# 14

# Kinetic Theory of Gases

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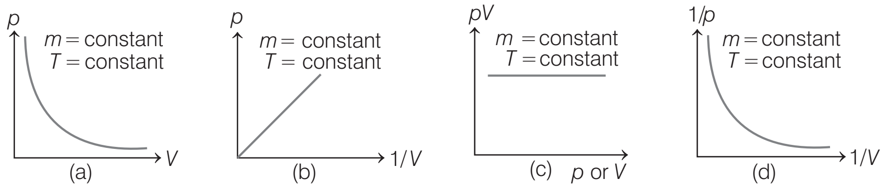
## Ideal Gas or Perfect Gas

A gas which strictly obeys the gas laws such as Boyle's law, Charles' law, Gay-Lussac's law, Avogadro's law, etc., is known as an *ideal gas* or *perfect gas*.

These law (gaseous laws) are given below

**Boyle's Law** According to this law, "for a given mass of an ideal gas at constant temperature (called *isothermal process*), the volume of a gas is inversely proportional to its pressure." *i.e.* $V \propto \frac{1}{p}$ (if $m$ and $T$ = constant)

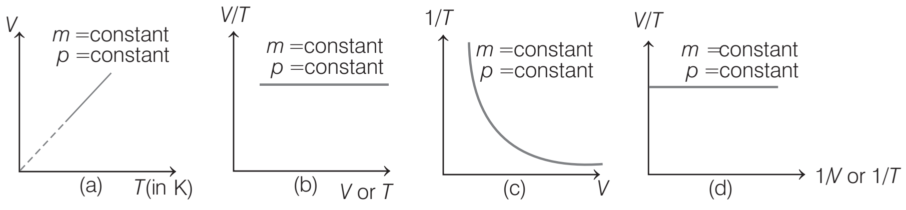
Graphical forms of such law are shown in following figure



**Charles' Law** According to this law, "for a given mass of an ideal gas at constant pressure (called **isobaric process**), volume of a gas is directly proportional to its absolute temperature."

*i.e.* $V \propto T$ (if $m$ and $p$ = constant)

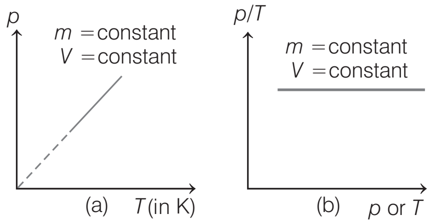
Graphical forms of such law are shown in following figure



**Gay-Lussac's Law or Pressure Law** According to this law, "for a given mass of an ideal gas at constant volume (called *isochoric process*), pressure of a gas is directly proportional to its absolute temperature."

*i.e.* $p \propto T$ (if $m$ and $V$ = constant)

Graphical forms of such law are shown in following figure



**Avogadro's Law** According to this law, "at same temperature and pressure, equal volumes of all the gases contain equal number of molecules."

*i.e.* $N_1 = N_2$

where, $N_1$ and $N_2$ are number at molecules at first and second, gas respectively.

## Equation of State of a Perfect Gas

In practice, the gases do not obey the gas laws at all values of temperature and pressure. It is because of the intermolecular forces between the gas molecules.

Ideal gas equation is a form of combined effect of above first four laws. Thus, the equation is given by

$$pV = nRT = \frac{m}{M} RT$$

Here, $n$ = number of moles of the gas = $\frac{m}{M}$,

$m$ = total mass of the gas,

$M$ = molecular mass

and $R$ = universal gas constant

$= 8.31 \text{ Jmol}^{-1} \text{ K}^{-1}$

$= 2.0 \text{ cal mol}^{-1}\text{K}^{-1}$.

Above equation is also known as equation of state of an ideal gas.

- In terms of density, the ideal gas equation may be expressed as

$$\frac{p}{\rho T} = \text{constant}$$

- In terms of number of molecules ($n$) per unit volume of a gas, the ideal gas equation may be expressed as

$$p = nkT$$

- If $n_1$ mole of a non-reactive gas in thermodynamical state $(p_1, V_1, T_1)$ be mixed with $n_2$ mole of another non-reactive gas at $(p_2, V_2, T_2)$ and the resultant gas mixture is at a state $(p, V, T)$, then

$$\frac{p_1 V_1}{T_1} + \frac{p_2 V_2}{T_2} = \frac{pV}{T}$$

## Work Done on Compressing a Gas

Work done on compressing a gas can be written as

$$W = \frac{mR(T_2 - T_1)}{1 - n}$$

where, $m$ = total mass of the gas,

$R$ = universal gas constant,

$n$ = number of moles of the gas,

$T_1$ and $T_2$ = initial and final temperature of the gas.

***Example 1.*** *During an experiment, an ideal gas is found to obey an additional law $Vp^2$ = constant. The gas is initially at temperature T and volume V. What will be the temperature of the gas when it expands to a volume 2V?*

(a) $\sqrt{3}\, T$ (b) $\sqrt{1/2}\, T$

(c) $\sqrt{2}\, T$ (d) $\sqrt{3}\, T$

***Sol.*** (c) Here it is given that $Vp^2$ = constant $K$ (say). Hence, we may write the gas equation as, $pV = nRT$

or $$\sqrt{\frac{K}{V}} \cdot V = nRT \quad \text{or} \quad \sqrt{V} = \frac{nR}{\sqrt{K}} T$$

$$\Rightarrow \quad \sqrt{\frac{V_1}{V_2}} = \frac{T_1}{T_2}$$

$$\therefore \quad T_2 = T_1 \sqrt{\frac{V_2}{V_1}} = T\sqrt{\frac{2V}{V}} = \sqrt{2} T$$

***Example 2.*** *A closed container of volume $0.02\, m^3$ contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of $1 \times 10^5\, Nm^{-2}$. The total mass of the mixture is 28 g. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal.*

*(Given, $R = 8.31\, Jmol^{-1}K^{-1}$)*

(a) 4 g, 24 g (b) 5 g, 25 g

(c) 6 g, 26 g (d) 7 g, 27 g

***Sol.*** (a) Let in the given container mass of neon be $m$ and mass of argon be $(28 - m)$ g, so that

$$n_{\text{Ne}} = \frac{m}{20}$$

and $$n_{\text{Ar}} = \frac{28 - m}{40}$$

$$n = n_{\text{Ne}} + n_{\text{Ar}} = \frac{m}{20} + \frac{(28 - m)}{40} = \frac{28 + m}{40} \quad \text{...(i)}$$

and using ideal gas equation for the mixture, we have

$$n = \frac{pV}{RT} = \frac{1 \times 10^5 \times 0.02}{8.314 \times 300} = 0.8 \quad \text{...(ii)}$$

Comparing Eqs.(i) and (ii), we get

$$\frac{28 + m}{40} = 0.8$$

$$\Rightarrow \quad m = 4 \text{ g}$$

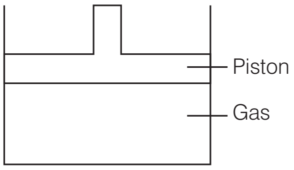
$$\therefore \quad m_{\text{Ne}} = 4 \text{ g}$$

and $$m_{\text{Ar}} = 28 - 4 = 24 \text{ g}$$

**Example 3.** *Two moles of an ideal gas is contained in a cylinder fitted with a frictionless movable piston, exposed to the atmosphere, at an initial temperature $T_0$. The gas is slowly heated so that its volume becomes four times the initial value. The work done by the gas is*

*(a) zero* *(b)* $2RT_0$ *(c)* $4RT_0$ *(d)* $6RT_0$

**Sol.** (*d*) Given that gas is slowly heated, which means it remains in equilibrium with the atmosphere, *i.e.* the process takes place at constant pressure.



