y = 10

(3) x = 0 (4) y = 0



Hint : **Ans : (1)**

Equation of path travelled by the man is x = 10

3. The straight line given by the equation *x* = 11 is

(1) parallel to X axis

(2) parallel to Y axis

(3) passing through the origin

(4) passing through the point (0,11)

Hint : **Ans : (2)**

Equation x = C is a line parallel to y - axis

4. If (5,7), (3,p) and (6,6) are collinear, then the value of p is

(1) 3 (2) 6 (3) 9 (4) 12

Hint : **Ans : (3)**

A (5, 7), B (3, p), C (6, 6) are collinear

$\therefore$ Slope of AB = Slope of BC

$$\frac{p-7}{-2}=\frac{6-p}{3}$$

$$\Rightarrow \quad 3p-21=-12+2p$$

$$\Rightarrow \quad p=9$$

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5. The point of intersection of $3x - y = 4$ and $x + y = 8$ is

(1) (5,3) (2) (2,4) (3) (3,5) (4) (4,4)

Hint : **Ans : (3)**

Substitute and check the point to satisfy the given lines.

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6. The slope of the line joining (12, 3) , (4,a) is $\frac{1}{8}$. The value of 'a' is

(1) 1 (2) 4 (3) – 5 (4) 2

Hint : **Ans : (4)**

$$\text{Slope of } (12,3),(4,a)=\frac{1}{8}$$

$$\Rightarrow \quad \frac{a-3}{-8}=\frac{1}{8}$$

$$\Rightarrow \quad a-3=-1$$

$$\Rightarrow \quad a=2$$

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7. The slope of the line which is perpendicular to a line joining the points (0,0) and (–8,8) is

(1) –1 (2) 1 (3) $\frac{1}{3}$ (4) – 8

Hint : **Ans : (2)**

Slope of the line joining (0, 0), (– 8, 8)

$$=\frac{8-0}{-8-0}$$

$$=-1$$

$\therefore$ Slope of the line perpendicular to it = 1.

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8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then slope of the perpendicular bisector of PQ is

(1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) 0

Hint : **Ans : (2)**

$$\text{Slope of PQ}=\frac{+1}{\sqrt{3}}$$

$$\text{Slope of its perpendicular bisector}=-\sqrt{3}$$

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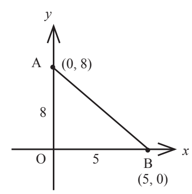
9. If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line AB is

(1) 8x + 5y = 40 (2) 8x −5y = 40

(3) x = 8 (4) y = 5

Hint : **Ans : (1)**



Here a = 5, b = 8

$$\therefore \text{Equn. of the line is } \frac{x}{5}-\frac{y}{8}=1$$

$$\Rightarrow \quad 8x+5y-40=0$$

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10. The equation of a line passing through the origin and perpendicular to the line 7x −3y + 4 = 0 is

(1) $7x$ −3y + 4 = 0 (2) $3x$ −7y + 4 = 0

(3) $3x$ + 7y = 0 (4) $7x$ −3y = 0

Hint : **Ans : (3)**

Equation of the line perpendicular to

7x −3y + 4 = 0 is

3x + 7y + k = 0

Since it passes through (0, 0), k = 0

$\therefore$ 3x + 7y = 0

11. **Consider four straight lines**

(i) l_2 : 3y = 4x + 5 (ii) l_2 : 4y = 3x – 1

(iii) l_3 : 4y + 3x + 7 (iv) l_4 : 4x + 3y = 2

Which of the following statement is true?

