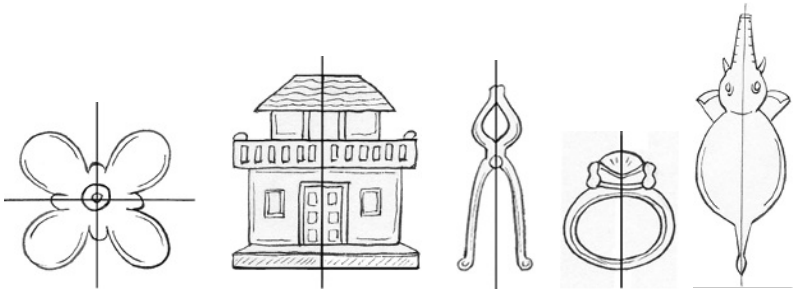
15.SYMMETRY

15.0 Introduction

Look around you. You will find that many objects around you are symmetrical. So are the objects that are drawn below.



All these objects are symmetrical as they can be divided in such a way that their two parts coincide with each other.

15.1 Line Symmetry

Let us take some more examples and understand what we mean. Trace the following figures on a tracing paper.

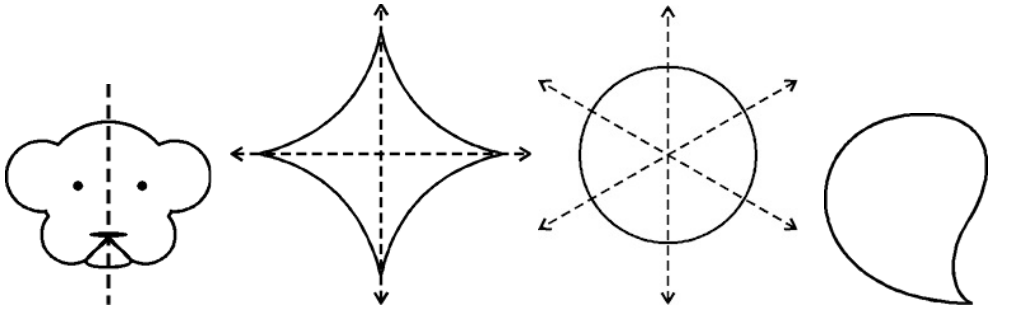


Figure 1 Figure 2 Figure 3 Figure 4

Fold Figure 1 along the dotted line. What do you observe?

You will find that the two parts coincide with each other. Is this true in Figure 2 and 3?

You will observe that in Figure 2, this is true along two lines and in Figure 3 along many lines. Can Figure 4 be divided in the same manner?

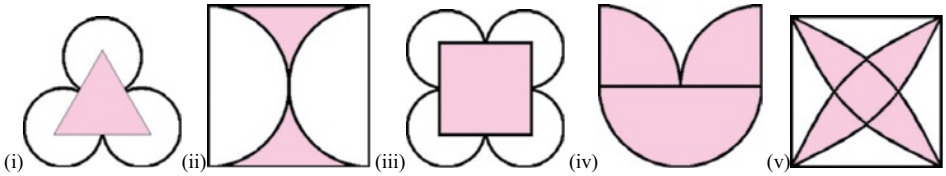
Figure 1, 2 and 3 have line symmetry as they can be divided in such a manner that two parts of the figure coincide with each other when they are folded along the line of symmetry. The dotted line which divides the figures into two equal parts is the line of symmetry or axis of symmetry. As you have seen, an object can have one or more than one lines of symmetry or axes of symmetry.