From the equation of ideal gas law,

$$pV = nRT$$

For infinitesimal change,

$$pdV = nR\,dT$$

or $$p\Delta V = nR\,\Delta T$$

Also, $p\Delta V =$ work done by the gas $= \Delta W$

$\therefore$ $$\Delta W = nR\Delta T$$

Also, $$\Delta V \propto \Delta T$$

$\therefore$ $$\Delta T \propto \Delta V \propto V_2 - V_1$$

Given, $$V_2 = 4V_1$$

$\therefore$ $$\Delta T \propto 4V_1 - V_1 \propto 3V_1 \propto 3T_0$$

Also, it is given that, $n = 2$ moles

The expression for work done becomes

$$\Delta W = nR\Delta T$$

$$\Delta W = 2R\,3T_0 = 6RT_0$$

**Example 4.** *One mole of an ideal gas passes through a process, where pressure and volume obey the relation* $p = p_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. *Here, $p_0$ and $V_0$ are constants. Calculate the change in the temperature of the gas if its volume changes from $V_0$ to $2V_0$.* **[JEE Main 2019]**

*(a)* $\frac{1}{2}\frac{p_0V_0}{R}$ *(b)* $\frac{1}{4}\frac{p_0V_0}{R}$ *(c)* $\frac{3}{4}\frac{p_0V_0}{R}$ *(d)* $\frac{5}{4}\frac{p_0V_0}{R}$

**Sol.** (*d*) Given process equation for 1 mole of an ideal gas is

$$p = p_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right] \qquad \ldots(\text{i})$$

Also, for 1 mole of ideal gas,

$$pV = RT$$

$\therefore$ $$p = \frac{RT}{V} \qquad \ldots(\text{ii})$$

So, from Eqs. (i) and (ii), we have

$$\frac{RT}{V} = p_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$$

$\therefore$ $$T = \frac{p_0V}{R}\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right] \qquad \ldots(\text{iii})$$

When volume of gas is $V_0$, then by substituting $V = V_0$ in Eq. (iii), we get

Temperature of gas is

$$T_1 = \frac{p_0V_0}{R}\left[1 - \frac{1}{2}\left(\frac{V_0}{V_0}\right)^2\right] = \frac{p_0V_0}{2R}$$

Similarly, at volume, $V = 2V_0$

Temperature of gas is

$$T_2 = \frac{p_0(2V_0)}{R}\left[1 - \frac{1}{2}\left(\frac{V_0}{2V_0}\right)^2\right] = \frac{7}{4}\frac{p_0V_0}{R}$$

So, change in temperature as volume changes from $V_0$ to $2V_0$ is

$$\Delta T = T_2 - T_1 = \left(\frac{7}{4} - \frac{1}{2}\right)\frac{p_0V_0}{R} = \frac{5}{4}\frac{p_0V_0}{R}$$

**Example 5.** *A vessel contains two non-reactive gases neon (monoatomic) and oxygen (diatomic). The ratio of their partial pressure is 3 : 2. The ratio of number of molecules is [Given atomic mass of Ne = 20.2 u, molecular mass of $O_2$ = 32 u]* **[NCERT]**

*(a)* $\frac{2}{3}$ *(b)* $\frac{3}{2}$ *(c)* $\frac{4}{3}$ *(d)* $\frac{3}{4}$

**Sol.** (*b*) Each gas (assumed ideal) obey the gas laws. Since, $V$ and $T$ are common to the two gases, we have

$$p_1V = \mu_1RT \quad \text{and} \quad p_2V = \mu_2RT$$

$$\frac{p_1}{p_2} = \frac{\mu_1}{\mu_2}$$

Here, 1 and 2 refer to neon and oxygen gas respectively.

Given, $$\frac{p_1}{p_2} = \frac{3}{2}$$

$$\frac{\mu_1}{\mu_2} = \frac{3}{2}$$

By definition, $$\mu_1 = \frac{N_1}{N_A} \quad \text{and} \quad \mu_2 = \frac{N_2}{N_A}$$

where, $N_1$ and $N_2$ are the number of molecules of two gases and $N_A$ is Avogadro's number.

$$\frac{N_1}{N_2} = \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

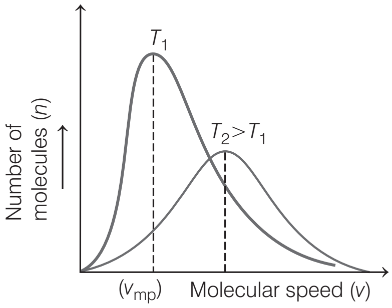
# Kinetic Theory of Gases

The kinetic theory of gases correlates the macroscopic properties of gases (*e.g.* pressure, temperature, etc) to the microscopic properties of gas molecules (*e.g.* speed, momentum, kinetic energy of molecules, etc). This theory attempts to develop a model of the molecular behaviour of gas, which results in the observed behaviour of an ideal gas. The kinetic theory of gases is based on the following assumptions

(i) A gas consists of a large number of tiny, identical, spherical and electrically neutral, stable elastic particles called *molecules*.

(ii) The space occupied by the molecules of a gas is extremely small as compared to the volume of the gas.

(iii) The molecules of a gas are in a state of continuous, random motion with all possible speeds ranging from zero to infinity in different possible directions. The speed distribution is in accordance with Maxwell's distribution law of molecular speeds and has been shown in figure.



(iv) Each molecule behaves as an independent entity. There is no force of attraction among the molecules. Thus, gas molecules have no potential energy but possess only kinetic energy which is directly proportional to temperature of the gas.

(v) The pressure of a gas is due to elastic collision of gas molecules with the walls of the container.

(vi) The dynamics of the particles is governed by Newton's laws of motion.

(vii) The time of contact of a moving molecule with the container walls at the time of collision is negligible as compared to the time between two successive collisions with the same wall of the container.

