7 Probability (Further Continued from Class IX)

KEY FACTS

1. Conditional Probability.

Suppose a red card is drawn from a pack of 52 cards, and is not put back, then the probability of drawing a red card in the first attempt is $\frac{26}{52}$ and in the second one it is $\frac{25}{51}$ as the red card is not replaced. Similarly in the above given case, if we draw a black card in the second attempt, then its probability = $\frac{26}{51}$ as number of black cards = 26 but total number of remaining cards = 51.

Hence the occurrence of the second event is fully dependent on the first event. Such events are called *conditional events*.

Definition: Let A and B be two events associated with a random experiment. Then, the probability of the occurrence of A under the condition that B has already occurred and $P(B) \neq 0$, is called the conditional **probability of *A* given *B*** and is written as $\boldsymbol{P(A/B)}$

How to evaluate P(A/B) or P(B/A)

If the event A occurs when B has already occurred, then $P(B) \neq 0$, then we may regard B as a new (reduced) sample space for event A. In that case, the outcomes favourable to the occurrence of event A are those outcomes which are favourable to B as well as favourable to A, *i.e*, the outcomes favourable to $A \cap B$ and probability of occurrence of A so obtained is the conditional probability of A under the condition that B has already occurred.

$$\therefore \quad P(A/B) = \frac{\text{Number of outcomes favourable to both } A \text{ and } B}{\text{Number of outcomes in sample space } (B, \text{ here})}$$

$$= \frac{n(A \cap B)}{n(B)} = \frac{\left\{\frac{n(A \cap B)}{n(S)}\right\}}{\left\{\frac{n(B)}{n(S)}\right\}} = \frac{P(A \cap B)}{P(B)},$$ where S is the sample space for the events A and B.

Similarly, $$P(B/A) = \frac{P(A \cap B)}{P(A)}, \ P(A) \neq 0$$

where $P(B/A)$ is the conditional probability of occurrence of B, knowing that A has already occurred.

Note: If A and B are mutually exclusive events, then,

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0 \quad \because P(A \cap B) = 0$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = 0 \quad \because P(A \cap B) = 0.$$

Ex. *Two coins are tossed. What is the conditional probability of two tails given that at least one coin shows a tail.*

Let A : Getting two tails, B : Getting at least one tail

Sample space $S = \{HH, HT, TH, TT\}$

$\Rightarrow \quad A = \{TT\}, B = \{HT, TH, TT\}. A \cap B = \{TT\}$

$P(A) = \frac{1}{4}, P(B) = \frac{3}{4}, (A \cap B) = \frac{1}{4}$

$\therefore$ Required probability $= P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = \mathbf{\frac{1}{3}}$.

2. Multiplication Theorem of Probability.

If A and B are two events in a random experiment such that $P(A) \neq 0$ and $P(B) \neq 0$, then the probability of the simultaneous occurrence of the events A and B *i.e.*, $P(A \cap B)$ is given by:

$$\mathbf{P(A \cap B) = P(A) \times P(B/A)} \text{ or } \mathbf{P(A \cap B) = P(B) \times P(A/B)}$$

(This follows directly from the formula given for conditional probability in Key Fact No. **1**)

Thus, the above given formulae hold true for dependent events.

Corollary 1: In case of **independent events,** occurrence of event B does not depend on the occurrence of A, hence $P(B/A) = P(B)$.

$$\therefore \quad \mathbf{P(A \cap B) = P(A) \times P(B)}$$

Thus, we can say **if $P(A \cap B) = P(A) \times P(B)$, then the events A and B are independent.**

Also, If A and B are two independent events associated with a random experiment having a sample space S, then

(a) $\overline{A}$ and B are also independent events. So,

$P(\overline{A} \cap B) = P(\overline{A}) \times P(B)$

(b) A and $\overline{B}$ are also independent events, so,

$P(A \cap \overline{B}) = P(A) \times P(\overline{B})$

(c) $\overline{A}$ and $\overline{B}$ are also independent events, so,

$P(\overline{A} \cap \overline{B}) = P(\overline{A}) \times P(\overline{B})$

Corollary 2: If $A_1, A_2, A_3, ..., A_n$ are n independent events associated with a random experiment, then

$P(A_1 \cap A_2 \cap A_3 ... \cap A_n) = P(A_1) \times P(A_2) \times P(A_3) ... \times P(A_n)$

Corollary 3: If $A_1, A_2, A_3, ..., A_n$ are n independent events associated with a random experiment, then

$P(A_1 \cup A_2 \cup A_3 .... \cup A_n) = 1 - P(\overline{A}_1) \times P(\overline{A}_2) \times P(\overline{A}_3) \times ... \times P(\overline{A}_n)$

Corollary 4: If the probability that an event will happen is p, the chance that **it will happen** in **any succession of *r* trials** is p^r. Also for **the *r* repeated non-occurrence** of the event we have the probability = $\mathbf{(1 - p)^r}$.

3. Law of Total Probability

Let $E_1, E_2, ..., E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ... or E_n, then

$P(A) = P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2) + ... + P(E_n) \times P(A/E_n)$.

Ex. ***There are two bags. One bag contains 4 white and 2 black balls. Second bag contains 5 white and 4 black balls. Two balls are transferred from first bag to second bag. Then one ball is taken from the second bag. Find the probability that it is white.***

Sol. There are three mutually exclusive and exhaustive ways in which 2 balls can be transferred from first bag to second bag and then a white ball be drawn from the second bag.