(1) l_1 and l_2 are perpendicular

(2) l_1 and l_4 are parallel

(3) l_2 and l_4 are perpendicular

(4) l_2 and l_3 are parallel

Hint : **Ans : (3)**

i) Slope of $l_1 = 4/3$

ii) Slope of $l_2 = 3/4$

iii) Slope of $l_3 = -3/4$

iv) Slope of $l_4 = -4/3$

Here l_1 and l_3 are perpendicular

l_2 and l_4 are perpendicular

But 3rd option is a contradiction

12. **A straight line has equation 8y = 4x +21. Which of the following is true**

(1) The slope is 0.5 and the y intercept is 2.6

(2) The slope is 5 and the y intercept is 1.6

(3) The slope is 0.5 and the y intercept is 1.6

(4) The slope is 5 and the y intercept is 2.6

Hint : **Ans : (1)**

Given equation is 8y = 4x + 21

$$\Rightarrow y = 1/2\, x + 21/8$$

$$\Rightarrow y = 0.5\, x + 2.6$$

$\therefore$ Slope = 0.5, y − int = 2.6

13. **When proving that a quadrilateral is a trapezium, it is necessary to show**

(1) Two sides are parallel.

(2) Two parallel and two non-parallel sides.

(3) Opposite sides are parallel.

(4) All sides are of equal length.

Hint : **Ans : (2)**

A quadrilateral is trapezoid if one pair of opposite sides are parallel and another pair is non parallel.

14. **When proving that a quadrilateral is a parallelogram by using slopes you must find**

(1) The slopes of two sides

(2) The slopes of two pair of opposite sides

(3) The lengths of all sides

(4) Both the lengths and slopes of two side

Hint : **Ans : (1)**

We should find the slopes of all the sides when proving a quadrilateral is a parallelogram.

15. **(2, 1) is the point of intersection of two lines.**

(1) x −y −3 = 0; 3x −y −7 = 0

(2) x +y = 3; 3x +y = 7

(3) 3x +y = 3; x +y = 7

(4) x + 3y −3 = 0; x −y −7 = 0

Hint : **Ans : (2)**

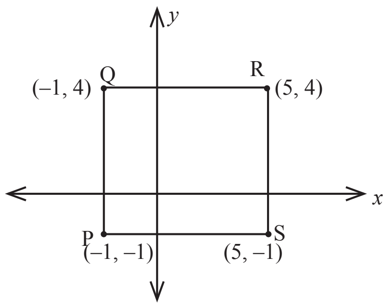
Substitute (2, 3) & check in all pair of lines.

UNIT EXERCISE - 5

1. PQRS is a rectangle formed by joining the points P(-1,-1), Q(-1, 4) ,R(5, 4) and S(5,-1) . A, B, C and D are the mid-points of PQ, QR, RS and SP respectively. Is the quadrilateral ABCD a square, a rectangle or a rhombus? Justify your answer.

Solution :

Given P = (−1, −1) Q (−1, 4), R (5, 4), S (5, −1)



$$A = \text{Mid point of } PQ = \left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$$

$$B = \text{Mid point of } QR = \left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = (2, 4)$$

$$C = \text{Mid point of } RS = \left(\frac{5+5}{2}, \frac{4-1}{2}\right) = \left(5, \frac{3}{2}\right)$$

$$D = \text{Mid point of } PS = \left(\frac{-1+5}{2}, \frac{-1-1}{2}\right) = (2, -1)$$

$$\text{Slope of } AB = \frac{4-3/2}{2+1} = \frac{5}{6}$$

$$\text{Slope of } BC = \frac{3/2-4}{5-2} = \frac{-5/2}{3} = \frac{-5}{6}$$

$$\text{Slope of } CD = \frac{-1-3/2}{2-5} = \frac{-5/2}{-3} = \frac{5}{6}$$

$$\text{Slope of } AD = \frac{-1-3/2}{2+1} = \frac{-5}{6}$$

∴ AB | | CD, BC | | AD

$$\text{Mid Point of } AC = \left(\frac{-1+5}{2}, \frac{3/2+3/2}{2}\right)$$

$$= \left(2, 3/2\right)$$

$$\text{Mid Point of } BD = \left(\frac{2+2}{2}, \frac{4-1}{2}\right)$$

$$= \left(2, 3/2\right)$$

∴ diagonals bisect each other & opposite sides are parallel.

ABCD is a rhombus.

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2. The area of a triangle is 5 sq.units. Two of its vertices are (2,1) and (3, –2). The third vertex is (*x, y*) where y = *x* + 3 . Find the coordinates of the third vertex.

