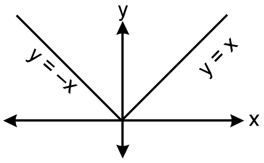
Fundamentals of Mathematics-II

He is unworthy of the name of man who is ignorant of the fact that the diagonal of square is incommensurable with its side ....**Plato**

Absolute value function / modulus function :

The symbol of modulus function is f (x) = $|x|$ and is defined as: y = $|x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$.



y = |x|

Properties of modulus : For any a, b ∈ R

(i) $|a| \geq 0$ (ii) $|a| = |-a|$

(iii) $|a| \geq a$, $|a| \geq -a$ (iv) $|ab| = |a|\,|b|$

(v) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ (vi) $|a + b| \leq |a| + |b|$; Equality holds when $ab \geq 0$

(vii) $|a - b| \geq ||a| - |b||$; Equality holds when $ab \geq 0$

Example # 1 : Solve the following linear equations

(i) $x\,|x| = 4$ (ii) $|x - 3| + 2|x + 1| = 4$

Solution : (i) $x|x| = 4$

If $x > 0$

$\therefore \quad x^2 = 4 \Rightarrow x = \pm 2$

$\therefore \quad x = 2 \quad (\because x \geq 0)$

If $x < 0 \Rightarrow -x^2 = 4 \Rightarrow x^2 = -4$ which is not possible

(ii) $|x - 3| + 2|x + 1| = 4$

case I : If $x \leq -1$

$\therefore \quad -(x - 3) - 2(x + 1) = 4$

$\Rightarrow \quad -x + 3 - 2x - 2 = 4 \Rightarrow -3x + 1 = 4$

$\Rightarrow \quad -3x = 3 \Rightarrow x = -1$

case II : If $-1 < x \leq 3$

$\therefore \quad -(x - 3) + 2(x + 1) = 4$

$\Rightarrow \quad -x + 3 + 2x + 2 = 4 \Rightarrow x = -1$ which is not possible

case III : If $x > 3$

$x - 3 + 2(x + 1) = 4$

$3x - 1 = 4 \Rightarrow x = 5/3$ which is not possible $\therefore$ $x = -1$ **Ans.**

Rational function :

A rational function is a function of the form, y = f (x) = $\frac{g(x)}{h(x)}$, where g (x) & h (x) are polynomial functions.

Irrational function :

An irrational function is a function y = f(x) in which the operations of addition, substraction, multiplication, division and raising to a fractional power are used.

For example y = $\frac{x^3 + x^{1/3}}{2x + \sqrt{x}}$ is an irrational function

(a) The equation $\sqrt{f(x)} = g(x)$, is equivalent to the following system

$f(x) = g^2(x) \quad \& \quad g(x) \geq 0$

(b) The inequation $\sqrt{f(x)} < g(x)$, is equivalent to the following system

$f(x) < g^2(x)$ & $f(x) \ge 0$ & $g(x) \ge 0$

(c) The inequation $\sqrt{f(x)} > g(x)$, is equivalent to the following system

$g(x) \le 0$ & $f(x) \ge 0$ or $g(x) \ge 0$ & $f(x) > g^2(x)$

Example # 2 : **Solve :** $x + 2 > 2\sqrt{1-x^2}$

Solution : $4(1 - x^2) < (x + 2)^2$ and $x + 2 \ge 0$ & $1 - x^2 \ge 0$

$x \in \left(-\infty, \frac{-4}{5}\right) \cup (0, \infty)$ ...(1)

$x \in [-2, \infty)$ ...(2)

$x \in [-1, 1]$ ...(3)

$(1) \cap (2) \cap (3)$

$\left[-1, -\frac{4}{5}\right) \cup (0, 1]$

Self Practice Problem :

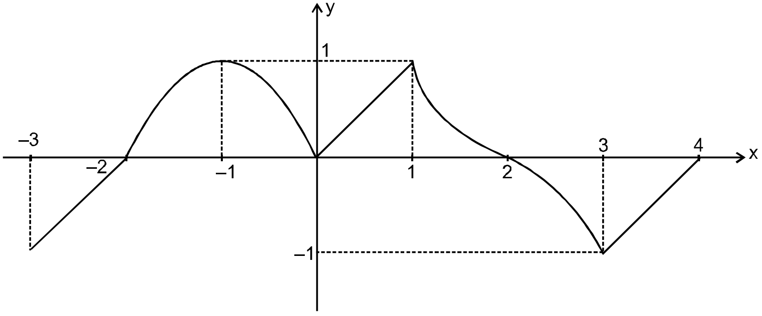
(1) $\sqrt{2x^2 + x - 6} < x$ (2) $\sqrt{5 - x} > x + 1$

(3) $x + 3 + \sqrt{x^2 + 4x - 5} > 0$ (4) $\sqrt{x} - \sqrt{4 - x} \ge 1$

Ans. (1) $\left[\frac{3}{2}, 2\right)$ (2) $(-\infty, 1)$ (3) $(-\infty, -1] \cup [5, \infty)$ (4) $\left[\frac{4+\sqrt{7}}{2}, 4\right]$

Graphs Related to modulus :

If graph of y = f(x) is

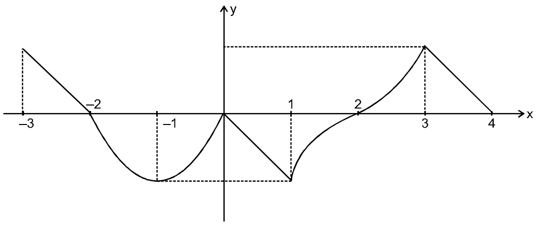


then draw graph of

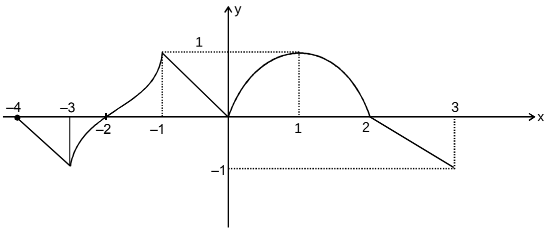
(a) y = – f(x) (b) y = f(–x) (c) y = f(|x|) (d) y = f(–|x|)

(e) y = |f(x)| (f) |y| = f(x) (g) |y| = – f(x)

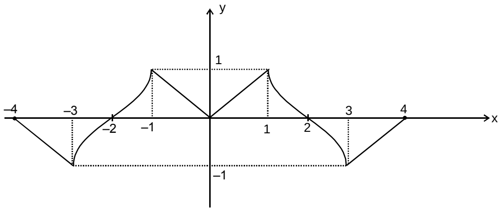
(a)



(b)



(c)



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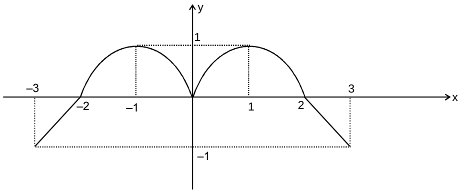

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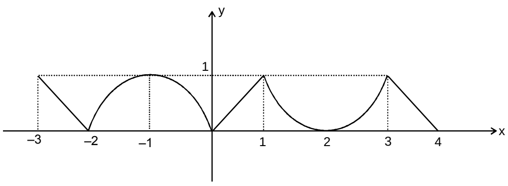
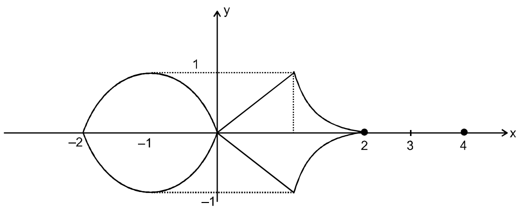
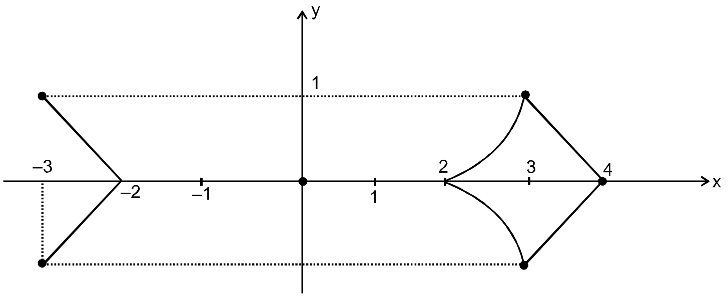


(f)

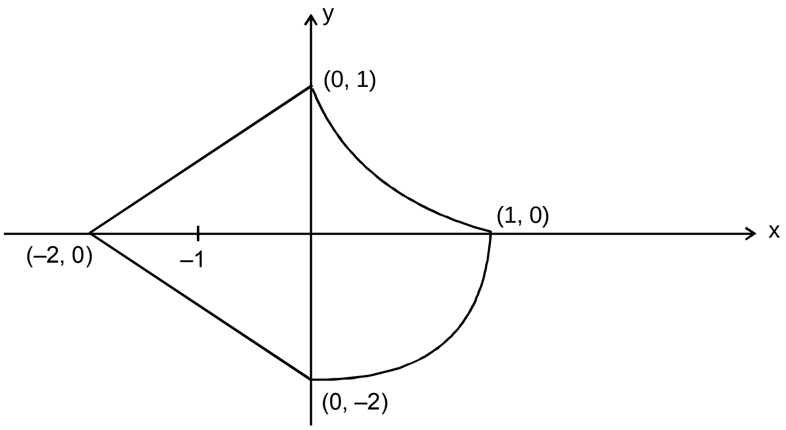


(g)



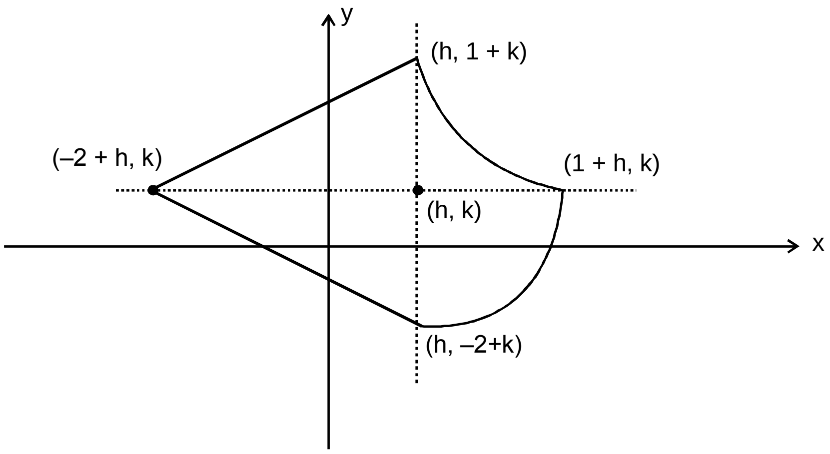
Graphical Trasformation :

If graph of y = f(x) is

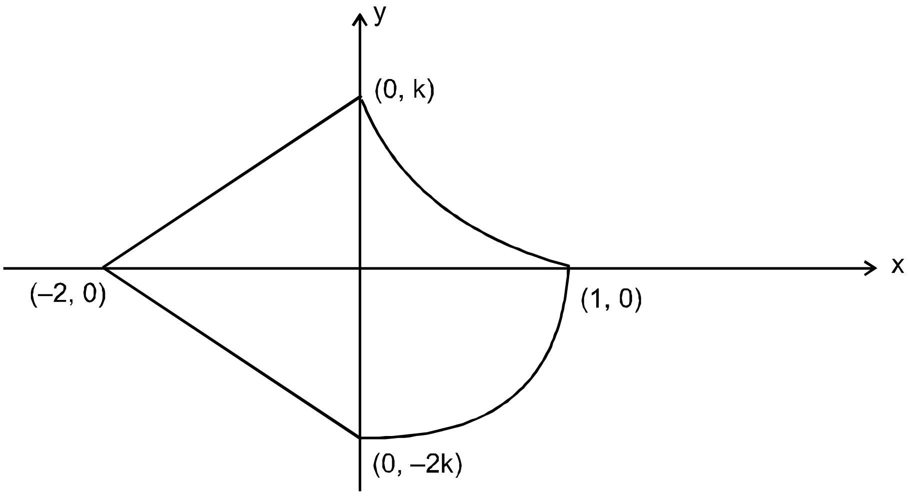


then graph of (a) y – k = f(x – h) (b) y = kf(x), (k > 0) (c) y = f(kx), (k > 0)

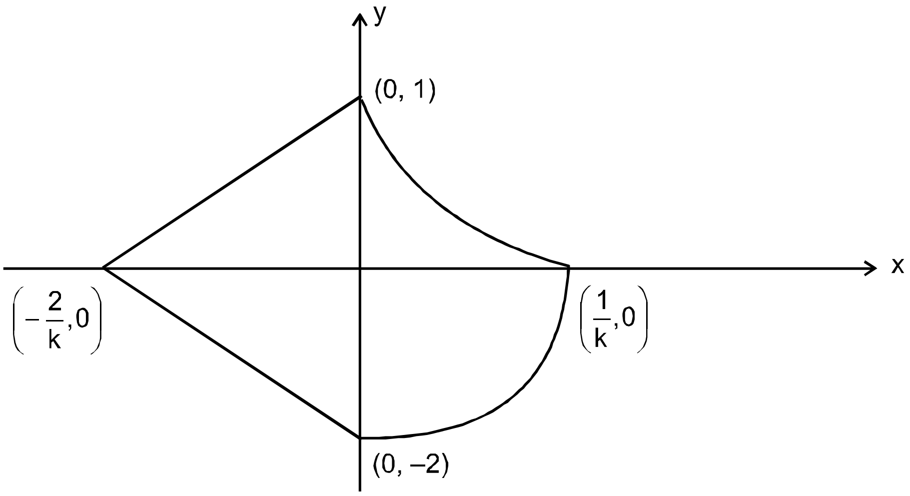
(a)



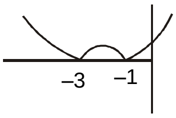
(b)



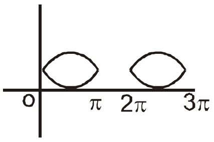
(c)



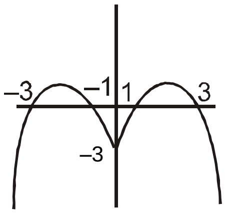
Example # 3 : $y = |x^2 + 4x + 3|$ **Solution :**



Example # 4 : $|y–1| = \sin x$ **Solution :**

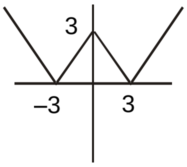


Example # 5 : $y = –x^2 + 4|x| – 3$ **Solution :**



Example # 6 : $y = ||x| - 3|$

Solution :



Example # 7 : $y = \sin\left(\frac{x}{3}\right)$

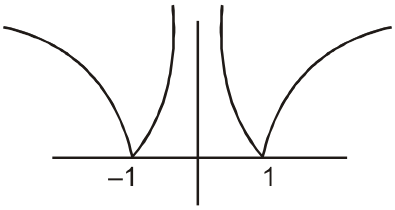
Solution : period is 6π

Example # 8 : $y = \left| \sin x - 3 \right|$

Solution : Graphical Transformation

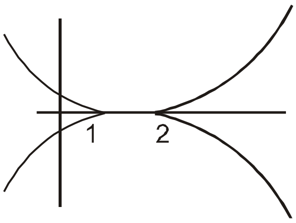
Example # 9 : $y = |-\ell n\, |-x||$

Solution :



Example # 10 : $|y| = x^2 - 3x + 2$

Solution :



Exercise-1

✍ **Marked questions are recommended for Revision.**

✍ चिन्हित प्रश्न दोहराने योग्य प्रश्न है।

PART - I : SUBJECTIVE QUESTIONS

भाग - I : विषयात्मक प्रश्न (SUBJECTIVE QUESTIONS)

Section (A) : Modulus Function & Equation

खण्ड (A) : मापांक फलन एवं समीकरण

A-1. Write the following expression in appropriate intervals so that they are bereft of modulus sign

नीचे दिये गये व्यंजकों को अन्तराल के रूप में लिखिये जो मापांक रहित हो **[16JM110001]**

(i)	$\|x^2 - 7x + 10\|$	Ans.	(i)	$x^2 - 7x + 10,\ x > 5 \text{ or } x \le 2$ $-(x^2 - 7x + 10),\ 2 < x \le 5$
(ii)	$\|x^3 - x\|$	Ans.	(ii)	$x^3 - x,\ x \in [-1, 0] \cup [1, \infty)$ $x - x^3,\ x \in (-\infty, -1) \cup (0, 1)$
(iii)	$\|2^x - 2\|$	Ans.	(iii)	$2^x - 2,\ x \ge 1$ $2 - 2^x,\ x < 1$
(iv)	$\|x^2 - 6x + 10\|$	Ans.	(iv)	$x^2 - 6x + 10,\ x \in R$
(v)	$\|x - 1\| + \|x^2 - 3x + 2\|$	Ans.	(v)	$x^2 - 2x + 1,\ x \ge 2$ $4x - x^2 - 3,\ 1 \le x < 2$ $x^2 - 4x + 3,\ x < 1$
(vi)	$\sqrt{x^2 - 6x + 9}$	Ans.	(vi)	$x - 3,\ x \ge 3$ $3 - x,\ x < 3$
(vii)	$2^{(x-1)} + \|x + 2\| - 3^{\|x+1\|}$	Ans.	(vii)	$2^{x-1} + x + 2 - 3^{x+1} \quad x \ge -1$ $2^{x-1} + x + 2 - 3^{-(x+1)} \quad -2 \le x < -1$ $2^{x-1} - x - 2 - 3^{-(x+1)} \quad x < -2$

A-2. Draw the labled graph of following **[16JM110002]**

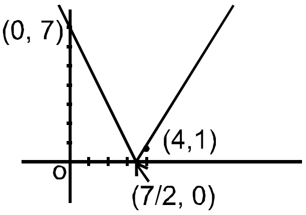
निम्नलिखित के आरेख बनाइये–

(i) $y = |7 - 2x|$ (ii) $y = |x - 1| - |3x - 2|$

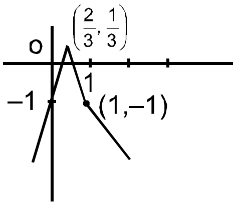
(iii) $y = |x - 1| + |x - 4| + |x - 7|$ (iv) $y = |4x + 5|$

(v) $y = |2x - 3|$

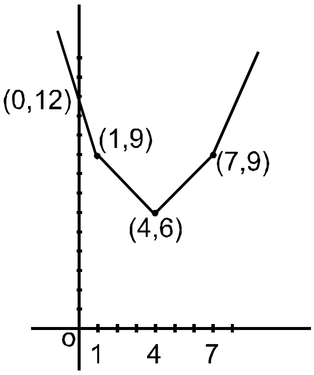
Ans. (i)



(ii)



(iii)



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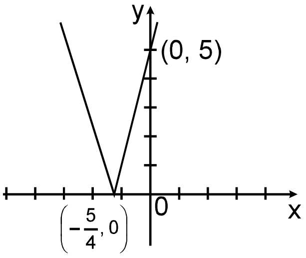
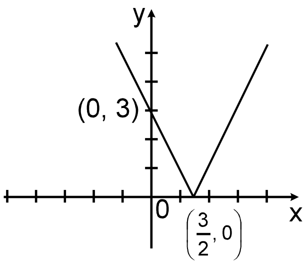


(v)



