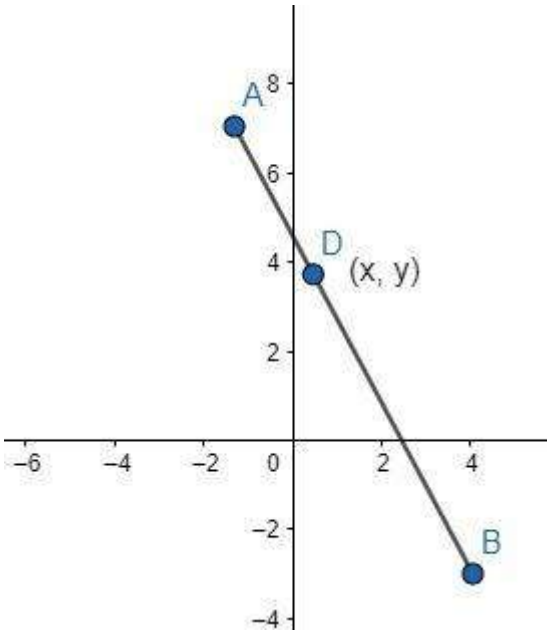
Chapter – 7
Coordinate Geometry

Exercise 7.2

Q. 1 Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2: 3

Answer:



If (x_1, y_1) and (x_2, y_2) are points that are divided in ratio m:n then,

$$(x, y) = \frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}$$

Let D(x, y) be the required point.

Now,

Using the section formula, we get

$$x = \frac{2\times4+3\times-1}{2+3}$$

$$x = \frac{8-3}{5}$$

$$x = 1$$

$$y = \frac{2\times-3+3\times7}{2+3}$$

$y = \frac{-6+21}{5}$

y = 3

Therefore, the point is (1, 3).

Q. 2 Find the coordinates of the points of trisection of the line segment joining (4,–1) and (–2, –3).

Answer:

The points of trisection means that the points which divide the line in three equal parts. From the figure C, and D are these two points.Let C (x_1, y_1) and D (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., BC = CD = DA
Let BC = CD = DA = kPoint C divides the BC and CA as:BC = kCA = CD + DA = k + k = 2kHence ratio between BC and CA is:

$\frac{BC}{CA} = \frac{k}{2k} = \frac{1}{2}$

Therefore, point C divides BA internally in the ratio 1:2

then by section formula we have that if a point P(x, y) divide two points P (x_1, y_1) and Q (x_2, y_2) in the ratio m:n then,
the point (x, y) is given by $(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Therefore C(x, y) divides B(–2, –3) and A(4,–1) in the ratio 1:2, then

$C(x, y) = \frac{(1\times4)+(2\times-2)}{1+2}, \frac{(1\times-1)+(2\times-3)}{1+2}$

$C(x, y) = \frac{4-4}{1+2}, \frac{-1-6}{1+2}$

$C(x, y) = 0, \frac{-7}{3}$

Point D divides the BD and DA as:DA = kBD = BC + CD = k + k = 2k Hence ratio between BD and DA is:

$$\frac{BD}{DA} = \frac{2k}{k} = \frac{2}{1}$$

The point D divides the line BA in the ratio 2:1
So now applying section formula again we get

$$D(x,y) = \frac{(2\times4)+(1\times-2)}{2+1}, \frac{(2\times-1)+(1\times-3)}{2+1}$$

$$D(x,y) = \frac{8-2}{3}, \frac{-2-3}{3}$$

$$D(x,y) = \frac{6}{3}, \frac{-5}{3}$$

$$D(x,y) = 2, \frac{-5}{3}$$

Q. 3 To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. 7.12. Niharika runs $\frac{1}{4}$th thedistance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

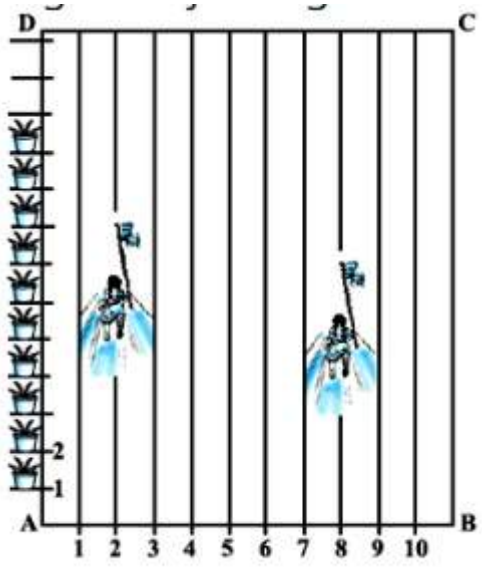


Fig. 7.12

Answer:

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., 1/4 × 100 = 25 m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25)

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., 1/5 x 100 = 20 m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20)

Now we have the positions of posts by Preet and Niharika
According to distance formula, distance between points A(x_1, y_1) and B(x_2, y_2) is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance between these flags by using distance formula, D

$= [(8 - 2)^2 + (25 - 20)^2]^{1/2}$

$= (36 + 25)^{1/2}$

$= \sqrt{61}m$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (x,y)

Now by midpoint formula,

$$(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

$$x = \frac{2+8}{2} = 5$$

$$y = \frac{25+20}{2} = 22.5$$

Hence, A (x, y) = (5, 22.5)

Therefore, Rashmi should post her blue flag at 22.5m on 5th line.