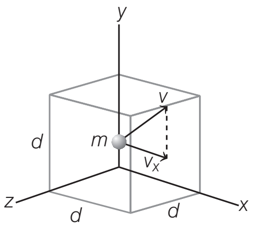
## Concept of Pressure

On the basis of above assumptions, we can perform mathematical calculations to find expression for pressure exerted by a gas. Accordingly, we find that

$$pV = \frac{1}{3} mN (\overline{v})^2$$

where, $m$ = mass of 1 gas molecule and $N$ = total number of gas molecules and $\overline{v}$ = root mean square velocity.

$$v_{\text{rms}} = \overline{v} = \left[\frac{v_1^2 + v_2^2 + \ldots + v_N^2}{N}\right]^{1/2}$$



A cubical box with sides of length $d$ containing an ideal gas. The molecule shown moves with velocity $v$.

Thus, pressure exerted by a gas $p = \frac{1}{3}\frac{mN}{V}\overline{v}^2 = \frac{1}{3}\rho\overline{v}^2$,

where $\rho = \frac{mN}{V}$ = density of given gas.

Now, $$p = \frac{1}{3} m \left(\frac{N}{V}\right)\overline{v}^2$$
$$= \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}m\overline{v}^2\right)$$

or $$pV = \frac{2}{3} N \text{ KE}$$

Also, $$p = \frac{1}{3}\rho\overline{v}^2 = \frac{2}{3}\left(\frac{1}{2}\rho\overline{v}^2\right)$$

Now, $\frac{1}{2}\rho\overline{v}^2$ = average kinetic energy of the gas per unit volume = $E$.

$\therefore$ $$p = \frac{2}{3}E$$

## Kinetic Energy and Temperature

According to kinetic theory of gases, $pV = \frac{1}{3} mN\overline{v}^2$ ...(i)

but according to equation of state for an ideal gas,

$$pV = nRT \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2}m\overline{v}^2 = \frac{3}{2}\frac{RT}{n} = \frac{3}{2}kT$$

where, $k$ is the Boltzmann's constant. Its value is $1.38 \times 10^{-23}$ J mol$^{-1}$ K$^{-1}$.

$\therefore$ **Mean translational kinetic energy** of a gas molecule $= \frac{3}{2}kT$, *i.e.* the mean translational kinetic energy of a gas molecule depends only on its temperature and is independent of its nature or mass, etc.

On this basis, we can define *absolute zero temperature* as the temperature at which translational kinetic energy of a gas molecule becomes zero, *i.e.* at which the molecular motion ceases altogether.

***Example 6.*** *At what temperature does the average translational kinetic energy of a molecule in a gas becomes equal to the kinetic energy of an electron accelerated from rest through 1 V potential difference* ($k = 1.38 \times 10^{-23}$ $JK^{-1}$)

(a) 7330 K (b) 7730 K
(c) 7530 K (d) 7430 K

***Sol.*** (*b*) Kinetic energy gained by an electron when accelerated by a potential difference of 1 V is $1\text{ eV} = 1.6 \times 10^{-19}$ J.

According to kinetic theory of gases, kinetic energy $= \frac{3}{2}kT$

As, $$\frac{3}{2}kT = 1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$$

$\Rightarrow$ $$T = \frac{2 \times 1.6 \times 10^{-19}}{3k}$$
$$= \frac{2 \times 1.6 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}} = 7730\text{ K}$$

## Critical Temperature, Pressure and Volume

Gases cannot be liquified above a temperature called *critical temperature* ($T_c$) however large the pressure may be. The pressure required to liquify the gas at critical temperature is called *critical pressure* ($p_c$) and the volume of the gas at critical temperature and pressure is called *critical volume* ($V_c$).

Value of critical constants in terms of Van der Waals' constants $a$ and $b$ are as under

$$V_c = 3b,$$

$$p_c = \frac{a}{27b^2}$$

and $$T_c = \frac{8a}{27Rb}$$

Further, $\frac{RT_c}{p_cV_c} = \frac{8}{3}$ is called **critical coefficient** and is same for all gases.

# Different Speeds of Gas Molecules

- **rms Speed** The square root of the mean of squares of the speeds of different molecules is known as root mean square (rms) speed of gas molecules.

  *i.e.* $$v_{\text{rms}} = \sqrt{\overline{v}^2}$$

  $$= \sqrt{(v_1^2 + v_2^2 + \ldots + v_N^2)/N}$$

  According to kinetic theory of gases, it is observed that

  $$v_{\text{rms}} = \sqrt{\overline{v}^2} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3pV}{M}}$$

  $$= \sqrt{\frac{3RT}{M}}$$

  $$= \sqrt{\frac{3kT}{m}}$$

  where, $M$ is the molar mass of gas, while $m$ is the mass of a single gas molecule.

- **Average Speed** It is the arithmetic mean of the speeds of molecules in a gas. Thus,

  $$v_{\text{av}} = \overline{v} = \frac{v_1 + v_2 + \ldots + v_N}{N}$$

  On the basis of kinetic theory, it is observed that

  $$v_{\text{av}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8p}{\pi\rho}} = \sqrt{\frac{8kT}{\pi m}}$$

- **Most Probable Speed** It is the speed possessed by maximum number of gas molecules in a given gas. Basis on kinetic theory, it is found that

  $$v_{\text{mp}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2p}{\rho}}$$

  Thus, we find that for a given gas, $v_{\text{mp}} < v_{\text{av}} < v_{\text{rms}}$ and $v_{\text{mp}} : v_{\text{av}} : v_{\text{rms}} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$.

***Example 7.*** *A flask contains argon and chlorine in the ratio of $2:1$ by mass. The temperature of the mixture is $27°C$. Ratio of root mean square speed $v_{rms}$ of the molecules of the two gases is [Given, atomic mass of argon = 39.9 u, molecular mass of chlorine = 70.9 u]* ***[NCERT]***

(a) 0.22 (b) 2.2 (c) 1.33 (d) 3.3

**Sol.** (c) The average kinetic energy (per molecule) of any (ideal) gas is always equal to $\frac{3}{2}k_BT$. It depends only on temperature and is independent of the nature of the gas. Since, argon and chlorine both have the same temperature in the flask, therefore the ratio of average kinetic energy (per molecule) of the two gases is 1 : 1.

Now, $$\frac{1}{2}mv_{\text{rms}}^2 = \text{average kinetic energy per molecule}$$

$$= \frac{3}{2}k_BT$$

$$\frac{(v_{\text{rms}}^2)_{\text{Ar}}}{(v_{\text{rms}}^2)_{\text{Cl}}} = \frac{(m)_{\text{Cl}}}{(M)_{\text{Ar}}} = \frac{(M)_{\text{Cl}}}{(M)_{\text{Al}}} = \frac{70.9}{39.9} = 1.77$$

where, $M$ denotes the molecular mass of the gas. Taking square root

$$\frac{(v_{\text{rms}})_{\text{Ar}}}{(v_{\text{rms}})_{\text{Cl}}} = 1.33$$

***Example 8.*** *For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at $127°$ C. At 2 atm pressure and at $227°$ C, the rms speed of the molecules will be*

(a) $100\sqrt{5}$ m/s (b) 80 m/s ***[JEE Main 2019]***

(c) 100 m/s (d) $80\sqrt{5}$ m/s

**Sol.** (a) For a gas molecule, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\therefore \quad v_{\text{rms}} \propto \sqrt{T}$$

Let unknown rms speed be $v_{\text{rms, 2}}$.