A-3. Solve the following equations

निम्नलिखित समीकरणों को हल कीजिए–

(i) $|x| + 2\,|x-6| = 12$ **Ans.** $x = 0, 8$

(ii) $||x+3| - 5| = 2$ **Ans.** $x = -10, -6, 0, 4$

(iii) $|||x-2| - 2\,| - 2| = 2$ **Ans.** $x = 0, \pm 4, 8$

(iv) $|4x+3| + |3x-4| = 12$ **Ans.** $x = -\frac{11}{7}, \frac{13}{7}$

Sol. **(1)** $|x| + 2|x-6| = 12$

Case-I : $x \geq 6$ $3x = 24 \Rightarrow x = 8$

Case-II : $0 \leq x < 6$

$x + 12 - 2x = 12 \Rightarrow x = 0$

Case-III : $x < 0$

$-x + 12 - 2x = 12 \Rightarrow x = 0$

so solution is इसलिए हल $x = 0, 8$

(2) $||x+3| - 5| = 2$

$\Rightarrow |x+3| - 5 = 2, -2$ $\Rightarrow |x+3| = 7$ or या $|x+3| = 3$

$\Rightarrow x + 3 = 7, -7$ or या $x + 3 = 3, -3$

$\Rightarrow x = 4, -10$ or या $x = 0, -6$

so इसलिए $x = -10, -6, 0, 4$

(3) $|||x-2| - 2\,| - 2| = 2$

$\Rightarrow ||x-2| - 2\,| - 2 = \pm 2$

either $||x-2| - 2\,| = 4$ or या 0

case-I : $||x-2| - 2| = 4$

$|x-2| - 2 = \pm 4 \Rightarrow |x-2| = 6$ or या $-2 \Rightarrow x - 2 = \pm 6 \Rightarrow x = 8$ or या -4

case-II : $||x-2| - 2| = 0$

$|x-2| - 2 = 0 \Rightarrow |x-2| = 2 \Rightarrow x - 2 = \pm 2 \Rightarrow x = 4$ or या 0

hence four solutions अतः चार हल : 0, –4, 4 & 83

(iv) $|4x+3| + |3x-4| = 12$

case-1 स्थिति **-1 :** $x < -\frac{3}{4}$

$-4x - 3 - 3x + 4 = 12 \Rightarrow -7x = 11 \Rightarrow x = -\frac{11}{7}$

case-2 : स्थिति**-2** $-\frac{3}{4} \leq x \leq \frac{4}{3} \Rightarrow 4x + 3 - 3x + 4 = 12$

$x = 5$, not acceptable. जो स्वीकार्य नहीं है।

case-3 : स्थिति-3 $x \geq \frac{4}{3}$

$4x + 3 + 3x - 4 = 12 \quad \Rightarrow \quad 7x = 13 \quad \Rightarrow \quad x = \frac{13}{7}$

$\therefore \quad x = -\frac{11}{7}, \ \frac{13}{7}.$

A-4. Solve the following equations : **[16JM110003]**
निम्न समीकरणों को हल कीजिए :

(i) $x^2 - 7|x| - 8 = 0$ **Ans.** ± 8
(ii) $|x^2 - x + 1| = |x^2 - x - 1|$ **Ans.** 0, 1
(iii) $|x^3 - 6x^2 + 11x - 6| = 6$ **Ans.** 0, 4
(iv) $|x^2 - 2x| + x = 6$ **Ans.** –2, 3
(v)✍ $|x^2 - x - 6| = x + 2.$ **Ans.** $x \in \{-2, 2, 4\}$

Sol. (i) $|x|^2 - 7|x| - 8 = 0 \quad \Rightarrow \quad (|x| - 8)(|x| + 1) = 0 \quad \Rightarrow |x| = 8 \quad \Rightarrow x = \pm 8$

(ii) Squaring both sides, we get दोनों तरफ वर्ग करने पर

$(x^2 - x + 1)^2 - (x^2 - x - 1)^2 = 0 \quad \Rightarrow (2x^2 - 2x)(2) = 0 \quad \Rightarrow \quad x = 0, 1$

(iii) $|x^3 - 6x^2 + 11x - 6| = 6$

$x^3 - 6x^2 + 11x - 6 = 6 \quad \Rightarrow \quad x^3 - 6x^2 + 11x - 12 = 0$

$\Rightarrow \quad (x - 4)(x^2 - 2x + 6) = 0 \quad \Rightarrow \quad x = 4$

or या $x^3 - 6x^2 + 11x - 6 = -6 \quad \Rightarrow \quad x(x^2 - 6x + 11) = 0 \quad \Rightarrow \quad x = 0$

(iv) Caseस्थिति -I : $x \in (-\infty, 0] \cup [2, \infty)$

$x^2 - 2x + x = 6 \quad \Rightarrow x^2 - x - 6 = 0 \quad \Rightarrow x = -2, 3$

Caseस्थिति -II : $x \in [0, 2]$

$2x - x^2 + x = 6 \quad \Rightarrow x^2 - 3x + 6 = 0$ ⇒ No real roots कोई वास्तविक मूल नहीं

(v) **Case-I :** $x \in [-2, 3]$

$6 + x - x^2 = x + 2 \quad \Rightarrow \quad x^2 = 4 \quad \Rightarrow \quad x = \pm 2$

Case-II : $x \in (-\infty, -2] \cup [3, \infty)$

$x^2 - x - 6 = x + 2 \quad \Rightarrow \quad x^2 - 2x - 8 = 0 \quad \Rightarrow \quad (x - 4)(x + 2) = 0$

$\Rightarrow \quad x = -2, 4$

A-5. Find the number of real roots of the equation
समीकरण के वास्तविक मूलों की संख्या ज्ञात कीजिए।

(i) $\left|x\right|^2 - 3\left|x\right| + 2 = 0$ **Ans.** **4**
(ii) $||x - 1| - 5| = 2$ **Ans.** **4**
(iii) $|2x^2 + x - 1| = |x^2 + 4x + 1|$ **Ans.** **4**

Sol. **(i)** $|x|^2 - 3|x| + 2 = 0$

$(|x| - 2)(|x| - 1) = 0$

$|x| = 2 \qquad |x| = 1$

$x = 2, -2 \qquad x = 1, -1$

for solutions अतः चार मूल होगें।

(ii) $|x - 1| - 5 = \pm 2 \quad \Rightarrow \quad |x - 1| = 7, 3 \quad \Rightarrow$ four values चार मान.

(iii) $(2x^2 + x - 1)^2 - (x^2 + 4x + 1)^2 \quad \Rightarrow \quad (3x^2 + 5x)(x^2 - 3x - 2) = 0$

$\Rightarrow \quad x = -\frac{5}{3}, 0, \frac{3 \pm \sqrt{17}}{2} \quad \Rightarrow$ four solutions चार हल

A-6. Find the sum of solutions of the following equations : **[DRN1172]**
निम्न दिए गये समीकरणों के हलों का योगफल ज्ञात कीजिए

(i) $x^2 - 5|x| - 4 = 0$ **Ans.** 0
(ii) $(x - 3)^2 + |x - 3| - 11 = 0$ **Ans.** 6
(iii) $|x|^3 - 15x^2 - 8|x| - 11 = 0$ **Ans.** 0
(iv) $||x - 3| - 4| = 1$ **Ans.** 12
(v) $2^{|x|} + 3^{|x|} + 4^{|x|} = 9$ **Ans.** 0

Sol. (i) $|x|^2 - 5|x| - 4 = 0 \quad \Rightarrow \quad |x| = \frac{5 + \sqrt{41}}{2} \quad \Rightarrow \quad x = \pm\left(\frac{5 + \sqrt{41}}{2}\right)$

Hence sum अतः योगफल = 0.

(ii) Let |x – 3| = t; equation becomes $t^2 + t - 11 = 0 \begin{matrix} \nearrow \alpha < 0 \\ \searrow \beta > 0 \end{matrix}$

so $|x - 3| = \beta \Rightarrow x = 3 \pm \beta$ ⇒ sum = 6

(iii) Let |x| = t, for any value of t satisfying this equation corresponding x = ±t ⇒ sum is zero.

(iv) $|x - 3| = 4 \pm 1$ ⇒ $|x - 3| = 3, 5$ ⇒ x = 6, 0, 8, –2 ⇒ sum योग = 12

(ii) माना |x – 3| = t ; समीकरण $t^2 + t - 11 = 0 \begin{matrix} \nearrow \alpha < 0 \\ \searrow \beta > 0 \end{matrix}$

इसलिए $|x - 3| = \beta \Rightarrow x = 3 \pm \beta$ ⇒ योगफल = 6

(iii) माना |x| = t, t के किसी मान के लिये समीकरण सन्तुष्ट होती है

x = ±t ⇒ योग शून्य है

(iv) $|x - 3| = 4 \pm 1$ ⇒ $|x - 3| = 3, 5$ ⇒ x = 6, 0, 8, –2 ⇒ योग = 12

(v) |x| = 1 is the solution एक हल है ⇒ x = ±1 ⇒ sum of roots मूलों का योग = 0

A-7. Find number of solutions of the following equations **[16JM110004]**

निम्न समीकरणों के हलों की संख्या ज्ञात कीजिए

(i) $|x - 1| + |x - 2| + |x - 3| = 9$

(ii) $|x - 1| + |x - 2| + |x - 3| + |x - 4| = 4$

(iii)✍ $|x| + |x + 2| + |x - 2| = p, p \in R$

Ans. (i) 2

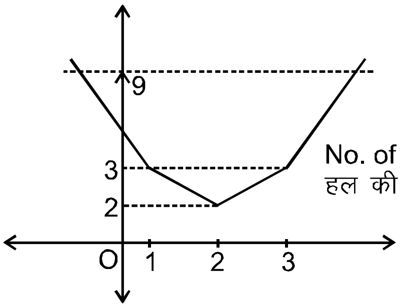
(ii) Infinite अनन्त

(iii) p < 4 no solution कोई हल नहीं

p = 4 one solution एक हल

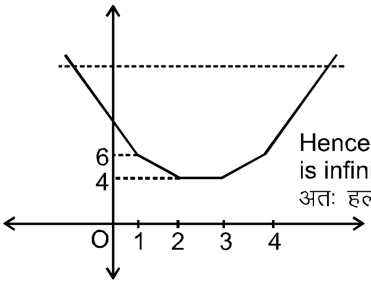
p > 4 Two solution दो हल

Sol. (i)



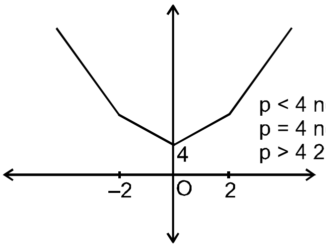
No. of solutions=2
हल की संख्या =2

(ii)



Hence no. of solutions is infinite
अतः हलों की संख्या अनन्त है

(iii)



p < 4 no solution
p = 4 no solution
p > 4 2 solution

p < 4 कोई हल नहीं
p = 4 कोई हल नहीं
p > 4, 2 हल

A-8. Find the minimum value of $f(x) = |x - 1| + |x - 2| + |x - 3|$ **[16JM110014]**

$f(x) = |x - 1| + |x - 2| + |x - 3|$ का न्यूनतम मान है–

Ans. 2

Sol. $$f(x) = |x - 1| + |x - 2| + |x - 3| = \begin{cases} -x+1-x+2-x+3 = 6-3x\ , & x \le 1 \\ x-1-x+2-x+3 = 4-x & 1 < x \le 2 \\ x-1+x-2-x+3 = x & 2 < x \le 3 \\ x-1+x-2+x-3 = 3x-6 & x > 3 \end{cases}$$

min न्यूनतम f(x) = 2.

A-9 If $x^2 - |x - 3| - 3 = 0$, then $|x|$ can be
यदि $x^2 - |x - 3| - 3 = 0$, तब $|x|$ हो सकता है

Ans. 2, 3

Sol. Let माना $x \geq 3$ $x^2 - (x - 3) - 3 = 0$ $\Rightarrow$ $x^2 - x = 0$ $\Rightarrow$ $x = 0, 1$ rejected अस्वीकार्य
If यदि $x < 3$ $x^2 + (x - 3) = 0 \Rightarrow$ $x^2 + x - 6 = 0$ $\Rightarrow$ $(x + 3)(x - 2) = 0$
$\Rightarrow$ $x = -3, 2$
Hence अतः $|x| = 2$ or या 3

A-10.✍ If $|x^3 - 6x^2 + 11x - 6|$ is a prime number then find the number of possible integral values of x.
यदि $|x^3 - 6x^2 + 11x - 6|$ अभाज्य संख्या है तब x के संभावित पूर्णांक मानों की संख्या है–

Ans. 0

Sol. $|(x - 1)(x - 2)(x - 3)|$ cannot be a prime integer for integer values of x as product of 3 consecutive integers cannot be prime.
$|(x - 1)(x - 2)(x - 3)|$ अभाज्य पूर्णांक नहीं हो सकता x के पूर्णांक मान के लिए क्योंकि 3 क्रमागत पूर्णांकों गुणक अभाज्य नहीं हो सकता।

Section (B) : Modulus Inequalities
खण्ड (B) : मापांकीय असमीकाऐं

B-1. Solve the following inequalities :
निम्नलिखित असमिकाओं को हल कीजिए–

(i) $|x - 3| \geq 2$ (ii) $||x - 2| - 3| \leq 0$

(iii) $||3x - 9| + 2| > 2$ (iv) $|2x - 3| - |x| \leq 3$
(v) $|x - 1| + |x + 2| \geq 3$
(vi)✍ $||x - 1| - 1| \leq 1$

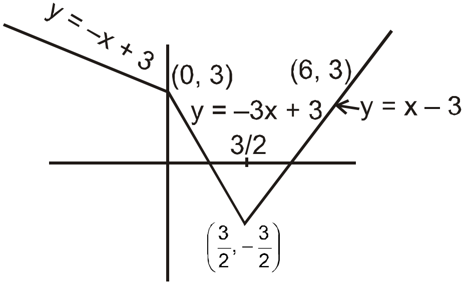
Ans. (i) $x \in (-\infty, 1] \cup [5, \infty)$ (ii) $x = 5$ or $x = -1$
(iii) $x \in R - \{3\}$ (iv) $x \in [0, 6]$ **(v)** R
(vi) $[-1, 3]$

Sol. (i) $|x - 3| \geq 2$
$x - 3 \geq 2$ or या $x - 3 \leq -2$
$x \geq 5$ or या $x \leq 1$

(ii) $|x - 2| - 3 = 0$
$|x - 2| = 3$
$x = 5$ or या $x = -1$

(iii) $|3x - 9| + 2 > 2$ or या $|3x - 9| + 2 < -2$
$|3x - 9| > 0$ or या $x \in \phi$
$x \in R - \{3\}$

(iv)



for $y \leq 3$ के लिये $x \in [0, 6]$

(v) $|x - 1|$ $|x + 2| \geq 3$

Now अब $|a| + |b| \geq |a - b|$ $\Rightarrow$ $|x + 2| + |x - 1| \geq 3$ $\forall\ x \in R$.

(vi) $-1 \leq |x - 1| - 1 \leq 1$ $\Rightarrow$ $0 \leq |x - 1| \leq 2$

$\therefore$ $0 \leq |x - 1|$ $\Rightarrow$ $x \in R$ ...(1)

and और $|x - 1| \leq 2$

$\Rightarrow$ $-2 \leq x - 1 \leq 2$ $\Rightarrow$ $-1 \leq x \leq 3$ ...(2)

$(1) \cap (2)$ से

$\Rightarrow$ $x \in [-1, 3]$.

B-2. Solve the following inequalities : **[16JM110005]**

निम्नलिखित असमिकाओं को हल कीजिए

(i) $\left|1+\frac{3}{x}\right| > 2$ **Ans.** $x \in (-1, 0) \cup (0, 3)$

(ii) $\left|\frac{3x}{x^2-4}\right| \leq 1$ **Ans.** $x \in (-\infty, -4] \cup [-1, 1] \cup [4, \infty)$

(iii) $\frac{|x+3|+x}{x+2} > 1$ **Ans.** $x \in (-5, -2) \cup (-1, \infty)$

(iv) $|x^2 + 3x| + x^2 - 2 \geq 0$ **Ans.** $x \in \left(-\infty,\ -\frac{2}{3}\right] \cup \left[\frac{1}{2},\ \infty\right)$

(v) $|x + 3| > |2x - 1|$ **Ans.** $x \in \left(-\frac{2}{3},\ 4\right)$

Sol. **(i)** $1 + \frac{3}{x} > 2$ or या $1 + \frac{3}{x} < -2$ $\Rightarrow$ $\frac{3-x}{x} > 0$ or या $\frac{x+1}{x} < 0$

$\Rightarrow$ $0 < x < 3$ or या $-1 < x < 0$ $\Rightarrow$ $x \in (-1, 0) \cup (0, 3)$

(ii) $-1 \leq \frac{3x}{x^2-4} \leq 1$ $\Rightarrow$ $\frac{3x+x^2-4}{x^2-4} \geq 0$ and और $\frac{3x-x^2+4}{x^2-4} \leq 0$

$\Rightarrow$ $\frac{(x+4)\ \ (x-1)}{(x-2)\ \ (x+2)} \geq 0$ and और $\frac{(x-4)\ \ (x+1)}{(x-2)\ \ (x+2)} \geq 0$

$x \in (-\infty, -4] \cup (-2, 1] \cup (2, \infty)$ and और $x \in (-\infty, -2) \in [-1, 2) \cup [4, \infty)$

Taking intersection we get उभयनिष्ठ लेने पर $x \in (-\infty, -4] \cup [-1, 1] \cup [4, \infty)$

(iii) **case-I:** स्थिति**-I:** $x \geq -3$ $\Rightarrow$ $\frac{2x+3-x-2}{x+2} > 0$

$\Rightarrow$ $\frac{x+1}{x+2} > 0 \Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$ But लेकिन $x \geq -3 \Rightarrow x \in [-3, -2) \cup (-1, \infty)$

case-II: स्थिति**-II :** $x < -3$ $\Rightarrow$ $\frac{-3-x-2}{x+2} > 0$ $\Rightarrow$ $\frac{x+5}{x+2} < 0$ $\Rightarrow -5 < x < -2$

But लेकिन $x < -3 \Rightarrow x \in (-5, -3)$ $\therefore$ $x \in (-5, -2) \cup (-1, \infty)$.

