5 Expansion

- **Let us remember :**
  - A polynomial having one term is called Monomial.
  - A polynomial having two terms is called Binomial.
  - A polynomial having three terms is called Trinomial.
- **Let us learn new :**
  - **Multiplication (product) of monomial with monomial :**

$2 \times 3 = 6$ similarly $a \times b = ab$

$2 \times 3x = 2 \times 3 \times x = 6x$

$5x \times 3y = 5 \times x \times 3 \times y = 5 \times 3 \times x \times y = 15xy$

$5x \times 3x^2 = 5 \times x \times 3 \times x^2 = 5 \times 3 \times x \times x^2 = 15x^3$

$3x \times (-5y) = 3 \times x \times (-5) \times y = 3 \times (-5) \times x \times y = (-15)xy$

$4x \times 6y^2 = 24xy^2$

Therefore, in multiplication (product) of two monomials their coefficients are multiplied with coefficients and variable is multiplied with variable. Hence here multiplication (product) of monomials with monomial is monomial only.

  - **Multiplication (product) of monomial with binomial :**

In the earlier class we have learnt about distribution of multiplication over addition.

$a \times (b + c) = (a \times b) + (a \times c)$ (Distribution of multiplication over addition)

$a \times (b + c) = ab + ac$

Now, if $a = 2x$, $b = 3y$ and $c = 5z$, then

$2x \times (3y + 5z) = (2x \times 3y) + (2x \times 5z)$

$= 6xy + 10xz$

There by using distributive law, the product on left hand side is expressed in the form of addition on right hand side. This process is called expansion. i.e. The process of getting $ab + ac$ from $a(b + c)$ is expansion. **"To express the product of polynomials as a single polynomial means expansion."** The product of monomial with binomial is a binomial.

Example 1 : $a \times (5a - 6b)$ **or** $= (a \times 5a) + [(a)(-6b)]$

$= (a \times 5a) - (a \times 6b)$ $= 5a^2 + (-6ab)$

$= 5a^2 - 6ab$ $= 5a^2 - 6ab$

Example 2 : $(2x - 4) \times (-3x)$

$= (-3x) \times (2x - 4)$

$= [(-3x) \times (2x)] - [(-3x) \times (4)]$

$= (-6x^2) - (-12x)$

$= (-6x^2) + 12x$

Example 3 : $2m(3m^2 + 5)$

$= (2m \times 3m^2) + (2m \times 5)$

$= 6m^3 + 10m$

- **Product (multiplication of binomial with binomial :**

To get product of binomial $(a + b)(c + d)$, both the terms of any one binomial is multiplied with other nonomial respectively.

$(a + b)(c + d)$ **or** $(a + b)(c + d)$

$= a(c + d) + b(c + d)$ $= (a + b)\ c + (a + b)d$

$= ac + ad + bc + bd$ $= ac + bc + ad + bd$

- **Expand :**

Example 4 : $(x + y)(2y + 5)$

$= x(2y + 5) + y(2y + 5)$

$= 2xy + 5x + 2y^2 + 5y$ (No two terms are like terms)

Example 5 : $(2a + 3b)(5x - 3y)$

$= 2a(5x - 3y) + 3b(5x - 3y)$

$= 10ax - 6ay + 15bx - 9by$ (No two terms are like terms)

Example 6 : $(5a - 7b)(3a - 2b)$

$= 5a(3a - 2b) - 7b(3a - 2b)$

$= 15a^2 - \underline{10ab - 21ab} + 14b^2$ (Two terms are like terms)

$= 15a^2 - 31ab + 14b^2$

Example 7 : $(x^2 - 5)(x^2 + 3)$

$= x^2(x^2 + 3) - 5(x^2 + 3)$

$= x^4 + \underline{3x^2 - 5x^2} - 15$ (Two terms are like terms)

$= x^4 - 2x^2 - 15$

Therefore, product of a binomial with a binomial is a polynomial.