Try This

1. Name a few things in nature, that are symmetric.
2. Name 5 man-made things that are symmetric.

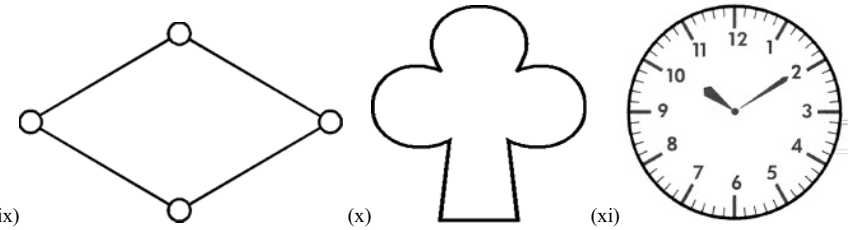
Exercise - 1

1. Given below are some fiugres. Which of them are symmetric? Draw the axes of symmetry for the symmetric figures.

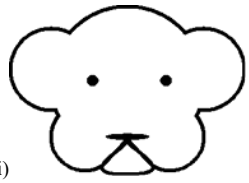
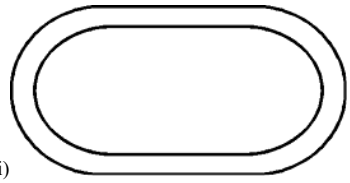
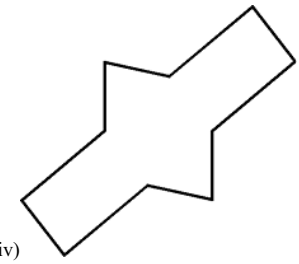


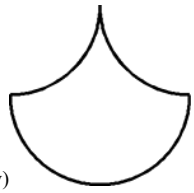
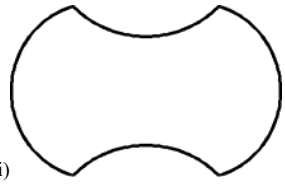
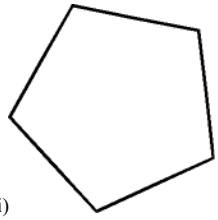
(i) (ii) (iii) (iv) (v)

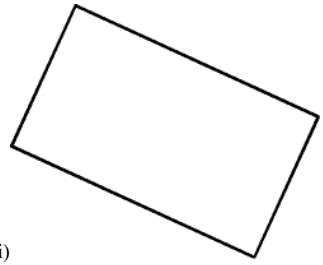
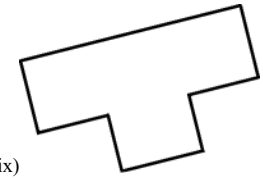
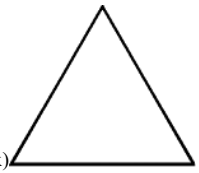
(vi) (vii) (viii)

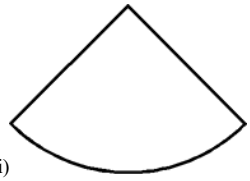
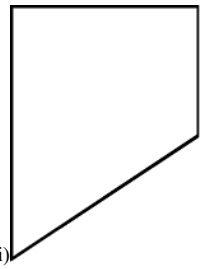
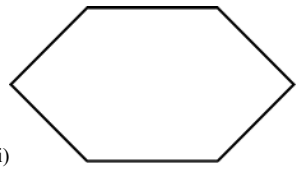


(ix) (x) (xi)

(xii)  (xiii)  (xiv) 

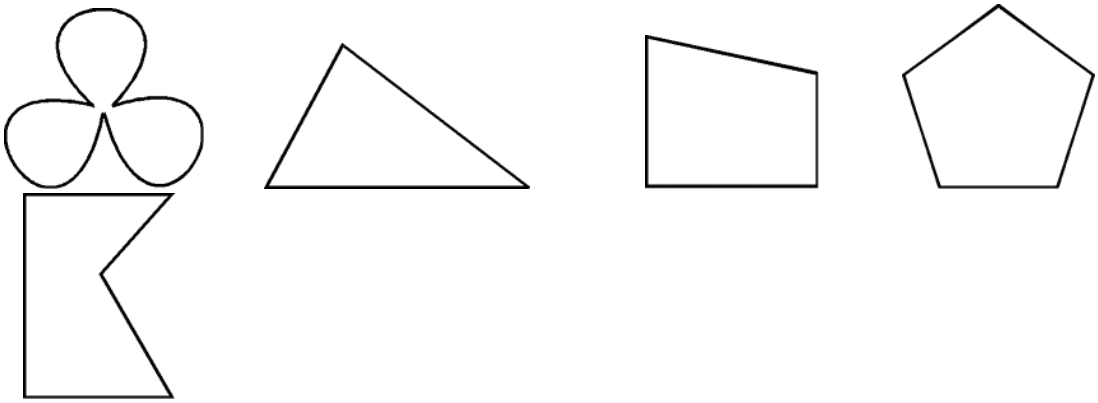
(xv)  (xvi)  (xvii) 