Q. 4 Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

Answer: Let the ratio in which the line segment joining (- 3, 10) and (6, - 8) is divided by point (- 1, 6) be k : 1

Using section formula

i.e. the coordinates of the points P(x, y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2,y_2)$, internally in the ratio m : n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$(-1, 6) = \left(\frac{k(6)+1(-3)}{k+1}, \frac{k(-8)+1(10)}{k+1}\right)$$

Therefore,

$$-1 = \frac{6k-3}{k+1}$$

$\Rightarrow$ -k – 1 = 6k – 3

$\Rightarrow$ - k -6k = -3 + 1

$\Rightarrow$ - 7k = -2

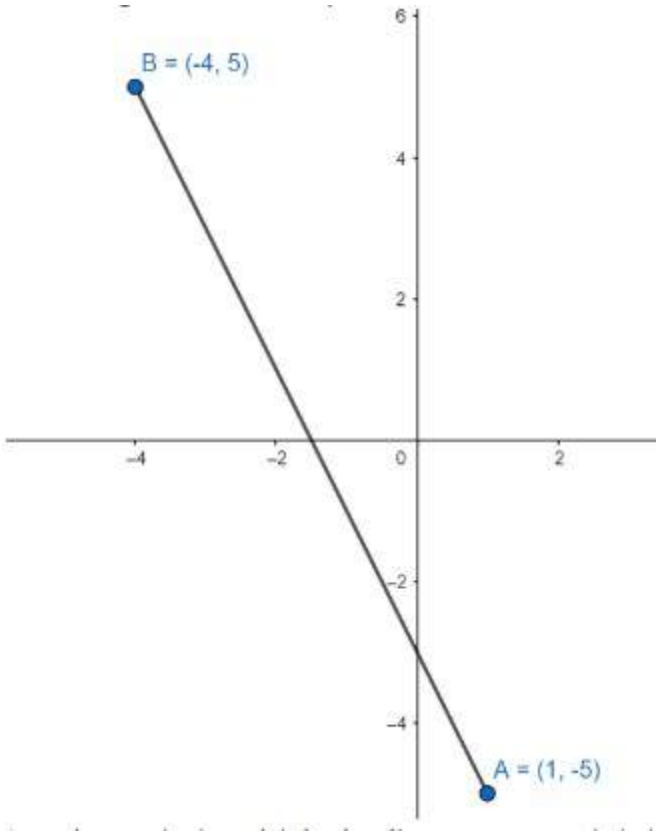
$\Rightarrow$ 7k = 2

$$\Rightarrow k = \frac{2}{7}$$

Therefore, the required ratio is 2: 7.

Q. 5 Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also, find the coordinates of the point of division.

Answer: The diagram for the question is



Let the ratio in which the line segment joining A (1, - 5) and B (- 4, 5) is divided by *x*-axis be k: 1

Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$

We know that *y*-coordinate of any point on *x*-axis is 0

Therefore,

$\frac{5k-5}{k+1} = 0$

k = 1

Therefore, *x*-axis divides it in the ratio 1:1.

Division point = $\left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right)$

$= \left(\frac{-4+1}{2}, \frac{5-5}{2}\right)$

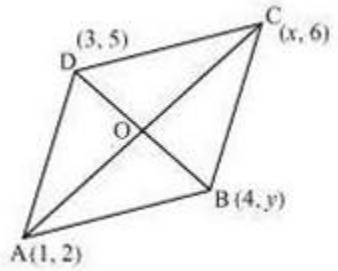
$= \left(\frac{-3}{0}, 0\right)$

Q. 6 If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer: Let (1, 2), (4, y), (x, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD.

Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.