Solution :

Given, area of triangle ABC is 5 sq.units and A (2, 1), B (3, −2), C (x, y) where y = x + 3

∴ Area of Δ

$$= \frac{1}{2}\begin{bmatrix} 2 & 3 & x & 2 \\ 1 & -2 & y & 1 \end{bmatrix} = 5$$

$$\Rightarrow \quad (-4 + 3y + x) - (3 - 2x + 2y) = 10$$

$$\Rightarrow \quad x + 3y - 4 - 3 + 2x - 2y = 10$$

$$\Rightarrow \quad 3x + y = 17 \quad \text{...... (1)}$$

Also given, $x - y = -3$ ...... (2)

Adding, $4x = 14$

$$x = 7/2$$

Sub, $x = 7/2$ in (2)

$$7/2 - y = -3$$

$$y = 7/2 + 3 = 13/2$$

∴ Third vertex is $\left(7/2, 13/2\right)$

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3. **Find the area of a triangle formed by the lines 3x +y −2 = 0 , 5x +2y −3 = 0 and 2x − y −3 = 0.**

Solution :

Given lines are

$3x + y - 2 = 0$ .... (1)

$5x + 2y - 3 = 0$ .... (2)

$2x - y - 3 = 0$ .... (3)

Solving (1) & (2)

$$\begin{array}{rcl} (1)\times 2 & \Rightarrow & 6x + 2y = 4 \\ (2) & \Rightarrow & \underline{5x + 2y = 3} \\ & & x \quad = 1 \end{array}$$

Sub. in (1)

$3 + y - 2 = 0$

$y = -1$

$\therefore$ A $(1, -1)$

Solving (1) & (2)

$$\begin{array}{r} 3x + y = 2 \\ \underline{2x - y = 3} \\ 5x \quad = 5 \end{array}$$

$x = 1$

$\therefore\ y = -1$

$\therefore$ B is $(1, -1)$

Solving (2) & (3)

$$\begin{array}{rcl} (2) & \Rightarrow & 5x + 2y = 3 \\ (3)\times 2 & \Rightarrow & \underline{4x - 2y = 6} \\ & & 9x \quad = 9 \end{array}$$

$x = 1$

$\therefore$ (3) $\Rightarrow 2 - y - 3 = 0$

$\Rightarrow \quad -y = 1$

$\therefore \quad y = -1$

$\therefore$ C is $(1, -1)$

$\therefore$ A $(1, -1)$, B $(1, -1)$, C $(1, -1)$

$\therefore$ All point line on the same line

$\therefore$ Area of $\Delta = 0$ sq. units

4. **If vertices of a quadrilateral are at A(-5,7) , B(-4,k) , C(-1,-6) and D(4,5) and its area is 72 sq.units. Find the value of k.**

Solution :

Given vertices of quadrilateral are

A $(-5, 7)$, B $(-4, k)$, C $(-1, -6)$, D $(4, 5)$ & its area = 72 sq.units

$$\Rightarrow \frac{1}{2}\begin{bmatrix} -5 & -4 & -1 & 4 & -5 \\ 7 & k & -6 & 5 & 7 \end{bmatrix} = 72$$

$(-5k+24-5+28) - (-28-k-24-25) = 144$

$(-5k + 47) - (-k - 77) = 144$

$-4k + 124 = 144$

$-4k = 20$

$k = -5$

5. **Without using distance formula, show that the points (-2,-1) , (4, 0) , (3, 3) and (-3,2) are vertices of a parallelogram.**

Solution :

Given vertices are

$(-2, -1)$ $(4, 0)$, $(3, 3)$, $(-3, 2)$

Let A $(-2, -1)$, B $(4, 0)$, C $(3, 3)$, D $(-3, 2)$

$$\text{Slope of } AB = \frac{0+1}{4+2} = \frac{1}{6}$$

$$\text{Slope of } CD = \frac{3-2}{3+3} = \frac{1}{6}$$

$\therefore$ AB & CD are parallel

$$\text{Slope of } AD = \frac{2+1}{-3+2} = \frac{3}{-1} = -3$$

$$\text{Slope of } BC = \frac{0-3}{4-3} = \frac{-3}{1} = -3$$

$\therefore$ AD & BC are parallel

$\therefore$ ABCD is a parallelogram

6. **Find the equations of the lines, whose sum and product of intercepts are 1 and –6 respectively.**