(xviii)  (xix)  (xx) 

(xxi)  (xxii)  (xxiii) 

15.1.1 Lines of symmetry for regular polygons

Look at the following closed figures.



A closed figure made from several line segments is called a ‘Polygon’. Which of the above figures are polygons?

Try This

1. Can we make a polygon with less than three line segments?
2. What is the minimum number of sides of a polygon?

Observe the different triangles below.

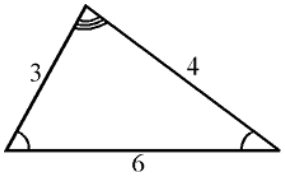


Figure 1

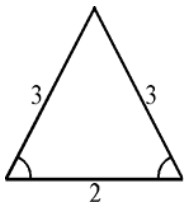


Figure 2

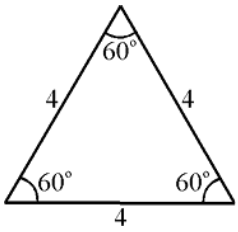
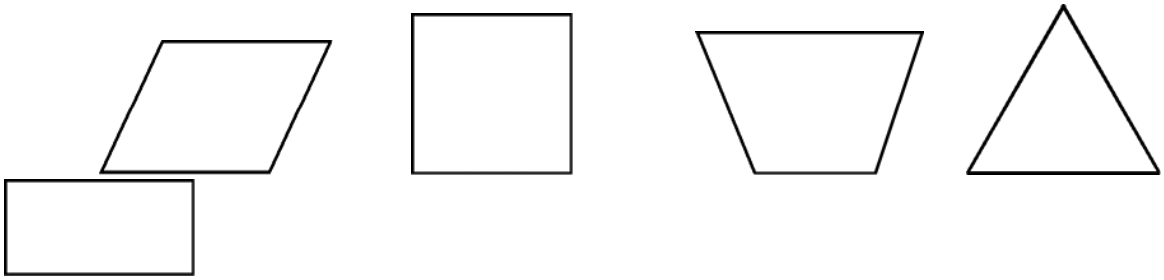


Figure 3

In Figure 3, the triangle has equal sides and congruent angles. It is thus called a regular polygon.

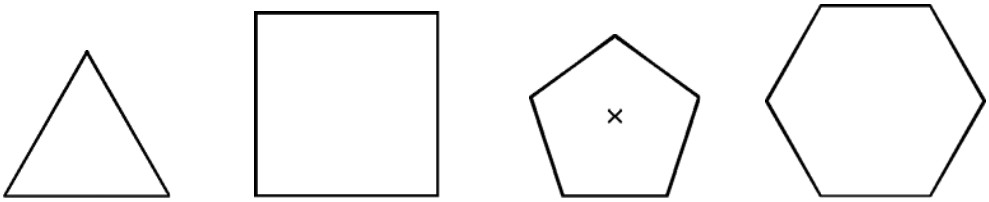
A polygon, with all sides and all angles equal is called a 'Regular Polygon'.

Which of the following polygons are regular polygons?



Parallelogram Square Trapezium Equilateral triangle Rectangle

Now draw axes of symmetry for the following regular polygons.