(i) **Two white** balls are transferred from first bag to second bag

(ii) **Two black balls** are transferred from first bag to second bag

(iii) **One white and one black** balls are transferred from first bag to second bag

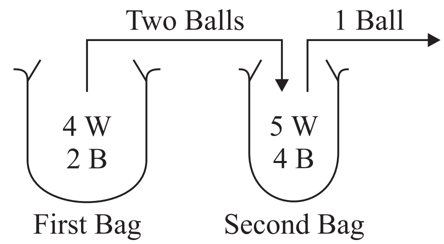
Let the events be described as under:

A : 2 white balls drawn from 1st bag, transferred to 2nd bag

B : 2 black balls drawn from 1st bag, transferred to 2nd bag

C : 1 white and 1 black ball drawn from 1st bag, transferred to 2nd bag

D : 1 white ball drawn from second bag.



$$\therefore \quad P(A) = \frac{^4C_2}{^6C_2} = \frac{4\times 3}{6\times 5} = \frac{6}{15}$$

$$P(B) = \frac{^2C_2}{^6C_2} = \frac{1}{15}$$

$$P(C) = \frac{^4C_1 \times {^2C_1}}{^6C_2} = \frac{4\times 2}{\frac{6\times 5}{2}} = \frac{8}{15}$$

Now **when A has occurred,** we have 7 white and 4 black balls in 2nd bag.

$\therefore$ P(Getting a white ball from 2nd bag) = $P(D/A) = \frac{7}{11}$

Similarly **when B has occurred,** we have 5 white and 6 black balls in 2nd bag

$\therefore$ P(Getting a white ball from 2nd bag) = $P(D/B) = \frac{5}{11}$

when **C has occurred,** we have 6 white and 5 black balls in 2nd bag,

$\therefore$ P(Getting a white ball from 2nd bag) = $P(D/C) = \frac{6}{11}$

$\therefore$ By law of total probability, ***P*(Ball drawn from 2nd bag is white)**

$$P(D) = P(A) \times P(D/A) + P(B) \times P(D/B) + P(C) \times P(D/C)$$

$$= \frac{6}{15}\times\frac{7}{11} + \frac{1}{15}\times\frac{5}{11} + \frac{8}{15}\times\frac{6}{11}$$

$$= \frac{42}{165} + \frac{5}{165} + \frac{48}{165} = \frac{95}{165} = \mathbf{\frac{19}{33}}.$$

4. Some Useful Facts and Formulae

1. *If $P_1, P_2, P_3, ..., P_n$ are the respective probabilities of the happening of certain n independent events,* then the **probability of the failure of all these events** is given by:

$$\mathbf{P = (1 - P_1)\,(1 - P_2)\,...(1 - P_n)}$$

2. Probability of the occurrence of at least one of the *n* independent events of a random experiment.

If $P_1, P_2, P_3, ..., P_n$ are the probabilities of the happening of 'n' independent events, then the **(probability that at least one of the events must happen)**

= 1 – Probability of failure of all events

$= 1 - (1 - P_1)\,(1 - P_2)\,(1 - P_3) \ldots (1 - P_n)$

Ex. ***A problem in mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the problem is solved?*** **(*AIEEE 2002, NDA 2002, SCRA 2002*)**

Sol. Let the respective events of solving the problem be denoted by A, B, C. Then

$$P(A) = \frac{1}{2},\ P(B) = \frac{1}{3},\ P(C) = \frac{1}{4}.$$

Clearly A, B, C are independent events and the problem will be considered to have been solved if at least one student solves it.

$\therefore$ Required probability = $P(A \text{ or } B \text{ or } C) = P(A \cup B \cup C) = 1 - P(\overline{A})\, P(\overline{B})\, P(\overline{C})$

$\overline{A}, \overline{B}, \overline{C}$ are the respective events of not solving the problem.

Also, $P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$, $P(\overline{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$, $P(\overline{C}) = 1 - P(C) = 1 - \frac{1}{4} = \frac{3}{4}$.

$\therefore$ Required probability $= 1 - P(\overline{A})\, P(\overline{B})\, P(\overline{C}) = 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \mathbf{\frac{3}{4}}$.

5. Baye's Theorem: Let $E_1, E_2, E_3, \ldots, E_n$ be n mutually exclusive events associated with a random experiment. If A is an event which occurs as a result of the events (cases) $E_1, E_2, E_3, \ldots, E_n$ then

$P\left(\frac{Ei}{A}\right)$ = Probability of occurrence of event A as a result of a particular cause (event) E_i

$$= \frac{P(E_i) \, . \, P(A)}{\sum_{i=1}^{n} P(E_i) \, . \, P(A / E_i)} \quad i = 1, 2, \ldots, n$$

Ex. ***Three boxes contain 6 white, 4 blue; 5 white, 5 blue and 4 white, 6 blue balls respectively. One of the box is selected at random and a ball is drawn from it. If the ball drawn is blue, find the probability that it is from the second box.***

Sol. Let A, B, C, D be the events defined as:

A : Selecting first box

B : Selecting second box

C : Selecting third box

D : Event of drawing a blue ball.

Since there are three boxes and each box has an equally likely chance of selection, $P(A) = P(B) = P(C) = \frac{1}{3}$

- If first box is chosen, *i.e.*, A has already occurred, then

  Probability of drawing a blue ball from $A = \frac{4}{10} \Rightarrow P(D/A) = \frac{4}{10}$

- If second box is chosen, *i.e.*, B has already occurred, then

  Probability of drawing a blue ball from $B = \frac{5}{10} \Rightarrow P(D/B) = \frac{5}{10}$

  Similarly $P(D/C) = \frac{6}{10}$

Now we are required to find the probability (B/D), *i.e.*, given that the ball drawn is blue, we need to find the probability that it is drawn from second box.