Solution :

Given, sum of intercepts = 1

$\Rightarrow a + b = 1$

$\therefore\ b = 1 - a$

Given, product of intercepts = − 6

$\Rightarrow ab = -6$

$$\therefore\ ab = -6 \Rightarrow a(1-a) = -6$$
$$\Rightarrow a - a^2 = -6$$
$$\Rightarrow a^2 - a - 6 = 0$$
$$\Rightarrow (a-3)(a+2) = 0$$
$$\therefore\ a = 3, -2$$

If a = 3, b = −2

If a = −2, b = 3

$$a = 3, b = -2 \Rightarrow \frac{x}{3} + \frac{y}{-2} = 1$$
$$\Rightarrow \frac{x}{3} - \frac{y}{2} = 1$$
$$\Rightarrow 2x - 3y - 6 = 0$$
$$a = -2, b = 3 \Rightarrow \frac{x}{-2} + \frac{y}{3} = 1$$
$$\Rightarrow \frac{-3x + 2y}{6} = 1$$
$$\Rightarrow -3x + 2y = 6$$
$$\Rightarrow 3x - 2y + 6 = 0$$

7. **The owner of a milk store finds that, he can sell 980 litres of milk each week at ₹14/ litre and 1220 litres of milk each week at ₹16/litre. Assuming a linear relationship-between selling price and demand, how many litres could he sell weekly at ₹17/ litre?**

Solution :

By data given,

the linear relationship between selling price per litre and demand is the equation of the line passing through the points

(14, 980) and (16, 1220) is

$$\therefore \quad \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$
$$\Rightarrow \frac{y - 980}{1220 - 980} = \frac{x - 14}{16 - 14}$$
$$\Rightarrow \frac{y - 980}{240} = \frac{x - 14}{2}$$
$$\Rightarrow y - 980 = 120(x - 14)$$
$$\Rightarrow y = 120(x - 14) + 980 \quad \text{........ (1)}$$

When x = Rs.17 / litre

y = 120 (17 - 14) + 980

y = 120 (3) + 980

= 360 + 980

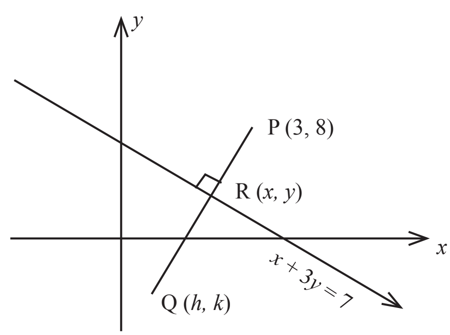
= 1340

$\therefore$ He can sell weekly 1340 litres at Rs.17/ litre

8. **Find the image of the point (3,8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.**

Solution :

To find the image of (3, 8) w.r.to the line x + 3y = 7



Let Q (h, k) be the image of P (3, 8) about the line x + 3y = 7 .... (1)

Since the line is assumed as a plane mirror P & Q are equidistant from R (x, y)

$\therefore$ R is the midpoint and PQ is a perpendicular bisector of (1)

$$\therefore (x, y) = \left(\frac{h+3}{2}, \frac{k+8}{2}\right)$$

$$\therefore \quad x = \frac{h+3}{2}, \; y = \frac{k+8}{2}$$

Since R (x, y) is a point on (1)

$$\left(\frac{h+3}{2}\right) + 3\left(\frac{k+8}{2}\right) = 7$$

$\Rightarrow \quad h + 3 + 3k + 24 = 14$

$\Rightarrow \quad h + 3k = -13$ .... (2)