At $T_2 = 227°\text{C (or 500 K)}$

and at $T_1 = 127°\text{C (or 400 K)}$

$v_{\text{rms,1}} = 200 \text{ m/s}$

$\therefore$ Using the relation $v_{\text{rms}} \propto \sqrt{T}$, we can write

$$\frac{v_{\text{rms, 2}}}{v_{\text{rms, 1}}} = \sqrt{\frac{T_2}{T_1}} \quad \ldots \text{(i)}$$

Substituting these given values in Eq. (i), we get

$$\therefore \quad v_{\text{rms, 2}} = \sqrt{\frac{500}{400}} \times 200 \text{ m/s}$$

$$= \frac{1}{2}\sqrt{5} \times 200 \text{ m/s} = 100\sqrt{5} \text{ m/s}$$

***Example 9.*** *The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to (Given, Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J/K, Avogadro number $N_A = 6.02 \times 10^{26}$/kg, radius of earth = $6.4 \times 10^6$ m and gravitational acceleration on earth = $10 \text{ ms}^{-2}$]*

(a) $10^4$ K (b) 650 K (c) $3 \times 10^5$ K (d) 800 K

***Sol.*** (*a*) Root mean square velocity of hydrogen molecule is given as

$$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}}$$

Escape velocity of hydrogen molecule from the earth is given as

$$v_e = \sqrt{2gR_e}$$

Given, $v_{\text{rms}} = v_e$

or $$\sqrt{\frac{3k_BT}{m}} = \sqrt{2gR_e}$$

$$\Rightarrow \quad T = \frac{2gR_em}{3 \times k_B}$$

Substituting the given values, we get

$$T = \frac{2 \times 10 \times 6.4 \times 10^6 \times 2}{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}} \approx 10^4 \text{ K}$$

**Note** Question gives Avogadro number, $N_A = 6.02 \times 10^{26}$/kg but we take $N_A = 6.02 \times 10^{26}$/kmol.

## Van der Waals' Gas Equation

The gases actually found in nature are called *real gases*. They do not obey the gas laws. A real gas behaves as ideal gas most closely at *low pressure and high temperature.*

Equation of state for real gases is given by Van der Waals' equation with two corrections in ideal gas (i) volume correction (ii) pressure correction.

Van der Waals' gas equation for 1 mole of gas is given by

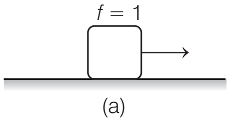
$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

For $n$ moles, $$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

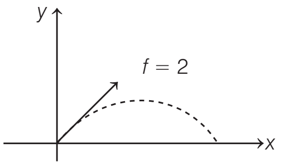
Here, $a$ and $b$ are constants called **Van der Waals' constants.**

# Degree of Freedom ($f$)

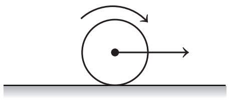
The term degree of freedom refers to the number of possible independent ways in which a system can have energy.



(a)



(b)



(c)

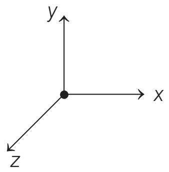
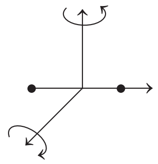
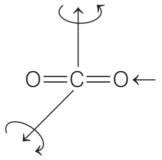
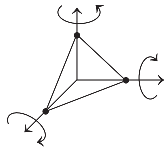
**For example** In Fig. (a), block has one degree of freedom, because it is confined to move in a straight line and has only one translational degree of freedom.

In Fig. (b), the projectile has two degrees of freedom because it is confined to move in a plane and so it has two translational degrees of freedom.

In Fig. (c), the sphere has two degrees of freedom, *i.e.* one rotational and another translational.

Similarly, a particle free to move in space will have three translational degrees of freedom.

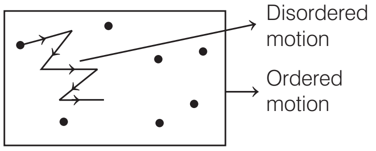
- Degrees of freedom for different gases according to atomicity of gas at low temperature is given below.

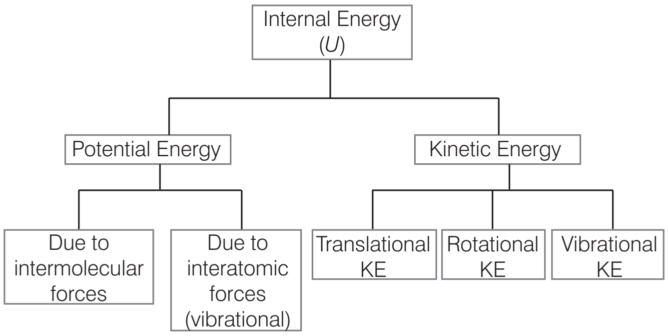
| Atomicity of Gas | Translational | Rotational | Total | |
|---|---|---|---|---|
| Monoatomic, *e.g.* Ar, Ne, ideal gas, etc. | 3 | 0 | 3 |  |
| Diatomic, *e.g.* $O_2$, $Cl_2$, $N_2$, etc. | 3 | 2 | 5 |  |
| Triatomic (linear) , *e.g.* $CO_2$, $C_2H_2$, etc. | 3 | 2 | 5 |  |
| Triatomic (non-linear) or Polyatomic, *e.g.* $H_2O$, $NH_3$, $CH_4$, etc. | 3 | 3 | 6 |  |

- Degrees of freedom of a diatomic and polyatomic gas depends on temperature and since there is no clear cut demarcation line above which vibrational energy become significant. Moreover, this temperature varies from gas to gas. On the other hand, for a monoatomic gas there is no such confusion.
- Degree of freedom here is 3 at all temperatures. Unless and until stated in the question you can take $f = 3$ for monoatomic gas, $f = 5$ for a diatomic gas and $f = 6$ for a non-linear polyatomic gas.
- When a diatomic or polyatomic gas dissociates into atoms it behaves as a monoatomic gas, whose degrees of freedom are changed accordingly.

## Internal Energy of an Ideal Gas

The internal energy ($U$) of the gas is concerned only with its disordered motion, *i.e.* it is not concerned with its ordered motion. When the temperature of the gas is increased, its disordered motion increases and hence its internal energy is increased.