Example 8 : $(2x + 3y)(2x - 3y)$

$= 2x(2x - 3y) + 3y(2x - 3y)$

$= 4x^2 - \underline{6xy + 6xy} - 9y^2$ (Two terms are like terms)

$= 4x^2 - 9y^2$

We can see from the examples in expansion that, the product of a binomial with a binomial, get (1) Four terms **or** (2) Three terms **or** (3) Two terms.

1. Complete the given Table by multiplying monomial with monomial :

1st monomial → / 2nd monomial ↓	$2x$	$-5y$	$3a^2$	$-4xy$	mn
$2x$	$4x^2$	$-10xy$			
$3y^2$			$9a^2y^2$		
$-2a$				$8axy$	
$3mn$		$-15mny$			
$5xy$				$-20x^2y^2$	

2. Expand :

(1) $2a(3x + 5y)$ (2) $3x^2(5x - 4y)$ (3) $5a(6a + 3b)$

(4) $(-6y)(5x - 7y^2)$ (5) $(-10a)(5a^2 + b)$ (6) $(-3ab)(2a + 3b)$

(7) $(6x^3y^3 - 1)(-2x^2)$ (8) $2ab(3ab - 1)$

3. Expand :

(1) $(2x + y)(a + 3b)$ (2) $(5x - 3)(2a + 5)$ (3) $(x - 2)(x^2 + 3)$

(4) $(5x^2 + 3)(2x^2 + 5)$ (5) $(x - 3)(x - 7)$ (6) $(2m^2 + 5)(3m + 1)$

(7) $(3x + 5)(2x - 4)$ (8) $(x - 3a)(4x + 5a)$

Expansion of square of binomial :

$(a + b)^2 = (a + b) \times (a + b)$

$= a(a + b) + b(a + b)$

$= a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$ $(\because\ ba = ab)$

$\boldsymbol{(a + b)^2 = a^2 + 2ab + b^2}$

(F.T. = First Term, S.T. = Second Term)

In words,

$(\text{F.T.} + \text{S.T.})^2 = (\text{F.T.})^2 + 2(\text{F.T.})(\text{S.T.}) + (\text{S.T.})^2$

$(a - b)^2 = (a - b) \times (a - b)$

$= a(a - b) - b(a - b)$

$= a^2 - ab - ba + b^2$

$= a^2 - ab - ab + b^2$ $(\because\ ba = ab)$

$\boldsymbol{(a - b)^2 = a^2 - 2ab + b^2}$

In words $(\text{F.T.} - \text{S.T.})^2 = (\text{F.T.})^2 - 2(\text{F.T.})(\text{S.T.}) + (\text{S.T.})^2$

Expand :

Example 8 : $(x + 3)^2$

$= (x)^2 + 2(x)(3) + (3)^2$

$= x^2 + 6x + 9$

Example 9 : $(a - 5)^2$

$= (a)^2 - 2(a)(5) + (5)^2$

$= a^2 - 10a + 25$

Example 10 : $(2x + 3y)^2$

$= (2x)^2 + 2(2x)(3y) + (3y)^2$

$= 4x^2 + 12xy + 9y^2$

Example 11 : $(2ab - y)^2$

$= (2ab)^2 - 2(2ab)(y) + (y)^2$

$= 4a^2b^2 - 4aby + y^2$

Find the values using expansion formula :

Example 12 : $(12)^2$

$= (10 + 2)^2$

$= (10)^2 + 2(10)(2) + (2)^2$

$= 100 + 40 + 4$

$= 144$

Example 13 : $(34)^2$

$= (30 + 4)^2$

$= (30)^2 + 2(30)(4) + (4)^2$

$= 900 + 240 + 16$

$= 1156$

Example 14 : $(26)^2$

$= (30 - 4)^2$

$= (30)^2 - 2(30)(4) + (4)^2$

$= 900 - 240 + 16$

$= 676$

or

$(26)^2$

$= (20 + 6)^2$

$= (20)^2 + 2(20)(6) + (6)^2$

$= 400 + 240 + 36$

$= 676$

1. Fill in the blanks :

(1) $(x - y)^2 = x^2 - 2xy +$ .....