Also, slope of PQ × Slope of (1) = −1

$$\Rightarrow \quad \frac{k-8}{h-3} \times \frac{-1}{3} = -1$$

$$\Rightarrow \quad \frac{k-8}{h-3} = 3$$

$$\Rightarrow \quad k - 8 = 3h - 9$$

$$\Rightarrow \quad 3h - k = 1 \quad \text{.........(3)}$$

Solving (2) & (3)

$$\begin{array}{lll} (2) & \Rightarrow & h + 3k = -13 \\ (3) \times 3 & \Rightarrow & 9h - 3k = 3 \\ \hline & & 10h = -10 \\ & & h = -1 \end{array}$$

Sub in (2)

$$-1 + 3k = -13$$

$$3k = -12$$

$$k = -4$$

$\therefore$ Q is (− 1, − 4), which is the image of P (3, 8)

9. Find the equation of a line passing through the point of intersection of the lines 4x + 7y −3 = 0 and 2x −3y +1 = 0 that has equal intercepts on the axes.

Solution :

First, we find the point of intersection of the lines

4x + 7y = 3 .... (1)

2x − 3y = −1 .... (2)

$$\begin{array}{lll} (1) & \Rightarrow & 4x + 7y = +3 \\ (2) \times 2 & \Rightarrow & 4x - 6y = -2 \\ \hline & & 13y = 5 \\ & & y = 5/13 \end{array}$$

Sub in (1)

$$4x + 35/13 = 3$$

$$\Rightarrow \quad 4x = 3 - 35/13$$

$$\Rightarrow \quad 4x = 4/13$$

$$\Rightarrow \quad x = 1/13$$

$\therefore$ The point is $\left(1/13, \; 5/13\right)$

Equation of plane in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1, \quad \text{where } a = b$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad x + y = a \quad \text{...........(1)}$$

Since (1) passes through $\left(1/13, \; 5/13\right)$

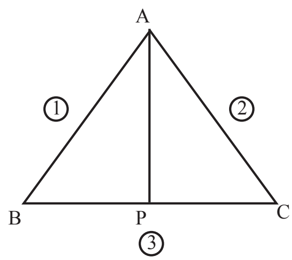
$$a = 1/13 + 5/13 = 6/13$$

$$\therefore \; x + y = 6/13$$

$$\Rightarrow \; \boxed{13x + 13y - 6 = 0}$$

10. A person standing at a junction (crossing) of two straight paths represented by the equations 2x −3y + 4 = 0 and 3x + 4y −5 = 0 seek to reach the path whose equation is 6x −7y + 8 = 0 in the least time. Find the equation of the path that he should follow.

Solution :



Given straight paths are

$2x - 3y + 4 = 0$ .... (1) (AB)

$3x + 4y - 5 = 0$ .... (2) (AC)

To reach the path

$6x - 7y + 8 = 0$ .... (3) (BC)

in the least time

To find : Equation of the path (AP)

A → Position of the person

Solving (1) & (2)

$$(1) \times 3 \Rightarrow 6x - 9y = -12$$
$$(2) \times 2 \Rightarrow 6x + 8y = 10$$
$$-17y = -22$$
$$\therefore y = \frac{22}{7}$$

Sub in (1)

$$2x - \frac{66}{17} = -4$$
$$\Rightarrow 2x = \frac{66}{17} - 4$$
$$\Rightarrow 2x = \frac{66-68}{17}$$
$$\Rightarrow 2x = \frac{-2}{7}$$
$$\Rightarrow x = \frac{-1}{7}$$
$$\therefore \text{ A is } \left(\frac{-1}{7}, \frac{22}{17}\right)$$

Also AP is perpendicular to BC, whose slope is $\frac{6}{7}$

$\therefore$ Slope of $AP = \frac{-7}{6}$

$\therefore$ Equation of the required path AP is

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - \frac{22}{17} = \frac{-7}{6}\left(x + \frac{1}{7}\right)$$
$$\Rightarrow \frac{17y-22}{17} = \frac{-7}{6}\left(\frac{7x+1}{7}\right)$$
$$\Rightarrow \frac{17y-22}{17} = \frac{-(7x+1)}{6}$$
$$\Rightarrow 6(17y-22) = -17(7x+1)$$
$$\Rightarrow 102y - 132 = -119x - 17$$
$$\Rightarrow 119x + 102y - 125 = 0$$

is the required path.

