Chapter : 3. INTRODUCTION TO EUCLID'S GEOMETRY

Exercise : 3A

Question: 1

A theorem is a statement that has been proven true while an axiom is a statement that is accepted as true.

Axiom is a statement that is assumed to be true without proof.

Theorem is a statement that has been proven through testing or calculation.

Question: 2

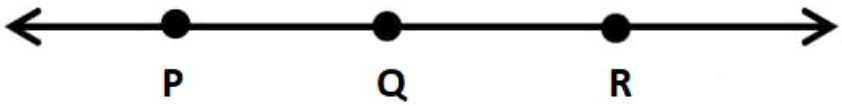
(i) In line segment two points are connected with a straight line.

A line segment has two end points with a definite length.

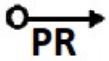
(ii) A ray is a part of line with one end point and infinitely extends in one direction. It can show by drawing an arrow at one end of the ray.

(iii) Parallel lines are lines that never cross one another. Parallel lines do not intersect to each other.

(iv) A straight line extends from a point indefinitely in one direction only. It is the set of all points on a line on a given side of a given point of the line.

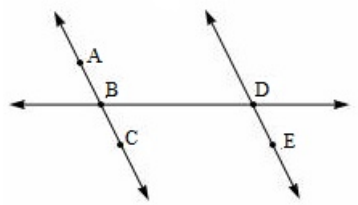


Notation:



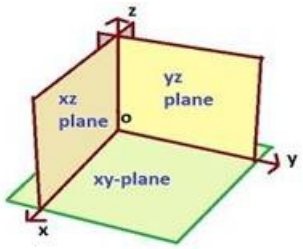
is the half-line.

(v) When three or more points lie on a straight line called collinear points. If the points lie on the same line then the points are called collinear points.



A, B and C the collinear points.

(vi) A plane is a flat surface with no thickness. A plane is two dimensional.



Question: 3

(i) A, B, C, D, E, F

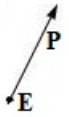
A point has location and it has no size.

(ii) $\overline{EG}$, $\overline{FH}$, $\overline{EF}$, $\overline{GH}$, $\overline{MN}$

A line segment has two end points with a definite length.

(iii) $\overrightarrow{EP}$, $\overrightarrow{GR}$, $\overrightarrow{GB}$, $\overrightarrow{HD}$

A ray is a part of line with one end point and infinitely extends in one direction.

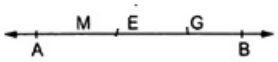


(iv) $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$

A line has no beginning point or end point.

(v) M, E, G, B

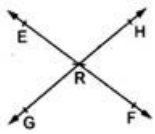
If three or more points lie on a straight line called collinear points.



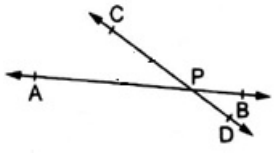
Question: 4

(i) {$\overleftrightarrow{EF}$, $\overleftrightarrow{GH}$, R},{$\overleftrightarrow{AB}$,$\overleftrightarrow{CD}$,P}

Lines EF and GH cross to each other at point R, therefore EF and GH are intersecting lines at point R.

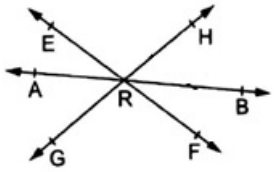


Similarly lines AB and CD cross to each other at point P, therefore EF and GH are intersecting lines at point P.



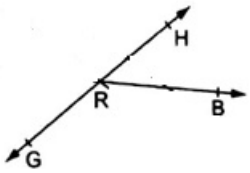
(ii) $\overleftrightarrow{AB}$,$\overleftrightarrow{EF}$,$\overleftrightarrow{GH}$,R

If the set of lines or curves intersect at the same point called concurrent lines. In this figure lines AB, EF and GH intersect at the point R.



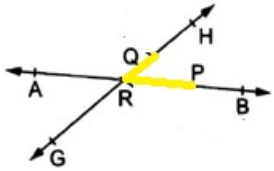
(iii) $\overline{RB}, \overline{RH}, \overline{RG}$

A ray is a part of line with one end point and infinitely extends in one direction.