Equilateral Triangle Square Regular Pentagon Regular Hexagon

Write down your conclusions in the table below.

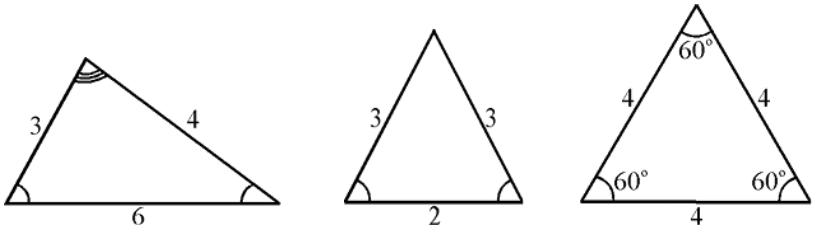
Regular Polygon	No. of sides	No. of axes of symmetry
Triangle	3	3
Square		
Pentagon		
Hexagon		

Did you find any relationship between the number of sides of a regular polygon and number of axes symmetry? You will find that the number of sides is equal to number of axes of symmetry.

You can verify your results by tracing out all the four figures on a paper, cuting them out and actually folding each figure to find the axes of symmetry.

Try This

1. Given below are three types of triangles. Do all the triangles have the same number of lines of symmetry? Which triangle has more?



2. Given below are different types of quadrilaterals. Do all of them have the same number of lines of symmetry? Which quadrilateral has the most?

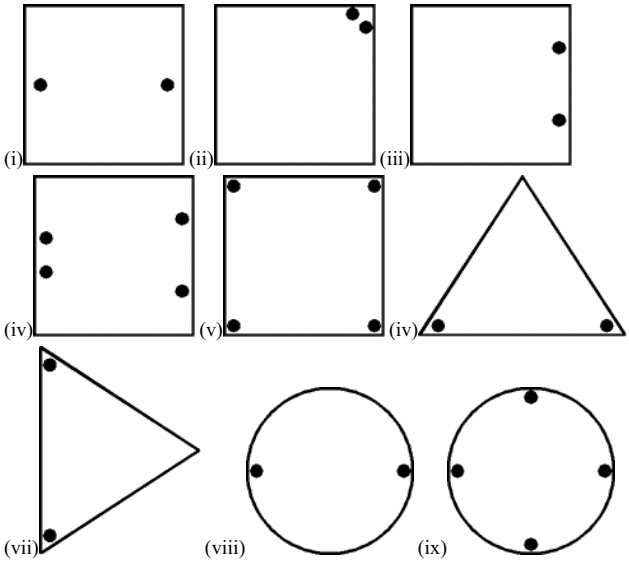
Rhombus Square Rectangle

Hint: You can trace the triangles and quadrilaterals on a tracing paper and actually fold each figure to find the axes of symmetry.

On the basis of (i) and (ii) can we say that a regular polygon has the maximum number of axes of symmetry.

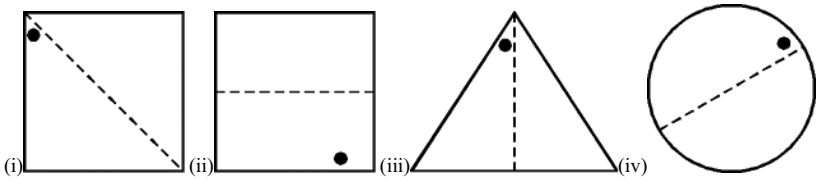
Exercise - 2

1. In the figures given below find the axes of symmetry such that on folding along the axis the two dots fall on each other.