By Baye's Theorem,

$$P(B/D) = \frac{P(B) \times P(D/B)}{P(A) \times P(D/A) + P(B) \times P(D/B) + P(C) \times P(D/C)}$$

$$= \frac{\frac{1}{3} \times \frac{5}{10}}{\frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10} + \frac{1}{3} \times \frac{6}{10}} = \frac{\frac{1}{3} \times \frac{5}{10}}{\frac{1}{3} \times \frac{15}{10}} = \mathbf{\frac{1}{3}}.$$

SOLVED EXAMPLES

Ex. 1. Two coins are tossed. Find the conditional probability of getting two heads given that at least one coin shows a head .

Sol. Let A : Getting two heads

B : At least one coin showing a head.

$S = \{HH, HT, TH, HH\}$

Then, $A = \{HH\}$, $B = \{HT, TH, HH\} \Rightarrow A \cap B = \{HH\}$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}, P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}, P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

Now, Required probability $= P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/4}{3/4} = \mathbf{\dfrac{1}{3}}.$

Ex. 2. Two integers are selected at random from integers 1 through 11. If the sum is even, find the probability that both numbers are odd.

Sol. The integers from 1 through 11 are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. Out of these, there are 5 even and 6 odd integers.

Let ***A*** : Both numbers chosen are odd

B : Sum of numbers is even at random

S : Choosing 2 numbers from 11 numbers.

Then, $n(S) = {}^{11}C_2$

$n(A) = {}^{6}C_2$ ($\because$ There are 6 odd integers)

As the sum of both chosen integers can be even if both are even or both are odd, so

$n(B) = {}^{6}C_2 + {}^{5}C_2$

and $n(A \cap B) = {}^{6}C_2$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{{}^{6}C_2}{{}^{11}C_2} = \frac{6 \times 5}{11 \times 10} = \frac{3}{11}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{{}^{6}C_2 + {}^{5}C_2}{{}^{11}C_2} = \frac{6 \times 5 + 5 \times 4}{11 \times 10} = \frac{5}{11}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{{}^{6}C_2}{{}^{11}C_2} = \frac{5}{11}$$

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{3/11}{5/11} = \mathbf{\frac{3}{5}}.$$

Ex. 3. A die is rolled. If the outcome is an odd number, what is the probability that it is a prime number ?

Sol. $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

Let A : Event of getting an odd number

B : Event of getting a prime number

$A = \{1, 3, 5\} \Rightarrow n(A) = 3$

$B = \{2, 3, 5\} \Rightarrow n(B) = 3$

$A \cap B = \{3, 5\} \Rightarrow n(A \cap B) = 2$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}, P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}, P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \quad P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/3}{1/2} = \mathbf{\frac{2}{3}}.$$

Ex. 4. A man speaks truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. (***DCE 2009***)

Sol. Let E_1, E_2 and A be the events defined as follows:

E_1 = Six occurs,

E_2 = Six does not occur

A = man reports it is a six

Then, $P(E_1) = \frac{1}{6}, P(E_2) = 1 - \frac{1}{6} = \frac{5}{6}$

$P(A/E_1)$ = Probability of man reporting it a six when six occurs = Probability of speaking truth = $\frac{3}{4}$

$P(A/E_2)$ = Probability of man reporting a six when six does not occur

= Probability of not speaking truth = $1 - \frac{3}{4} = \frac{1}{4}$

$\therefore$ P(Throw is actually a six) = $\dfrac{P(E_1) \times P(A/E_1)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)}$ (Baye's Theoram)

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{\frac{3}{24}}{\frac{8}{24}} = \mathbf{\frac{3}{8}}.$$

Ex. 5. A person goes to office either by car, scooter, bus or train, the probabilities of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car. (***IIT 2005***)

Sol. Let the events E_1, E_2, E_3, E_4 and A be defined as follows:

E_1 : Event that the person goes to office by car

E_2 : Event that the person goes to office by scooter

E_3 : Event that the person goes to office by bus

E_4 : Event that the person goes to office by train.

A : Event that the person reaches office in time.

Then, $P(E_1) = \frac{1}{7}, P(E_2) = \frac{3}{7}, P(E_3) = \frac{2}{7}, P(E_4) = \frac{1}{7}$

$P(A/E_1) = P$(Person reaches office in time if he goes by car)

= 1 – P(Person reaches office late if he goes by car)

$= 1 - \frac{2}{9} = \frac{7}{9}$

$P(A/E_2) = P$(Person reaches office in time if he goes by scooter)

= 1 – P(Person reaches office late if are goes by scooter)

$= 1 - \frac{1}{9} = \frac{8}{9}$

$P(A/E_3) = P$(Person reaches office in time if he goes by bus)

= 1 – P(Person reaches office late if he goes by bus)

$= 1 - \frac{4}{9} = \frac{5}{9}$

$P(A/E_4) = P$(person reaches office in time if he goes by train)

$= 1 - P$(person reaches office late if he goes by train)

$$= 1 - \frac{1}{9} = \frac{8}{9}$$

Now, P(person travelled by car if he reached office in time)

$$P(E_1/A) = \frac{P(E_1) \times P(A/E_1)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2) + P(E_3) + P(A/E_3) + P(E_4) \times P(A/E_4)} \quad \text{(Baye's Theoram)}$$

$$= \frac{\frac{1}{7} \times \frac{7}{9}}{\frac{1}{7} \times \frac{7}{9} + \frac{3}{7} \times \frac{8}{9} + \frac{2}{7} \times \frac{5}{9} + \frac{1}{7} \times \frac{8}{9}} = \frac{\frac{7}{63}}{\frac{7}{63} + \frac{24}{63} + \frac{10}{63} + \frac{8}{63}} = \frac{\frac{7}{63}}{\frac{49}{63}} = \frac{7}{49} = \mathbf{\frac{1}{7}}$$

Ex. 6. Box *A* contains 2 black and 3 red balls, while box *B* contains 3 black and 4 red balls. Out of these two boxes one is selected at random; and the probability of choosing box *A* is double that of box *B*. If a red ball is drawn from the selected box, then find the probability that it has come from box *B*.

