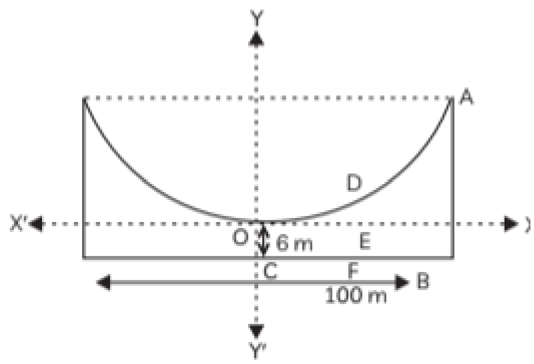
Conic Sections

Case Study Based Questions

1. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway, which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. A supporting wire is also attached to the roadway 18 meter from the middle. (see figure below)



Based on the above information, answer the following questions.

(A) Find the value of a in the standard equation.

(a) $\frac{24}{625}$ (b) $\frac{18}{625}$

(c) $\frac{625}{24}$ (d) $\frac{625}{18}$

(B) Find the length of a supporting wire attached to the roadway 18 m from the middle.

(a) 3.11 m

(b) 6 m

(c) 9.11 m

(d) none of these

(C) What are the coordinates of point A?

(a) (50, 30)

(b) (50, 24)

(c) (30,50)

(d) (24,50)

(D) What is a particular equation of parabola?

(a) $x^2 = 25y$

(b) $x^2 = 24y$

(c) $x^2 = 625y$

(d) $6x^2 = 625y$

(E) What is the parabola in this case? standard equation of

(a) $y^2 = 4ax$

(b) $y^2 = -4ax$

(c) $x^2 = 4ay$

(d) $x^2=-4ay$

Ans. (A)

(c) $\frac{625}{24}$

Explanation: Since, A(50, 24) is a point on the parabola.

$$(50)^2 = 4a(24)$$

$$\Rightarrow \quad a = 50\times\frac{50}{4}\times 24$$

$$= \frac{625}{24}$$

(B) (c) 9.11 m

Explanation: The x-coordinate of point D is 18.

Hence, at x = 18

$6(18)^2=625y$

$$\Rightarrow \quad y = 6\times 18\times\frac{18}{625}$$

$$\Rightarrow \quad y = 3.11 \text{ (approx.)}$$

$$\therefore \quad DE = 3.11 \text{ m}$$

$$DF = DE + EF$$

$$= 3.11 \text{ m} + 6 \text{ m}$$

$$= 9.11 \text{ m}$$

(C) (b) (50, 24)

Explanation: Here, AB = 30 m, OC = 6 m,

and $BC = \frac{100}{2} = 50$ m

The coordinates of point A are (50, 30 -6) = (50, 24)

(D) (d)$6x^2 = 625y$

Explanation: Equation of the parabola,

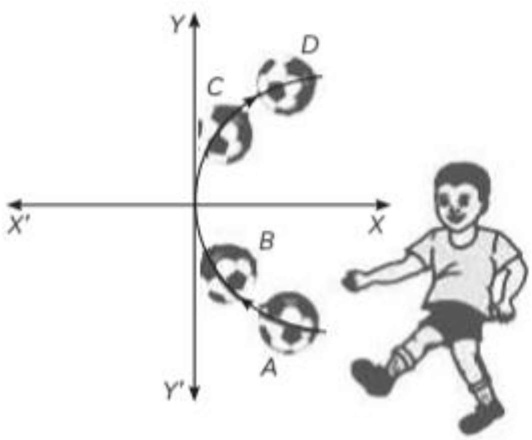
$$x = 4 \times \frac{625}{24} \times y$$

or $$6x^2 = 625y$$

(E) (c) $x^2 = 4ay$

Explanation: The equation of the parabola is of the form $x^2 = 4ay$ (as it is opening upwards).

2. Arun was playing a football match. When he kicked the football, the path formed by the football from ground level is parabolic, which is shown in the following graph. Consider the coordinates of point A as (3,-2).



(A) The equation of path formed by the football is:

(a) $y^2=x+1$

(b) $3x^2 = 4y$

(c) $3y^2 = 4x$

(d) $x^2=y-1$

(B) The equation of directrix of path formed by football is:

(a) $x - \frac{4}{3} = 0$ (b) $x + \frac{2}{3} = 0$

(c) $x + 3 = 0$ (d) $x + \frac{1}{3} = 0$

(C) The extremities of latus rectum of given curve are:

(a) $\left(\frac{1}{3}, \pm\frac{2}{3}\right)$ (b) $\left(\frac{2}{3}, \pm\frac{1}{3}\right)$

(c) $\left(\pm\frac{1}{3}, 1\right)$ (d) $\left(\pm\frac{1}{3}, \frac{4}{3}\right)$

(D) The length of latus rectum of a given curve is:

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$

(c) 3 (d) $\frac{4}{3}$

(E) The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is:

(a) $x = 1$ (b) $x = -1$

(c) $x = \frac{3}{2}$ (d) $x = -\frac{3}{2}$

Ans. (A) (c) $3y^2 = 4x$

Explanation: The path formed by football is in the shape of parabola. We know that general equation of parabola is $y^2 = 4ax$.