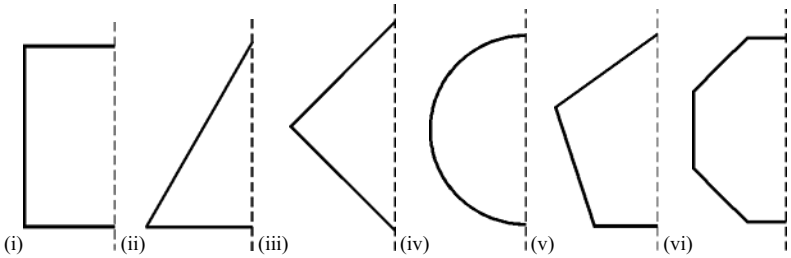
(i) (ii) (iii) (iv) (v) (iv) (vii) (viii) (ix)

2. Given the line of symmetry, find the other dot.



(i) (ii) (iii) (iv)

3. In the following incomplete figures, the mirror line (i.e. the line of symmetry) is given as a dotted line. Complete each figure, performing reflection on the dotted (mirror) line. (You might perhaps place a mirror along the dotted line and look into the mirror for the image). Are you able to recall the name of the figure you complete?



(i) (ii) (iii) (iv) (v) (vi)

4. State whether the following statements are true or false.

(i) Every closed figure has an axis of symmetry. ( )

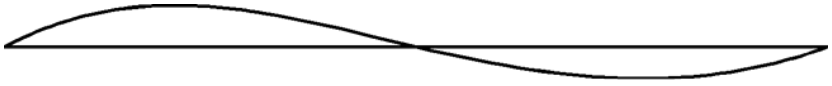
(ii) A figure with at least one axis of symmetry is called a symmetric figure. ( )

(iii) A regular polygon of 10 sides will have 12 axes of symmetry. ( )

5. Draw a square and construct all its axes of symmetry. Measure the angles between each pair of successive axes of symmetry. What do you notice? Does the same rule apply for other regular polygons?

15.2 Rotational Symmetry

Activity 1 : Trace the following diagram onto a tracing paper.



Try to fold the diagram so that its two parts coincide. Is this diagram symmetric?

Now, let us try to match the different positions of the diagram in another way. Draw the above diagram on a piece of paper. Mark a point ‘o’at the centre and name the four edges of the paper A,B,C,D as shown in Figure 1.

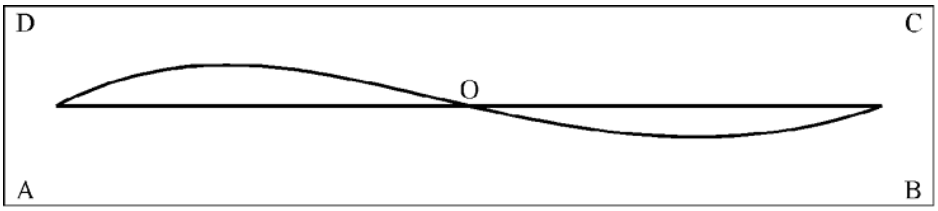


Figure 1

Rotate the paper around the marked point for 180º.

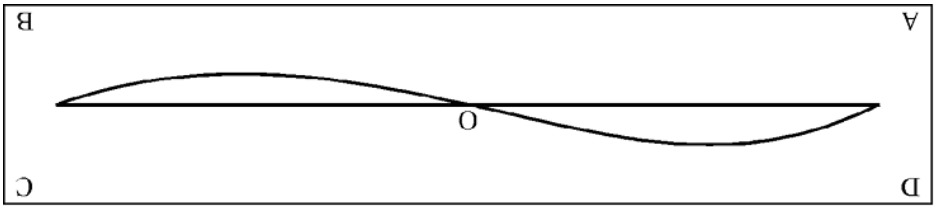


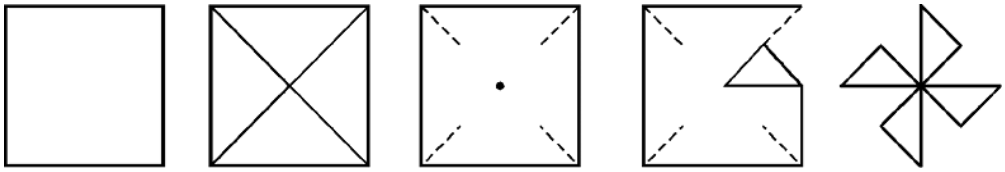
Figure 2

What do you notice? Does this diagram look different from the previous one?