(EAMCET 2005)

Sol. Let the events be defined as:

A : Selection of box A

B : Selection of box B

R : Drawing a red ball.

Let $P(B) = p$. Then, according to given condition $P(A) = 2P(B) = 2p$

$$P\left(\frac{R}{A}\right) = \frac{{}^3C_1}{{}^5C_1} = \frac{3}{5}, \quad P\left(\frac{R}{B}\right) = \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7}$$

$$\therefore \quad P\left(\frac{B}{R}\right) = \frac{P(B) \, . \, P\left(\frac{R}{B}\right)}{P(A) \, . \, P\left(\frac{R}{A}\right) + P(B) \, . \, P\left(\frac{R}{B}\right)} = \frac{p \, . \, \frac{4}{7}}{2p \, . \, \frac{3}{5} + p \, . \, \frac{4}{7}} = \frac{4/7}{6/5 + 4/7} = \frac{4/7}{\frac{42+20}{35}} = \frac{4/7}{62/35} = \mathbf{\frac{10}{31}}.$$

Ex. 7. *A* and *B* are two independent witnesses (*i.e.*, there is no collision between them) in a case. The probability that *A* will speak the truth is *x* and the probability that *B* will speak the truth is *y*. *A* and *B* agree in a certain statement. What is the probability that the statement is true? ***(BITSAT 2004)***

Sol. A and B will agree in a certain statement if both speak truth or both tell a lie. Now, let us define the events as follows:

E_1 : A and B both speak the truth $\Rightarrow P(E_1) = xy$

E_2 : A and B both tell a lie $\Rightarrow P(E_2) = (1 - x)(1 - y)$

E : A and B agree on a certain statement.

Clearly, $P(E/E_1) = P(E/E_2) = 1$

Now, we are required to find $P(E_1/E)$.

$$P(E_1/E) = \frac{P(E_1). \, P(E/E_1)}{P(E_1). \, P(E/E_1) + P(E_2). \, P(E/E_2)} = \frac{xy \, . 1}{xy \, . 1 + (1-x)(1-y) \, . 1} = \mathbf{\frac{xy}{1 - x - y + 2xy}}.$$

PRACTICE SHEET

1. If A and B are events such that $P(A \cup B) = 0.5$, $P(\overline{B}) = 0.8$ and $P(A/B) = 0.4$, then what is $P(A \cap B)$ equal to ?

(*a*) 0.08 (*b*) 0.02 (*c*) 0.2 (*d*) 0.8

(*NDA/NA 2011*)

2. If $P(S) = 0.3$, $P(T) = 0.4$ and S and T are independent events, then $P(S/T)$ is equal to

(*a*) 0.12 (*b*) 0.2 (*c*) 0.3 (*d*) 0.4

(*Orissa JEE 2011*)

3. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Then $P(B)$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{3}$ (*c*) $\frac{1}{6}$ (*d*) $\frac{1}{3}$

(*AIEEE 2008*)

4. Let X and Y be two events such that $P(X/Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is/are correct?

(*a*) $P(X \cup Y) = 2/3$

(*b*) X and Y are independent

(*c*) X and Y are not independent

(*d*) $P(X^C \cap Y) = \frac{1}{3}$ **(*IIT JEE 2012*)**

5. Given, $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.3$, then $P\left(\frac{A'}{B'}\right)$ is equal to

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{3}{4}$

(*MHCET 2009*)

6. A die is rolled. If the outcome is an odd number, what is the probability that it is a prime number ?

(*a*) $\frac{3}{8}$ (*b*) $\frac{7}{9}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{5}{6}$

7. Two dice are thrown. Find the probability that the sum is 8 or greater than 8, if 3 appears on the first die.

(*a*) $\frac{3}{8}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{7}{8}$

8. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ... 49. Then the probability that the sum of the digits selected is 8, given that the product of these digits is zero is equal to

(*a*) 1/14 (*b*) 1/7 (*c*) 5/14 (*d*) 1/50

(*AIEEE 2009*)

9. One Indian and four American men and their wives are to be seated randomly around a circular table. Then, the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{5}$ (*d*) $\frac{1}{5}$

(*IIT JEE 2007*)

10. A bag contains 6 red and 9 blue balls. Two successive drawing of four balls are made such that the balls are not replaced before the second draw. Find the probability that the first draw gives 4 red balls and the second draw gives 4 blue balls.

(*a*) $\frac{3}{715}$ (*b*) $\frac{7}{715}$

(*c*) $\frac{15}{233}$ (*d*) None of these

11. Two numbers are selected at random from the integer 1 through 9. If the sum is even, find the probability that both numbers are odd.

(*a*) 5/8 (*b*) 3/8

(*c*) 3/10 (*d*) None of these

12. A card is drawn from a well shuffled deck of cards. What is the probability of getting a king, given that the card drawn is black.