(2) $(a + 7)^2 =$ ..... $+ 14a + 49$

(3) $(2m - n)^2 = 4m^2 -$ ..... $+ n^2$

(4) $(x + 1)^2 =$ .....

(5) $(2a - 3)^2 =$ .....

(6) $(m + 2)^2 =$ .....

2. Expand :

(1) $(x + 7)^2$

(2) $(m - 4)^2$

(3) $(x + 3y)^2$

(4) $(8x + 5y)^2$

(5) $(2a - 3b)^2$

(6) $(4ab - 3xy)^2$

(7) $(3xy - 7)^2$

(8) $(2xy - 3z)^2$

3. Find the value of the following using expansion formulae of $(a + b)^2$ or $(a - b)^2$:

(1) $(43)^2$ (2) $(82)^2$ (3) $(67)^2$ (4) $(48)^2$

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- Expansion of $(a + b)(a - b)$: $(a + b)\,(a - b)$

$= a(a - b) + b(a - b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

$(a + b)(a - b) = a^2 - b^2$

In words : (F.T. + S.T.) (F.T. − S.T.) = (F.T.)² − (S.T.)²

Expand :

Example 15 : $(x + 5)\ (x - 5)$

$= (x)^2 - (5)^2$

$= x^2 - 25$

Example 16 : $(2a - 3b)\ (2a + 3b)$

$= (2a)^2 - (3b)^2$

$= 4a^2 - 9b^2$

Example 17 : Find the value of 22×18 using expansion formula.

$22 \times 18 = (20 + 2)\ (20 - 2)$

$= (20)^2 - (2)^2$

$= 400 - 4$

$= 396$

1. Make pairs :

	Section 'A'	Section 'B'
(1)	$(m + n)(m - n)$	(a) $m^2 - 49$
(2)	$(m + 7)(m - 7)$	(b) $25 - m^2$
(3)	$(5 + m)(5 - m)$	(c) $9m^2 - 1$
(4)	$(3m - 1)(3m + 1)$	(d) $m^2 - n^2$

2. Expand :

(1) $(4x + 1)(4x - 1)$ (2) $(3x - 7y)(3x + 7y)$

(3) $(6 - x)(6 + x)$ (4) $(a + 8b)(a - 8b)$

(5) $(11 + 3xy)(11 - 3xy)$ (6) $(2mn + 5)(2mn - 5)$

3. Find values using expansion formula :

(1) 41×39 (2) 56×64 (3) 73×67 (4) 33×27

*

- **Expansion of $(x + a)(x + b)$:**

$$(x + a)(x + b) = x(x + b) + a(x + b)$$
$$= x^2 + xb + ax + ab$$
$$= x^2 + ax + bx + ab$$
$$\therefore \quad (x + a)(x + b) = x^2 + (a + b)x + ab$$

- **Expand :**

Example 18 : $(x + 3)(x + 2)$

$$= (x)^2 + (3 + 2)(x) + (3)(2)$$
$$= x^2 + 5x + 6$$

Example 19 : $(x + 3)(x - 5)$

$$= (x)^2 + (3 - 5)(x) + (3)(-5)$$
$$= x^2 - 2x - 15$$

Example 20 : $(x + 8)(x - 3)$

$$= (x)^2 + (8 - 3)(x) + (8)(-3)$$
$$= x^2 + 5x - 24$$

Example 21 : $(x - 4)(x - 3)$

$$= (x)^2 + (-4 - 3)(x) + (-4)(-3)$$
$$= x^2 - 7x + 12$$

Example 22 : $(3x + 5y)(3x - 2y)$

$$= (3x)^2 + (5y - 2y)(3x) + (5y)(-2y)$$
$$= 9x^2 + (3y)(3x) + (-10y^2)$$
$$= 9x^2 + 9xy - 10y^2$$

- **Find the values using expansion formula :**

Example 23 : 26×32

$$= (30 - 4)(30 + 2)$$
$$= (30)^2 + (-4 + 2)(30) + (-4)(2)$$
$$= 900 + (-2)(30) + (-8)$$
$$= 900 - 60 - 8$$
$$= 832$$