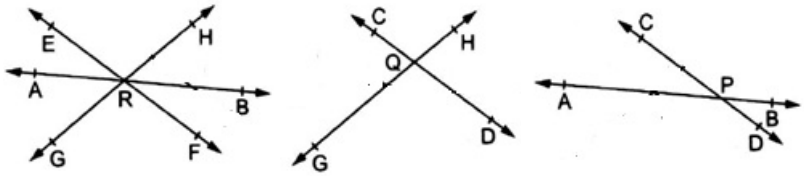
(iv) $\overline{RQ}, \overline{RP}$

A line segment has two end points with a definite length.



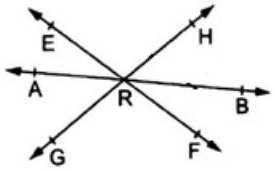
(v) { AB, EF, GH}, { CD, GH}, { AB, CD}

When two or more lines meet at a point, these are called intersecting lines. Intersecting lines share exactly one point.



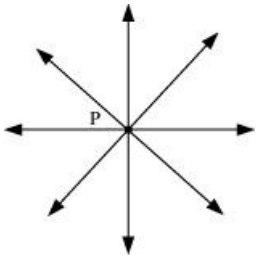
(vi) $\overline{AB}, \overline{EF}, \overline{GH}$

If the set of lines or curves intersect at the same point, it is called concurrent lines.

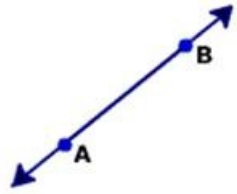


Question: 5

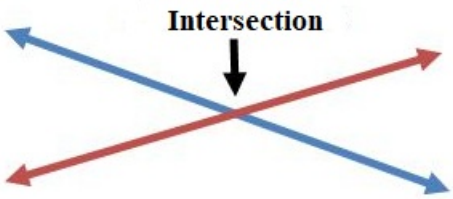
(i) Infinite number of lines can be drawn to pass through a given point.



(ii) Only one line can be drawn to pass through two given points.



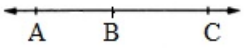
(iii) Two lines intersect in at most one point.



(iv) $\overline{AB}$, $\overline{BC}$, $\overline{AC}$

A line segment has two end points with a definite length.

If three or more points lie on a straight line, it is called collinear points.



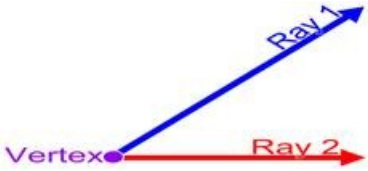
Question: 6

(i) False

Because a line segment has two end points with a definite length.

(ii) False

Because a ray has one end point and infinitely extends in one direction.



(iii) False

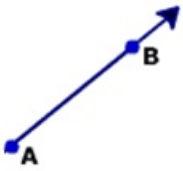
Because a line has no beginning point or end point therefore a line has not definite length.

(iv) True

Because a line has no beginning point or end point therefore line $\overline{AB}$ is the same as line $\overline{BA}$.

(v) False

Because a ray has one end point and infinitely extends in one direction therefore ray $\overrightarrow{AB}$ is not same as the ray $\overrightarrow{BA}$.



In this ray $\overline{AB}$ has the end point A.

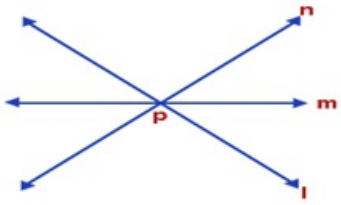
In this ray $\overline{BA}$ has the end point B.

(vi) True

Because a line has no beginning point or end point.

(vii) True

Because the set of lines intersect at the same point called concurrent lines.



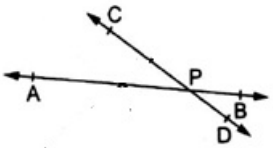
Lines l, m, and n are concurrent at the common point P.

(viii) True

Two lines in a plane that do not intersect or touch each other at any point, they are called parallel lines.

(ix) True

Because parallel lines never intersect or cross one another.