(iv) $|x^2 + 3x| + x^2 - 2 \geq 0$

case-I: स्थिति-I : यदि $x < -3$

$\Rightarrow$ $2x^2 + 3x - 2 \geq 0 \Rightarrow$ $(2x - 1)(x + 2) \geq 0$ $\Rightarrow$ $x \in (-\infty, -2) \cup \left[\frac{1}{2},\ \infty\right)$

But लेकिन $x < -3$ $\Rightarrow$ $x \in (-\infty, -3)$ ....(i)

case-II: स्थिति-II : यदि $-3 \leq x < 0$

$\Rightarrow$ $3x + 2 \leq 0$ $\Rightarrow$ $x \leq -\frac{2}{3}$

But लेकिन $-3 \leq x < 0$ $\Rightarrow$ $x \in \left[-3,\ \frac{-2}{3}\right]$ ....(ii) case-III: स्थिति-III : यदि $x \geq 0$

$\Rightarrow \quad 2x^2 + 3x - 2 \geq 0 \Rightarrow \quad x \in \left[\frac{1}{2}, \infty\right)$ ....(iii)

union of (i), (ii) and (iii) gives (i) ∪ (ii) ∪ (iii) से

$x \in \left(-\infty, -\frac{2}{3}\right) \cup \left[\frac{1}{2}, \infty\right)$

(v) $|x + 3| > |2x - 1| \quad \Rightarrow \quad x^2 + 9 + 6x > 4x^2 + 1 - 4x$

$\Rightarrow \quad 3x^2 - 10x - 8 < 0 \quad \Rightarrow \quad \left(x + \frac{2}{3}\right)(x - 4) < 0 \quad \Rightarrow \quad -\frac{2}{3} < x < 4$

B-3. Solve the following inequalities
निम्न असमिकाओं को हल कीजिए

(i) $|x^3 - 1| \geq 1 - x$ **Ans.** $x \in (-\infty, -1] \cup [0, \infty)$

(ii) $\left|x^2 - 4x + 4\right| \geq 1$ **Ans.** $x \in (-\infty, 1] \cup [3, \infty)$

(iii) $\frac{|x+2| - x}{x} < 2$ **Ans.** $x \in (-\infty, 0) \cup (1, \infty)$

(iv) $\frac{|x-2|}{x-2} > 0$ **Ans.** $x \in (2, \infty)$

(v) $|x - 2| > |2x - 3|$ **Ans.** (1, 5/3)

(vi) $|x + 2| + |x - 3| < |2x + 1|$ **Ans.** $(2, \infty)$

Sol. **(i)** $|x^3 - 1| \geq 1 - x \quad \Rightarrow \quad |(x - 1)|\,(x^2 + x + 1) \geq 1 - x$

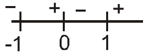
case स्थिति I $x \geq 1 \quad \Rightarrow \quad (x - 1)(x^2 + x + 1) + (x - 1) \geq 0$

$(x - 1)(x^2 + x + 2) \geq 0 \quad \Rightarrow \quad (x - 1) \geq 0$

$x \geq 1 \quad \Rightarrow \quad x \in [1, \infty)$

case स्थिति II $x < 1$

$[-(x - 1)(x^2 + x + 1)] + (x - 1) \geq 0 \quad \Rightarrow \quad -(x - 1)[x^2 + x + 1 - 1] \geq 0$



$(x - 1)(x^2 + x) \leq 0 \;= x(x - 1)(x + 1) \leq 0$

$x \in (-\infty, -1] \cup [0, 1)$

Taking Union of both the cases, we get $x \in (-\infty, -1] \cup [0, \infty)$ **Ans.**

दोनों स्थितियों का संघ लेने पर $x \in (-\infty, -1] \cup [0, \infty)$ **Ans.**

(ii) $|(x - 2)^2| \geq 1 \quad \Rightarrow \quad (x - 2)^2 \geq 1$

$(x - 2 + 1)(x - 2 - 1) \geq 0 \quad \Rightarrow \quad (x - 1)(x - 3) \geq 0$

$x \in (-\infty, 1] \cup [3, \infty)$ **Ans.**

(iii) $\frac{|x+2| - x}{x} < 2$

case स्थिति I $x \leq -2 \quad \Rightarrow \frac{-x - 2 - x}{x} - 2 < 0 \quad \Rightarrow \frac{-4x - 2}{x} < 0$

$\frac{4x + 2}{x} > 0 \quad \Rightarrow \quad \frac{2x + 1}{x} > 0$ i.e. $x \in \left(-\infty, -\frac{1}{2}\right) \cup (0, \infty)$

i.e. $x \in (-\infty, -2]$

case स्थिति II $x > -2$

$\frac{x + 2 - x}{x} < 2 \quad \Rightarrow \quad \frac{1}{x} - 1 < 0 \quad \Rightarrow \quad \frac{1 - x}{x} > 0$

i.e. $x \in (-\infty, 0) \cup (1, \infty)$ (Intersection with the given case सर्वनिष्ठ लेने पर)

i.e. $x \in (-2, 0) \cup (1, \infty)$

दोनों स्थितियों का संघ लेने पर $x \in (-\infty, 0) \cup (1, \infty)$ **Ans.**

(iv) $\frac{|x-2|}{x-2} > 0$

caseस्थिति I____ $x > 2$

caseस्थिति II $x < 2$

$x \in (2, \infty)$ $-1 > 0$ Not possibleसंभव नहीं

$x \in (2, \infty)$ **Ans.**

(v) Squaring वर्ग करने पर

$x^2 - 4x + 4 > 4x^2 - 12x + 9$

$3x^2 - 8x + 5 < 0 \Rightarrow (x-1)(3x-5) < 0 \Rightarrow x \in \left(1, \frac{5}{3}\right)$

(vi) **caseस्थिति I :** $x < -2$

$-x - 2 - x + 3 < -2x - 1 \Rightarrow 1 < -1$ Not possible संभव नहीं

caseस्थिति II : $-2 \le x < -\frac{1}{2}$

$x + 2 - x + 3 < -2x - 1 \Rightarrow 2x < -6 \Rightarrow x < -3$ Not possible संभव नहीं

caseस्थिति -III : $-\frac{1}{2} \le$ **x < 3**

$x + 2 - x + 3 < 2x + 1 \Rightarrow x > 2$

caseस्थिति -IV : $x \ge 3$

$x + 2 + x - 3 < 2x + 1 \Rightarrow 1 > -1$ Hence अतः $x \in (2, \infty)$

B-4. Solve the following equations **[16JM110006]**

निम्न समीकरणों को हल करने पर

(i) $|x^3 + x^2 + x + 1| = |x^3 + 1| + |x^2 + x|$ **Ans.** $\{-1\} \cup [0, \infty)$

(ii) $|x^2 - 4x + 3| + |x^2 - 6x + 8| = |2x - 5|$ **Ans.** $[1, 2] \cup [3, 4]$

(iii) $|x^2 + x + 2| - |x^2 - x + 1| = |2x + 1|$ **Ans.** $x \in \left[-\frac{1}{2}, \infty\right)$

(iv). $|x^2 - 2x - 8| + |x^2 + x - 2| = 3\,|x + 2|$ **Ans.** $[1, 4] \cup \{-2\}$

(v). $|2x - 3| + |x + 5| \le |x - 8|$ **Ans.** $\left[-5, \frac{3}{2}\right]$

Sol. (i) $|a + b| = |a| + |b| \Rightarrow ab \ge 0 \Rightarrow (x^2 + 1)(x^2 + x) \ge 0$

$\Rightarrow (x + 1)^2\, x(x^2 - x + 1) \ge 0 \Rightarrow x \in \{-1\} \cup [0, \infty)$

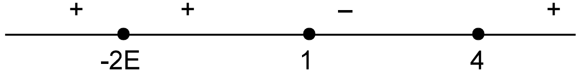
(ii) $|a| + |b| = |a - b| \Rightarrow ab \le 0 \Rightarrow (x-1)(x-2)(x-3)(x-4) \le 0$

$\Rightarrow x \in [1, 2] \cup [3, 4]$

(iii) $|a| - |b| = |a - b| \Rightarrow |a| \ge |b| \Rightarrow 2x + 1 \ge 0$ (as क्योंकि $a, b \ge 0$) $\Rightarrow x \in \left[-\frac{1}{2}, \infty\right)$

(iv) Since $(x^2 + x - 2) - (x^2 - 2x - 8) = 3x + 6 = 3(x+2)$ $\therefore$ $(x^2 - 2x - 8)(x^2 + x - 2) \le 0$

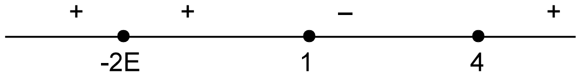
i.e. $(x - 4)(x + 2)(x + 2)(x - 1) \le 0$



$\therefore$ Solution set is $[1,4] \cup \{-2\}$

Hindi चूँकि $(x^2 + x - 2) - (x^2 - 2x - 8) = 3x + 6 = 3(x+2)$ $\therefore$ $(x^2 - 2x - 8)(x^2 + x - 2) \le 0$

अर्थात् $(x - 4)(x + 2)(x + 2)(x - 1) \le 0$



$\therefore$ हल समुच्चय $[1,4] \cup \{-2\}$ है।

(v) $| a | + | b | \le | a - b |$ i.e. $| a | + | -b | \le | a + (-b) |$

$| a | + | -b | \le | a + (-b) | \Rightarrow | a | + | -b | = | a + (-b) | \Rightarrow a (- b) \ge 0$ i.e. $ab \le 0$

$\therefore$ solution set is given by $(2x - 3) (x + 5) \le 0$

i.e. $- 5 \le x \le 3/2$.

Hindi $| a | + | b | \le | a - b |$ अर्थात् $| a | + | -b | \le | a + (-b) |$

$| a | + | -b | \le | a + (-b) | \Rightarrow | a | + | -b | = | a + (-b) | \Rightarrow a (- b) \ge 0$ अर्थात् $ab \le 0$

$\therefore$ हल समुच्चय $(2x - 3) (x + 5) \le 0$ द्वारा दिया जाता है।

अर्थात् $- 5 \le x \le 3/2$.

B-5. Find the solution set of the inequalities $|x^2 + x - 2| \le 0$ and $|x^2 - x + 2| \ge 0$ **[16JM110019]**

असमिकाओं $|x^2 + x - 2| \le 0$ और $|x^2 - x + 2| \ge 0$ का हल समुच्च्य ज्ञात कीजिए।

Ans. $\{-2, 1\}$

Sol. $|x^2 + x - 2| \le 0 \Rightarrow (x + 2)(x - 1) = 0 \Rightarrow x = -2, 1$

$|x^2 - x + 2| = 0 \Rightarrow x \in R$ Hence अत: $x \in \{-2, 1\}$

Section (C) : Miscellaneous Modulus Equations & Inequations

खण्ड (C) : विविध मापांकीय समीकरण एवं असमीकाऐं

C-1. Write the following expression in appropriate intervals so that they are bereft of modulus sign

नीचे दिये गये व्यंजकों को अन्तराल के रूप में लिखिये जो मापांक रहित हो

(i) $|\log_{10}x| + |2^{x-1} - 1|$ (ii) $|(\log_2x)^2 - 3(\log_2x) + 2|$ (iii) $| 5^{x^2-4x+5} - 25 |$

Ans+Sol. (i) $\log_{10}x + 2^{x-1} - 1$ $x \ge 1$

$-(\log_{10}x + 2^{x-1} - 1)$ $0 < x < 1$

(ii) $(\log_2x)^2 - 3(\log_2x) + 2$ $x \in (0, 2] \cup [4, \infty)$

$-((\log_2x)^2 - 3(\log_2x) + 2)$ $x \in (2, 4)$

(iii) $5^{x^2-4x+5} - 25$ $x \in (-\infty, 1] \cup [3, \infty)$

$25 - 5^{x^2-4x+5}$ $x \in (1, 3)$

C-2. Solve the equations $\log_{100} | x + y | = 1/2$, $\log_{10}y - \log_{10}|x| = \log_{100}4$ for x and y. **[16JM110007]**

समीकरणों $\log_{100} | x + y | = 1/2$, $\log_{10}y - \log_{10}|x| = \log_{100}4$ को हल करके x एवं y के मान ज्ञात कीजिए।

Ans. $x = 10/3, y = 20/3$ & $x = - 10, y = 20$

Sol. $\log_{100} |x + y| = \frac{1}{2}$

$|x + y| = 10$

this gives $x + y = 10$ .....(i)

or $x + y = - 10$ .....(ii)

$\log_{10} y - \log_{10} |x| = \log_{100}4 \Rightarrow \frac{y}{|x|} = 2$

for $x < 0$, we get $y = - 2x$ .....(ii)

for $x > 0$, we get $y = 2x$ .....(iv)

on solving (i) and (iii), we get $x = - 10, y = 20$ and

on solving (i) and (iv), we get $x = \frac{10}{3}, y = \frac{20}{3}$

Hindi $\log_{100} |x + y| = \frac{1}{2} \Rightarrow |x + y| = 10$

$\Rightarrow$ $x + y = 10$ .....(i)

या $x + y = - 10$ .....(ii)

एवं $\log_{10} y - \log_{10} |x| = \log_{100}4 \Rightarrow \frac{y}{|x|} = 2$

$x < 0$ के लिए $y = -2x$ .....(ii)

$x > 0$ के लिए $y = 2x$ .....(iv)

(i) और (iii), को हल करने पर $x = -10,\ y = 20$ और

(i) और (iv) को हल करने पर $x = \frac{10}{3},\ y = \frac{20}{3}$

C-3. Solve the inequality असमिका को हल कीजिए

(i) $(\log_2 x)^2 - |(\log_2 x) - 2| \geq 0$ **Ans.** $x \in \left(0, \frac{1}{4}\right] \cup [2, \infty)$

(ii) $2\,|\log_3 x| + \log_3 x \geq 3$ **Ans.** $\left(0, \frac{1}{27}\right] \cup [3, \infty)$

(iii).✎ Find the complete solution set of $2^x + 2^{|x|} \geq 2\sqrt{2}$

$2^x + 2^{|x|} \geq 2\sqrt{2}$ का सम्पूर्ण हल समुच्चय ज्ञात कीजिए।

Ans. $(-\infty, \log_2(\sqrt{2}-1)] \cup \left[\frac{1}{2}, \infty\right)$

Sol. **(i)** Let माना $\log_2 x = t$

$t^2 - |t - 2| \geq 0$

Caseस्थिति-I $t \geq 2$

$t^2 - t + 2 \geq 0 \Rightarrow t \in R$

Hence अतः $t \in [2, \infty)$

Caseस्थिति-II $t < 2$

$t^2 + t - 2 \geq 0 \Rightarrow (t + 2)(t - 1) \geq 0$

$\Rightarrow t \in (-\infty, -2] \cup [1, 2)$

Hence अतः $t \in (-\infty, -2] \cup [1, 2)$

From **Caseस्थिति-I** & **Caseस्थिति-II** $t \in (-\infty, -2] \cup [1, \infty)$

$\Rightarrow \log_2 x \in (-\infty, -2] \cup [1, \infty)$

$\Rightarrow x \in \left(0, \frac{1}{4}\right] \cup [2, \infty)$

(ii) Let $\log_3 x \geq 0 \Rightarrow x \geq 1$

Inequation become $\log_3 x \geq 1 \Rightarrow x \geq 3$

If $\log_3 x \leq 0 \Rightarrow x \in [0, 1]$

Inequation becomes $-\log_3 x \geq 3 \Rightarrow 0 < x \leq \frac{1}{27}$ so $x \in \left(0, \frac{1}{27}\right] \cup (3, \infty)$

माना $\log_3 x \geq 0 \Rightarrow x \geq 1$

असमिका $\log_3 x \geq 1 \Rightarrow x \geq 3$

यदि $\log_3 x \leq 0 \Rightarrow x \in [0, 1]$

असमिका $-\log_3 x \geq 3 \Rightarrow 0 < x \leq \frac{1}{27}$ इसलिए $x \in \left(0, \frac{1}{27}\right] \cup (3, \infty)$

(iii) Caseस्थिति-I : $x \geq 0 \Rightarrow 2^{x+1} \geq 2^{3/2} \Rightarrow x \geq \frac{1}{2}$

Caseस्थिति-II : $x \leq 0 \Rightarrow 2^x + 2^{-x} \geq 2^{3/2}$

Let माना $2^x = y \Rightarrow y \Rightarrow y^2 - 2\sqrt{2}\,y + 1 \geq 0$

$\Rightarrow y = \frac{2\sqrt{2} - 2}{2}$ or या $2^x \geq \sqrt{2} + 1$ (projected as $x < 0$)

$\Rightarrow x \leq \log_2(\sqrt{2} - 1)$

C-4. Find the number of real solution(s) of the equation $|x-3|^{3x^2-10x+3} = 1$

समीकरण $|x-3|^{3x^2-10x+3} = 1$ के वास्तविक हलों की संख्या है –

Ans. 3

Sol. $|x-3|^{3x^2-10x+3} = 1$

$|x-3| = 1$

$x-3 = 1$ & $x-3 = -1$

$x = 4$ & $x = 2$

$3x^2 - 10x + 3 = 0$

$3x^2 - 9x - x + 3 = 0$

$(3x-1)(x-3) = 0$

$\Rightarrow x = \frac{1}{3}, 3$

but $x \neq 3$

$\therefore$ three real solutions.

Hindi. $|x-3|^{3x^2-10x+3} = 1$

या तो $|x-3| = 1$

$\Rightarrow$ $x-3 = 1$ और $x-3 = -1$

$\Rightarrow$ $x = 4$ और $x = 2$

या $3x^2 - 10x + 3 = 0$

$\Rightarrow$ $3x^2 - 9x - x + 3 = 0$

$\Rightarrow$ $(3x-1)(x-3) = 0$

$\Rightarrow$ $x = \frac{1}{3}, 3$

लेकिन $x \neq 3$

अतः तीन वास्तविक हल होगें।

C-5. If x, y are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then find the value of $|x + y|$ **[16JM110021]**

यदि समीकरण $2x^2 - 3xy - 2y^2 = 7$ के पूर्णांक हल x, y हो, तो $|x + y|$ का मान है –

Ans. 4

Sol. $2x^2 - 4xy + xy - 2y^2 = 7$

$2x(x-2y) + y(x-2y) = 7$

$(x-2y)(2x+y) = 7$

x, y are integers $\Rightarrow$ $x - 2y$, $2x + y$ are also integers

Four cases are possible

Case I $x - 2y = 1, 2x + y = 7$ $\Rightarrow$ $x = 3, y = 1$

$|x + y| = 4$

Case II $x - 2y = 7, 2x + y = 1$ $\Rightarrow$ $x = \frac{9}{5}$ rejected

Case III $x - 2y = -1, 2x + y = -7$

$\Rightarrow$ $x = -3, y = -1$

$|x + y| = 4$

Case IV $x - 2y = -7, 2x + y = -1$ $\Rightarrow$ $x = -\frac{9}{5}$ rejected

Hence $|x + y| = 4$

Hindi $2x^2 - 4xy + xy - 2y^2 = 7$

$2x(x-2y) + y(x-2y) = 7$

$(x-2y)(2x+y) = 7$

x, y पूर्णांक हैं $\Rightarrow$ $x - 2y$, $2x + y$ भी पूर्णांक हैं

चार स्थितियाँ संभव है

स्थिति I $x - 2y = 1, 2x + y = 7$ $\Rightarrow$ $x = 3, y = 1$

$|x + y| = 4$

स्थिति II $x - 2y = 7, 2x + y = 1$ $\Rightarrow$ $x = \frac{9}{5}$ (निरस्त)

स्थिति III $x - 2y = -1, 2x + y = -7$

$\Rightarrow$ $x = -3, y = -1$

$|x + y| = 4$

स्थिति IV $x - 2y = -7,\ 2x + y = -1 \quad \Rightarrow \quad x = -\frac{9}{5}$ (निरस्त)

अतः $|x + y| = 4$

C-6. If x, |x + 1|, |x – 1| are three terms of an A.P., then find the number of possible values of x

Ans. 2

यदि x, |x + 1|, |x – 1| किसी समान्तर श्रेढ़ी के तीन पद हो, तो x के सम्भावित मानों की संख्या होगी–

Sol. since x , |x + 1|, |x – 1| are in A.P.

so $2\,|x + 1| = x + |x - 1|$ .... (i)

Case-I If x < – 1, then (i) becomes

$$-2\,(x + 1) = x - (x - 1) \quad \Rightarrow \quad x = -\frac{3}{2}$$

Case-II If $-1 \le x \le 1$, then (i) becomes

$$2\,(x + 1) = x - (x - 1) \quad \Rightarrow x = -1/2 \qquad \text{then series } -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$$

Case-III If $x \ge 1$, then (i) becomes

$2\,(x + 1) = x + x - 1$

$2 = -1$ impossible.