Internal energy of an ideal gas depends only on its absolute temperature ($T$) and is directly proportional to $T$.

or $$U \propto T$$

## Law of Equipartition of Energy

According to the law of equipartition of energy, an ideal gas distributes its internal energy equally in all degrees of freedom. In each degree of freedom, energy of one mole of an ideal gas is $\frac{1}{2}RT$, where $T$ is the absolute temperature of the gas. Thus, if $f$ be the number of degrees of freedom, then the internal energy of 1 mole of the gas will be $\frac{f}{2}RT$ or internal energy of $n$ moles of the gas will be $\frac{n}{2}fRT$. Thus,

$$U = \frac{n}{2}fRT \qquad \ldots\text{(i)}$$

For a monoatomic gas, $f = 3$.

Therefore, $$U = \frac{3}{2}RT$$

(for 1 mole of a monoatomic gas)

For a diatomic and linear polyatomic gas at low temperatures, $f = 5$, so,

$$U = \frac{5}{2}RT \qquad \text{(for 1 mole)}$$

and for non-linear polyatomic gas at low temperatures, $f = 6$, so

$$U = \frac{6}{2}RT = 3RT \qquad \text{(for 1 mole)}$$

***Example 10.*** *A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is*

*(a)* 15 *(b)* 13
*(c)* 11 *(d)* 20

***Sol.*** (a) Internal energy of a gas mixture = Sum of internal energies of its components.

Also, internal energy of a gas is given by

$$U = \frac{f}{2}(nRT)$$

Here, gases are oxygen (diatomic, $f = 5$) and argon (monoatomic, $f = 3$).

So, total internal energy of the mixture is

$$U = \left(\frac{f}{2}nRT\right)_{O_2} + \left(\frac{f}{2}nRT\right)_{Ar}$$
$$= \frac{5}{2} \times 3 \times RT + \frac{3}{2} \times 5 \times RT = 15RT$$

## Molar Specific Heat of the Gases

Consider a container containing $m$ gram of gas of molecular mass $M$. If $n$ is the number of moles of gas in container, $\Delta Q$ is the heat supplied and rise in temperature is $\Delta T$, then

$$c = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

Further, $$n = \frac{m}{M} \quad \text{or} \quad m = nM$$

so $$c = \frac{1}{nM}\frac{\Delta Q}{\Delta T}$$

Thus, molar specific heat,

$$C = Mc = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)$$

where, $C$ = molar specific heat of the mass
and $c$ = gram specific heat of the gas.

Molar specific heat is of two types

(i) **Specific heat at constant volume** ($C_V$) When heat is supplied to the gas at constant volume, the entire heat supplied just increases the internal energy of gas and does nothing else.

$$C_V = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)_V$$

Thus, $$C_V = \frac{1}{n}\left(\frac{\Delta U}{\Delta T}\right)$$

(ii) **Specific heat at constant pressure** ($C_p$) When heat is supplied to the gas at constant pressure, a part of it increases the internal energy of the gas and remaining does an external work.

So, specific heat at constant pressure,

$$C_p = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)_p$$

- At constant pressure, to increase the internal energy of the gas by the same amount (as in case of heat supplied at constant volume), more amount of heat is supplied. Hence, we conclude that $C_p > C_V$
- The relation between $C_p$ and $C_V$ is given by Mayer's relation which is $C_p - C_V = R$

  Here, $R$ is gas constant.
- $C_p$ and $C_V$ in terms of degrees of freedom $f$ can be written as $C_V = \frac{f}{2}R$

  and $$C_p = C_V + R = \frac{f}{2}R + R = \left(\frac{f}{2} + 1\right)R$$
- Ratio of specific heats $C_P$ and $C_V$ is $\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$

## Specific Heat Capacity of Monoatomic, Diatomic and Polyatomic Gases

**Monoatomic Gases** The molar specific heat at constant volume $C_V$ is

$$C_V \text{ (monoatomic gas)} = \frac{dU}{dT} = \frac{3}{2}R$$

From Mayer's formula, $C_p - C_V = R$

where, $C_p$ is molar specific heat at constant pressure.

Thus, $C_p = \frac{5}{2}R$

Ratio of specific heat, $\gamma = \frac{C_p}{C_V} = \frac{5}{3}$

**Diatomic Gases** A diatomic molecule has 5 degrees of freedom, *i.e.* 3 translational and 2 rotational. Using the law of equipartition of energy, the total internal energy of a mole of such a gas is

$$U = \frac{5}{2}k_BT \times N_A = \frac{5}{2}RT$$

The molar specific heats are then given by

$$C_V \text{ (rigid diatomic)} = \frac{5}{2}R, C_p = \frac{7}{2}R$$

$$\gamma \text{ (rigid diatomic)} = \frac{7}{5}$$

If the diatomic molecule is not rigid but has an additional vibrational mode, then

$$U = \left(\frac{5}{2}k_BT + k_BT\right)N_A = \frac{7}{2}RT$$

$$C_V = \frac{7}{2}R, C_p = \frac{9}{2}R, \gamma = \frac{9}{7}R$$

**Polyatomic Gases** A polyatomic molecule has 3 translational, 3 rotational degrees of freedom and a certain number $(f)$ of vibrational modes.

From law of equipartition of energy, one mole of such a gas has

$$U = \left(\frac{3}{2}\ k_BT + \frac{3}{2}k_BT + f\,k_BT\right)N_A$$

$$C_V = (3 + f)R$$
$$C_p = (4 + f)R$$
$$\gamma = \frac{(4 + f)}{(3 + f)}$$

**Note** Specific heat of lighter elements is higher than that of heavier elements and *vice versa*. Specific heat of the same substance in different states (solid, liquid and vapour) is different. *For example*, specific heat of water is $1 \text{ cal g}^{-1\circ}\text{C}^{-1}$ and that of ice is $0.5 \text{ cal g}^{-1\circ}\text{C}^{-1}$.

***Example 11.*** *Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy U of a mole of this gas and the value of* $\gamma\left(= \frac{C_p}{C_V}\right)$ *are given respectively, by* **[JEE Main 2020]**

(a) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$
(b) $U = 5RT$ and $\gamma = \frac{7}{5}$
(c) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$
(d) $U = 5RT$ and $\gamma = \frac{6}{5}$

***Sol.*** (*c*) Given that, translational degree of freedom, $f_{\text{trans}} = 3$

Rotational degree of freedom, $f_{\text{rot}} = 2$

Total degree of freedom,

$$f = f_{\text{trans}} + f_{\text{rot}} = 3 + 2 = 5$$

At $T$ temperature, total internal energy of an ideal gas is given by $U = \frac{f}{2}\mu RT$

$$U = \frac{5}{2}RT \qquad (\because \mu = 1)$$

$$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f} = 1 + \frac{2}{5}$$

$$\gamma = \frac{7}{5}$$

***Example 12.*** *A cylinder of fixed capacity* 44.8 *L contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperrature of the gas in the cylinder by* $15°\text{C}$ *is (Given,* $R = 8.31\, J\, mol^{-1}K^{-1}$*)*

(*a*) 45 *J*
(*b*) 374 *J*
(*c*) 273 *J*
(*d*) 432 *J*

***Sol.*** (*b*) From ideal gas law, $pV = \mu RT$

1 mole of any (ideal) gas at standard temperature (273 K) and pressure ($1 \text{ atm} = 1.01 \times 10^5$ Pa) occupies a volume of 22.4 L. This universal volume is called molar volume. Thus, the cylinder in this example contains 2 mole of helium. Further, since helium is monoatomic, its predicted (and observed) molar specific heat at constant volume $C_V = \frac{3}{2}R$ and molar specific heat at constant pressure.

$$C_p = \frac{3}{2}R + R = \frac{5}{2}R$$

Since, volume of the cylinder is fixed, the heat required is determined by $C_V$.

