# RELATIONS

## 1. INTRODUCTION :

Let A and B be two sets. Then a relation R from A to B is a subset of $A \times B$.

thus, R is a relation from A to B $\Leftrightarrow R \subseteq A \times B$.

**Total Number of Relations :** Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So total number of subsets of $A \times B$ is $2^{mn}$.

**Domain and Range of a relation :** Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called to domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R.

Thus, Domain (R) = $\{a : (a, b) \in R\}$

and, Range (R) = $\{b : (a, b) \in R\}$

It is evident from the definition that the domain of a relation from A to B is a subset of A and its range is a subset of B.

**Inverse Relation :** Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by $R^{-1}$, is a relation from B to A and is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}$$

Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$

Also, Domain(R) = Range($R^{-1}$) and Range (R) = Domain($R^{-1}$)

**Note :** Relation on a set : If R is a relation from set A to A itself then R is called Relation on set A.

## 2. TYPES OF RELATIONS :

In this section we intend to define various types of relations on a given set A.

**Void Relation :** Let A be a set. Then $\phi \subseteq A \times A$ and so it is a relation on A. This relation is called the void or empty relation on A.

**Universal Relation :** Let A be a set. Then $A \times A \subseteq A \times A$ and so it is a relation on A. This relation is called the universal relation on A.

**Identity Relation :** Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A.

In other words, a relation $I_A$ on A is called the identity relation if every element of A is related to itself only.

**Reflexive Relation :** A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R on a set A is not reflexive if there exists an element $A \in A$ such that $(a, a) \notin R$.

Every Identity relation is reflexive but every reflexive relation is not identity.

**Symmetric Relation :** A relation R on a set A is said to be a symmetric relation iff

$$(a, b) \in R \Leftrightarrow (b, a) \in R$$

i.e. $a R b \Leftrightarrow bRa$

**Transitive Relation :** Let A be any set. A relation R on A is said to be a transitive relation iff

$$(a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R$$

i.e. $a R b$ and $b R c \Rightarrow a R c$

**Antisymmetric Relation :** Let A be any set. A relation R on set A is said to be an antisymmetric relation iff

$$(a, b) \in R \text{ and } (b, a) \in R \Rightarrow a = b$$

**Equivalence Relation :** A relation R on a set A is said to be an equivalence relation on A iff

(i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$

(ii) it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$

(iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

It is not necessary that every relation which is symmetric and transitive is also reflexive.