Hindi चूँकि x , |x + 1|, |x – 1| समान्तर श्रेढी में है।

so $2\,|x + 1| = x + |x - 1|$ .... (i)

Case-I यदि x < – 1, तो (i) होगा

$$-2\,(x + 1) = x - (x - 1) \quad \Rightarrow \quad x = -\frac{3}{2}$$

Case-II यदि $-1 \le x \le 1$, तब (i) होगा

$2\,(x + 1) = x - (x - 1) \quad \Rightarrow \quad x = -1/2$

तो श्रेढी है $-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

Case-III

यदि $x \ge 1$, तब (i) होगा

$2\,(x + 1) = x + x - 1$

$2 = -1$ संभव नहीं

Section (D) : Irrational Inequations

खण्ड (D) : अपरिमेय असमिकाऐं

D-1. Solve the following inequalities :

निम्नलिखित असमिकाओं को हल कीजिए –

(i) $\frac{\sqrt{2x-1}}{x-2} < 1$ **Ans.** $\left[\frac{1}{2}, 2\right) \cup (5, \infty)$

(ii) $x - \sqrt{1 - |x|} < 0$ **Ans.** $[-1, (\sqrt{5} - 1)/2)$

(iii) $\sqrt{x^2 - x - 6} < 2x - 3$ **Ans.** $x \in [3, \infty)$

(iv) $\sqrt{x^2 - 6x + 8} \le \sqrt{x+1}$ **Ans.** $x \in \left[\frac{7-\sqrt{21}}{2},\ 2\right] \cup \left[4,\ \frac{7+\sqrt{21}}{2}\right]$

(v)✍ $\sqrt{x^2 - 7x + 10} + 9\log_4\left(\frac{x}{8}\right) \ge 2x + \sqrt{14x - 20 - 2x^2} - 13$ **Ans.** $x = 2$

(vi) $x - 3 < \sqrt{x^2 + 4x - 5}$ **Ans.** $(-\infty, -5]\ U\ [1, \infty)$

(vii) $\sqrt{x^2 - 5x - 24} > x + 2$ **Ans.** $(-\infty, -3]$

(viii) $\sqrt{4-x^2} \geq \frac{1}{x}$ **Ans.** $[-2, 0) \cup [\sqrt{2-\sqrt{3}}, \sqrt{2+\sqrt{3}}]$

(ix) $\frac{\sqrt{x+7}}{x+1} > \sqrt{3-x}$ **Ans.** $(-1, 1) \cup (2, 3]$

Sol. (i) $\frac{\sqrt{2x-1}}{x-2} < 1$

Case-I **स्थिति-I :** $x - 2 < 0 \quad \Rightarrow \quad x < 2$ ....(i)

$2x - 1 > (x - 2)^2$

$x \in (-\infty, 1) \cup (5, \infty)$ ....(ii)

$x \in$ (i) $\cap$ (ii) $\quad x \in (-\infty, 1)$ ....(A)

Case-II स्थिति-II **:** $x - 2 > 0 \quad \Rightarrow \quad x > 2$ ....(iii)

$2x - 1 < (x - 2)^2$

$2x - 1 < x^2 - 4x + 4$

$x^2 - 6x + 5 > 0$

$\Rightarrow \quad x \in (-\infty, 1) \cup (5, \infty)$ ....(iv)

$x \in$ (iii) $\cap$ (iv)

$x \in (5, \infty)$ ....(B)

$x \in$ (A) $\cup$ (B)

$x \in (-\infty, 1) \cup (5, \infty)$

(ii) $x < \sqrt{1 - |x|}$

Case-I स्थिति-I **:** $x < 0$ ......(i)

$1 - |x| \geq 0 \quad \Rightarrow \quad 1 + x \geq 0 \quad \Rightarrow \quad x \geq -1$ ....(ii)

$x \in$ (i) $\cap$ (ii) $\quad \Rightarrow \quad x \in [-1, 0)$ ....(A)

Case-II स्थिति-II **:** $x \geq 0$ .....(i)

$1 - x \geq 0 \quad \Rightarrow \quad x \leq 1$ ....(ii)

$x^2 < 1 - x$

$\Rightarrow \quad x^2 + x < 1 \quad \Rightarrow \quad x^2 + x + \frac{1}{4} < \frac{5}{4}$

$\Rightarrow \quad \left(x+\frac{1}{2}\right)^2 < \frac{5}{4}$

$\frac{-1-\sqrt{5}}{2} < x < \frac{\sqrt{5}-1}{2}$ ......(iii)

$x \in$ (i) $\cap$ (ii) $\cap$ (iii) $\quad \Rightarrow \quad x \in \left[0, \frac{\sqrt{5}-1}{2}\right)$ ....(B)

$x \in$ (A) $\cup$ (B) $\quad \Rightarrow \quad x \in \left[-1, \frac{\sqrt{5}-1}{2}\right)$

(iii) $\sqrt{x^2-6x+8} \leq \sqrt{x+1}$

Domain प्रान्त $x + 1 \geq 0 \Rightarrow x \geq -1$

$x^2 - 6x + 8 \geq 0 \Rightarrow (x - 2)(x - 4) \geq 0$

$\Rightarrow x \leq 2$ or $x > 4$

$\Rightarrow$ Domain प्रान्त $\Rightarrow x \in [-1, 2] \cup [4, \infty)$

squaring वर्ग करने पर $x^2 - 6x + 8 \leq x + 1 \Rightarrow x^2 - 7x + 7 \leq 0$

$\left(x-\frac{7}{2}\right)^2 - \frac{21}{4} \leq 0 \Rightarrow x \in \left[\frac{7-\sqrt{21}}{2}, \frac{7+\sqrt{21}}{2}\right]$

Ans. $x \in \left[\frac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7+\sqrt{21}}{2}\right]$

(iv) $\sqrt{8+2x-x^2} > 6-3x$

(a) $8 + 2x - x^2 \geq 0 \quad \Rightarrow \quad x \in [-2, 4]$ .... (i)

case स्थिति - I

when जब (i) $6 - 3x \geq 0 \quad \Rightarrow \quad x \leq 2$ ... (ii)

so इसलिए $8 + 2x - x^2 > 36 + 9x^2 - 36x$

$\Rightarrow \quad 10x^2 - 38x + 28 < 0$

$\Rightarrow \quad 5x^2 - 19x + 14 < 0$

$\Rightarrow \quad (5x - 14)(x - 1) < 0$

$x \in \left(1, \frac{14}{5}\right)$ .... (iii)

by (1) and (2) and (3)

(1) तथा (2) तथा (3) की सहायता से

$x \in (1, 2]$

Case स्थिति -II

$6 - 3x < 0 \quad \Rightarrow \quad x > 2$

+ ve > –ve

so अतः $x > 2$ .... (iv)

by (1) and (4)

(1) तथा (4) की सहायता से

$x \in (2, 4]$

so by case (1) and (2) $x \in (1, 4]$

स्थिति (1) तथा (2) की सहायता से $x \in (1, 4]$

Hindi (iv) $\sqrt{8+2x-x^2} > 6-3x$

(a) $8 + 2x - x^2 \geq 0 \Rightarrow x \in [-2, 4]$ .... (i)

स्थिति-I

(i) यदि $6 - 3x \geq 0$ हो, तो $x \leq 2$ ... (ii)

अतः $8 + 2x - x^2 > 36 + 9x^2 - 36x$

$\Rightarrow \quad 10x^2 - 38x + 28 < 0$

$\Rightarrow \quad 5x^2 - 19x + 14 < 0$

$\Rightarrow \quad (5x - 14)(x - 1) < 0$

$\Rightarrow \quad x \in \left(1, \frac{14}{5}\right)$ .... (iii)

(i), (ii) एवं (iii) से–

$x \in (1, 2]$

स्थिति-II

$6 - 3x < 0 \quad \Rightarrow \quad x > 2$

धनात्मक संख्या > ऋणात्मक संख्या

अतः $x > 2$ .... (iv)

(i) व (iv) से–

$x \in (2, 4]$

अतः स्थिति-I एवं स्थिति-II से

$x \in (1, 4]$

(v) $x^2 - 7x + 10 \geq 0$ and $14x - 20 - 2x^2 \geq 0$

$(x - 2)(x - 5) \geq 0$ and $(x - 2)(x - 5) \leq 0$ ....(i)

so $x = 2$ or $x = 5$

now check for $x = 2$

$9 \log_4 \left(\frac{1}{4}\right) \geq -9$

$-9 \geq -9$

which is true hence $x = 2$ is a solution

now check $x = 5$

$$\frac{9}{2} \log \left(\frac{5}{8}\right) \geq -3$$

$$\log_2 \left(\frac{5}{8}\right) \geq -\frac{2}{3}$$

$(1.6)^3 \leq 4$

$4.096 \leq 4$

which is false

so only solution is $x = 2$

Hindi (v) $x^2 - 7x + 10 \geq 0$ एवं $14x - 20 - 2x^2 \geq 0$

$(x - 2)(x - 5) \geq 0$ एवं $(x - 2)(x - 5) \leq 0$ ......(i)

अतः $x = 2$ या $x = 5$

$x = 2$ के लिए जाँच करने पर

$$9 \log_4 \left(\frac{1}{4}\right) \geq -9$$

$-9 \geq -9$

जोकि सत्य है अतः $x = 2$ एक हल है।

अब $x = 5$ के लिए जाँच करने पर

$$\frac{9}{2} \log \left(\frac{5}{8}\right) \geq -3$$

$$\log_2 \left(\frac{5}{8}\right) \geq -\frac{2}{3}$$

$(1.6)^3 \leq 4$

$4.096 \leq 4$

जोकि असत्य है।

अतः केवल $x = 2$ हल है।

(vi) **Case I :**

If $x - 3 < 0$, then we have

$x^2 + 4x - 5 \geq 0 \Rightarrow x^2 + 5x - x - 5 \geq 0 \Rightarrow (x - 1)(x + 5) \geq 0$

$x \in (-\infty, -5] \cup [1, \infty) \quad \therefore \quad x \in (-\infty, -5] \cup [1, 3)$ ... (i)

Case II :

if $x - 3 \geq 0$, then we have

$$(x - 3)^2 < (x^2 + 4x - 5) \Rightarrow x^2 - 6x + 9 < x^2 + 4x - 5 \Rightarrow x > \frac{7}{5}$$

$\therefore \quad x \in [3, \infty)$ ... (ii)

$\therefore \quad$ (i) $\cup$ (ii) is

$x \in (-\infty, -5] \cup [1, \infty)$

Hindi. **स्थिति I :**

यदि $x - 3 < 0$ तब

$x^2 + 4x - 5 \geq 0 \Rightarrow x^2 + 5x - x - 5 \geq 0 \Rightarrow (x - 1)(x + 5) \geq 0$

$x \in (-\infty, -5] \cup [1, \infty) \quad \therefore \quad x \in (-\infty, -5] \cup [1, 3)$ ... (i)

स्थिति II :

यदि $x - 3 > 0$ तब

$$(x - 3)^2 < (x^2 + 4x - 5) \Rightarrow x^2 - 6x + 9 < x^2 + 4x - 5 \Rightarrow x > \frac{7}{5}$$

$\therefore \quad x \in [3, \infty)$ ... (ii)

$\therefore$ (i) $\cup$ (ii) से $x \in (-\infty, -5] \cup [1, \infty)$

(vii) $\sqrt{x^2 - 5x - 24} > x + 2$

$x^2 - 5x - 24 \geq 0 \Rightarrow$ $x \in (-\infty, -3] \in [8, \infty)$

Case-I : $x \in (-\infty, -3]$, LHS ≥ 0 & RHS < 0, hence inequality holds

Case-II : $x \in [8, \infty)$ squaring both sides

$x^2 - 5x - 24 > x^2 + 4x + 4 \quad \Rightarrow \quad x < -\frac{28}{9}$ (Not possible)

Hence $x\left(-\infty,-3\right]$

$\sqrt{x^2 - 5x - 24} > x + 2$

$x^2 - 5x - 24 \geq 0 \Rightarrow$ $x \in (-\infty, -3] \in [8, \infty)$

स्थिति -I : $x \in (-\infty, -3]$, LHS ≥ 0 तथा RHS < 0, अतः असमिका सन्तुष्ट होती है

स्थिति -II : $x \in [8, \infty)$ वर्ग करने पर

$x^2 - 5x - 24 > x^2 + 4x + 4 \quad \Rightarrow \quad x < -\frac{28}{9}$ (Not possibleसंभव नही)

Hence अतः $x \left(-\infty,-3\right]$

(viii) $\sqrt{4 - x^2} \quad \Rightarrow \quad -2 \leq x \leq 2$

Case I if $-2 \leq x < 0$, then $\sqrt{4-x^2} \geq \frac{1}{x}$ holds

$\therefore$ [–2, 0) are solutions

Case II If $0 < x \leq 2$, then

$$\sqrt{4-x^2} \geq \frac{1}{x} \quad \Rightarrow \quad 4 - x^2 \geq \frac{1}{x^2} \quad \Rightarrow \quad x^2 (4 - x^2) \geq 1$$

$$\Rightarrow \quad x^4 - 4x^2 + 1 \leq 0 \quad \Rightarrow \quad 2 - \sqrt{3} \leq x^2 \leq 2 + \sqrt{3}$$

$$\Rightarrow \quad \sqrt{2-\sqrt{3}} \leq x \leq \sqrt{2+\sqrt{3}}$$

HINDI $\sqrt{4 - x^2} \quad \Rightarrow \quad -2 \leq x \leq 2$

Case I यदि $-2 \leq x < 0$ हो, तो $\geq$ सत्य है

$\therefore$ [–2, 0) हल है

Case II यदि $0 < x \leq 2$ हो, तो

$$\sqrt{4-x^2} \geq \frac{1}{x} \quad \Rightarrow \quad 4 - x^2 \geq \frac{1}{x^2} \quad \Rightarrow \quad x^2 (4 - x^2) \geq 1$$

$$\Rightarrow \quad x^4 - 4x^2 + 1 \leq 0 \quad \Rightarrow \quad 2 - \sqrt{3} \leq x^2 \leq 2 + \sqrt{3}$$

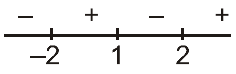
$$\Rightarrow \quad \sqrt{2-\sqrt{3}} \leq x \leq \sqrt{2+\sqrt{3}}$$

(ix) $$\frac{\sqrt{x+7}}{x+1} > \sqrt{3-x} \Rightarrow \left.\begin{array}{l} x+7 \geq 0 \quad \Rightarrow \quad x \geq -7 \\ 3-x \geq 0 \quad \Rightarrow \quad x \leq 3 \end{array}\right\} x \in [-7, \quad 3]$$

Caseस्थिति-I :

$x < -1$

No solution कोई हल नहीं



Caseस्थिति-II :

$x > -1$

$\Rightarrow \quad (x + 7) > (x + 1)^2 (3 - x)$

$\Rightarrow \quad (x + 7) + (x - 3)(x^2 + 2x + 1) > 0$

$\Rightarrow \quad x + 7 + x^3 + 2x^2 + x - 3x^2 - 6x - 3 > 0$

$\Rightarrow \quad x^3 - x^2 - 4x + 4 > 0$

$\Rightarrow \quad (x + 2) (x - 2) (x - 1) > 0$

Ans. $(-2, 1) \cup (2, \infty)$

C-II $[-7, 3] \Rightarrow (-1, 1) \cup (2, 3]$

D-2. Solve the equation $\sqrt{a(2^x - 2) + 1}$ = 1 – 2^x for every value of the parameter a. **[16JM110008]**

प्राचल a के प्रत्येक मान के लिए समीकरण $\sqrt{a(2^x - 2) + 1}$ = 1 – 2^x को हल कीजिए।

Ans. $x = \log_2 a$ where, $a \in (0, 1]$

Ans. $x = \log_2 a$ जहाँ , $a \in (0, 1]$

Sol. Let $y = 2^x$. Then, $\sqrt{a(2^x - 2) + 1} = 1 - 2^x$

$\Rightarrow \quad \sqrt{a(y-2)+1} = 1 - y \quad \Rightarrow \quad a(y-2) + 1 = (1-y)^2$

$\Rightarrow \quad y^2 - 2y = a(y-2) \quad \Rightarrow \quad (y-2)(y-a) = 0$

$\Rightarrow \quad y = 2$ or $y = a$

Now, $y = 2 \quad \Rightarrow \quad 2^x = 2 \quad \Rightarrow \quad x = 1$

But, x = 1 is not a solution of the given equation, because for x = 1, LHS = 1 and RHS = – 1

$y = a$

$\Rightarrow \quad 2^x = a \quad \Rightarrow \quad a > 0$ and $x = \log_2 a \qquad [\because 2^x > 0$ for all $x \in R]$

Putting $2^x = a$ in the given equation, we get $\sqrt{a(a-2)+1} = 1 - a$

$\Rightarrow \quad \sqrt{(a-1)^2} = 1 - a \quad \Rightarrow \quad |a - 1| = 1 - a \quad \Rightarrow \quad a - 1 \le 0$

$\Rightarrow \quad a \le 1$

Also, a > 0. Therefore, $a \in (0, 1]$

Hence, $x = \log_2 a$ is the solution of the given equation for all $a \in (0, 1]$. For $a \le 0$ and for $a > 1$, the equation has no solution

Hindi. माना $y = 2^x$. तो $\sqrt{a(2^x - 2) + 1} = 1 - 2^x$

$\Rightarrow \quad \sqrt{a(y-2)+1} = 1 - y \quad \Rightarrow \quad a(y-2) + 1 = (1-y)^2$

$\Rightarrow \quad y^2 - 2y = a(y-2) \quad \Rightarrow \quad (y-2)(y-a) = 0$

$\Rightarrow \quad y = 2$ या $y = a$

अब, $y = 2 \quad \Rightarrow \quad 2^x = 2 \quad \Rightarrow \quad x = 1$

लेकिन x = 1 दी गई समीकरण का एक हल नहीं हैं क्योंकि x = 1 के लिए LHS = 1 तथा RHS = – 1

$y = a$

$\Rightarrow \quad 2^x = a \quad \Rightarrow \quad a > 0$ और $x = \log_2 a \qquad [\because 2^x > 0 \;\; \forall \; x \in R]$

दी गई समीकरण में $2^x = a$ रखने पर हमें मिलता है $\sqrt{a(a-2)+1} = 1 - a$

$\Rightarrow \quad \sqrt{(a-1)^2} = 1 - a \quad \Rightarrow \quad |a - 1| = 1 - a \quad \Rightarrow \quad a - 1 \le 0$

$\Rightarrow \quad a \le 1$

साथ ही $a > 0$ इसलिए $a \in (0, 1]$

इस प्रकार $x = \log_2 a$ सभी $a \in (0, 1]$ के लिए दी गई समीकरण का एक हल है। $a \le 0$ तथा $a > 1$ के लिए समीकरण का कोई हल नहीं है।

Section (E) : Transformation of curves
खण्ड (E) : वक्रो का रूपान्तरण

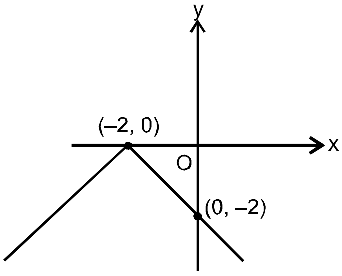
E-1. Draw the graph of followings — **[16JM110009]**

निम्नलिखित वक्रों के आलेख खींचिए–

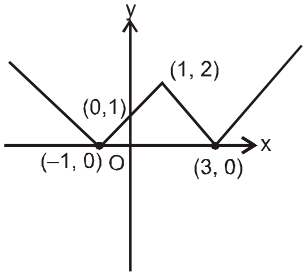
(i) $y = -|x + 2|$ (ii) $y = ||x - 1| - 2|$

(iii) $y = |x + 2| + |x - 3|$ (iv) $|y| + x = -1$

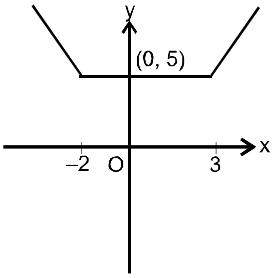
Ans (i)



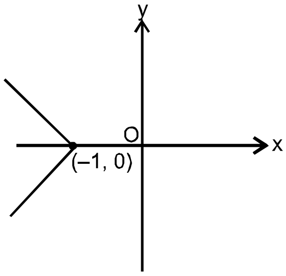
(ii)



(iii)



(iv)



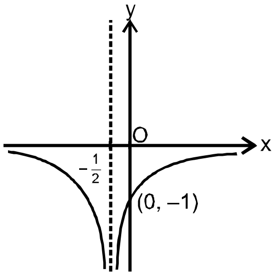
E-2. Draw the graphs of the following curves :
निम्नलिखित वक्रों के आलेख खींचिए–

(i) $y = -\dfrac{1}{|2x+1|}$

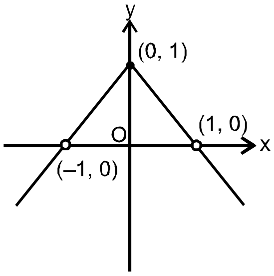
(ii) $\dfrac{y}{|x|-1} = -1$ (iii) $|y-3| = |x-1|$

(iv) $y = \dfrac{|x^2-1|}{(x^2-1)}\ell nx$

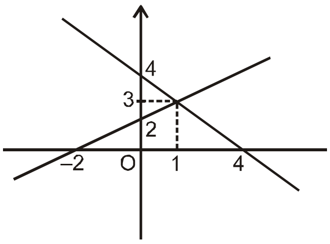
Ans. (i)



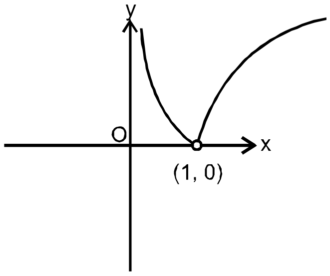
(ii)



(iii)

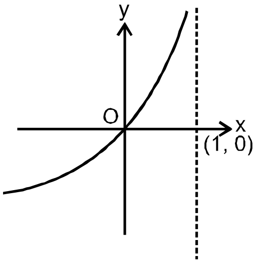


(iv)



E-3. Draw the graph of $y = \log_{1/2}(1-x)$. **[16JM110010]**
$y = \log_{1/2}(1-x)$ का आरेख खींचीएं।

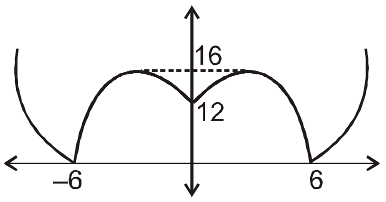
Ans.