Due to the rotation, the points A,B,C,D have changed position however the diagram seems to be unchanged. This is because the diagram has rotational symmetry.

Activity 2 : Lets make a wind wheel

- Take a paper and cut it into the shape of a square.
- Fold it along the diagonals.
- Starting from one corner, cut the paper along the diagonals towards the centre, up to one fourth of the length of the diagonal. Do the same from the remaining corners.
- Fold the alternate corners towards the centre.
- Fix the mid point to a stick with a pin so that the paper rotates freely.
- Face it in the opposite direction of the wind. You will find it rotates



Now, let us rotate the wind-wheel by 90º. After each rotation you will see that the wind-wheel looks exactly the same. The wind-wheel has rotational symmetry.

Thus, if we rotate a figure, about a fixed point by a certain angle and the figure looks exactly the same as before, we say that the figure has rotational symmetry.

15.2.1 Angle of Rotational Symmetry

We know that the square has line symmetry and 4 axes of symmetry. Now, let us see if the square has rotational symmetry.

Consider a square as in Figure (i) with P as one of its corners.

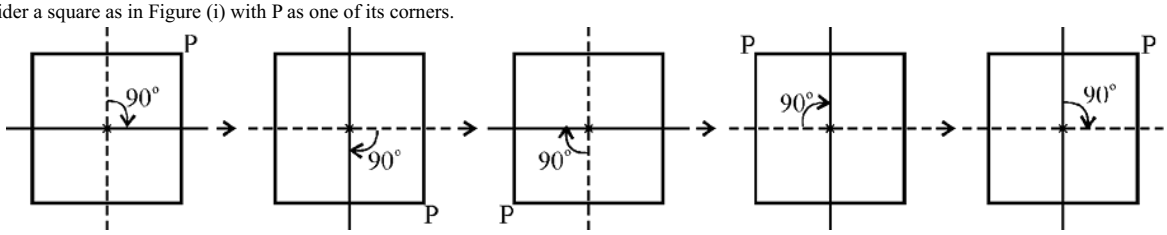


Figure 1 Figure 2 Figure 3 Figure 4

Figure 5

Figure 1 represent the initial position of square.

Rotate the square by 90 degrees about the centre. This quarter turn will lead to Figure 2. Note the position of P. In this way, rotate the square again through 90 degrees and you get Figure 3. When we complete four quarter turns, the square reaches its original position. After each turn, the square looks exactly like it did in its original position. This can also be seen with the help of the position taken by P.

In the above activity all the positions in figure 2, figure 3, figure 4 and figure 5 obtained by the rotation of the first figure through 90^0 , 180^0, 270^0 and 360^0 look exactly like the original
figure 1. Minimum of these i.e., 90^0 is called the angle of rotational symmetry.

The minimum angle rotation of a figure to get exactly the same figure as original is called the "angle of rotational symmetry" or "angle of rotation".

Do This

1. What is the angle of rotational symmetry of a square?
2. What is the angle of rotational symmetry of a parallelogram?
3. What is the angle of rotational symmetry of a circle?

15.2.2 Order of rotational symmetry

In the above activity, the angle of rotational symmetry of square is 90^0 and the figure is turned through the angle of rotational symmetry for 4 times before it comes to original position. Now we say that the square has rotational symmetry of order 4.

Consider an equilateral triangle. Its angle of rotational symmetry is 120^0. That means it has to be rotated about its centre for 3 times to get exactly the same position as the original one. So the order of rotational symmetry of a equilateral triangle is 3.