∴ Heat required = Number of moles × Molar specific heat at constant volume × Rise in temperature

$$= 2 \times 1.5 R \times 15 = 45 R$$

$$= 45 \times 8.31 = 374 \text{ J}$$

## Mean Free Path

Every gas consists of a very large number of molecules. These molecules are in a state of continuous rapid and random motion. They undergo perfectly elastic collision against one another. Therefore, path of a single gas molecule consists of a series of short *zig-zag* paths of different lengths. The mean free path of a gas molecule is the average distance between two successive collisions.

Mathematically, it is expressed as $\lambda = \dfrac{1}{\sqrt{2}\,\pi d^2 n_V}$

where, $d$ = diameter of molecules,

$n_V$ = number of molecules per unit volume.

The number of molecules per unit volume can be determined from Avogadro's number and the ideal gas law leading to

$$n_V = \frac{nN_A}{V} = \frac{nN_A}{\dfrac{nRT}{p}} = \frac{N_A p}{RT}$$

$$\therefore \quad \lambda = \frac{RT}{\sqrt{2}\,\pi d^2 N_A p}$$

One mole of an ideal gas will occupy a volume of 22.4L at STP.

Avogadro's number, $N_A = 6.0221367 \times 10^{23}$ mol

**Note** Standard temperature 0° C = 273.15 K

Standard pressure = 1 atmosphere = 760 mm of Hg = 101.3 kPa

Standard volume of 1 mole of an ideal gas at STP = 22.4 L.

***Example 13.*** *In a dilute gas at pressure p and temperature T, the mean time between successive collisions of a molecule varies with T as*

(a) $T$ (b) $\dfrac{1}{\sqrt{T}}$ (c) $\dfrac{1}{T}$ (d) $\sqrt{T}$

**Sol.** (*b*) The expression of mean time ($\tau$) between successive collisions of a molecule at pressure $p$ and temperature $T$ is given by

$$\tau = \frac{\lambda}{v_{\text{avg}}}$$

As, mean free path of gas molecule, $\lambda = \dfrac{1}{\sqrt{2}\pi d^2 n_V}$

where, $n_V$ = number of molecules volume.

$$\Rightarrow \quad \lambda \propto (T)^0$$

Also, average speed, $v_{\text{avg}} = \sqrt{\dfrac{8RT}{\pi M}}$

$$\Rightarrow \quad v_{\text{avg}} \propto \sqrt{T} \ \therefore \ \tau \propto \frac{1}{\sqrt{T}}$$

***Example 14.*** *A vessel contains a mixture of 7 g of nitrogen and 11g of carbon dioxide at temperature T = 300 K. If the pressure of the mixture is 1 atm ($1 \times 10^5$ N/m²), its density is (Given, gas constant, R = 2513 J/mol K)*

(a) 0.72 kg/m³ (b) 1.44 kg/m³

(c) 2.88 kg/m³ (d) 5.16 kg/m³

**Sol.** (*b*) The expression for density of a mixture of gases is given by, $\rho_{\text{mix}} = \dfrac{pM_{\text{mix}}}{RT}$

where, $M_{\text{mix}}$ is the mass of mixture of non-reactive gases, $p$ is pressure, $R$ is gas constant and $T$ is temperature.

$$M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

Given, Mass of nitrogen gas, $m_{N_2} = 7 \text{ g} = 7 \times 10^{-3}$ kg

Mass of carbon dioxide, $m_{CO_2} = 11 \text{g} = 11 \times 10^{-3}$ kg

Molecular weight of nitrogen gas, $m_{N_2} = 28 \times 10^{-3}$ kg

Molecular weight of carbon dioxide, $M_{CO_2} = 44 \times 10^{-3}$ kg

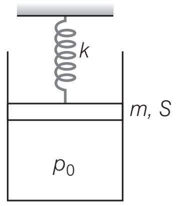
$$M_{\text{mix}} = \frac{n_{N_2} M_{N_2}}{n_{N_2} + n_{CO_2}} = \frac{\dfrac{m_{N_2}}{M_{N_2}} M_{N_2} + \dfrac{m_{CO_2}}{M_{CO_2}} M_{CO_2}}{\dfrac{m_{N_2}}{M_{N_2}} + \dfrac{m_{CO_2}}{M_{CO_2}}}$$

$$M_{\text{mix}} = \frac{m_{N_2} + m_{CO_2}}{\dfrac{m_{N_2}}{M_{N_2}} + \dfrac{m_{CO_2}}{M_{CO_2}}} = \frac{(7+11) \times 10^{-3}}{\left(\dfrac{7 \times 10^{-3}}{28 \times 10^{-3}} + \dfrac{11 \times 10^{-3}}{44 \times 10^{-3}}\right)}$$

$$M_{\text{mix}} = \frac{18 \times 10^{-3}}{\dfrac{1}{4} + \dfrac{1}{4}} = 36 \times 10^{-3} \text{ kg}$$

$$\therefore \quad \rho = \frac{(1 \times 10^5)\,(36 \times 10^{-3})}{\dfrac{25}{3} \times 300} = 1.44 \text{ kg/m}^3$$

***Example 15.*** *In the arrangement shown in the figure gas is thermally insulated. An ideal gas is filled in the cylinder having pressure $p_0$ (> atmospheric pressure $p_a$). The spring of force constant k is initially unstretched. The piston of mass m and area S is frictionless. In equilibrium, the piston rises up a distance $x_0$, then the decrease in internal energy of the gas is given by*



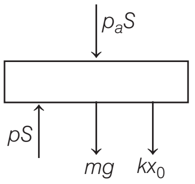
(a) $p_a S x_0 + \dfrac{1}{2} k x_0^2 + mg x_0$ (b) $p_a S x_0 + \dfrac{4}{5} x_0^2 + 2 mg x_0$

(c) $2\, p_a S x_0 + \dfrac{4}{5} x_0^2 + \dfrac{2}{3} mg x_0$ (d) $p_a S x_0 + 2\, k x_0^2 + 2\, mg x_0$

**Sol.** (a) Equilibrium of piston gives

$$pS = p_a S + mg + kx_0$$

$$p = p_a + \frac{mg}{S} + \frac{kx_0}{S} \qquad (p = \text{final pressure of gas})$$



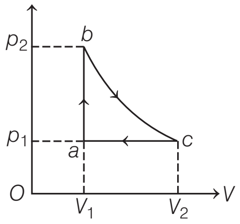
Work done by the gas is equal to work done against atmospheric pressure + elastic potential energy stored in the spring + increase in gravitational potential energy of the piston.

$$= p_a \Delta V + \frac{1}{2} k x_0^2 + mgx_0$$

$$= p_a S x_0 + \frac{1}{2} k x_0^2 + mgx_0$$

This is also the decrease in internal energy of the gas, because the gas is thermally insulated and this work is done at the expense of internal energy of the gas.