E-4.✍ Find the set of values of λ for which the equation $|x^2 - 4|x| - 12| = \lambda$ has 6 distinct real roots.
λ का मानों का समुच्चय ज्ञात कीजिए जिसके लिए समीकरण $|x^2 - 4|x| - 12| = \lambda$ के 6 भिन्न–भिन्न वास्तविक मूल है।

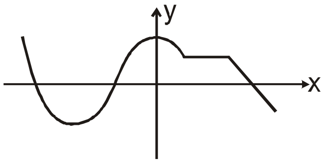
Ans. $\lambda \in (12, 16)$

Sol.



Hence for 6 distinct real roots $\lambda \in (12, 16)$
अतः 6 भिन्न–भिन्न वास्तविक मूलों के लिए $\lambda \in (12, 16)$

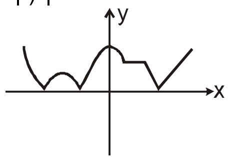
E-5. If y = f(x) has following graph
यदि y = f(x) का आलेख निम्नानुसार हो, –



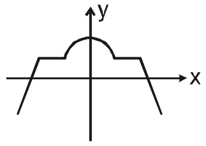
Then draw the graph of तब आरेख खीचिंए।

(i) $y = |f(x)|$
(ii) $y = f(|x|)$
(iii)✍ $y = f(-|x|)$
(iv) $y = |f(|x|)|$

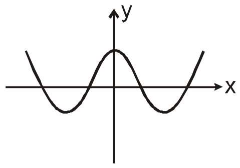
Ans. (i)



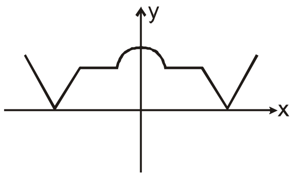
(ii)



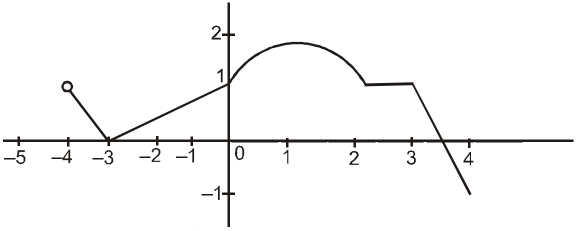
(iii)



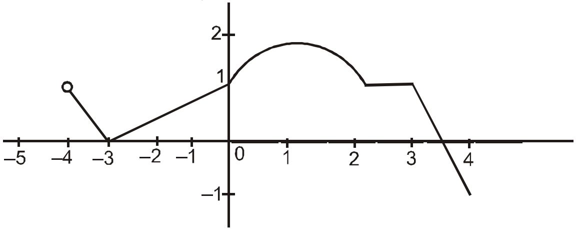
(iv)



E-6. If y = f(x) is shown in figure given below, then plots the graph for
(A) $y = f(|x + 2|)$ (B) $|y - 2| = f(-3x)$.

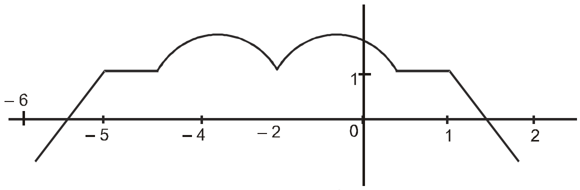


y = f(x) का आरेख निम्न चित्रानुसार हो,
(A) $y = f(|x + 2|)$ (B) $|y - 2| = f(-3x)$.

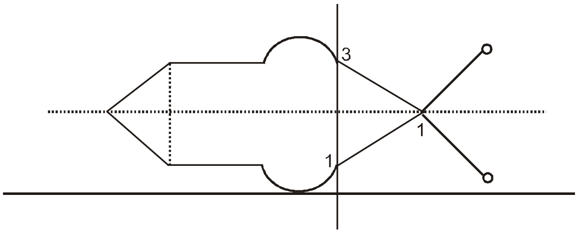


तो निम्नलिखित के आरेख बनाइये

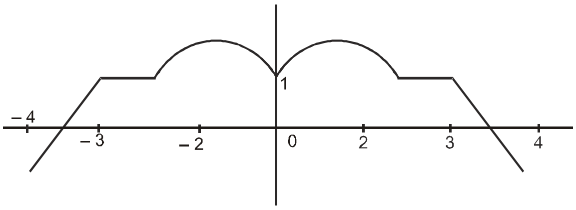
Ans. **(A)**



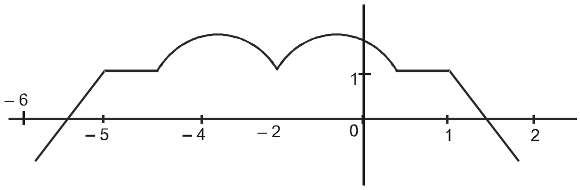
(B)



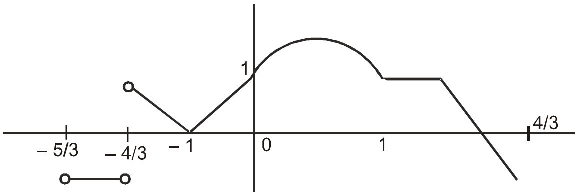
Sol. (A) $y = f(|x|)$



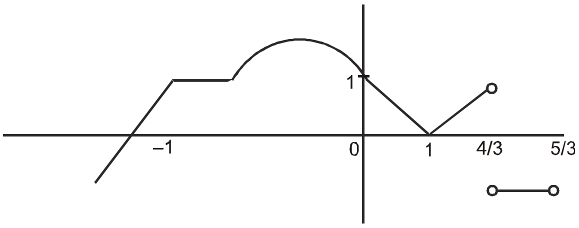
replace x by x + 2 we get
graph of
$y = f(|x + 2|)$



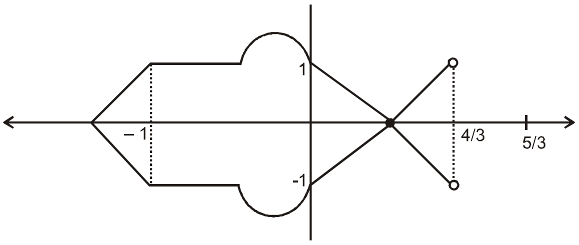
(B) $y = f(3x)$



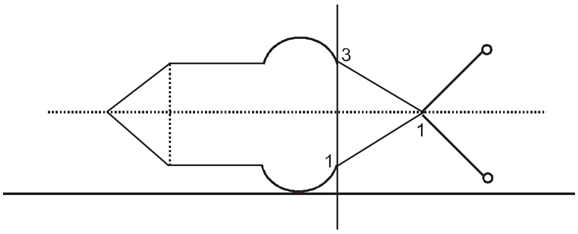
$y = f(– 3x)$



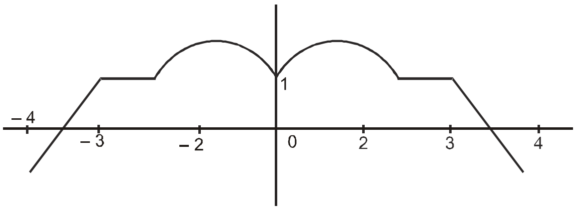
$|y| = f(– 3x)$



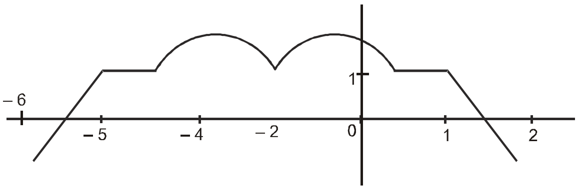
$|y – 2| = f(– 3x)$



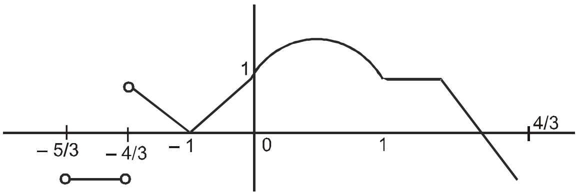
Hindi. (A) $y = f(|x|)$



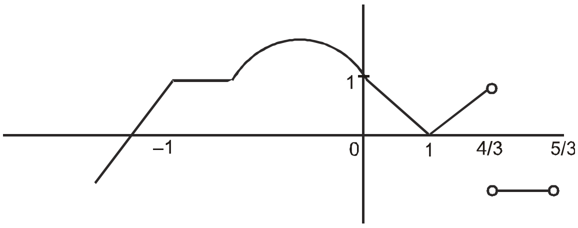
x को x + 2 से प्रतिस्थापित करने पर
$y = f(|x + 2|)$ का आरेख



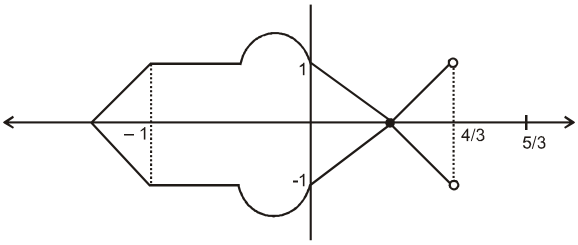
(B) $y = f(3x)$



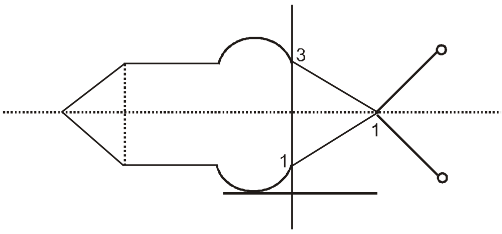
$y = f(– 3x)$



$|y| = f(– 3x)$



$|y – 2| = f(– 3x)$

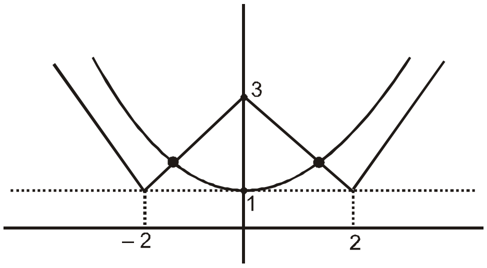


E-7. Find the number of roots of equation
समीकरणों के मूलों की संख्या ज्ञात कीजिए।

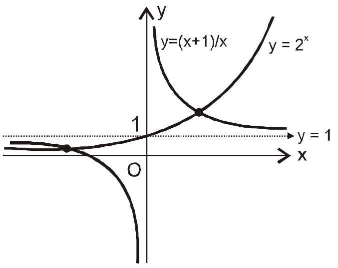
(i) $3^{|x|} - |2 - |x|| = 1$ Ans. 2 **[DRN1093]**

(ii) $x + 1 = x \cdot 2^x$ Ans. 2 **[16JM110025]**

Sol. **(i)** $3^{|x|} = 1 + ||x| - 2|$



(ii)



$1 + \ = 2^x$

Draw graphs of both sides. दोनों तरफ का आरेख

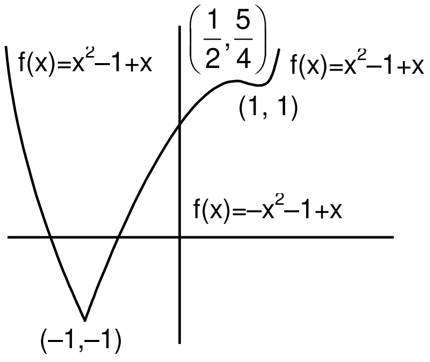
E-8 Find values of k for which the equation $|x^2 - 1| + x = k$ has
k का मान होगा जबकि समीकरण $|x^2 - 1| + x = k$ रखता है।

(i) 4 solution हल (ii) 3 solutions हल (iii) 1 solution हल (iv) 2 solutions हल

Ans. (i) $k \in \left(1, \frac{5}{4}\right)$ (ii) $k = 1, \frac{5}{4}$

(iii) $k = -1$ (iv) $k \in \left(\frac{5}{4}\infty\right) \cup (-1, 1)$

Sol. Graph of $f(x) = |x^2 - 1| + x$ is $f(x) = x^2 - 1 \ + x$
f(x) का आरेख $= |x^2 - 1| + x$, $f(x) = x^2 - 1 \ + x$



$$f(x) = \begin{cases} x^2 + x + 1 & x \in (-\infty, -1] \cup [1, \infty) \\ -x^2 + x + 1 & x \in [-1, 1] \end{cases}$$

Exercise-2

✍ **Marked questions are recommended for Revision.**

✍ **चिन्हित प्रश्न दोहराने योग्य प्रश्न है।**

* **Marked Questions may have more than one correct option.**

* **चिन्हित प्रश्न एक से अधिक सही विकल्प वाले प्रश्न है।**

1.✍ Number of integral values of 'x' satisfying the equation $3^{|x+1|} - 2.3^x = 2.|3^x - 1| + 1$ are

समीकरण $3^{|x+1|} - 2.3^x = 2.|3^x - 1| + 1$ को सन्तुष्ट करने वाले x के पूर्णांक मानों की संख्या है

(A) 1 (B*) 2 (C) 3 (D) 4

Sol. $3^{|x+1|} - 2.3^x = 2.|3^x - 1| + 1$

critical points are $x = -1,\ 3^x - 1 = 0 \Rightarrow x = 0$

Case-I : $x < -1$

$$3^{-(x+1)} - 2.3^x = -2(3^x - 1) + 1 \Rightarrow \frac{3^{-x}}{3} - 2.3^x = -2.3^x + 2 + 1$$

$$3^{-x} = 9 \Rightarrow x = -2$$

Case-II $-1 \le x < 0$

$$3^{(x+1)} - 2.3^x = -2(3^x - 1) + 1 \Rightarrow 3^x = 1 \Rightarrow x = 0$$

Not in solution.

Case-III : $x \ge 0$

$$3^{x+1} - 2.3^x = 2.3^x - 2 + 1 \Rightarrow x = 0$$

so x = 0, 2 satisfy the equation.

$3^{|x+1|} - 2.3^x = 2.|3^x - 1| + 1$

क्रांतिक बिन्दु $x = -1,\ 3^x - 1 = 0 \Rightarrow x = 0$

स्थिति-I : $x < -1$

$$3^{-(x+1)} - 2.3^x = -2(3^x - 1) + 1 \Rightarrow \frac{3^{-x}}{3} - 2.3^x = -2.3^x + 2 + 1$$

$$3^{-x} = 9 \Rightarrow x = -2$$

स्थिति-II $-1 \le x < 0$

$$3^{(x+1)} - 2.3^x = -2(3^x - 1) + 1 \Rightarrow 3^x = 1 \Rightarrow x = 0$$

Not in solution हल में नहीं

स्थिति **-III :** $x \ge 0$

$$3^{x+1} - 2.3^x = 2.3^x - 2 + 1 \Rightarrow x = 0$$

इसलि x = 0, 2 समीकरण को सन्तुष्ट करता है

2. $|x^2 + 6x + p| = x^2 + 6x + p\ \forall\ x \in R$ where p is a prime number then least possible value p is

$|x^2 + 6x + p| = x^2 + 6x + p\ \forall\ x \in R$ जहाँ p एक अभाज्य संख्या है तब p का न्यूनतम संम्भावित मान है–

(A) 7 (B*) 11 (C) 5 (D) 13 **[16JM110029]**

Sol. $|f(x)| = f(x)\ \forall x \Rightarrow f(x) \ge 0 \Rightarrow x^2 + 6x + p \ge 0\ \forall x \in p \Rightarrow D \le 0 \Rightarrow 4(9 - p) \le 0$

Hence अतः $p \ge 9 \Rightarrow p_{min} = 11$ (as p is prime) (चूंकि P अभाज्य है।)

3. If $(\log_{10}x)^2 - 4|\log_{10}x| + 3 = 0$, the product of roots of the equation is :

यदि $(\log_{10}x)^2 - 4|\log_{10}x| + 3 = 0$ हो, तो समीकरण के मूलों का गुणनफल है

(A) 3 (B) 10^4 (C) 10^8 (D*) 1

Sol. Let माना $|\log_{10}x| = y$

$$y^2 - 4y + 3 = 0 \Rightarrow y = 1, 3 \Rightarrow \log_{10}x = \pm1, \pm3 \Rightarrow x = 10, \frac{1}{10}, 10^3, \frac{1}{10^3}$$

Hence product अतः गुणनफल = 1

4.✍ The equaiton $||x - 1| + a| = 4$ can have real solutions for x if a belongs to the interval
समीकरण $||x - 1| + a| = 4$ का x के लिए वास्तविक हल हो सकता है जबकि a का अन्तराल है–**[16JM110030]**

(A*) $(-\infty, 4]$ (B) $(4, \infty)$ (C) $(-4, \infty)$ (D) $(-\infty, -4) \cup (4, \infty)$

Sol. $||x - 1| + a| = 4$
$|x - 1| = -a + 4, -a - 4$
For this equation to have solutions $-a + 4 \geq 0 \Rightarrow a \leq 4$
इस समीकरण के हल होने के लिए $-a + 4 \geq 0 \Rightarrow a \leq 4$

5. The number of values of x satisfying the equation $| 2x + 3 | + | 2x - 3 | = 4x + 6$, is
समीकरण $| 2x + 3 | + | 2x - 3 | = 4x + 6$ को सन्तुष्ट करने वाले x के मानों की संख्या है–

(A*) 1 (B) 2 (C) 3 (D) 4

Sol. $|2x + 3| + |2x - 3| = 4x + 6$

Caseस्थिति -I : $x \geq \frac{3}{2}$ $\Rightarrow$ $2x + 3 + 2x - 3 = 4x + 6 \Rightarrow$ No solution हल नहीं

Caseस्थिति -II : $-\frac{3}{2} \leq x \leq \frac{3}{2}$ $\Rightarrow$ $2x + 3 + 3 - 2x = 4x + 6 \Rightarrow$ $x = 0$

Caseस्थिति -III : $x \leq -\frac{3}{2}$ $\Rightarrow$ $-2x - 3 - 2x + 3 = 4x + 6 \Rightarrow$ No solution हल नहीं

6. Number of prime numbers satisfying the inequality $\log_3 \frac{|x^2 - 4x| + 3}{x^2 + |x - 5|} \geq 0$ is equal to **[16JM110031]**

असमिका $\log_3 \frac{|x^2 - 4x| + 3}{x^2 + |x - 5|} \geq 0$ को सन्तुष्ट करने वाली अभाज्य संख्याओं की संख्या है–

(A*) 1 (B) 2 (C) 3 (D) 4

Sol. $\log_3 \frac{|x^2 - 4x| + 3}{x^2 + |x - 5|} \geq 0$ $\Rightarrow$ $|x^2 - 4x| + 3 \geq x^2 + |x - 5|$

Caseस्थिति-I $x \geq 5$ $x^2 - 4x + 3 \geq x^2 + x - 5$ $\Rightarrow$ $-5x + 8 \geq 0$

$\Rightarrow$ $x \leq \frac{8}{5}$ (No solution हल नहीं)

Caseस्थिति-II $x \in (-\infty, 0] \cup [4, 5]$

$x^2 - 4x + 3 \geq x^2 + x - 5$ $\Rightarrow$ $-3x \geq 2$ $\Rightarrow$ $x \leq \frac{-2}{3}$

Caseस्थिति-III $x \in [0, 4]$

$4x - x^2 + 3 \geq x^2 - x + 5$ $\Rightarrow$ $2x^2 - 5x + 2 \leq 0 \Rightarrow$ $x \in \left[\frac{1}{2}, 2\right]$

7.✍ If $|x + 2| + y = 5$ and $x - |y| = 1$ then the value of $(x + y)$ is
यदि $|x + 2| + y = 5$ तथा $x - |y| = 1$ तब $x + y$ का मान ज्ञात कीजिए।

(A) 1 (B) 2 (C*) 3 (D) 4

Sol. $|x + 2| + y = 5$ for $x < -2$
we get $-x + y = 7$ ...(1)
& for $x \geq -2$
we get $x + y = 3$ ...(2)
$x - |y| = 1$ for $y < 0$
we get $x + y = 1$ ...(3)
& for $y \geq 0$
we get $x - y = 1$ ...(4)

solving (2) & (4)
$x = 2$ & $y = 1$

Hindi $|x + 2| + y = 5$
$x < -2$ के लिए
$-x + y = 7$ ...(1)
और $x \geq -2$ के लिए
$x + y = 3$ ...(2)
$x - |y| = 1$
$y < 0$ के लिए
$x + y = 1$ ...(3)
और $y \geq 0$ के लिए
$x - y = 1$ ...(4)
समीकरण (2) व (4) को हल करने पर
$x = 2$ तथा $y = 1$

8. The number of value of x satisfying the equation $|x - 1|^A = (x - 1)^7$, where $A = \log_3 x^2 - 2 \log_x 9$ समीकरण $|x - 1|^A = (x - 1)^7$, जहाँ $A = \log_3 x^2 - 2 \log_x 9$ को सन्तुष्ट करने वाले x के मानों की संख्या है–

[16JM110037]

(A) 1 (B*) 2 (C) 0 (D) 3

Sol. $|x - 1|^A = (x - 1)^7$
case (i) $x - 1 = 1 \Rightarrow x = 2$
case (ii) $\log_3 x^2 - 2 \log_x 9 = 7 \Rightarrow 2 \log_3 x - 4 \log_x 3 = 7$
let $\log_3 x = y$

$$\therefore \quad 2y - \frac{4}{y} = 7 \Rightarrow 2y^2 - 7y - 4 = 0 \Rightarrow (2y + 1)(y - 4) = 0$$

$$\Rightarrow \quad y = -\frac{1}{2} \quad \& \quad y = 4$$

$$\Rightarrow \quad \log_3 x = -\frac{1}{2} \qquad \Rightarrow \log_3 x = 4$$

$x = 3^{-1/2}$ $\Rightarrow x = 81$

$\frac{1}{\sqrt{3}} =$ for $x = \frac{1}{\sqrt{3}}$
$x - 1 < 0$
$\therefore$ not acceptable
$\therefore$ $x = 2$ or 81.