(*a*) $\frac{1}{13}$ (*b*) $\frac{4}{13}$ (*c*) $\frac{6}{13}$ (*d*) $\frac{7}{13}$

13. A bag A contains 2 white and 2 red balls and another bag B contains 4 white and 5 red balls. A ball is drawn and is found to be red. The probability that it was drawn from bag B is:

(*a*) $\frac{5}{19}$ (*b*) $\frac{21}{52}$ (*c*) $\frac{10}{19}$ (*d*) $\frac{25}{52}$

(*AMU 2010*)

14. A bag A contains 4 green and 3 red balls and bag B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted to be green. The probability that it comes from bag B is

(*a*) $\frac{2}{7}$ (*b*) $\frac{2}{3}$ (*c*) $\frac{3}{7}$ (*d*) $\frac{1}{3}$

(*DCE 2005*)

15. In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing is

(*a*) $\frac{37}{40}$ (*b*) $\frac{1}{37}$ (*c*) $\frac{36}{37}$ (*d*) $\frac{1}{9}$

(*Kerala CEE 2004, EAMCET 2012*)

16. In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it is $\frac{1}{8}$. The probability that his answer is correct, given that he guessed it is $\frac{1}{4}$. The probability that they knew the answer to the questions given that he correctly answered it is

(*a*) $\frac{24}{31}$ (*b*) $\frac{31}{24}$ (*c*) $\frac{24}{29}$ (*d*) $\frac{29}{24}$

(*J&K CET 2004, IIT*)

17. In four schools B_1, B_2, B_3 and B_4 the percentage of girl students is 12, 20, 13 and 17 respectively. From a school selected at random, one student is picked up at random and it is found that the student is a girl. The probability that the girl selected from school B_2 is

(*a*) $\frac{6}{31}$ (*b*) $\frac{10}{31}$ (*c*) $\frac{13}{62}$ (*d*) $\frac{17}{62}$

(*UPSEE 2000*)

18. An architecture company built 200 bridges, 400 hospitals and 600 hotels. The probability of damage due to an earthquake of a bridge, a hospital and a hotel is 0.01, 0.15, 0.03 respectively. One of the construction gets damaged in an earthquake. What is the probability that it is a hotel?

(*a*) $\frac{1}{26}$ (*b*) $\frac{1}{40}$ (*c*) $\frac{7}{52}$ (*d*) $\frac{9}{40}$

19. A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. *A* third box B_3 contains 3 white balls, 4 red balls and 5 black balls. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(*a*) $\frac{116}{181}$ (*b*) $\frac{126}{181}$ (*c*) $\frac{65}{181}$ (*d*) $\frac{55}{181}$

(*IIT JEE 2013*)

20. Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(*a*) $\frac{17}{23}$ (*b*) $\frac{11}{23}$ (*c*) $\frac{15}{23}$ (*d*) $\frac{12}{23}$

(*IIT JEE 2011*)

ANSWERS

1. (*a*)	**2.** (*c*)	**3.** (*d*)	**4.** (*a*) and (*b*)	**5.** (*c*)	**6.** (*c*)	**7.** (*c*)	**8.** (*a*)	**9.** (*c*)	**10.** (*a*)
11. (*a*)	**12.** (*a*)	**13.** (*c*)	**14.** (*c*)	**15.** (*b*)	**16.** (*c*)	**17.** (*b*)	**18.** (*d*)	**19.** (*d*)	**20.** (*d*)

HINTS AND SOLUTIONS

1. $P(A \cup B) = 0.5$, $P(B) = 0.8$, $P(A/B) = 0.4$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A/B) \times P(B)$$

$$\Rightarrow P(A \cap B) = (1 - P(\bar{B})) \times P(A/B)$$

$$= (1 - 0.8) \times 0.4 = 0.2 \times 0.4 = \mathbf{0.8.}$$

2. *S* and *T* being independent events,

$$P(S \cap T) = P(S) \times P(T) = 0.3 \times 0.4$$

$$P(S/T) = \frac{P(S \cap T)}{P(T)} = \frac{0.3 \times 0.4}{0.4} = \mathbf{0.3.}$$

3. Given, $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$$\Rightarrow P(A \cap B) = P(B/A) \times P(A) = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$$

Now $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow \frac{1}{2} = \frac{1}{6} \times \frac{1}{P(B)} \Rightarrow P(B) = \frac{2}{6} = \mathbf{\frac{1}{3}}.$$

4. $P(X/Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$, $P(X \cap Y) = \frac{1}{6}$

$$\therefore P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)}$$

$$\Rightarrow \frac{1}{2} = \frac{1/6}{P(Y)} \Rightarrow P(Y) = \frac{1}{3} \quad ...(i)$$

Now $P\left(\frac{Y}{X}\right) = \frac{P(Y \cap X)}{P(X)} = \frac{P(X \cap Y)}{P(X)}$

$$\Rightarrow \frac{1}{3} = \frac{1}{6} \times \frac{1}{P(X)} \Rightarrow P(X) = \frac{1}{2} \quad ...(ii)$$

$$\therefore P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3} \quad ...(iii)$$

$$P(X \cap Y) = \frac{1}{6} \text{ and } P(X) \,.\, P(Y) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$\Rightarrow P(X \cap Y) = P(X) \,.\, P(Y)$

$\Rightarrow$ *X* and *Y* are independent events

Now $P(X^C \cap Y) = P(Y) - P(X \cap Y) = \frac{1}{3} - \frac{1}{6} = \mathbf{\frac{1}{6}}.$

5. $$P\left(\frac{A'}{B'}\right) = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{P(B')}$$

$$= \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - \{P(A) + P(B) - P(A \cap B)\}}{1 - P(B)}$$

$$= \frac{1 - \{0.5 + 0.4 - 0.3\}}{1 - 0.4} = \frac{0.4}{0.6} = \mathbf{\frac{2}{3}}.$$

6. Let S be the sample space of rolling a dice. Then,

$S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

Let A : Event of rolling an odd number and

B : Event of rolling a prime number.