Since, it passes through (3,-2)

$(-2)^2 = 4 \times a \times 3$

$\Rightarrow \quad a = \frac{1}{3}$

⇒ Hence, required equation of path formed by football is $y^2 = \frac{4x}{3}$

$\Rightarrow \quad 3y^2 = 4x$

(B)

(d) $x + \frac{1}{3} = 0$

Explanation: Since, $a = \frac{1}{3}$. Therefore, the equation of its directrix is $x + \frac{1}{3} = 0$.

(C)

(a) $\left(\frac{1}{3}, \pm\frac{2}{3}\right)$

Explanation: The extremities of latus rectum are

$(a, \pm 2a) \equiv \left(\frac{1}{3}, \pm\frac{2}{3}\right)$

(D)

(d) $\frac{4}{3}$

Explanation: The length of latus rectum

$$= 4a = 4 \times \frac{1}{3} = \frac{4}{3}$$

(E)

(c) $x = \frac{3}{2}$

Explanation: Given equation:

$y^2 + 4y 4x 2 = 0$

Rearranging the equation, we get

$(y + 2)^2 = -4x+2$

$$(y+2)^2 = -4\left(x - \left(\frac{1}{2}\right)\right)$$

Let $\quad y = y + 2 \text{ and } x - \left(\frac{1}{2}\right)$

So, $y^2 = -4X$ ...(i)

Hence, equation (i) is of the form

y2 = -4ax ...(ii)

By comparing (i) and (ii), we get a = 1.

We know that equation of directrix is x = a

Now, substitute $a = 1$ and $x = \ x = x - \left(\frac{1}{2}\right)$ in

the directrix equation.

$$x - \left(\frac{1}{2}\right) = 1$$

$$x = 1 + \left(\frac{1}{2}\right) = \frac{3}{2}.$$

Therefore, the equation of the directrix of

the parabola $y^2 + 4y + 4x + 2 = 0$ is $\frac{3}{2}$

3. Karan, the student of class XI was studying in his house. He felt hungry and found that his mother was not at home. So, he went to the nearby shop and purchased a packet of chips. While eating the chips, he observed that one piece of the chips is in the shape of hyperbola. Consider the vertices of the hyperbola at (+ 5, 0) and focii at (± 7,0).

(A) Find the equation of hyperbolic curve and the length of conjugate axis formed by a given piece of chip.

(B) Find the eccentricity and length of latus rectum of the hyperbolic curve formed by a given piece of chips.

(C) Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola $3x^2 - y^2 = 4$.

Ans. (A) We have, a = 5 and ae = 7

Now, $b^2 = a^2 e^2 - a2$

= 49-25

= 24

So, equation is $\frac{x^2}{25} - \frac{y^2}{24} = 1$

Length of conjugate axis = 2b

=2×2√6

= 4√6

(B) Eccentricity,

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{24}{25}} = \sqrt{\frac{49}{25}} = \frac{7}{5}$$

$$\text{Length of latus-rectum} = \frac{2b^2}{a} = \frac{48}{5} = 9.6$$

(C) Given,

The equation $3x^2\ y^2 = 4$

The equation can be expressed as:

$$\frac{3x^2}{4} - \frac{y^2}{4} = 1$$

$$\frac{x^2}{\frac{4}{3}} - \frac{y^2}{4} = 1$$

$$\frac{x^2}{\left(\frac{2}{\sqrt{3}}\right)^2} - \frac{y^2}{(2)^2} = 1$$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a = \frac{2}{\sqrt{3}}$ and $b = 2$

Eccentricity is given by:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$= \sqrt{1 + \frac{4}{\frac{4}{3}}}$$

$$= \sqrt{1+3}$$

$$= \sqrt{4}$$

$$= 2$$

Foci: The coordinates of the foci are (+ ae, 0)

$$(\pm ae, 0) = \pm\left(\frac{2}{\sqrt{3}}\right)(2) = \pm\frac{4}{\sqrt{3}}$$

$$(\pm ae, 0) = \left(\pm\frac{4}{\sqrt{3}}, 0\right)$$

The equation of directrices is given as:

$$x = \pm\frac{a}{e}$$

$$\Rightarrow \quad x = \pm\frac{\frac{2}{\sqrt{3}}}{2}$$

$$\Rightarrow \quad x = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}x = \pm 1$$

The length of the latus-rectum is given as:

$$\frac{2b^2}{a} = \frac{2(4)}{\left[\frac{2}{\sqrt{3}}\right]}$$

$$= 4\sqrt{3}$$