By mid point formula, If (x, y) is the midpoint of the line joining points $A(x_1, y_1)$ and $B(x_2, y_2)$ Then,

$(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

If O is the mid-point of AC, then the coordinates of O are:

$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{x+1}{2}, 4\right)$

If O is the mid-point of BD, then the coordinates of O are:

$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$

Since both the coordinates are of the same point O

Comparing the x coordinates we get,

$\frac{x+1}{2} = \frac{7}{2}$

$x + 1 = 7$

$x = 6$

And,

Comparing the y coordinates we get,

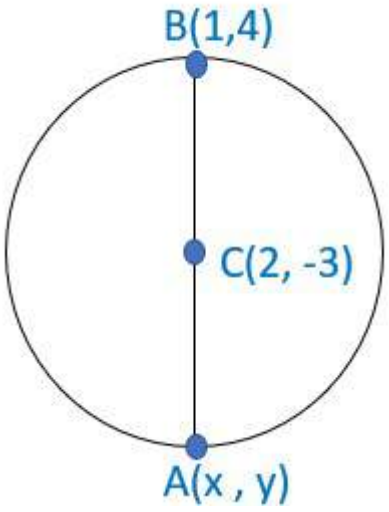
$\frac{5+y}{2} = 4$

5 + y = 8

y = 3

Hence, x = 6 and y = 3.

Q. 7 Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4)



For points A(x_1, y_1) and B(x_2, y_2), the midpoints (x, y) is given by

$(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

Let the coordinates of point A be (*x*, *y*)

Mid-point of AB is (2, - 3), which is the center of circle and point B is (1, 4)

Therefore,

$(2, -3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$

Equating the coordinates we get,

$\frac{x+1}{2} = 2, \frac{y+4}{2} = -3$

x = 4 – 1, y = -6 – 4

x= 3, y = -10

Therefore point is A (3, -10)

Therefore, the coordinates.

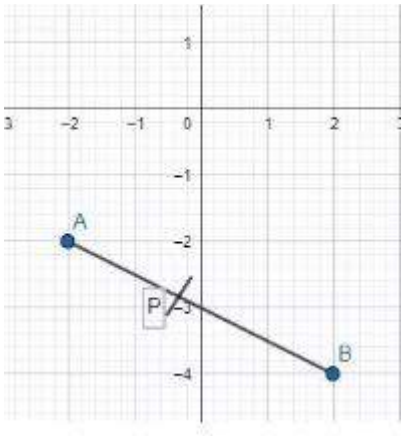
Q. 8 If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB

Answer: To find: The coordinates of P

Given: Points A(-2, -2) and B(2, -4) and ratio AP:AB = 3:7

The coordinates of point A and B are (- 2, - 2) and (2, - 4) respectively

AP = $\frac{3}{7}$ AB



Therefore, AP: PB = 3:4

Point P divides the line segment AB in the ratio 3:4

By section formula,
If a point divides the point (x_1, y_1) and (x_2, y_2) in the ratio m:n

Then, $(X, Y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

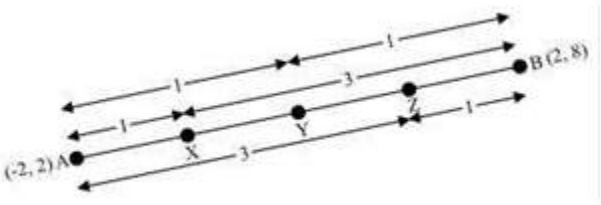
$$(X, Y) = \left(\frac{3\times2+4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right)$$

$$(X, Y) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$$

(-2/7, -20/7) is the point which divides line in the ratio of 3:4.

Q. 9 Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

Answer: It can be observed from the figure that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1respectively



Coordinate of X $= \left(\frac{1*2+3*-2}{1+3}, \frac{1*8+3*2}{1+3}\right)$

$= \left(-1, \frac{7}{2}\right)$

Coordinate of Y $= \left(\frac{2\pm2}{2}, \frac{2+8}{2}\right)$

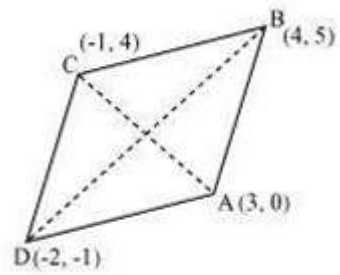
= (0, 5)

Coordinates of Z = $\left(\frac{3*2+1*-2}{3+1}, \frac{3*8+1*2}{3+1}\right)$

$= \left(1, \frac{13}{2}\right)$

Q. 10 Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order. [Hint: Area of a rhombus $= \frac{1}{2}$ (product of its diagonals)]

Answer: Let (3, 0), (4, 5), (- 1, 4) and (- 2, - 1) are the vertices A, B, C, D of a rhombus ABCD



Length of diagonal AC = $[(3 + 1)^2 + (0 – 4)^2]^{1/2}$

$= \sqrt{16 + 16}$

$= 4\sqrt{2}$

Length of diagonal BD = $[(4 + 2)^2 + (5+ 1)^2]^{1/2}$

$= \sqrt{36 + 36}$

$= 6\sqrt{2}$

Therefore,

Area of rhombus = 1/2 (Product of the length of the diagonals)

Therefore,

Area of rhombus ABCD = $1/2 \times 4\sqrt{2} \times 6\sqrt{2} = 1/2 \times 48$

= 24 square units