Then, $A = \{1, 3, 5\} \Rightarrow n(A) = 3$

$B = \{2, 3, 5\} \Rightarrow n(B) = 3$

$A \cap B = \{3, 5\} \Rightarrow n(A \cap B) = 2$

$\therefore \quad P(A) = \frac{3}{6} = \frac{1}{2},\ P(B) = \frac{3}{6} = \frac{1}{2},\ P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

Now, P(Rolling a prime number, if the outcome is an odd number)

$$= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/3}{1/2} = \mathbf{\frac{2}{3}}.$$

7. Let A : Event of getting a sum of 8 or greater than 8 in a throw of two dice

B : Event of getting 3 on the first die.

Then, $A = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\}$.

$\Rightarrow \quad n(A) = 13$

$B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)\}$.

$\Rightarrow \quad n(B) = 6$

$A \cap B = \{(3, 5), (3, 6)\} \quad \Rightarrow n(A \cap B) = 2.$

$\therefore \quad P$(Event of getting sum = 8 or > 8 when 3 appears on first die)

$\Rightarrow$ Occurence of event A on the satisfaction of condition B

$$= P(A/B) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{n(A \cap B)}{n(B)} = \frac{2}{6} = \mathbf{\frac{1}{3}}.$$

8. Let $S = \{00, 01, 02, \ldots, 48, 49\}$.

$n(S) = 50$

Let A be the event that sum of the digits on the selected ticket is 8, then

$A = \{08, 17, 26, 35, 44\} \Rightarrow n(A) = 5$

Let B be the event that the product of the digits is zero. Then,

$B = \{00, 01, 02, \ldots, 08, 09, 10, 20, 30, 40\} \Rightarrow n(B) = 14$

$\therefore A \cap B = \{08\} \Rightarrow n(A \cap B) = 1$

$\therefore$ Required probability $= P\left(\frac{A}{B}\right)$

$$= \frac{P(A \cap B)}{P(B)} = \frac{1/50}{14/50} = \mathbf{1/14.}$$

9. Let A : Event that Indian man is seated adjacent to his wife.

Let B : Event that each American is seated adjacent to his wife.

Consider each couple as one entity. Thus, there are 5 entities to be arranged and husbands and wife can interchange their seats in 2! ways.

$\therefore \quad P(A \cap B) = \frac{4!\,(2!)^5}{9!}$

Next consider each American couple as an entity. Thus, there are 6 entities to be arranged including the Indian and his wife.

$\therefore \quad P(B) = \frac{5!\,(2!)^4}{9!}$

$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{4!\,(2!)^5}{5!\,(2!)^4} = \mathbf{\frac{2}{5}}.$

10. Let

A : Event of drawing 4 red balls in the first draw and

B : Event of drawing 4 blue balls in the second draw without replacement of the balls drawn in A.

Then, $P(A) = \frac{n(A)}{n(S)}$

$$= \frac{\text{No. of ways of drawing 4 red balls out of 6 red balls}}{\text{No. of ways of drawing 4 balls out of 15 balls}}$$

$$= \frac{{}^6C_4}{{}^{15}C_4} = \frac{\frac{\lfloor 16}{\lfloor 2 \lfloor 4}}{\frac{\lfloor 15}{\lfloor 11 \lfloor 4}} = \frac{\frac{6 \times 5}{2}}{\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}} = \frac{15}{1365} = \frac{1}{91}$$

$$P\left(\frac{B}{A}\right) = \frac{n(B)}{n(S)} = \frac{\text{No. of ways of drawing 4 blue balls out of 9 blue balls}}{\text{No. of ways of drawing 4 balls out of 11 balls remaining after the 1st draw}}$$

$$= \frac{{}^9C_4}{{}^{11}C_4} = \frac{\frac{\lfloor 9}{\lfloor 5 \lfloor 4}}{\frac{\lfloor 11}{\lfloor 7 \lfloor 4}} = \frac{9 \times 8 \times 7 \times 6}{11 \times 10 \times 9 \times 8} = \frac{42}{110} = \frac{21}{55}$$

> **Note:** This second event is denoted by B/A as it depends on condition A.

$\therefore \quad$ Required probability $= P(A) \times P(B/A)$

$$= \frac{1}{91} \times \frac{21}{55} = \mathbf{\frac{3}{715}}.$$

11. In the set of integers from 1 to 9, there are four even numbers 2, 4, 6, 8 and 5 odd numbers 1, 3, 5, 7, 9.

Let A : Event of choosing odd numbers

$\Rightarrow \quad n(A) = {}^5C_2$

($\because$ *2 numbers are chosen from 5 odd numbers*)

B : Event of getting the sum as even number.

$\Rightarrow \quad n(B) = {}^4C_2 + {}^5C_2$ ($\because$ *The sum is even if both the numbers chosen are even or both are odd*)

$\therefore \quad n(A \cap B) = {}^5C_2$

(Event of getting sum as even if both the numbers are odd)

$\therefore \quad P$(Selecting both odd numbers or getting an even sum)

$$= P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(B)} = \frac{{}^5C_2}{{}^4C_2 + {}^5C_2} = \frac{10}{16} = \mathbf{\frac{5}{8}}.$$

12. Let S be the sample space of drawing a card out of 52 cards. Then,

$n(S) = 52$

Let A : Event of drawing a king $\Rightarrow n(A) = 4$

(*A pack has 4 kings*)

B : Event of drawing a black card $\Rightarrow n(B) = 26$

(*A pack has 26 black cards*)

$A \cap B$: Event of drawing a king of a black card

$\Rightarrow n(A \cap B) = 2$ (*A pack has 2 black kings*)

$\therefore P(A) = \frac{4}{52} = \frac{1}{13}, P(B) = \frac{26}{52} = \frac{1}{2}, P(A \cap B) = \frac{2}{52} = \frac{1}{26}$

P(Getting a king, given card drawn is black

$$= P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/26}{1/2} = \frac{2}{26} = \mathbf{\frac{1}{13}}.$$

($\because$ Event A depends on B)

13. Let the events E_1, E_2 and A be defined as follows:

E_1 = Choosing bag A

E_2 = Choosing bag B

A = Choosing red ball.