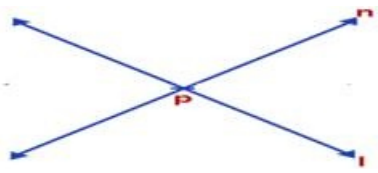


(x) False

Because half-line is the set of all points on a line on a given side of a given point of the line while ray is a part of line with one end point and infinitely extends in one direction.

(xi) False

Because two lines intersect at only one point.



Lines l and n intersect at only one point P.

(xii) False

Two lines are said to be parallel when

(a) they never meet

(b) they are coplanar

Exercise : CCE QUESTIONS

Question: 1

In ancient India,

Solution:

In ancient India, the shapes of altars used for household rituals were Squares and circles.

The geometry of the Vedic period originated with the construction of altars (or vedis) and fireplaces for performing Vedic rites. Square and circular altars were used for household rituals, while altars, whose shapes were combinations of rectangles, triangles and trapeziums, were required for public worship.

Question: 2

The number of int

Solution:

The number of interwoven isosceles triangles in Sriyantra is 9.

The Sri Yantra is a form of mystical diagram, known as a yantra. It consists of nine interlocking triangles that radiate out from the central point.

Question: 3

Thales belongs to

Solution:

Thales belongs to the country Greece. He was a Greek philosopher, mathematician and astronomer. He was one of the seven sages of Greece.

Question: 4

Euclid belongs to

Solution:

Euclid belongs to the country Greece

Euclid was born in 300 BCE, Alexandria, Egypt. He was the most prominent mathematician of Greece and was best known for his treatise on geometry.

Question: 5

Pythagoras was a

Solution:

Pythagoras was a student of Thales

Pythagoras was born in about 570 BC on the Greek island of Samos. His father was a merchant. Pythagoras was taught mathematics by Thales, who brought mathematics to the Greeks from Ancient Egypt.

Question: 6

In Indus Valley C

Solution:

In Indus Valley Civilization (about 300 BC) the bricks used for construction work were having dimensions in the ratio 4:2:1

In Indus Valley Civilization, the bricks used for construction work were having dimensions in the ratio length : breadth : thickness = 4 : 2 : 1.

Question: 7

Which of the foll

Solution:

A theorem needs a proof.

A theorem is a mathematical statement proved by different steps of mathematical reasoning.

Question: 8

Axioms are assume

Solution:

Axioms are assumed to be universal truths in all branches of mathematics.

Axioms are the derived and accepted true statements.

Question: 9

'Lines are parall

Solution:

Lines are parallel if they do not intersect' is stated in the form of a definition.

Definition is a formal statement of the meaning of a word or a set of words.

Question: 10

Euclid stated tha

Solution:

Euclid stated that 'all right angles are equal to each other', in the form of an axiom.

A right angle means 90°. Thus, All right angles are equal to 90°. Therefore, all right angles ae equal is an axiom. This is because it is derived from a true statement.

Question: 11

Greeks emphasized

Solution:

Greeks emphasized ondeductive reasoning.

The Greeks were interested in establishing the truth of the statements they discovered using deductive reasoning.

Question: 12

A solid has

Solution:

A solid has 3 dimensions.

A solid figure is three dimensional because it has length, width and height.

Question: 13

A surface has

Solution:

A surface has 2 dimensions.

A surface is two dimensional because it has length and width only.

Question: 14

A point has

Solution:

A point has 0 dimension.

A point has no dimensions, only position

Question: 15

Boundaries of sol

Solution:

Boundaries of solids are surfaces.

The solids are three dimensional but their surfaces are two dimensional.

Question: 16

Boundaries of sur

Solution:

Boundaries of surfaces are curves.

Boundaries of surfaces are curves because surfaces are two dimensional figures and their boundaries are one-dimensional i.e. curves.

Question: 17

The side faces of

Solution:

The side faces of a pyramid are triangles

A pyramid is a figure with triangular surfaces which converge to one single point.

Question: 18

The base of a pyr

Solution:

The base of a pyramid is any polygon.

In geometry, a pyramid is a polyhedron formed by connecting a polygonal base and a point, called the apex.

Question: 19

The number of pla

Solution:

The number of planes passing through three non-collinear points is 1.

If the points are collinear then an infinite number of planes can be made to pass through them. If three distinct points are non-collinear then exactly one plane passes through them.

