6. Indices

- We use exponents to write very large numbers.

  For example, 1000000000 can be written as $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

  It is read as ten raised to the power nine, where 10 is known as base and 9 as the exponent.

  The number 10^9 is known as exponential form of 1000000000.

- If the exponent of a negative base is odd, then the value of the exponential form is negative. However, if the exponent of a negative base is even, then the value of the exponential form is positive.

  For example, $(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$
  $(-1)^6 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = 1$

- **Laws of exponents** (Here, *a* and *b* are non-zero integers and *m* and *n* are integers)
  1. $a^m \times a^n = a^{m+n}$
  2. $\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$
  3. $(a^m)^n = a^{mn}$
  4. $a^m \times b^m = (ab)^m$
  5. $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
  6. $a^{-m} = \frac{1}{a^m}$
  7. $a^0 = 1 \quad (a \neq 0)$

  For example, $\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + \left(\frac{1}{11}\right)^{-1}$ can be simplified using laws of exponents as:

$\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + \left(\frac{1}{11}\right)^{-1}$

$= \frac{1^{-2}}{6^{-2}} + \frac{1^{-1}}{7^{-1}} + \frac{1^{-1}}{11^{-1}} \qquad \left(\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right)$

$= \frac{6^2}{1^2} + \frac{7^1}{1^1} + \frac{11^1}{1^1} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$

$= 36 + 7 + 11$

$= 54$

Prime factorization method of finding the square roots of numbers

The square root of 67600 can be found by prime factorization method as follows:
The number 67600 can be prime factorized as:
$\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5} \times \underline{13 \times 13}$
The numbers, 2, 2, 5, 13, occur in pairs. Therefore,
$\sqrt{67600} = 2 \times 2 \times 5 \times 13 = 260$

Example: Find the smallest number by which 252 can be multiplied to make it a perfect square.
Solution: We have, $252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$
The number 7 does not occur in pair. Therefore, if we multiply 252 by 7, then it will become a perfect square.
Therefore, $252 \times 7 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$
That is, 1764 is a perfect square and $\sqrt{1764} = 42$

Finding square root of perfect squares by division method

The steps of finding the square root of 1369 by division method are as follows:
Step1: Firstly, place bars over every pair of digits starting from the digit at ones place. We obtain $\overline{13}\,\overline{69}$.
Step2: Find the largest number whose square is less than or equal to the number under the extreme left bar.
Take this number as the divisor and the number under the extreme left bar as the dividend. Divide and obtain the remainder.

	3
3	13.69
	–9
9	4

Step3: Bring down the number under the next bar to the right of the remainder.
Therefore, the new dividend is 469.
Double the divisor and enter it with the blank on its right.

	3.
3	13.69
	–9
9	4.69

Step 4: Guess the largest possible digit to fill the blank, which becomes the new digit in the quotient, such that when
the new digit is multiplied to the new quotient, the product is less than or equal to the dividend.
In this case, $97 \times 7 = 469$
Therefore, the quotient is 7.
Also, the remainder becomes 0 and no bar is left.
Therefore, $\sqrt{1369} = 3.7$