Then, $P(E_1) = P(E_2) = \frac{1}{2}$

($\because$ There are two bags that have an equally likely chance of being chosen)

$P(A/E_1) = P$(Drawing red ball from bag A) $= \frac{2}{4} = \frac{1}{2}$

($\because$ *2 red out of 4 balls*)

$P(A/E_2) = P$(Drawing red ball from bag B) $= \frac{5}{9}$

(*5 red out of 9 balls*)

$\therefore P$(Red balls drawn from bag B)

$$= P(E_2 / A) = \frac{P(E_2) \times P(A/E_2)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{5}{9}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{5}{9}} = \frac{\frac{5}{18}}{\frac{1}{4} + \frac{5}{18}} = \frac{5/18}{19/36} = \mathbf{\frac{10}{19}}.$$

14. P(Drawing a green ball from bag A) $= P\left(\frac{G}{A}\right) = \frac{4}{7}$

P(Drawing a green ball from bag B) $= P\left(\frac{G}{B}\right) = \frac{3}{7}$

$\therefore$ Required probability $P\left(\frac{B}{G}\right)$

$$= \frac{P(B) \,.\, P\left(\frac{G}{B}\right)}{P(A) \,.\, P\left(\frac{G}{A}\right) + P(B) \,.\, P\left(\frac{G}{B}\right)}$$

$$= \frac{\frac{1}{2} \cdot \frac{3}{7}}{\frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{3}{7}} = \frac{3/14}{4/14 + 3/14} = \frac{3/14}{7/14} = \mathbf{\frac{3}{7}}.$$

15. We define the given events as:

A_1 : Student knows the answer

A_2 : Student does not know the answer

E : He gets the correct answer.

$P(A_1) = \frac{9}{10}, P(A_2) = 1 - \frac{9}{10} = \frac{1}{10}$

$\therefore P(E/A_1) = P$(Student gets the correct answer when he knows the answer) = 1

$P(E/A_2) = P$(Student gets the correct answer when he does not know the correct answer) = 1/4

$\therefore$ Required probability

$$= P(A_2/E) = \frac{P(A_2) \,.\, P(E/A_2)}{P(A_1) \,.\, P(E/A_1) + P(A_2) \,.\, P(E/A_2)}$$

$$= \frac{\frac{1}{10} \cdot \frac{1}{4}}{\frac{9}{10} \cdot 1 + \frac{1}{10} \cdot \frac{1}{4}} = \frac{\frac{1}{40}}{\frac{37}{40}} = \mathbf{\frac{1}{37}}.$$

16. Let,

E_1 : Examinee guesses the answer

E_2 : Examinee copies the answer

E_3 : Examinee knows the answer

E : Event examinee answers correctly.

Given, $P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{6}$

$\therefore P(E_3) = 1 - (P(E_1) + P(E_3)) = 1 - \left(\frac{1}{3} + \frac{1}{6}\right) = \frac{1}{2}$

Given, $P(E/E_1) = \frac{1}{4}, P(E/E_2) = \frac{1}{8}$ and $P(E/E_3) = 1$

$\therefore$ Required probability $= P(E_3/E)$

$$= \frac{P(E_3) \cdot P(E/E_3)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3)}$$

$$= \frac{\frac{1}{2} \cdot 1}{\frac{1}{3} \times \frac{1}{4} + \frac{1}{6} \times \frac{1}{8} + \frac{1}{2} \times 1} = \frac{\frac{1}{2}}{\frac{1}{12} + \frac{1}{48} + \frac{1}{2}}$$

$$= \frac{\frac{1}{2}}{\frac{4 + 1 + 24}{48}} = \frac{\frac{1}{2}}{\frac{29}{48}} = \mathbf{\frac{24}{29}}.$$

17. Let E_1, E_2, E_3, E_4 and A be the events defined as follows:

E_1 = Event of selecting school B_1

E_2 = Event of selecting school B_2

E_3 = Event of selecting school B_3

E_4 = Event of selecting school B_4

A = Event of selecting a girl.

Since there are four schools and each school has an equal chance of being chosen,

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \dfrac{1}{4}$

Now, P(Girl chosen is from school B_1) = $P\left(\dfrac{A}{E_1}\right) = \dfrac{12}{100}$

Similarly $P\left(\dfrac{A}{E_2}\right) = \dfrac{20}{100}, P\left(\dfrac{A}{E_3}\right) = \dfrac{13}{100}, P\left(\dfrac{A}{E_4}\right) = \dfrac{17}{100}$

$\therefore$ P(Girl chosen is from school B_2)

$$= \frac{P(E_2) \times P(A/E_2)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2) + P(E_3) \times P(A/E_3) + P(E_4) \times P(A/E_4)}$$

(Using Bayes Th.)

$$= \frac{\frac{1}{4} \times \frac{20}{100}}{\frac{1}{4} \times \frac{12}{100} + \frac{1}{4} \times \frac{20}{100} + \frac{1}{4} \times \frac{13}{100} + \frac{1}{4} \times \frac{17}{100}}$$

$$= \frac{\frac{1}{4} \times \frac{20}{100}}{\frac{1}{4} \times \frac{62}{100}} = \frac{20}{62} = \mathbf{\frac{10}{31}}.$$

18. Let E_1, E_2, E_3 and A be the events defined as follows:

E_1 = Construction chosen is a bridge.