Hindi $|x - 1|^A = (x - 1)^7$
स्थिति (i) यदि $x - 1 = 1$ हो, तो $x = 2$
स्थिति (ii) यदि $\log_3 x^2 - 2 \log_x 9 = 7$ हो, तो
$2 \log_3 x - 4 \log_x 3 = 7$
माना $\log_3 x = y$

$$\therefore \quad 2y - \frac{4}{y} = 7 \Rightarrow 2y^2 - 7y - 4 = 0$$

$$(2y + 1)(y - 4) = 0 \Rightarrow y = -\frac{1}{2}, 4 \Rightarrow \log_3 x = -\frac{1}{2}, 4$$

$$x = 3^{-1/2}, 3^4 = \frac{1}{\sqrt{3}}, 81$$

$x = \frac{1}{\sqrt{3}}$ के लिये $x - 1 < 0$ $\therefore$ जो स्वीकार्य नहीं है। $\therefore$ $x = 2$ या 81

9. The number of integral value of x satisfying the equation $\left|\log_{\sqrt{3}} x - 2\right| - \left|\log_3 x - 2\right| = 2$

समीकरण $\left|\log_{\sqrt{3}} x - 2\right| - \left|\log_3 x - 2\right| = 2$ को सन्तुष्ट करने वाला x का पूर्णांक मानों की संख्या है–

(A*) 1 (B) 2 (C) 3 (D) 4

Sol. $\left|\log_{\sqrt{3}} x - 2\right| - \left|\log_3 x - 2\right| = 2 \Rightarrow |2\log_3 x - 2| - |\log_3 x - 2| = 2$

case I If $\log_3 x - 2 \geq 0 \Rightarrow \log_3 x \geq 2$

Then $2\log_3 x - 2 - \log_3 x + 2 = 2 \Rightarrow \log_3 x = 2$

$\therefore \quad \log_3 x = 2 \Rightarrow x = 3^2 = 9 \Rightarrow x = 9$

case II $1 \leq \log_3 x < 2$

$\therefore \quad 2\log_3 x - 2 + \log_3 x - 2 = 2 \Rightarrow 3\log_3 x = 6 \Rightarrow \log_3 x = 2$

which is not possible

case III If $\log_3 x < 1$ the $-2\log_3 x + 2 + \log_3 x - 2 = 2$

$-\log_3 x = 2 \Rightarrow \log_3 x = -2$

$\therefore \quad x = 3^{-2} = \frac{1}{9} \Rightarrow x = 9, \frac{1}{9}$

Hindi. $\left|\log_{\sqrt{3}} x - 2\right| - \left|\log_3 x - 2\right| = 2 \Rightarrow |2\log_3 x - 2| - |\log_3 x - 2| = 2$

स्थिति-I यदि $\log_3 x - 2 \geq 0 \Rightarrow \log_3 x \geq 2$

तब $2\log_3 x - 2 - \log_3 x + 2 = 2 \Rightarrow \log_3 x = 2$

$\therefore \quad \log_3 x = 2 \Rightarrow x = 3^2 = 9 \Rightarrow x = 9$

स्थिति-II $1 \leq \log_3 x < 2$

$\therefore \quad 2\log_3 x - 2 + \log_3 x - 2 = 2 \Rightarrow 3\log_3 x = 6 \Rightarrow \log_3 x = 2$

जो कि सम्भव नहीं है।

स्थिति-III यदि $\log_3 x < 1$

$-2\log_3 x + 2 + \log_3 x - 2 = 2$

$-\log_3 x = 2 \Rightarrow \log_3 x = -2$

$\therefore \quad x = 3^{-2} = \frac{1}{9} \Rightarrow x = 9, \frac{1}{9}$

10. The sum of all possible integral solutions of equation **[16JM110038]**

$||x^2 - 6x + 5| - |2x^2 - 3x + 1|| = 3|x^2 - 3x + 2|$ is

समीकरण $||x^2 - 6x + 5| - |2x^2 - 3x + 1|| = 3|x^2 - 3x + 2|$ के सभी संभावित पूर्णांक हलों का योगफल है–

(A) 10 (B) 12 (C) 13 (D*) 15

Sol. $||A| - |B|| = |A + B|$ iff $AB \leq 0$

$(x^2 - 6x + 5)(2x^2 - 3x + 1) \leq 0 \Rightarrow x \in \left[\frac{1}{2}, 5\right]$

Hindi $||A| - |B|| = |A + B|$ यदि और केवल यदि $AB \leq 0$

$(x^2 - 6x + 5)(2x^2 - 3x + 1) \leq 0 \Rightarrow x \in \left[\frac{1}{2}, 5\right]$

11. The complete solution set of the inequality $(|x - 1| - 3)(|x + 2| - 5) < 0$ is $(a, b) \cup (c, d)$ then the value of $|a| + |b| + |c| + |d|$ is

असमिका $(|x - 1| - 3)(|x + 2| - 5) < 0$ का सम्पूर्ण हल समुच्चय $(a, b) \cup (c, d)$ है, तब $|a| + |b| + |c| + |d|$ का मान है।

(A) 14 (B) 15 (C*) 16 (D) 17

Sol. $(|x - 1| - 3)(|x + 2| - 5) < 0$

Case-I स्थिति-I : $x \leq -2$

$(x + 2)(x + 7) < 0 \Rightarrow x \in (-7, -2)$ ....(i)

Case-II स्थिति-II : $-2 < x \leq 1$

$(x - 4)(-x - 2) < 0 \Rightarrow (x - 4)(x + 2) > 0$

$\Rightarrow \quad x \in (-\infty, -2) \cup (4, \infty)$

No solution कोई हल नहीं

Case-III स्थिति-III : $\quad x > 1$

$(x - 4)(x - 3) < 0$

$x \in (3, 4) \quad \quad ....(ii)$

$x \in (i) \cup (ii) \quad \Rightarrow \quad x \in (-7, -2) \cup (3, 4)$

12. The product of all the integers which do not belong to the solution set of the inequality $\left|\dfrac{3\,|x|\,-\,2}{|x|\,-1}\right| \geq 2$ is **[16JM110039]**

सभी पूर्णाकों जो असमिका $\left|\dfrac{3\,|x|\,-\,2}{|x|\,-1}\right| \geq 2$ के हल समुच्चय में स्थित नहीं है, का गुणनफल है–

(A*) –1 (B) –4 (C) 4 (D) 0

Sol. Let माना |x| = y

$\left|\dfrac{3y-2}{y-1}\right| \geq 2 \quad \Rightarrow \quad (3y-2)^2 \geq 4(y-1)^2 \ \& \ y \neq 1$

$\Rightarrow \quad 5y^2 - 4y \geq 0 \quad \Rightarrow \quad y \in (-\infty, 0] \cup \left[\dfrac{4}{5}, \infty\right)$

Hence अतः $|x| \in \{0\} \quad \cup \left[\dfrac{4}{5}, 1\right) \cup (1, \infty)$

So इसलिए $x \neq \pm 1$ if $x \in z \Rightarrow$ Product गुणनफल = –1

13. Let f(x) = |x – 2| and g(x) = |3 – x| and

A be the number of real solutions of the equation f(x) = g(x)

B be the minimum value of h(x) = f(x) + g(x)

C be the area of triangle formed by f(x) = |x – 2|, g(x) = |3 – x| and x-axis and $\alpha < \gamma < \beta < \delta$ where $\alpha < \beta$ are the roots of f(x) = 4 and $\gamma < \delta$ are the roots of g(x) = 4, then the value of sum of digits of $\dfrac{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}{ABC}$.

माना f(x) = |x – 2| और g(x) = |3 – x| तथा f(x) = g(x) के वास्तविक हलों की संख्या A हैं। h(x) = f(x) + g(x) का न्यूनतम मान B है। f(x) = |x – 2|, g(x) = |3 – x| तथा x-अक्ष से परिबद्ध त्रिभुज का क्षेत्रफल C हैं तथा $\alpha < \gamma < \beta < \delta$ जहाँ f(x) = 4 के मूल α एवं β इस प्रकार है कि $\alpha < \beta$ है। और g(x) = 4 के मूल γ एवं δ इस प्रकार है कि $\gamma < \delta$ है, तब $\dfrac{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}{ABC}$ के अंको के योग का मान ज्ञात कीजिए–

(A) 7 (B) 8 (C) 11 (D*) 9

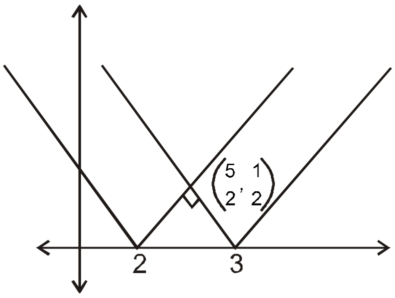
Sol. f(x) = |x – 2|, g(x) = |x – 3|

$f(x) = g(x) \quad \Rightarrow \quad |x - 2|^2 = |x - 3|^2 \quad \Rightarrow \quad x^2 - 4x + 4 = x^2 - 6x + 9$

$\Rightarrow \quad x = \dfrac{5}{2}$

Hence अतः A = 1

$h(x) = |x - 2| + |x - 3| \geq |(x - 2) - (x - 3)| = 1 \quad \Rightarrow \quad B = 1$



hence अत:C = $\frac{1}{4}$

$|x - 2| = 4 \quad \Rightarrow \quad x = 2 \pm 4 \quad \Rightarrow \quad \alpha = -2$ and तथा $\beta = 6$

$|x - 3| = 4 \quad \Rightarrow \quad x = 3 \pm 4 \quad \Rightarrow \quad \gamma = -1$ and तथा $\delta = 7$

$$\frac{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}{ABC} = 4(4 + 36 + 1 + 49) = 360$$

Sum of digits अंकों का योगफल = 9

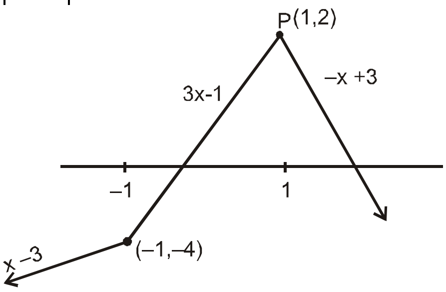
14*. If $f(x) = |x + 1| - 2\,|x - 1|$ then

(A*) maximum value of f(x) is 2.
(B*) there are two solutions of f(x) = 1.
(C*) there is one solution of f(x) = 2.
(D) there are two solutions of f(x) = 3.

यदि $f(x) = |x + 1| - 2\,|x - 1|$ हो, तो

(A*) f(x) का अधिकतम मान 2 है।
(B*) यहाँ f(x) = 1 के दो हल है।
(C*) यहाँ f(x) = 2 का एक हल है।
(D) यहाँ f(x) = 3 के दो हल है।

Sol. $f(x) = |x + 1| - 2\,|x - 1|$



(i) $x < -1$

$f(x) = -x - 1 + 2x - 2 = x - 3$

(ii) $-1 \le x \le 1$

$f(x) = x + 1 + 2x - 2 = 3x - 1$

(iii) $x > 1$

$f(x) = x + 1 - 2x + 2 = -x + 3$

15*. The solution set of inequality $|x| < \frac{a}{x}$, $a \in R$, is **[16JM110044]**

(A*) $\left(-\sqrt{-a}, 0\right)$ if $a < 0$
(B*) $\left(0, \sqrt{a}\right)$ if $a > 0$
(C*) ϕ if $a = 0$
(D) (0, a) if $a > 0$

असमिका $|x| < \frac{a}{x}$, $a \in R$ का हल समुच्चय है–

(A*) $\left(-\sqrt{-a}, 0\right)$ यदि $a < 0$
(B*) $\left(0, \sqrt{a}\right)$ यदि $a > 0$
(C*) ϕ यदि $a = 0$
(D) (0, a) यदि $a > 0$

Sol. $\underline{x < 0}$ $\quad$ $\underline{x > 0}$ $\quad$ $\underline{x \neq 0}$

$-x < \frac{a}{x}$ $\qquad x < \frac{a}{x}$ $\qquad \therefore x \in \phi$

$-x^2 > a$ $\qquad x^2 < a$

$x^2 < -a$ $\qquad x \in (-\sqrt{a}, \sqrt{a})$

$\therefore \quad x \in (-\sqrt{-a}, \sqrt{-a})$ $\qquad x \in (0, \sqrt{a})$

Let माना $\quad x < 0 \quad \Rightarrow x \in (-\sqrt{-a}, 0)$

16*. If a and b are the solutions of equation : $\log_5\left(\log_{64}|x| - \frac{1}{2} + 25^x\right) = 2x$, then

यदि समीकरण $\log_5\left(\log_{64}|x| - \frac{1}{2} + 25^x\right) = 2x$ के हल a तथा b है, तो –

(A*) $a + b = 0$ $\qquad$ (B*) $a^2 + b^2 = 128$ $\qquad$ (C) $ab = 64$ $\qquad$ (D) $a - b = 8$

Sol. $\log_5\left(\log_{64}|x| - \frac{1}{2} + 25^x\right) = 2x$

$\log_{64}|x| - \frac{1}{2} = 0 \Rightarrow \qquad |x| = (64)^{1/2}$

$\Rightarrow \qquad |x| = 8 \quad \Rightarrow \qquad x = \pm 8$

$a = 8, b = -8$

17. The number of solution of the equation $\log_3|x-1| \,.\, \log_4|x-1| \,.\, \log_5|x-1|$
$= \log_5|x-1| + \log_3|x-1| \,.\, \log_4|x-1|$ are

समीकरण $\log_3|x-1| \,.\, \log_4|x-1| \,.\, \log_5|x-1| = \log_5|x-1| + \log_3|x-1| \,.\, \log_4|x-1|$ के हलों की संख्या है–

(A) 3 $\qquad$ (B) 4 $\qquad$ (C) 5 $\qquad$ (D*) 6

Sol. Let $|x-1| = t$

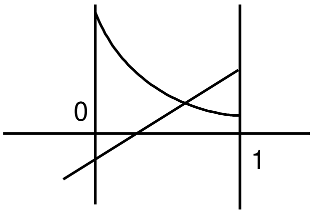
then $\log_3 t \log_4 t \; \log_5 t = \log_5 t + (\log_3 t \log_4 t)$

$$\Rightarrow \frac{1}{\log_t 3 \log_t 5} = \frac{1}{\log_t 5} + \frac{1}{\log_t 3 \log_t 4}$$

$\Rightarrow \log_t 5 + \log_t 3 \log_t 4 = 1$

$$\Rightarrow \qquad 4\log_t 3 = \frac{t}{5} \quad \Rightarrow 4^{\frac{\ell n^3}{\ell nt}} = \frac{t}{5}$$

$t \in (0,1)$



one solution between (0,1)

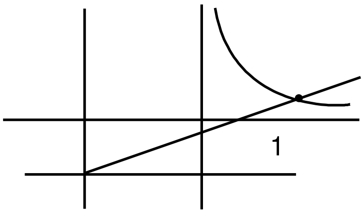
one solution is 1

$t > 1$

one solution is greater than 1

$\Rightarrow |x-1|$ has 3 positive sol.

$\Rightarrow$ x has 6 solution



Hindi. माना $|x-1| = t$

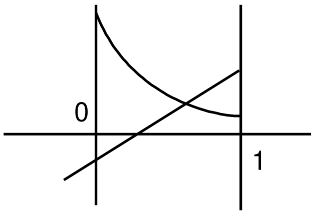
तब $\log_3 t \log_4 t \ \log_5 t = \log_5 t + (\log_3 t \log_4 t)$

$\Rightarrow$ $\frac{1}{\log_t 3 \log_t 5} = \frac{1}{\log_t 5} + \frac{1}{\log_t 3 \log_t 4}$

$\Rightarrow$ $\log_t 5 + \log_t 3 \log_t 4 = 1$

$\Rightarrow$ $4\log_t 3 = \frac{t}{5} \Rightarrow 4^{\frac{\ell n^3}{\ell nt}} = \frac{t}{5}$

$t \in (0,1)$



(0,1) मे एक हल

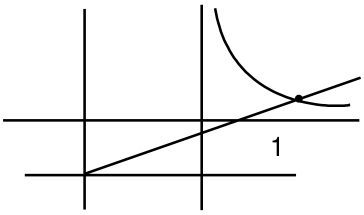
एक हल 1 है।

$t > 1$

1 से बड़ा एक हल

$\Rightarrow |x - 1|$ के तीन धनात्मक हल

$\Rightarrow$ x के 6 हल है।



18. Find the number of all the integral solutions of the inequality $\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \le 0$

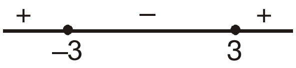
असमिका $\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \le 0$ के सभी पूर्णांक हलों की संख्या है–

(A) 1 (B*) 2 (C) 3 (D) 4

Sol. $x^2 - 16 \ge 0$

$\therefore$ $(x - 4)(x + 4) \ge 0$ $\therefore$ $x \in (-\infty, -4] \cup [4, \infty)$ ....(1)

Now अब $\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x-3)(x+3)} \le 0$



$x \in (-3, 3)$ ....(2)

By (1) and (2) $x \in \{-4, 4\}$

(1) व (2) से $x \in \{-4, 4\}$

19. Find the complete solution set of the inequality $\frac{1-\sqrt{21-4x-x^2}}{x+1} \ge 0$ **[16JM110032]**

असमिका $\frac{1-\sqrt{21-4x-x^2}}{x+1} \ge 0$ का सम्पूर्ण हल समुच्चय है–

(A) $\left[2\sqrt{6}-2,\ 3\right]$
(B) $\left[-2\ \ -2\sqrt{6},\ -1\right)$

(C) $\left[-2-2\sqrt{6}, -1\right] \cup \left[2\sqrt{6}-2,\ 3\right]$
(D*) $\left[-2\ \ -2\sqrt{6},\ -1\right) \cup \left[2\sqrt{6}-2,\ 3\right]$

Sol. For domain प्रान्त के लिए $21 - 4x - x^2 \ge 0$

$\Rightarrow \quad x^2 + 4x - 21 \le 0 \Rightarrow \quad (x + 7)(x - 3) \le 0 \quad \Rightarrow \quad x \in [-7, 3]$

case-I : (स्थिति I) **:** $-7 \le x < -1$ then तब $1-\sqrt{21-4x-x^2} - \le 0$

$\Rightarrow \quad 1 \le \sqrt{21-4x-x^2} \quad \Rightarrow \quad x^2 + 4x - 20 \le 0 \Rightarrow \quad (x + 2)^2 - 24 \le 0$

$\Rightarrow \quad (x + 2 + 2\sqrt{6})\,(x + 2 - 2\sqrt{6}) \le 0 \quad \Rightarrow \quad x \in [-2 - 2\sqrt{6}, 2\sqrt{6} - 2]$

$\therefore \quad x \in [-2 -2\sqrt{6}, -1)$

case-II : (स्थिति II) **:** $-1 < x \le 3$ then तब $1 \ge 21 - 4x - x^2$

$\Rightarrow \quad x^2 + 4x - 20 \ge 0 \Rightarrow \quad x \in (-\infty, -2 - 2\sqrt{6}\,] \cup [2\sqrt{6} - 2, \infty)$

$\therefore \quad x \in \left[2\sqrt{6}-2,\ 3\right]$

$x \in \left[-2\ \ -2\sqrt{6},\ -1\right) \cup \left[2\sqrt{6}-2,\ 3\right]$

20. The solution set of the inequality $\frac{|x+2|-|x|}{\sqrt{4-x^3}} \ge 0$ is

असमिका $\frac{|x+2|-|x|}{\sqrt{4-x^3}} \ge 0$ का हल समुच्चय है–

(A*) $[-1, \sqrt[3]{4})$
(B) $[1, \sqrt[3]{4})$
(C) $[-1, \sqrt[3]{2})$
(D) $[0, \sqrt[3]{4})$

Sol. $\frac{|x+2|-|x|}{\sqrt{4-x^3}} \ge 0 \Rightarrow 4 - x^3 > 0 \Rightarrow x^3 - 4 < 0 \Rightarrow x < 4^{1/3}$ ...(i)

$\therefore \ |x + 2| - |x| \ge 0$

case-I : स्थिति-**I :** $x \le -2$ then तब $-x - 2 + x \ge 0 \Rightarrow -2 \ge 0$ no solution कोई हल नहीं

case-II : स्थिति-**II :** $-2 < x \le 0$ then तब $x + 2 + x \ge 0 \Rightarrow \quad x + 1 \ge 0 \Rightarrow x \ge -1$

$\therefore \ x \in [-1, 0]$ ...(ii)

case-III : स्थिति-**III :** $x > 0$ then तब

$x + 2 - x \ge 0 \Rightarrow 2 \ge 0 \quad \therefore \ x \in R^+$ ...(iii)

$\therefore$ (i) $\cap$ (ii) $\cap$ (iii) is से $x \in [-1, 4^{1/3})$.