Question: 20

Euclid divided hi

Solution:

Euclid divided his book ‘Elements’ into 13 chapters.

The book name is: “The Elements”

It is divided into thirteen chapters. The chapters are as follows: -

Books VII-IX -- Theory of Numbers

Book X -- Incommensurables

Book XI-XIII -- Solid Geometry

Question: 21

Which of the foll

Solution:

The floor and the ceiling of a room are parallel planes.

The floor and the ceiling of the room are parallel planes.

Question: 22

Which of the foll

Solution:

If two circles are equal, then their radii are equal.

A circle is formed by taking a radius. If the radius of two circles are same, means the two circles are equal.

Question: 23

Which of the foll

Solution:

Ray AB =ray BA.

Because ray is a part of a line that has one endpoint and extends in one direction without ending.

Question: 24

A point C is call

Solution:

A point C is called the midpoint of a line segment AB, if C is an interior point of AB such that AC=CB

Question: 25

A point C is said

Solution:

A point C is said to lie between the points A and B if points A, C and B are collinear.

Question: 26

The question cons

Solution:

Let us consider, a line segment AB.Assume that it has two midpoints say C and D

Recall that the midpoint of a line segment divides it into two equal partsThat is AC = BC and AD = DBSince C is midpoint of AB, we have A, C and B are collinear.∴ AC + BC = AB → (1)Similarly, we get AD + DB = AB → (2)From (1) and (2), we getAC + BC = AD + DB2 AC = 2AD.∴ AC = ADThis is a contradiction unless C and D coincide.Therefore our assumption that a line segment AB has two midpoints is incorrect.Thus every line segment has one and only one midpoint.

Question: 27

The question cons

Solution:

Let us consider, a line segment AB.A and B are two different points from which infinite number of lines can be drawn.

Question: 28

The question cons

Solution:

A sentence that can be judged to be true or false is called a statement.

3+7 = 9 is false statement.

Question: 29

The question cons

Solution:

A line goes without end in both directions but a ray has one endpoint and goes without end in one direction.

The line AB is same as the line BA. But the ray AB is different than the ray BA.

Question: 30

The question cons

Solution:

A rectilinear figure is a figure all of whose edges meet at right angles.

So, Assertion false but the reason is true.

Question: 31

Match the followi

Solution:

(A)-(q), (B)-(r), (C)-(p), (D)-(s)

(A)-(q): A line segment has definite length.

Its length can be measured. A line segment AB has two end points A and B. It starts from point A and ends at point B. One and only one line-segment can be between two given points A and B.

(B) – (r): A ray $\overline{BA}$ has the end point “B”

A ray is a line with one end. It starts at a given point and goes off in a certain direction forever. Here, the end point is B and it extends infinitely in the direction A.

(C)-(p) Through a point, infinitely many lines can be drawn.

In 1 dimension, only one line can be drawn passing through a single point. However, in 2 or more dimension, uncountably many lines can be draw which pass through one point.

(D)-(s) Only one lines can be drawn to pass through two given points.

Through two given points only one unique line can be drawn.

Question: 32

Fill in the blank

Solution:

(A) Concurrent lines passthrough a given point.

If three or more lines pass through the same point then they are called concurrent lines and the common point is called the point of concurrency or concurrent point.

(B) Two distinct lines in a plane cannot have more than one point in common.

Let us suppose that the two lines intersect at two distinct points P and Q. But this assumption clashes with the axiom that only one line can pass through two distinct points. So, the assumption that we started with, that two lines can pass through two distinct points is wrong.

(C) Two distinct points in a plane determine a unique line.

For any two distinct points in space there is a unique line that passes through both of them.

(D) A line segment has two end points

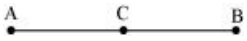
A line segment has definite length. Its length can be measured. A line segment AB has two end points A and B. It starts from point A and ends at point B. One and only one line-segment can be between two given points A and B.

Question: 33

A point C lies be

Solution:

According to question, C lies between points A and B and AC = BC



Adding AC both side we get,

AC + AC = BC + AC

According to definition of Euclid, if equals are added to equals, whole will equal.