Example 24 : 35×33

$= (30 + 5)(30 + 3)$

$= (30)^2 + (5 + 3)(30) + (5)(3)$

$= 900 + (8)(30) + (15)$

$= 900 + 240 + 15$

$= 1155$

1. Expand :

(1) $(y + 2)(y + 4)$ (2) $(m + 6)(m - 2)$

(3) $(2a - 5)(2a + 3)$ (4) $(4x - 2y)(4x + y)$

(5) $(a - 3b)(a + 2b)$ (6) $(5ab - 3)(5ab + 2)$

(7) $(6x + 3)(6x + 5)$ (8) $(7a + 4)(7a + 3)$

2. Find the values using expansion formula :

(1) 43×42 (2) 68×73 (2) 52×51 (2) 24×19 (2) 23×18 (2) 27×32

1. Multiply (find product) :

(1) $2a(-3a^2)$ (2) $(-4ab)(6a^2b)$

(3) $(2m)(3m + n)$ (4) $(-4n)(6n + 5m)$

(5) $(5a + 3b)(6a - 2b)$ (6) $(2x + 3y)(6x - 2y)$

(7) $(6xy + 1)(2xy - 3)$ (8) $(a - 2b)(2a - b)$

2. Expand :

(1) $(a + 5)^2$ (2) $(m - 7)^2$

(3) $(3m + 2n)^2$ (4) $(4xy - 3)^2$

(5) $(5ab + 3c)^2$ (6) $(4b^2 + 3)^2$

(7) $(2x - 7)^2$ (8) $(5 - 3mn)^2$

3. Expand :

(1) $(x - 7)(x + 7)$ (2) $(2a + 3b)(2a - 3b)$

(3) $(2m + 5)(2m - 5)$ (4) $(2mn + 3)(2mn - 3)$

4. Expand :

(1) $(a + 3)(a + 2)$ (2) $(m - 2)(m - 5)$

(3) $(x - 9)(x + 2)$ (4) $(x + 6y)(x - 2y)$

(5) $(5x - 2y)(5x - 4y)$ (6) $(2m + 3n)(2m + 5n)$

(7) $(xy - 7)(xy + 4)$ (8) $(x^2 - 5)(x^2 + 3)$

5. Find the values using expansion formula :

(1) 62^2 (2) 57^2 (3) 43×37

(4) 97×103 (5) 16×22

What did we learn ?

- Product of monomial with monomial.
- Product of monomial with binomial.
- Product of binomial with binomial.
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- $(x + a)(x + b) = x^2 + (a + b)(x) + ab$

Answers

Practice 1

1. (1) $6ax + 10ay$ (2) $15x^3 - 12x^2y$ (3) $30a^2 + 15ab$

(4) $-30xy + 42y^3$ (5) $-50a^3 - 10ab$ (6) $-6a^2b - 9ab^2$

(7) $-12x^5y^3 + 2x^2$ (8) $6a^2b^2 - 2ab$

2.

1st Monomial → / 2nd Monomial ↓	$2x$	$-5y$	$3a^2$	$-4xy$	mn
$2x$	$4x^2$	$-10xy$	$6xa^2$	$-8x^2y$	$2xmn$
$3y^2$	$6xy^2$	$-15y^3$	$9a^2y^2$	$-12xy^3$	$3y^2mn$
$-2a$	$-4ax$	$10ay$	$-6a^3$	$8axy$	$-2amn$
$3mn$	$6mnx$	$-15mny$	$9a^2mn$	$-12xymn$	$3m^2n^2$
$5xy$	$10x^2y$	$-25xy^2$	$15a^2xy$	$-20x^2y^2$	$5xymn$

3. (1) $2xa + 6xb + ya + 3yb$ (2) $10xa + 25x - 6a - 15$ (3) $x^3 + 3x - 2x^2 - 6$