**Example 16.** *Carbon monoxide is carried around a closed cycle abc, in which bc is an isothermal process, as shown.*



*The gas absorbs 7000 J of heat as its temperature is increased from 300 K to 1000 K is going from a to b. The quantity of heat ejected by the gas.  process ca is*

(a) 4200 *J* (b) 5000 *J*
(c) 9800 *J* (d) 3500 *J*

**Sol.** (c) Given that gas absorbs 7000 J of energy, hence

$$(\Delta Q)_{ab} = \mu C_V \Delta T$$

$$\therefore \qquad +7000 = \mu C_V (1000 - 300)$$

For the process *ca*, $T_a = 300$ K

$$T_c = T_b = 1000 \text{ K}$$

$$(\Delta Q)_{ca} = \mu C_p \Delta T$$

$$= \mu C_p (300 - 1000)$$

$$= -\mu C_p \times 700$$

Also, $C_p - C_V = R$

$$\therefore \qquad C_p = R + C_V$$

$$(\Delta Q)_{ca} = -\mu (C_V + R) 700$$

For carbon monoxide, $\gamma = 1 + \frac{2}{n} = 1 + \frac{2}{5} = \frac{7}{5}$

$$C_V = \frac{R}{\gamma - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5R}{2}$$

Hence, we have $(\Delta Q)_{ab} = \mu C_V 700$

$$(\Delta Q)_{ab} = \mu \frac{5R}{2} \times 700 = 7000$$

or $\mu R = \frac{20}{5} = 4$

$$\therefore \qquad (\Delta Q)_{ca} = -(7000 + 4 \times 700) = -9800 \text{ J}$$

Negative sign shows that heat is ejected.

# Practice Exercise

## ROUND I Topically Divided Problems

### Ideal Gas, Kinetic Theory of Gases and Degree of Freedom

**1.** An oxygen cylinder of volume 30 L has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder.
(Take, $R = 8.31 \text{J} \,\text{mol}^{-1} \text{K}^{-1}$ and molecular mass of $O_2 = 32 \,\text{u}$) **[NCERT]**

(a) 0.14 g (b) 0.02 g (c) 0.14 kg (d) 0.014 kg

**2.** At room temperature (27°C), the rms speed of the molecules of a certain diatomic gas is found to be 1920 $\text{ms}^{-1}$.The gas is

(a) $Cl_2$ (b) $O_2$ (c) $N_2$ (d) $H_2$

**3.** The rms velocity of gas molecules is 300 $\text{ms}^{-1}$. The rms velocity of molecules of gas with twice the molecular mass and half the absolute temperature is

(a) 300 $\text{ms}^{-1}$ (b) 600 $\text{ms}^{-1}$
(c) 75 $\text{ms}^{-1}$ (d) 150 $\text{ms}^{-1}$

**4.** 8 g of $O_2$, 14 g of $N_2$ and 22 g of $CO_2$ is mixed in a container of 10 L capacity at 27°C. The pressure exerted by the mixture in terms of atmospheric pressure is ($R$ = 0.082 L atm $\text{K}^{-1}$ $\text{mol}^{-1}$)

(a) 1.4 atm (b) 2.5 atm
(c) 3.7 atm (d) 8.7 atm

**5.** At a certain temperature, the ratio of the rms velocity of $H_2$ molecules to $O_2$ molecule is

(a) 1 : 1 (b) 1 : 4
(c) 4 : 1 (d) 16 : 1

**6.** Two containers of equal volume contain the same gas at the pressures $p_1$ and $p_2$ and absolute temperatures $T_1$ and $T_2$ respectively. On joining the vessels, the gas reaches a common pressure $p$ and a common temperature $T$. The ratio $p/T$ is equal to

(a) $\frac{p_1T_2 + p_2T_2}{T_1 \times T_2}$ (b) $\frac{p_1T_2 + p_2T_1}{T_1 + T_2}$
(c) $\frac{1}{2}\left(\frac{p_1T_2 + p_2T_1}{T_1T_2}\right)$ (d) $\frac{p_1T_2 - p_2T_1}{T_1 \times T_2}$

**7.** The volume $V$ of an enclosure contains a mixture of three gases : 16g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature $T$. Consider $R$ as universal gas constant. The pressure of the mixture of gases is **[JEE Main 2021]**

(a) $\frac{88\,RT}{V}$ (b) $\frac{3\,RT}{V}$ (c) $\frac{5}{2} \cdot \frac{RT}{V}$ (d) $\frac{4\,RT}{V}$

**8.** rms velocity of a particle is $c$ at pressure $p$. If pressure is increased two times, then rms velocity becomes

(a) 0.5 $c$ (b) $c$ (c) 2 $c$ (d) 3 $c$

**9.** Calculate the rms speed of smoke particles each of mass $5 \times 10^{-17}$kg in their Brownian motion in air at NTP ($k = 1.38 \times 10^{-23}$ $\text{JK}^{-1}$)

(a) 1.5 mm $\text{s}^{-1}$ (b) 1.5 $\text{ms}^{-1}$
(c) 1.5 $\text{cms}^{-1}$ (d) 1.5 $\text{kms}^{-1}$

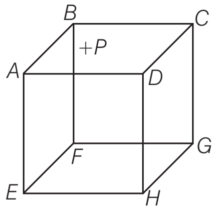
**10.** If $c_s$ is the velocity of sound in air and $c$ is the rms velocity, then

(a) $c_s < c$ (b) $c_s = c$
(c) $c_s = c\left(\frac{\gamma}{3}\right)^{1/2}$ (d) None of these

**11.** In a diatomic molecule, the rotational energy at a given temperature **[NCERT Exemplar]**

(a) do not obey Maxwell's distribution
(b) have the same value for all molecules
(c) equals the translational kinetic energy for each molecule
(d) is (2/3)rd the translational kinetic energy for each molecule

**12.** $ABCDEFGH$ is hollow cube made of an insulator. Face $ABCD$ has positive charge on it. Inside the cube, we have ionized hydrogen.