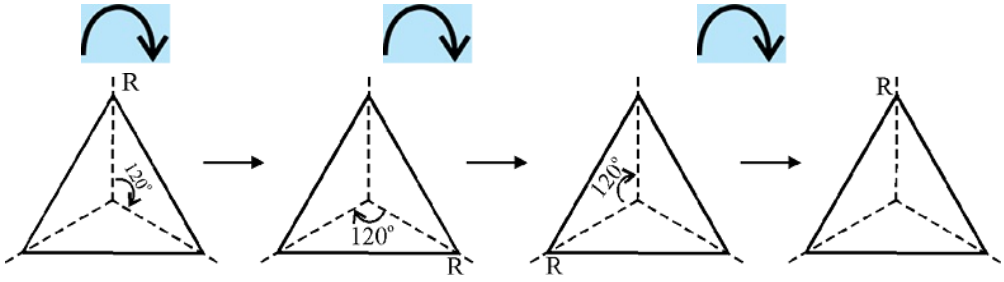
By these examples we conclude that the number of times a figure, rotated through its angle of rotational symmetry before it comes to original postion is called order of rotational symmetry.

Let us conclude from the above examples

- The centre of rotational symmetry of a square is its intersection point of its diagonals.
- The angle of rotational symmetry for a square is 90^0.
- The order of rotational symmetry for a sqaure is 4.

Try This

1. (i) Can you now tell the order of rotational symmetry for an equilateral triangle.



(i) (ii) (iii) (iv)

(ii) How many lines of semetry?

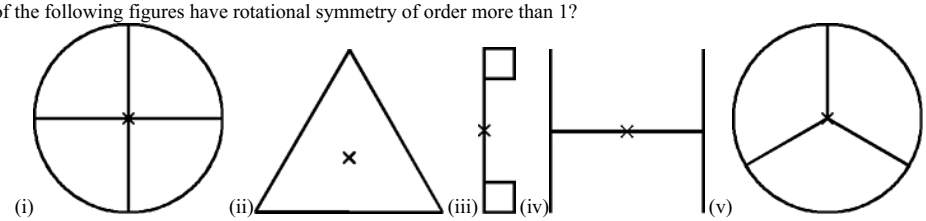
(iii) What is the angle between every adjacent axes?

2. Look around you. Name five objects which have rotational symmetry

Note: It is important to understand that all figures have rotational symmetry of order 1, as can be rotated completely through 360º to come back to its original position. So we say that an object has rotational symmetry, only when the order of symmetry is more than 1.

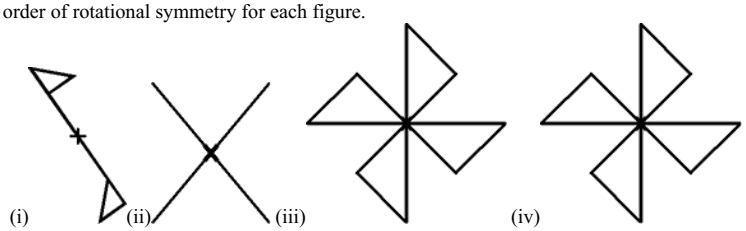
Exercise - 3

1. Which of the following figures have rotational symmetry of order more than 1?

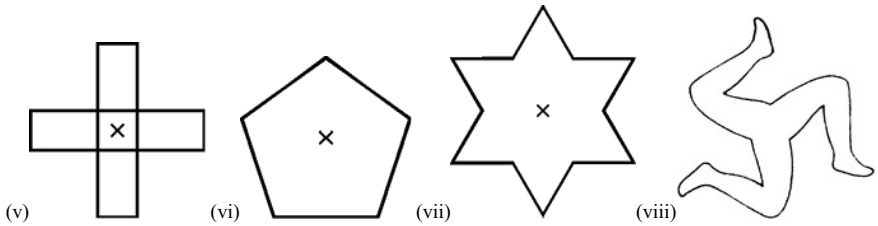


(i) (ii) (iii) (iv) (v)

2. Give the order of rotational symmetry for each figure.



(i) (ii) (iii) (iv)



(v) (vi) (vii) (viii)