PROBLEMS FOR PRACTICE

1. If P $\left(\frac{a}{2}, 4\right)$ is the mid point of the line joining the points A (−6, 5), B (−2, 3), then find 'a'. **(Ans : a = −8)**

2. Find the area of ΔABC whose vertices are

   i) A (3, 8), B (− 4, 2), C (5, −1) **(Ans : 37.5)**

   ii) A (1, 2), B (−3, 4), C (−5, −6) **(Ans : 22)**

   iii) A (0, 1), B (2, 3), C (3, 4) **(Ans : 0)**

3. If the area of Δ is 12 sq. units with vertices (a, −3), (3, a) (−1, 5) find 'a'. **(Ans : 1, 3)**

4. If the area of Δ formed by (x, y), (1, 2), (2, 1) is 6 sq.units, prove that x + y = 15.

5. For what values of k, are the points (8, 1), (3, −2k), and (k, −5) are collinear.

   $\left(\text{Ans} : k = 2, \frac{11}{2}\right)$

6. Find the value of 'p' if the area of Δ formed by $(p + 1, 2p - 2)$, $(p - 1, p)$ and $(p - 3, 2p - 6)$ is 0. **(Ans : p = 4)**

7. Find the area of quadrilateral whose vertices are,

   i) A (3, −1), B (9, −5), C (14, 0), D (9, 19) **(Ans : 132)**

   ii) P (−5,−3), Q (−4, −6), R (2, −3), S (1, 2) **(Ans : 28)**

   iii) E (−3,2), F (5, 4), G (7, −6), H (−5, −4) **(Ans : 80)**

   iv) A (−4, 5), B (0, 7), C (5,−5), D (−4, −2) **(Ans : 60.5)**

8. If (3, 3), (6, y), (x, 7) and (5, 6) are the vertices of a parallelogram taken in order, find x and y. **(Ans : x = 8, y = 4)**

9. If the points (p, q), (m, n), (p − m, q − n) are collinear, show that pn = qm.

10. Three vertices of a parallelogram ABCD are (1, 2), (4, 3), (6, 6). Find the 4th vertex D. **(Ans : 3, 5)**

11. The line joining A (0, 5) and B (4, 2) is perpendicular to the line joining C (−1, −2), D (5, b) find 'b'. **(Ans : b = 6)**

12. Find the equation of the line passing through (9, −1) having its x-intercept thrice as its y-intercept. **(Ans : x + 3y − 6 = 0)**

13. Find the slope and y-intercept of the line $10x + 15y + 6 = 0$. $\left(\text{Ans}: m = \frac{-2}{5}, c = \frac{-2}{5}\right)$

14. Find whether the lines drawn through the two pair of points are parallel (or) perpendicular.

    i) (5, 2), (0, 5) and (0, 0), (−5, 3) **(Ans : parallel)**

    ii) (4, 5), (0, −2) and (−5, 1), (2, −3) **(Ans : perpendicular)**

15. A line passing through the points (2, 7) and (3, 6) is parallel to the line joining (9, a) and (11, 2) find 'a'. **(Ans : −2 (or) 4)**

16. Find the equation of a straight line whose slope is $2/3$ and passing through the point (5, −4) **(Ans : 2x − 3y − 22 = 0)**

17. Without using distance formula, show that the points P (3, 2), Q (0, −3), R (−3, −2) and S (0, 1) are the vertices of a parallelogram.