Here, (BC + AC) will coincides with AB.

2AC = AB

So, AC = $\frac{1}{2}$ AB

Question: 34

Prove that every

Solution:

Let us consider, a line segment AB.Assume that it has two midpoints say C and D

Midpoint of a line segment divides it into two equal partsSo, AC = BC and AD = DBSince, C is midpoint of AB, we have A, C and B are collinearThus, AC + BC = AB ..... (i)

Similarly, we get AD + DB = AB .....(ii)

From eq(i) and (ii), we getAC + BC = AD + DB2 AC = 2ADAC = ADThis is a contradiction unless C and D coincide.Therefore our assumption that a line segment AB has two midpoints is incorrect.Thus every line segment has one and only one midpoint.

Question: 35

In the given figu

Solution:

From the above figure we get that,

AC = AB + BC

BD = BC + CD

And it is given is that AC = BD

So, AB + BC = BC + CD .....(i)

According to Euclid's axiom, when equals are subtracted from equals, the remainders are also equal.

Subtracting BC from both side in eq(i), we get

AB + BC − BC = BC + CD − BC

AB = CD

Question: 36

L, M, N are three

Solution:

Assume L will not intersect N.

Then, L ||N and it is given that M || N.

According to our assumption, L || M which is contradictory to given statement that L intersects M.

So, our assumption is wrong.

Thus, L will intersect N also.

Question: 37

Find the measure

Solution:

Let x be the angle.

According to question, x is 20° more than its complement.

So, another angle is x - 20°

As we know that, sum of compliment is 90°

So,

x + x - 20° = 90°

2x = 110°

x =55°

Question: 38

Find the measure

Solution:

Let x be the angle.

According to question, x is 20° less than its supplement.

So, another angle is x + 20°

As we know that, sum of compliment is 180°

So,

x + x + 20° = 180°

2x = 160°

x =80°

Question: 39

Find the measure

Solution:

Let x be the angle.

According to question,

(90 - x)° is the complementary angle.

(180 - x)° is the supplementary angle.

According to question,

5 X (90 - x) =2 X (180 -x) -12

450 – 5x = 360 – 2x – 12

3x = 450 – 348

3x = 102

x = 34°

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

Which of the foll

Solution:

A theorem needs a proof.

Question: 2

Number of planes

Solution:

If three distinct points are non-collinear then exactly one plane passes through them

Question: 3

How many lines ca

Solution:

(A) and (B)

(A) Infinite lines can be drawn through a given point.

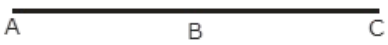
(B) Only a single line can be drawn through two given points.

Question: 4

A, B and C are th

Solution:

Three, AB, BC, AC



If A, B and C are three collinear points. Then there will be only 3 line segments.

AB, BC and AC.

Question: 5

In the given figu

Solution:

From the above figure we get that,

AC = AB + BC

BD = BC + CD

And it is given is that AC = BD

So, AB + BC = BC + CD .....(i)

According to Euclid’s axiom,when equals are subtracted from equals, the remainders are also equal.

Subtracting BC from both side in eq(i), we get

AB + BC − BC = BC + CD − BC

AB = CD

Question: 6

Show that every l

Solution:

Let us consider, a line segment AB.Assume that it has two midpoints say C and D

Midpoint of a line segment divides it into two equal partsSo, AC = BC and AD = DBSince, C is midpoint of AB, we have A, C and B are collinearThus, AC + BC = AB ..... (i)

Similarly, we get AD + DB = AB .....(ii)

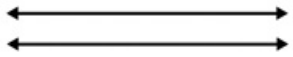
From eq (i) and (ii), we getAC + BC = AD + DB2 AC = 2ADAC = ADThis is a contradiction unless C and D coincide.Therefore our assumption that a line segment AB has two midpoints is incorrect.Thus every line segment has one and only one midpoint.

Question: 7

Define the follow

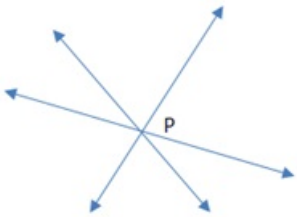
Solution:

(A) Parallel Lines: Two lines in a plane that do not intersect or touch each other at any point are said to be parallel lines.