3. Draw each of the shapes given below and fill in the blanks.

Shape	Centre of Rotation (Intersection of axes of symmetry)	Angle of Rotation	Order of Rotation
Square			
Rectangle			
Rhombus			
Equilateral Triangle			
Regular Hexagon			
Circle			

Semi-circle

15.3 Line symmetry and rotational symmetry

By now you must have realised that some shapes only have line symmetry and some have only rotational symmetry (of order more than 1) and some have both. Squares and equilateral triangles have both line and rotational symmetry. The circle is the most perfect symmetrical figure, because it can be rotated about its centre through any angle and it will look the same. A circle also has unlimited lines of symmetry.

Example 1 : Which of the following shapes have line symmetry? Which have rotational symmetry?

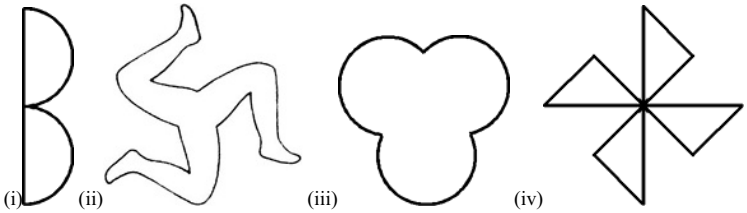


Figure	Line symmetry	Rotational symmetry
1.	Yes	No
2.	No	Yes
3.	Yes	Yes
4.	No	Yes

Activity 3 :

- Take a square shaped paper.
- Fold it vertically first, then horizontally.
- Then fold along a diagnal such that the paper takes a triangular shape (Figure 4).
- Cut the folded edges as shown in the figure or as you wish (Figure 5).
- Now open the piece of paper.

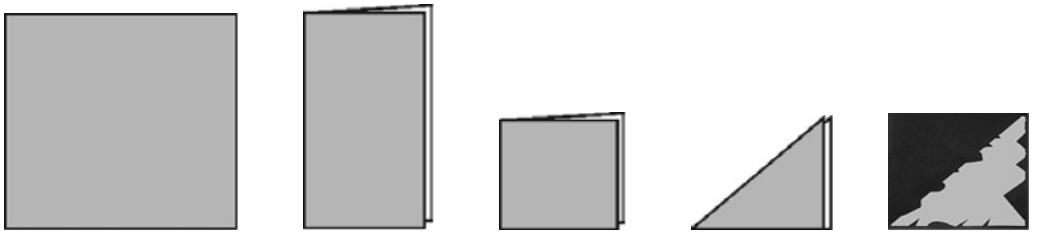


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

(i) Does the paper have line symmetry?

(ii) Does the paper have rotational symmetry?

Exercise - 4

1. Some english alphabets have fascinating symmetrical structures. Which capital letters have only one line of symmetry (like E))? Which capital letters have rotational symmetry of order 2 (like I)?

   Fill the following table, thinking along such lines.

Alphabets	Line symmetry	Number of lines symmetry	Rotational symmetry	Order of rotational symmetry
Z	No	0	Yes	2
S				
H				

O E N C	Yes	1	No	-

Home Project

Collect pictures of symmetrical figures from newspapers, magazines and advertisement pamphlets. Draw the axes of symmetry over them. Classify them.

Looking Back

- The line which divides a figure into two identical parts is called the line of symmetry or axis of symmetry.
- An object can have one or more than one lines of symmetry or axes of symmetry.
- If we rotate a figure, about a fixed point by a certain angle and the figure looks exactly the same as before, we say that the figure has rotational symmetry.
- The angle of turning during rotation is called the angle of rotation.
- All figures have rotational symmetry of order 1, as can be rotated completely through 360º to come back to their original position. So we say that an object has rotational symmetry only when the order of symmetry is more than 1.
- Some shapes only have line symmetry and some have only rotational symmetry and some have both. Squares, equilateral triangles and circles have both line and rotational symmetry.