18. A triangle has vertices at (3, 4), (1, 2), (−5, −6). Find the slopes of the medians. $\left(\text{Ans}: 6/5, 3/2, 9/7\right)$

19. Find the equation of altitude from A of a Δ ABC whose vertices are (1, −3), (−2, 5), (−3, 4). **(Ans : x + y + 2 = 0)**

20. Find the values of 'p' of the straight lines $8px + (2 - 3p)y + 1 = 0$ and $px + 8y - 7 = 0$ are perpendicular to each other. **(Ans : p = 1, 2)**

21. Find the equation of the straight line passing through (1, 4) and having intercepts in the ratio 3 : 5 **(Ans : 5x + 3y = 17)**

22. Find the area of the triangle formed by sides $x + 4y - 9 = 0$, $9x + 10y + 23 = 0$, $7x + 2y - 11 = 0$ **(Ans : 26 sq.units)**

23. Find the equation of the line through the point of intersection of the lines $2x + y - 5 = 0$, $x + y - 3 = 0$ and bisecting the line segment joining the points (3, −2) (−5, 6) **(Ans : x + 3y − 5 = 0)**

24. Find the image of the point (−2, 3) w.r.to the line $x + 2y - 9 = 0$ **(Ans : 0, 7)**

25. The equation of the diagonals of a rectangle are $4x - 7y = 0$, $8x - y = 26$ and one of its sides is $2x + 3y = 0$, find the equation of the other sides.

(Ans : 2x+3y−26=0, 3x−2y−13=0, 3x−2y = 0)

OBJECTIVE TYPE QUESTIONS

1. The point whose the line $3x - y + 6 = 0$ meets the x - axis is

a) (0, 6) b) (−2, 0)

c) (−1, 3) d) (2, 0) **Ans : (b)**

2. The point of intersection of the lines $2x + y - 3 = 0$, $5x + y - 6 = 0$ lies in the quadrant

a) I b) II c) IV d) III

Ans : (a)

3. The value of 'k' if the lines $3x + 6y + 7 = 0$ and $2x + ky - 5 = 0$ are perpendicular is

a) 1 b) −1 c) 2 d) $\frac{1}{2}$

Ans : (b)

4. The slope of the line which is parallel to the line joining the points (0, 0) and (−5, 5) is

a) 1 b) −1 c) 2 d) −2

Ans : (a)

5. The area of triangle formed by (0, 4) (4, 0) and origin is

a) 8 b) 16 c) 2 d) 4 **Ans : (a)**

6. The equation of a straight line which has the y-intercept 5 and slope 2 is

a) $2x + y + 5 = 0$ b) $2x - y + 5 = 0$

c) $2x - y - 5 = 0$ d) $2x + y - 5 = 0$

Ans : (b)

7. If the point (a, a) lies on the line $3x + 4y - 14 = 0$ then 'a' is

a) 2 b) −2 c) 1 d) 0 **Ans : (a)**

8. Equation of the line parallel to y-axis and passing through (−2, 3) is

a) $x = 3$ b) $y = -2$

c) $x = -2$ d) $y = 3$ **Ans (c)**

9, The x-intercept of the line $3x - 2y + 12 = 0$ is

a) 6 b) −6 c) 4 d) −4

Ans : (d)

10. AB is parallel to CD. If A and B are (2, 3) and (6, 9) the slope of CD is

a) $\frac{4}{9}$ b) $\frac{3}{2}$ c) $\frac{2}{3}$ d) $\frac{9}{4}$

Ans : (b)

11. If (1, 2), (4, 6), (x, 6) and (3, 2) are the vertices of a parallelogram taken in order, then x is

a) 6 b) 2 c) 1 d) 3

Ans : (a)

12. If the slope of a line is $-\sqrt{3}$, then the angle of inclimation is

a) 60^0 b) 30^0 c) 120^0 d) 150^0

Ans : (c)

13. The value of a for which (−a, a) is collinear with the points (2, 0), (0, 1) is

a) 1 b) 2 c) −2 d) −1

Ans : (b)

14. The x - coordinates of the point of intersection of the lines $x - 7y + 5 = 0$, $3x + y = 0$ is

a) $\frac{15}{22}$ b) $\frac{5}{22}$ c) $\frac{-5}{22}$ d) $\frac{-10}{22}$

Ans : (c)

15. The equations of the 4 sides of a rectangle are $x = 1$, $y = 2$, $x = 4$, $y = 5$. One vertex of the rectangle is at

a) (2, 4) b) (5, 1)

c) (2, 5) d) (4, 2)

Ans : (d)