(4) $10x^4 + 31x^2 + 15$ (5) $x^2 - 10x + 21$ (6) $6m^3 + 2m^2 + 15m + 5$

(7) $6x^2 - 2x - 20$ (8) $4x^2 - 7xa - 15a^2$

Practice 2

1. (1) y^2 (2) a^2 (3) $4mn$

(4) $x^2 + 2x + 1$ (5) $4a^2 - 12a + 9$ (6) $m^2 + 4m + 4$

2. (1) $x^2 + 14x + 49$ (2) $m^2 - 8m + 16$

(3) $x^2 + 6xy + 9y^2$ (4) $64x^2 + 80xy + 25y^2$

(5) $4a^2 - 12ab + 9b^2$ (6) $16a^2b^2 - 24abxy + 9x^2y^2$

(7) $9x^2y^2 - 42xy + 49$ (8) $4x^2y^2 - 12xyz + 9z^2$

3. (1) 1849 (2) 6724 (3) 4489 (4) 2304

Practice 3

1. (1) d (2) a (3) b (4) c

2. (1) $16x^2 - 1$ (2) $9x^2 - 49y^2$ (3) $36 - x^2$

(4) $a^2 - 64b^2$ (5) $121 - 9x^2y^2$ (6) $4m^2n^2 - 25$

3. (1) 1599 (2) 3584 (3) 4891 (4) 891

Practice 4

1. (1) $y^2 + 6y + 8$ (2) $m^2 + 4m - 12$ (3) $4a^2 - 4a - 15$

(4) $16x^2 - 4xy - 2y^2$ (5) $a^2 - ab - 6b^2$ (6) $25a^2b^2 - 5ab - 6$

(7) $36x^2 + 48x + 15$ (8) $49a^2 + 49a + 12$

2. (1) 1806 (2) 4964 (3) 2652 (4) 456 (5) 414 (6) 864

Exercise

1. (1) $-6a^3$ (2) $-24a^3b^2$ (3) $6m^2 + 2mn$

(4) $-24n^2 - 20mn$ (5) $30a^2 + 8ab - 6b^2$ (6) $12x^2 + 14xy - 6y^2$

(7) $12x^2y^2 - 16xy - 3$ (8) $2a^2 - 5ab + 2b^2$

2. (1) $a^2 + 10a + 25$ (2) $m^2 - 14m + 49$ (3) $9m^2 + 12mn + 4n^2$

(4) $16x^2y^2 - 24xy + 9$ (5) $25a^2b^2 + 30abc + 9c^2$

(6) $16b^4 + 24b^2 + 9$ (7) $4x^2 - 28x + 49$ (8) $25 - 30mn + 9m^2n^2$

3. (1) $x^2 - 49$ (2) $4a^2 - 9b^2$ (3) $4m^2 - 25$ (4) $4m^2n^2 - 9$

4. (1) $a^2 + 5a + 6$ (2) $m^2 - 7m + 10$ (3) $x^2 - 7x - 18$

(4) $x^2 + 4xy - 12y^2$ (5) $25x^2 - 30xy + 8y^2$ (6) $4m^2 + 16mn + 15n^2$

(7) $x^2y^2 - 3xy - 28$ (8) $x^4 - 2x^2 - 15$

5. (1) 3844 (2) 3249 (3) 1591 (4) 9991 (5) 352

Relations between units :

- 1 inch = 2.54 cm
  1 foot = 30.48 cm
  1 foot = 12 inch
  3 feet = 1 bar = 36 inch
  1 metre = 39.37 inch = 3.280 feet = 1.0936 bar
- 100 m^2 = 1 acre
  100 acre = 1 hectare
  100 hectore = 1 sq km
- 9 sq feet = 1 sq bar
  121 sq. bar = 1 Guntha
  40 guntha = 1 Acre = 4840 sq bar
- 1 cubic foot = 1728 cubic inch
  1 cubic bar = 27 cubic feet
  100 cubic feet = 1 brass
- 1 gallon = 4.546 litre
  1 litre = 0.22 gallon
- 1 qusek = 1 cubic foot/second
  = 6.25 cubic gallon/second