21. The number of integers satisfying the inequality $\sqrt{\log_{1/2}^2 x + 4\log_2\sqrt{x}} < \sqrt{2}\ (4 - \log_{16}x^4)$ are

असमिका $\sqrt{\log_{1/2}^2 x + 4\log_2\sqrt{x}} < \sqrt{2}\ (4 - \log_{16}x^4)$ को सन्तुष्ट करने वाले पूर्णाकों की संख्या है– **[16JM110040]**

(A) 2
(B*) 3
(C) 4
(D) 5

Sol. Domain प्रान्त $x > 0$

$\log^2{}_2x + 2\log_2x \geq 0$

$\log_2x\,(\log_2x + 2) \geq 0$

+ve -ve +ve

–2 0

$\log_2x \leq -2$ or या $\log_2x \geq 0$

$0 < x \leq \frac{1}{4}$ or या $x \geq 1$

$x \in \left(0, \frac{1}{4}\right] \cup [1, \infty)$ ....(i)

Case-I $4 - \log_2x < 0$

positive <negative (false)

Caseस्थिति -II $4 - \log_2x \geq 0 \Rightarrow \log_2x \leq 4$

$\Rightarrow \log_2{}^2x\ 2\log_2x < 2\,(4 - \log_2x)^2$

Letमाना $\log_2x = t$

$t^2 + 2t < 2\,(4 - t)^2$

$t^2 - 18t + 32 > 0$

$(t - 16)\,(t - 2) > 0$

$\Rightarrow t < 2 \cup t > 16$

$\log_2x < 2 \cup \log_2x > 16$ (Rejected)

$\log_2x < 2$

$x < 4$ ....(ii)

by (i) and (ii)

$$x \in \left(0, \frac{1}{4}\right] \cup [1, 4)$$

Hindi प्रान्त $x > 0$

$\log^2{}_2x + 2\log_2x \geq 0$

$\log_2x\,(\log_2x + 2) \geq 0$

+ve -ve +ve

–2 0

$\log_2x \leq -2$ या $\log_2x \geq 0$

$0 < x \leq \frac{1}{4}$ या $x \geq 1$

$x \in \left(0, \frac{1}{4}\right] \cup [1, \infty)$ ....(i)

स्थिति-I $4 - \log_2x < 0$

धनात्मक < ऋणात्मक (असत्य)

स्थिति-II $4 - \log_2x \geq 0 \Rightarrow \log_2x \leq 4$

$\Rightarrow \log_2{}^2x\ 2\log_2x < 2\,(4 - \log_2x)^2$

माना $\log_2x = t$

$t^2 + 2t < 2\,(4 - t)^2$

$t^2 - 18t + 32 > 0$

$(t - 16)\,(t - 2) > 0$

$\Rightarrow t < 2 \cup t > 16$

$\log_2x < 2 \cup \log_2x > 16$ (निरस्त)

$\log_2x < 2$

$x < 4$ ....(ii)

(i) एवं (ii) से

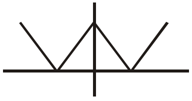
$$x \in \left(0, \frac{1}{4}\right] \cup [1, 4)$$

22. If $f_1(x) = ||x| - 2|$ and $f_n(x) = |f_{n-1}(x) - 2|$ for all $n \geq 2$, $n \in N$, then number of solution of the equation $f_{2015}(x) = 2$ is **[16JM110033]**

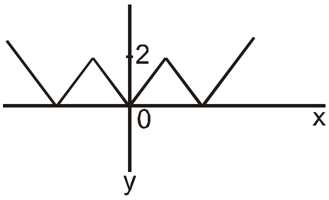
यदि $f_1(x) = ||x| - 2|$ और $f_n(x) = |f_{n-1}(x) - 2|$ सभी $n \geq 2$, $n \in N$, के लिए $f_{2015}(x) = 2$ समीकरण के हलों की संख्या है

(A) 2015 (B) 2016 (C*) 2017 (D) 2018

Sol. $f_1(x) = ||x| - 2|$



$f_1(x) = 2 \rightarrow 3$ solution हल

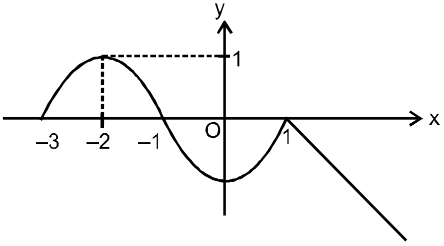


$f_2(x) = |||x| - 2| - 2|$ $f_2(x) = 2 \rightarrow 4$ solution हल

$f_n(x) = |f_{n-1}(x) - 2|$ के have n + 2 solution हल

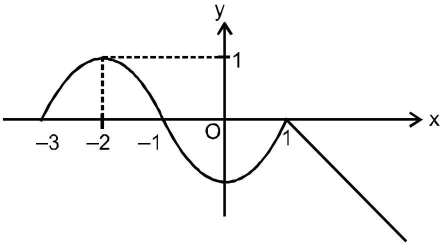
$f_{2015}(x) = 2$ के have 2017 solutions हल

23. If graph of y = f(x) in (–3,1), is as shown in the following figure



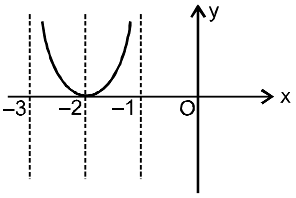
and g(x) = ℓn(f(x)), then the graph of y = g(– |x|) is

यदि y = f(x) का आलेख, f(x) अन्तराल (–3,1), निम्नलिखित चित्र द्वारा दर्शाया जाता है–

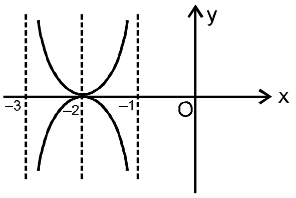


तथा g(x) = ℓn(f(x)) है, तो y = g(– |x|) का आलेख है।

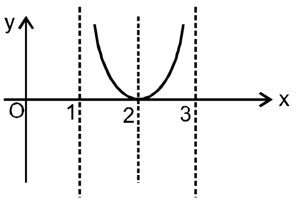
(A)



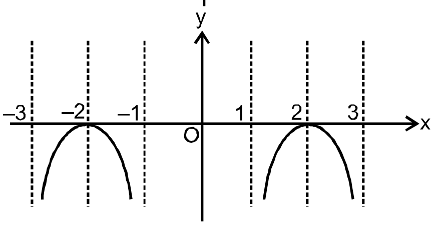
(B)



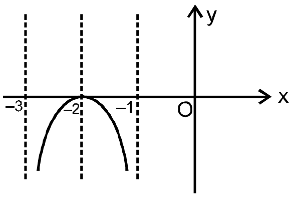
(C)



(D*)



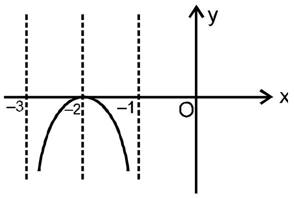
Sol. g(x) = ℓn(f(x)) domain is (–3, –1) range is (–∞, 0] graph of $y = g(x)$ is



$\therefore$ graph of $y = g(-|x|)$ is as shown in option (D)

Hindi. $g(x) = \ell n(f(x))$ प्रान्त $(-3, -1)$ है, परिसर $(-\infty, 0]$ है।

$y = g(x)$ का आलेख



है।

$\therefore$ $y = g(-|x|)$ का आलेख विकल्प (D) में दर्शाया गया है।

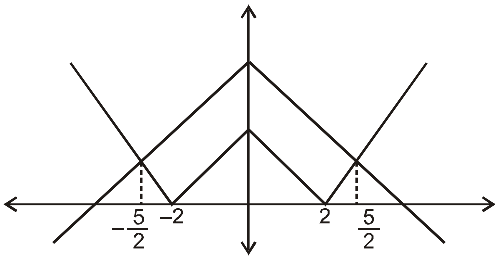
24*. Solution set of inequality $||x| - 2| \le 3 - |x|$ consists of : **[16JM110045]**

(A) exactly four integers
(B*) exactly five integers
(C) Two prime natural number
(D*) One prime natural number

असमिका $||x| - 2| \le 3 - |x|$ का हल समुच्चय रखता है

(A) ठीक चार पूर्णांक
(B*) ठीक पाँच पूर्णांक
(C) दो अभाज्य प्राकृत संख्या
(D*) एक अभाज्य प्राकृत संख्या

Sol.



Solution set is हल समुच्चय है $\left[-\frac{5}{2}, \frac{5}{2}\right]$

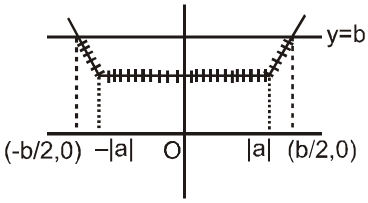
25*. If $a \ne 0$, then the inequation $|x - a| + |x + a| < b$ **[DRN1229]**

(A*) has no solutions if $b \le 2|a|$
(B*) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b > 2|a|$
(C) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b < 2|a|$
(D) All above

यदि $a \ne 0$, तो असमिका $|x - a| + |x + a| < b$ का

(A) कोई हल नहीं होगा यदि $b \le 2|a|$
(B) हल समुच्चय $\left(\frac{-b}{2}, \frac{b}{2}\right)$ होगा, यदि $b > 2|a|$
(C) हल समुच्चय $\left(\frac{-b}{2}, \frac{b}{2}\right)$ होगा यदि $b < 2|a|$
(D) उपरोक्त सभी

Sol.

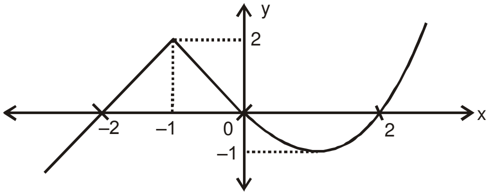


26. The equation $||x - a| - b| = c$ has four distinct real roots, then **[16JM110046]**
समीकरण $||x - a| - b| = c$ के चार भिन्न–भिन्न मूल है, तब

(A) $a > b - c > 0$
(B) $c > b > 0$
(C) $a > c + b > 0$
(D*) $b > c > 0$

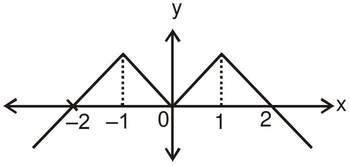
Sol. $||x - a| - b| = c \Rightarrow |x - a| = b + c, b - c$
for four solutions चार हल के लिए $b > c > 0$.

27*. If graph of $y = f(x)$ is as shown in figure

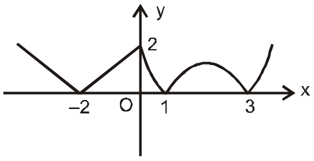


then which of the following options is/are correct ?

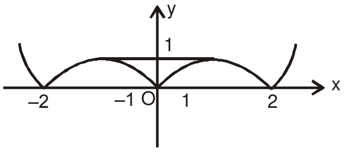
(A*) Graph of $y = f(-|x|)$ is



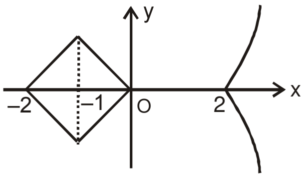
(B) Graph of $y = f(|x|)$ is



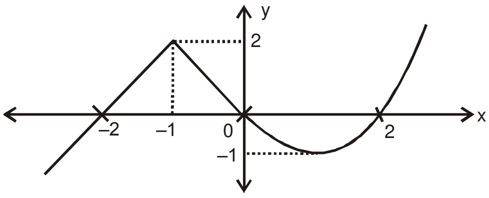
(C*) Graph of $y = |f(|x|)|$ is



(D*) Graph of $|y| = f(x)$ is

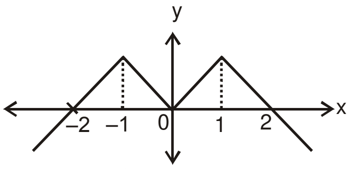


दिए गए चित्र में $y = f(x)$ का आरेख दर्शाया गया है।



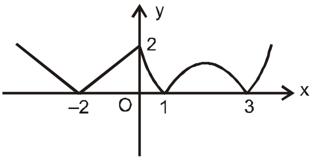
तब निम्न में से कौनसा/कौनसे विकल्प सही है?

(A*) $y = f(-|x|)$ का आरेख



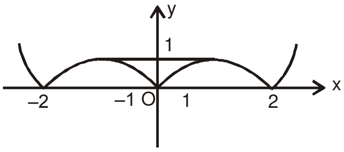
है।

(B) $y = f(|x|)$ का आरेख



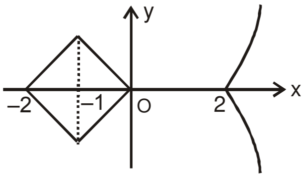
है।

(C*) $y = |f(|x|)|$ का आरेख



है।

(D*) $|y| = f(x)$ का आरेख



है।

Ans. (ACD)

Sol. Obviously (स्पष्ट है।) by graphical transformation. ग्राफीय रूपान्तरण से

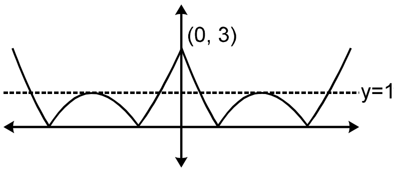
28*. Consider the equation $\left|x^2 - 4|x| + 3\right| = p$ **[16JM110047]**

(A*) for p = 2 the equation has four solutions
(B) for p = 2 the equation has eight solutions
(C*) there exists only one real value of p for which the equation has odd number of solutions
(D*) sum of roots of the equation is zero irrespective of value of p

माना कि समीकरण $\left|x^2 - 4|x| + 3\right| = p$

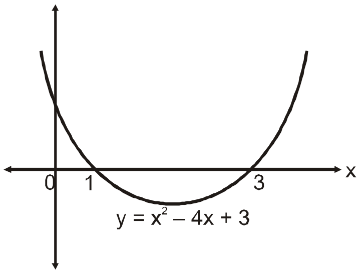
(A*) p = 2 के लिए समीकरण चार हल रखती है।
(B) p = 2 के लिए समीकरण आठ हल रखती है।
(C*) p का केवल एक वास्तविक मान है जिसके लिए समीकरण विषम संख्या में हल रखता है।
(D*) समीकरण के मूलों का योगफल शून्य है p के मान के संदर्भ में।

Ans.

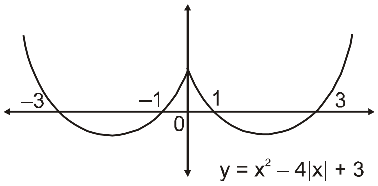


$a \in (1, 3) \cup \{0\}$

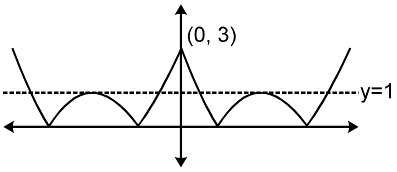
Sol.



$\Rightarrow$



$\Rightarrow$



Clearly for p = 2 there are four solutions. For p = 3 there three solutions and sum of solutions of the equations is zero.
स्पष्टतया p = 2 के लिए चार हल, p = 3 के लिए 3 हल अतः समीकरण के हलों का योगफल शून्य है

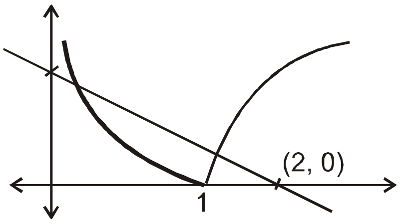
29*. Consider the equation $|\ell nx| + x = 2$, then

(A*) The equation has two solutions
(B*) Both solutions are positive
(C*) One root exceeds one and other in less than one
(D) Both roots exceed one

माना कि समीकरण $|\ell nx| + x = 2$, तब

(A*) समीकरण के दो हल है
(B*) दोनों हल धनात्मक है
(C*) एक मूल एक से बड़ा है और दूसरा मूल एक से छोटा है
(D) दोनों मूल एक से बड़े है

Sol.

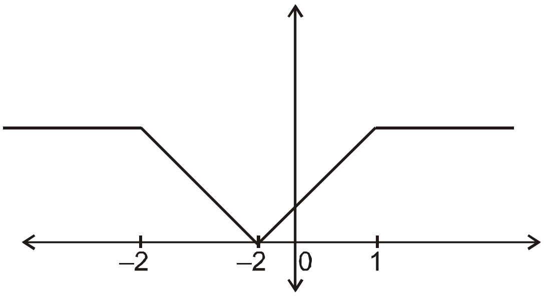


Two solutions one greater than unity and other less than unity.
दो हलों में एक इकाई से बड़ा और एक इकाई से छोटा

30*. Consider the equation $||x - 1| - |x + 2|| = p$. Let p_1 be the value of p for which the equation has exactly one solution. Also p_2 is the value of p for which the equation has infinite solution. Let α be the sum of all the integral values of p for which this equation has solution then
माना कि समीकरण $||x - 1| - |x + 2|| = p$ समीकरण के ठीक एक हल के लिए p का मान p_1 है तथा समीकरण के अनन्त हल के लिए p का मान p_2 है। माना समीकरण के हल होने के लिए p के सभी पूर्णांक मानों का योगफल α है। तब

(A*) $p_1 = 0$
(B*) $p_2 = 3$
(C*) $\alpha = 6$
(D) $p_1 + p_2 = 4$

Sol.