E_2 = Construction chosen is a hospital.

E_3 = Construction chosen is a hotel.

A = Construction gets damaged.

Since there are (200 + 400 + 600) = 1200 constructions,

$P(E_1) = \dfrac{200}{1200} = \dfrac{1}{6},\ P(E_2) = \dfrac{400}{1200} = \dfrac{1}{3},\ P(E_3) = \dfrac{600}{1200} = \dfrac{1}{2}$

Given, Probability that the construction that gets damaged is a bridge = $P(A/E_1) = 0.01$

Similarly, $P(A/E_2) = 0.15$ and $P(A/E_3) = 0.03$

$\therefore$ Probability that a hotel gets damaged in an earthquake

$= P\left(\dfrac{E_3}{A}\right)$

$$= \frac{P(E_3) \times P(A/E_3)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2) + P(E_3) \times P(A/E_3)}$$

(*Using Bayes Th.*)

$$= \frac{\frac{1}{2} \times 0.03}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.15 \times \frac{1}{2} \times 0.03}$$

$$= \frac{\frac{1}{2} \times 0.03}{\frac{1}{6} \times (0.01 + 0.3 \times 0.09)} = \frac{6}{2} \times \frac{0.03}{0.4} = \frac{6 \times 3 \times 10}{2 \times 4 \times 100} = \mathbf{\frac{9}{40}}.$$

19. Let E : Event of selecting red and one white ball

Probability of selecting a box = $P(B_1) = P(B_2) = P(B_2) = \dfrac{1}{3}$

Probability of selecting 1 Red and 1 White ball from box B_1

$$= P\left(\frac{E}{B_1}\right) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1 \times 3 \times 2}{6 \times 5} = \frac{1}{5}$$

$$P\left(\frac{E}{B_2}\right) = \frac{{}^2C_1 \times {}^3C_1}{{}^9C_2} = \frac{2 \times 3 \times 3}{9 \times 8} = \frac{1}{6}$$

$$P\left(\frac{E}{B_3}\right) = \frac{{}^3C_1 \times {}^4C_1}{{}^{12}C_2} = \frac{3 \times 4 \times 2}{12 \times 11} = \frac{2}{11}$$

$$\therefore\ P\left(\frac{B_2}{E}\right) = \frac{P(B_2) \times P\left(\frac{E}{B_2}\right)}{P(B_1) \times P\left(\frac{E}{B_1}\right) + P(B_2) \times P\left(\frac{E}{B_2}\right) + P(B_3) \times P\left(\frac{E}{B_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{6}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{2}{11}} = \frac{\frac{1}{6}}{\frac{66 + 55 + 30}{330}} = \frac{\frac{1}{6}}{\frac{181}{330}} = \mathbf{\frac{55}{181}}.$$

20. The movement of balls from Urn 1 to Urn 2 on the condition that head or tail appears on the coin can be shown as:

Original → U_1: [3 W, 2R] U_2: [1 W]

Head appears U_1: [2 W, 2R] —1 White→ U_2: [2 W] or

U_1: [3 W, 1R] —1 Red→ U_2: [1 W, 1R] ...(*i*)

Tail appears U_1: [1 W, 2R] —2 White→ U_2: [3 W] or

U_1: [3 W, 0R] —2 Red→ U_2: [1 W, 2R] or

U_1: [2 W, 1R] —1 R 1 W→ U_2: [2 W, 1R] ...(*ii*)

Let the events be defined as:

W : Drawing a white ball from U_2

H : Tossing a head

T : Tossing a tail.

$\therefore\ P(H) = P(T) = \dfrac{1}{2}$

$P(W/H)$ = Probability of drawing a white ball from Urn 2, when head is tossed

$$= \frac{{}^3C_1}{{}^5C_1} \times \frac{{}^2C_1}{{}^2C_1} + \frac{{}^2C_1}{{}^5C_1} \times \frac{{}^1C_1}{{}^2C_1} \quad \text{From } (i)$$

$$= \frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{2} = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

$P(W/T)$ = Probability of drawing a white ball from Urn 2, when tail is tossed

$$= \frac{{}^3C_2}{{}^5C_2} \times \frac{{}^3C_1}{{}^3C_2} + \frac{{}^2C_2}{{}^5C_2} \times \frac{{}^1C_1}{{}^3C_1} + \frac{{}^3C_1 \times {}^2C_1}{{}^5C_2} \times \frac{{}^2C_1}{{}^3C_2}$$

(see diag. (*ii*))

$$= \frac{3 \times 2}{5 \times 4} \times 1 + \frac{1 \times 2}{5 \times 4} \times \frac{1}{3} + \frac{3 \times 2 \times 2}{5 \times 4} \times \frac{2}{3}$$

$$= \frac{3}{10} + \frac{1}{30} + \frac{2}{5} = \frac{9 + 1 + 12}{30} = \frac{22}{30}$$

$$P\left(\frac{H}{W}\right) = \frac{P(H)\,.\,P(W/H)}{P(H)\,.\,P(W/H) + P(T)\,.\,P(W/T)}$$

$$= \frac{\frac{1}{2} \times \frac{4}{5}}{\frac{1}{2} \times \frac{4}{5} + \frac{1}{2} \times \frac{22}{30}}$$

$$= \frac{4/5}{4/5 + 22/30} = \frac{4/5}{\frac{24 + 22}{30}} = \frac{4/5}{46/30} = \frac{4}{5} \times \frac{30}{46} = \mathbf{\frac{12}{23}}.$$