(B) Intersecting Lines: Two lines in a plane that intersect or cross each other at any point are said to be intersecting lines.

(C) Concurrent Lines: If three or more lines intersect each other at a single point then they are said to be concurrent.



Question: 8

If L, M and N are

Solution:

L || M M|| L.

Now, M||L and L||N M||N.

Assume M will intersect N.

It is given that L ||M and L ||N

According to our assumption, M will intersect N which is contradictory to parallel axiom.

So, our assumption is wrong.

Thus, M|| N

Question: 9

Which of the foll

Solution:

If three or more lines pass through the same point then they are called concurrent lines and the common point is called the point of concurrency or concurrent point.

Question: 10

Which is true?

Solution:

Ray is a part of a line that has one endpoint.

Question: 11

Which is false?

Solution:

Infinite number of lines can be passed through a single point.

Question: 12

From the given fi

Solution:

(A) $\overline{AB}, \overline{PQ}, \overline{RS}$

(A) Lines will be AB, PQ and RS.

(B) CEFG

A rectilinear figure is a figure all of whose edges meet at right angles.

(C) Concurrent Points: A, E, F, B

If a set of lines pass through the same point then they are called concurrent lines and the common point is called the point of concurrency or concurrent point.

Question: 13

A point C is the<

Solution:

(C)

A point C is called the midpoint of a line segment AB, if C is an interior point of AB such that AC=CB

Question: 14

Is D the mid-poin

Solution:

(C)

From the above figure we get that,

AE = AD + DE

CB = CD + DB

And it is given is that AE = CB

So,

AD + DE = CD + DB

AD + CD= CD + DB.....(i) [DE = CD as given]

According to Euclid’s axiom, when equals are subtracted from equals, the remainders are also equal.

Subtracting CD from both side in eq (i), we get

AD + CD − CD = CD + DB − CD

AD = DB

So, D is the mid-point of line segment AB.

Question: 15

Given 4 distinct

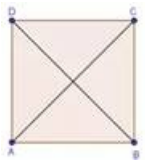
Solution:

(A) one

If all the 4 points are collinear then exactly one line can be drawn from them

(B)

6 lines can be drawn when no three of the four lines are collinear.



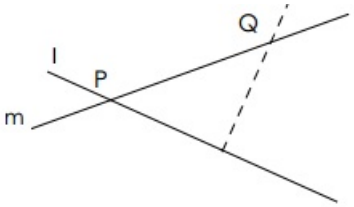
Question: 16

Prove that two di

Solution:

Suppose lines “l” and “m” intersect at two points P and Q. Then, line P must contain both the points P and Q.

Also, line m must contain both the points P and Q.



But only one line can pass through two different points.

Thus, the assumption we started with that two lines can pass through two distinct point is wrong.

Question: 17

Let us define a s

Solution:

(B)

It is given in the question as hint that: (A) and (C) are false sentences and (D) is a true sentence.

Then, “Kunal is a tall boy” is not a statement because this sentence is true for someone and it is false for other one. So, it is not a statement.

Question: 18

State Euclid’s ax

Solution:

The basic facts which are taken for granted without proof are called axioms.

Some Euclid's axioms are:

1. The things which are equal to the same thing are equal to one another.

2. If equals be added to the equals, the wholes are equal.

3. If equals be subtracted from equals, the remainders are equals.

4. Things which coincide with one another are equal to one another.

5. The whole is greater than the part.

6. Things which are double of the same thing are equal to one another.

7. Things which are halves of the same thing are equal to one another

Question: 19

Match the followi

Solution:

(A)-(q)

An infinite number of lines can be drawn to pass through a given point.

(B)-(p)

Only one line can be drawn to pass through two given point.

(C)-(s)

A line is a straight set of points that extend in opposite directions without ending.

(D)-(r)

Line segment $\overline{AB}$ has two end points A and B

Question: 20 A

The question cons

Solution:

A rectilinear figure is a figure all of whose edges meet at right angles.

So, Assertion& Reason both are true.

Question: 20 B

The question cons

Solution:

According to Euclid’s Fourth Postulate Assertion is right but reason given is not linked to right angle.