For one solution p = 0
For infinite solution p = 3
for solutions $p \in [0, 3]$, sum of integers = 0 + 1 + 2 + 3 = 6
एक हल के लिए p = 0
अनन्त हल के लिए p = 3
हल होने के लिए $p \in [0, 3]$, पूर्णांकों का योग = 0 + 1 + 2 + 3 = 6

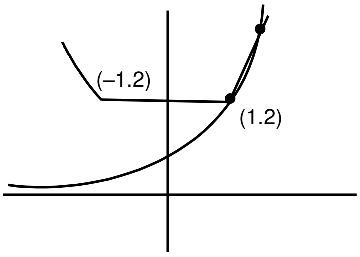
31. Number of the solution of the equation $2^x = |x - 1| + |x + 1|$ is

(A) 0
(B) 1
(C*) 2
(D) ∞

समीकरण $2^x = |x - 1| + |x + 1|$ के हलों की संख्या है –

(A) 0
(B) 1
(C*) 2
(D) ∞

Sol.



⇒ Two solutions

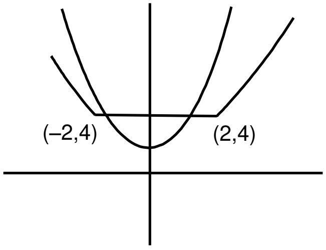
32. Number of the solution of the equation $x^2 = |x - 2| + |x + 2| - 1$ is

(A) 0 (B) 3 (C*) 2 (D) 4

समीकरण $x^2 = |x - 2| + |x + 2| - 1$ के हलों की संख्या है –

(A) 0 (B) 3 (C*) 2 (D) 4

Sol. $|x - 2| + |x + 2| = x^2 + 1$

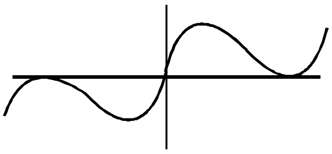


33. f(x) is polynomial of degree 5 with leading coefficient = 1, f(4) = 0. If the curve $y = |f(x)|$ and $y = f(|x|)$ are same, then the value of f(5) is

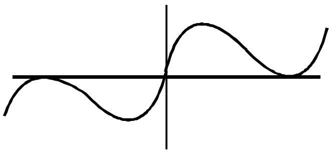
5 घात का बहुपदीय फलन f(x) जिसका अग्रग गुणांक 1 है। f(4) = 0. यदि वक्र $y = |f(x)|$ और $y = f(|x|)$ समान है तब f(5) का मान ज्ञात कीजिए।

(A*) 405 (B) –405 (C) 45 (D) –45

Sol. Because $|f(x)| \geq 0$ and graph of $f(|x|)$ and $|f(x)|$ is same so $f(|x|) \geq 0 \ \forall \ x \geq 0$
and 4 is repeated root of $f|x| = 0$. Hence – 4 is also repeated root
$f(x) = x (x - 4)^2 (x + 4)^2$
$f(5) = 405$



Hindi क्योंकि $|f(x)| \geq 0$ तथा $f(|x|)$ और $|f(x)|$ का आरेख समान है क्योंकि $f(|x|) \geq 0 \ \forall \ x \geq 0$
तथा 4, $f|x| = 0$ का पुनरावृत्ति मूल है अतः– 4 भी पुनरावृत्ति मूल है
$f(x) = x (x - 4)^2 (x + 4)^2$
$f(5) = 405$

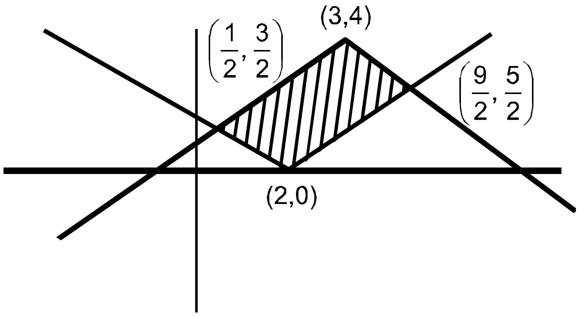


34. The area bounded by the curve $y \geq |x - 2|$ and $y \leq 4 - |x - 3|$ is

$y \geq |x - 2|$ और $y \leq 4 - |x - 3|$ से परिबद्ध क्षेत्र का क्षेत्रफल है –

(A) $\frac{13}{2}$ (B) 7 (C*) $\frac{15}{2}$ (D) 8

Sol. Area by (क्षेत्रफल) $y \geq |x - 1|$ and (और) $y \leq 4 - |x - 3|$



Area of rectange आयत का क्षेत्रफल = $\frac{5}{\sqrt{2}} \times \frac{3}{\sqrt{2}} = \frac{15}{2}$

Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

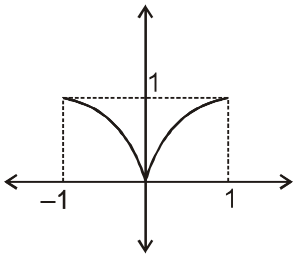
भाग - I : JEE (ADVANCED) / IIT-JEE (पिछले वर्षो) के प्रश्न

*** Marked Questions may have more than one correct option.**

*** चिन्हित प्रश्न एक से अधिक सही विकल्प वाले प्रश्न है -**

1. Draw the graph of $y = |x|^{1/2}$ for $-1 \le x \le 1$.
$-1 \le x \le 1$ के लिए $y = |x|^{1/2}$ का आरेख खीचिंए।

Sol.



2. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is :
समीकरण $|x|^2 - 3|x| + 2 = 0$ के वास्तविक हलों की संख्या है –
(A*) 4 (B) 1 (C) 3 (D) 2

Sol. $|x|^2 - 3|x| + 2 = 0 \Rightarrow |x| = 1, 2 \Rightarrow x = \pm1, \pm2$

3. If p, q, r are any real numbers, then
(A) max (p, q) < max (p, q, r)
(B*) min (p, q) = $\frac{1}{2}$ (p + q – |p – q|)
(C) max (p, q) < min(p, q, r)
(D) None of these

यदि p, q, r कोई वास्तविक संख्याएं है तब
(A) max (p, q) < max (p, q, r)
(B*) min (p, q) = $\frac{1}{2}$ (p + q – |p – q|)
(C) max (p, q) < min(p, q, r)
(D) None of theseइनमें से कोई नहीं

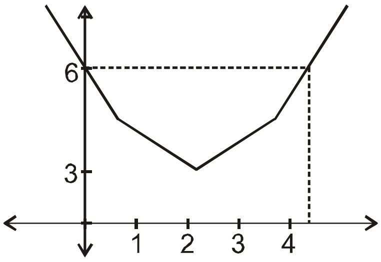
Sol. If यदि $p \ge q$ $\frac{(p+q)-|p-q|}{2} = \frac{p+q-p+q}{2} = q$

Hence correct option is (B) अतः सही विकल्प (B)

4. Let f(x) = |x – 1|. Then
माना f(x) = |x – 1|. तब
(A) $f(x^2) = (f(x))^2$
(B) f(x + y) = f(x) + f(y)

(C) f(|x|) = |f(x)| (D*) None of these इनमें से कोई नहीं

5. If x satisfies $|x-1| + |x-2| + |x-3| \geq 6$, then
यदि x, असमिका $|x-1| + |x-2| + |x-3| \geq 6$ को सन्तुष्ट करता है
(A) $0 \leq x \leq 4$ (B) $x \leq -2$ or या $x \geq 4$
(C*) $x \leq 0$ or या $x \geq 4$ (D) None of these इनमें से कोई नहीं



Hence correct option is (C)
अतः सही विकल्प (C)

6. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$.
$|x^2 + 4x + 3| + 2x + 5 = 0$ को हल कीजिए।

Ans. $x = -1 - \sqrt{3}$ or -4

Sol. $|x^2 + 4x + 3| + 2x + 5 = 0$

Caseस्थिति -I : $x \in (-\infty, -3] \cup [-1, \infty)$

$x^2 + 6x + 8 = 0 \Rightarrow x = -4, -2$ (rejected अस्वीकार्य)

Caseस्थिति -II : $x \in [-3, -1]$

$-x^2 - 2x + 2 = 0 \Rightarrow x^2 + 2x - 2 = 0 \Rightarrow x = -1 - \sqrt{3}, -1 + \sqrt{3}$ (rejectedअस्वीकार्य)

Hence अतः $x = -1 - \sqrt{3}$ or -4

7. If p, q, r are positive and are in A.P., then roots of the quadratic equation $px^2 + qx + r = 0$ are real for
(A*) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ (B) $\left|\frac{r}{p} - 7\right| < 4\sqrt{3}$
(C) all p and r (D) no p and r

यदि p, q, r धनात्मक और समान्तर श्रेणी में है तब द्विघात समीकरण $px^2 + qx + r = 0$ के मूल वास्तविक होने के लिए
(A*) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ (B) $\left|\frac{r}{p} - 7\right| < 4\sqrt{3}$
(C) सभी p और r (D) p और r के लिए नहीं

Sol. $2q = p + r$

moreover तथा $q^2 - 4pr \geq 0 \Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$

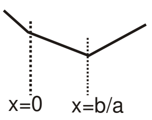
$\Rightarrow p^2 - 14pr + r^2 \geq 0 \Rightarrow \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 \geq 0$

$\Rightarrow \left(\frac{r}{p} - 7\right)^2 \geq 48 \Rightarrow \left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$

8. The function $f(x) = |ax - b| + c\,|x|\ \forall\ x \in (-\infty, \infty)$, where $a > 0, b > 0, c > 0$, assumes its minimum value only at one point if
फलन $f(x) = |ax - b| + c\,|x|\ \forall\ x \in (-\infty, \infty)$, जहाँ $a > 0, b > 0, c > 0$ का न्यूनतम मान केवल एक ही बिन्दु पर मिलेगा यदि

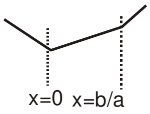
(A) $a \neq b$ (B*) $a \neq c$ (C) $b \neq c$ (D) $a = b = c$

Sol. $f(x) = \begin{cases} b-(a+c)x, & x<0 \\ b+(c-a)x, & 0 \le x < \frac{b}{a} \\ (a+c)x-b, & x \ge \frac{b}{a} \end{cases}$



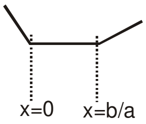
c<a

(i)



C>a

(ii)



C=a

(iii)

These figures clearly indicate that for exactly one point of minima, $a \neq c$
उपरोक्त चित्रों से स्पष्ट है कि केवल एक बिन्दु पर निम्निष्ठ होने के लिए $a \neq c$ होगा।

9.✍ Find the set of all solutions of the equation $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$
समीकरण $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$ के सभी हलों का समुच्चय ज्ञात कीजिए–

Ans. $\{-1\} \cup [1, \infty)$

Sol. $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$ .....(i)

(i) if $y \ge 1$, then equation (i) becomes
$2^y - (2^{y-1} - 1) = 2^{y-1} + 1$
$2^y = 2^y$ always true.

$\therefore$ $y \in [1, \infty)$

(ii) if $0 \le y < 1$, then equation (i) becomes
$2^y + 2^{y-1} = 2^{y-1} + 2$
$2^y = 2 \Rightarrow y = 1$ but $y \in [0, 1)$

$\therefore$ $y = 1$ is not acceptable

(iii) if $y < 0$, then equation (i) becomes
$2^{-y} + 2^{y-1} - 1 = 2^{y-1} + 1$
$2^{-y} = 2 \Rightarrow y = -1$ and $\because$ $y < 0$
$\therefore$ $y = -1$ acceptable $\therefore$ $y \in \{-1\} \cup [1, \infty)$

$2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$ .....(i)

(i) यदि $y \ge 1$, तब समीकरण (i) से
$2^y - (2^{y-1} - 1) = 2^{y-1} + 1$
$2^y = 2^y$ सदैव सत्य.

$\therefore$ $y \in [1, \infty)$

(ii) यदि $0 \le y < 1$, तब समीकरण (i) से
$2^y + 2^{y-1} = 2^{y-1} + 2$
$2^y = 2 \Rightarrow y = 1$ लेकिन $y \in [0, 1)$

$\therefore$ $y = 1$ स्वीकार्य नहीं है।

(iii) यदि $y < 0$, तब समीकरण (i) से
$2^{-y} + 2^{y-1} - 1 = 2^{y-1} + 1$
$2^{-y} = 2 \Rightarrow y = -1$ तथा $\because$ $y < 0$

$\therefore$ $y = -1$ स्वीकार्य

$\therefore$ $y \in \{-1\} \cup [1, \infty)$

10. The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is _______.

समीकरण $|x-2|^2 + |x-2| - 2 = 0$ के सभी वास्तविक मूलों का योग _______है।

Ans. 4

Sol. Let माना $|x - 2| = t$

$t^2 + t - 2 = 0$

$(t + 2)(t - 1) = 0$

$\Rightarrow$ $t = 1$ or या $t = -2$ not possible संभव नहीं $\Rightarrow$ $|x - 2| = 1$

$x - 2 = \pm 1$ $\Rightarrow$ $x = 3 ; x = 1$

Sum of roots मूलों का योग $= 3 + 1 = 4$

11. If α & β $(\alpha < \beta)$ are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then

यदि समीकरण $x^2 + bx + c = 0$ (जहाँ $c < 0 < b$) के मूल α एवं β $(\alpha < \beta)$ हो, तो–

(A) $0 < \alpha < \beta$

(B*) $\alpha < 0 < \beta < |\alpha|$

(C) $\alpha < \beta < 0$

(D) $\alpha < 0 < |\alpha| < \beta$

Sol. $x^2 + bx + c = 0$ $\Rightarrow$ α, β

$\because$ $\alpha + \beta = -b$

$\alpha\beta = c$

$\because$ Sum is +ve and product is – ve. मूलों का योग धनात्मक है और गुणनफलन ऋणात्मक है

$\therefore$ $\alpha < 0 < \alpha\beta < |\alpha|$

12. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min $f(x) >$ max $g(x)$, then the relation between b and c, is

(A) no relation

(B) $0 < c < b/2$

(C) $|c| < \sqrt{2}\,|b|$

(D*) $|c| > \sqrt{2}\,|b|$

यदि $f(x) = x^2 + 2bx + 2c^2$ एवं $g(x) = -x^2 - 2cx + b^2$ इस प्रकार है ताकि min $f(x) >$ max $g(x)$ हो, तो b, c, में सम्बन्ध है–

(A) कोई सम्बन्ध नहीं

(B) $0 < c < b/2$

(C) $|c| < \sqrt{2}\,|b|$

(D) $|c| > \sqrt{2}\,|b|$

Sol. min $f(x) >$ min $g(x)$

$-b^2 + 2c^2 > b^2 + c^2$

$c^2 > 2b^2 \Rightarrow$ $|c| > |b|$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

भाग - II : JEE (MAIN) / AIEEE (पिछले वर्षो) के प्रश्न

1. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$

(1) is always positive

(2) is always negative

(3*) does not exist

(4) none of these

समीकरण $t^2x^2 + |x| + 9 = 0$ के वास्तविक मूलों का गुणनफल है–

(1) सदैव धनात्मक हैं।

(2) सदैव ऋणात्मक है।

(3*) विद्यमान नहीं है।

(4) इनमें से कोई नहीं

Sol. Product of real roots $\frac{9}{t^2} > 0, \forall\ t \in R$

वास्तविक मूलों का गुणनफल $\frac{9}{t^2} > 0, \forall\ t \in R$

2. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is

(1) 3 (2) 2 (3*) 4 (4) 1

समीकरण $x^2 - 3|x| + 2 = 0$ के वास्तविक हलों की संख्या है–

(1) 3 (2) 2 (3*) 4 (4) 1

Sol. $x^2 - 3|x| + 2 = 0 \Rightarrow |x^2| - 3|x| + 2 = 0$

$(|x| - 2)(|x| - 1) = 0$

$|x| = 1, 2$ or $x = \pm 1, \pm 2$

$\therefore$ No. of solution = 4 हलों की संख्या = 4

3. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is :

समीकरण $x^2 + |2x - 3| - 4 = 0$ के मूलों का योग है–

(1) $-\sqrt{2}$ (2*) $\sqrt{2}$ (3) –2 (4) 2

Sol. Case-I : $x \geq \frac{3}{2}$ then $x^2 + 2x - 3 - 4 = 0 \Rightarrow x = -1 + 2\sqrt{2}$

Case-II : $x < \frac{3}{2}$ then $x^2 - 2x + 3 - 4 = 0 \Rightarrow x = 1 - \sqrt{2}$

$\Rightarrow$ sum of roots मूलों का योगफल $= \sqrt{2}$

4. The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real , has :

(1) exactly four solutions (2) exactly one solutions

(3) exactly two solutions (4*) no solution

समीकरण $\sqrt{3x^2 + x + 5} = x - 3$ जहाँ x वास्तविक है, रखता है–

(1) ठीक चार हल (2) ठीक एक हल

(3) ठीक दो हल (4*) कोई हल नहीं

Sol. $x \geq 3$ & $3x^2 + x + 5 = x^2 - 6x + 9$

$\Rightarrow x \geq 3$ & $2x^2 + 7x - 4 = 0 \quad \Rightarrow \quad x \in \phi$

5. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is :

फलन $f(x) = \frac{1}{\sqrt{|x| - x}}$ का प्रांत है :

(1) $(-\infty, \infty)$ (2) $(0, \infty)$ (3*) $(-\infty, 0)$ (4) $(-\infty, \infty) - \{0\}$

Sol. **(3)**

$f(x) = \frac{1}{\sqrt{|x| - x}}$ $\Rightarrow$ $|x| - x > 0$ $\Rightarrow$ $|x| > x$ $\Rightarrow$ $x < 0$

$\therefore$ $x \in (-\infty, 0)$ **Ans.**

6. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to

यदि x समीकरण $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$ का हल है तब $\sqrt{4x^2-1}$ बराबर है–

(1) 2 (2*) $\frac{3}{4}$ (3) $2\sqrt{2}$ (4) $\frac{1}{2}$

Sol. $\sqrt{2x+1} = 1 + \sqrt{2x-1}$

Squaring on both sides दोनों तरफ वर्ग करने पर

$2x + 1 = 1 + 2x - 1 + 2\sqrt{2x-1}$

$1 = 2\sqrt{2x-1}$

$1 = 4\sqrt{2x-1}$

x = 5/8

Now अब $\sqrt{4x^2-1}$ at x = 5/8 पर मान = $\sqrt{4 \times \frac{25}{64} - 1} = \frac{3}{4}$

7. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in the A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is :

माना α तथा β समीकरण $px^2 + qx + r = 0$, $p \neq 0$ के मूल है। यदि p, q ,r समान्तर श्रेढ़ी में हैं तथा $\frac{1}{\alpha} + \frac{1}{\beta} = 4$ है, तो $|\alpha - \beta|$ का मान है–

(1) $\frac{\sqrt{34}}{9}$ (2*) $\frac{2\sqrt{13}}{9}$ (3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

Sol. **Ans. (2)**

$px^2 + qx + r = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$; p, q, r $\rightarrow$ A.P. ; $2q = p + r$

$\frac{1}{\alpha} + \frac{1}{\beta} = 4$; $\frac{\alpha+\beta}{\alpha\beta} = 4$ $\Rightarrow$ $\frac{-q}{r} = 4$

$q = -4r$ ..... (i)

$\therefore$ $-8r = p + r$

$p = -9r$ ..... (ii)

$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{\frac{q^2}{p^2} - \frac{4r}{p}}$ by (i) और and (ii) से

$= \frac{\sqrt{q^2 - 4pr}}{|p|} = \frac{\sqrt{16r^2 + 36r^2}}{|-9r|} = \frac{2\sqrt{13}}{9}$

8. Let S = {$x \in R : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$}. Then S :

(1*) contains exactly two elements.

(2) contains exactly four elements.

(3) is an empty set.

(4) contains exactly one element

माना S = {$x \in R : x \geq 0$ तथा $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$} तो S :

(1*) में मात्र दो अवयव हैं।

(2) में मात्र चार अवयव हैं।

(3) एक रिक्त समुच्चय हैं।

(4) में मात्र एक ही अवयव हैं।

Sol. $2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0$

case स्थिति-i $\sqrt{x} \geq 3 \Rightarrow 2\sqrt{x} - 6 + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 4\sqrt{x} = 0$

$\Rightarrow \sqrt{x} = 4 \quad \Rightarrow \quad x = 16$

case स्थिति -ii $\sqrt{x} < 3 \Rightarrow -2\sqrt{x} + 6 + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \quad \Rightarrow \left(\sqrt{x} - 6\right)\left(\sqrt{x} - 2\right) = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$