The usual kinetic theory expression for pressure **[NCERT Exemplar]**

(a) will be valid
(b) will be valid since the ions would experience forces larger than due to collisions with the walls
(c) will not be valid since collisions with walls would not be elastic
(d) will not be valid because isotropy is lost

**13.** Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be (Take, molecular weight of oxygen = 32 g/mol and $R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$) **[JEE Main 2021]**

(a) $\sqrt{\frac{3}{8}}$ (b) $\sqrt{\frac{8}{3}}$
(c) $\sqrt{\frac{3\pi}{8}}$ (d) $\sqrt{\frac{8\pi}{3}}$

**14.** $N$ molecules, each of mass $m$, of gas $A$ and $2N$ molecules, each of mass $2m$, of gas $B$ are contained in the same vessel which is maintained at a temperature $T$. The mean square velocity of molecules of $B$ type is denoted by $V_2$ and the mean square velocity of $A$ type is denoted by $V_1$, then $\frac{V_1}{V_2}$ is

(a) 2 (b) 1
(c) 1/3 (d) 2/3

**15.** If $10^{22}$ gas molecules each of mass $10^{-26}$ kg collide with a surface (perpendicular to it) elastically per second over an area 1 $\text{m}^2$ with a speed $10^4$ m/s, the pressure exerted by the gas molecules will be of the order of **[JEE Main 2019]**

(a) $10^4 \text{N/m}^2$ (b) $10^8 \text{N/m}^2$
(c) $10^0 \text{N/m}^2$ (d) $10^{16} \text{N/m}^2$

**16.** To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 cal of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 cal of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)? **[JEE Main 2020]**

(a) 5 (b) 7
(c) 6 (d) 3

**17.** The specific heats, $C_p$ and $C_V$ of a gas of diatomic molecules, $A$ are given (in units of $\text{J mol}^{-1} \text{K}^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules $B$, has the corresponding values of $C_p$ and $C_V$ given as 30 and 21, respectively. If they are treated as ideal gases, then respectively. **[JEE Main 2019]**

(a) $A$ has a vibrational mode but $B$ has none
(b) both $A$ and $B$ have a vibrational mode each
(c) $A$ has one vibrational mode and $B$ has two
(d) $A$ is rigid but $B$ has a vibrational mode

**18.** A $25 \times 10^{-3} \text{ m}^3$ volume cylinder is filled with 1 mole of $O_2$ gas at room temperature (300 K). The molecular diameter of $O_2$ and its root mean square speed are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an $O_2$ molecule? **[JEE Main 2019]**

(a) $\sim 10^{10}$ (b) $\sim 10^{12}$
(c) $\sim 10^{11}0$ (d) $\sim 10^8$

**19.** Consider two ideal diatomic gases $A$ and $B$ at some temperature $T$. Molecules of the gas $A$ are rigid and have a mass $m$. Molecules of the gas $B$ have an additional vibrational mode and have a mass $\frac{m}{4}$.

The ratio of the specific heats ($C_V^A$ and $C_V^B$) of gas $A$ and $B$ respectively is **[JEE Main 2020]**

(a) 5 : 9 (b) 7 : 9
(c) 3 : 5 (d) 5 : 7

**20.** The average kinetic energy of a gas molecule at 27°C is $6.21 \times 10^{-21}$ J. Its average kinetic energy at 127°C will be

(a) $12.2 \times 10^{-21}$ J
(b) $8.28 \times 10^{-21}$ J
(c) $10.35 \times 10^{-21}$ J
(d) $11.35 \times 10^{-21}$ J

**21.** The root mean square velocity of the molecules in a sample of helium is 5/7th that of the molecules in a sample of hydrogen. If the temperature of the hydrogen as is 0°C, that of helium sample is about

(a) 0°C (b) 4 K
(c) 273°C (d) 100°C

## Internal Energy, Specific Heat Capacity and Mean Free Path

**22.** The value of molar specific heat at constant volume for 1 mole of polyatomic gas having $n$ number of degrees of freedom at temperature $T$ K is
($R$ = universal gas constant)

(a) $\frac{nR}{2T}$ (b) $\frac{nR}{2}$
(c) $\frac{nRT}{2}$ (d) $2nRT$

**23.** For a gas, if the ratio of specific heats at constant pressure and constant volume is $\gamma$, then the value of degrees of freedom is

(a) $\frac{\gamma + 1}{\gamma - 1}$ (b) $\frac{\gamma - 1}{\gamma + 1}$
(c) $\frac{(\gamma - 1)}{2}$ (d) $\frac{2}{\gamma - 1}$

**24.** Two moles of monoatomic gas is mixed with three moles of a diatomic gas. The molar specific heat of the mixture at constant volume is

(a) 1.55 $R$ (b) 2.10 $R$
(c) 1.63 $R$ (d) 2.20 $R$

**25.** An ideal gas occupies a volume of $2 \text{ m}^3$ at a pressure of $3 \times 10^6$ Pa. The energy of the gas is **[JEE Main 2019]**

(a) $6 \times 10^4$ J (b) $10^8$ J
(c) $9 \times 10^6$ J (d) $3 \times 10^2$ J

**26.** Calculate the value of mean free path $\lambda$ for oxygen molecules at temperature 27°C and pressure $1.01 \times 10^5$ Pa. Assume the molecular diameter 0.3 nm and the gas is ideal.
(Take, $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$) **[JEE Main 2021]**

(a) 58 nm (b) 32 nm
(c) 86 nm (d) 102 nm

**27.** The value of molar specific heat at constant pressure for one mole of triatomic gas (triangular arrangement) at temperature $T$ K is ($R$ = universal gas constant)

(a) $3R$ (b) $\frac{2}{7}R$
(c) $\frac{5}{2}R$ (d) $4R$

**28.** The diameter of a gas molecule is $2.4 \times 10^{-10}$ m. The mean free path of gas molecule at NTP is
($k = 1.38 \times 10^{-23} \text{ JK}^{-1}$)

(a) $1.46 \times 10^{-7}$ m (b) $2.46 \times 10^{-6}$ m
(c) $1.46 \times 10^{-6}$ m (d) $2.46 \times 10^{-7}$ m

**29.** Match the $C_p/C_V$ ratio for ideal gases with different type of molecules **[JEE Main 2020]**

| | Molecule type | $C_p/C_V$ |
|---|---|---|
| (A) | Monatomic molecules | I. 7/5 |
| (B) | Diatomic rigid molecules | II. 9/7 |
| (C) | Diatomic non-rigid molecules | III. 4/3 |
| (D) | Triatomic rigid molecules | IV. 5/3 |

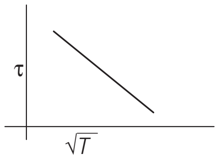
| | A | B | C | D | | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| (a) | IV | I | II | III | (b) | III | IV | II | I |
| (c) | II | III | I | IV | (d) | IV | II | I | III |

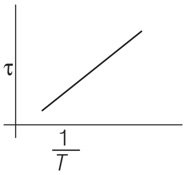
**30.** An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true? **[JEE Main 2020]**

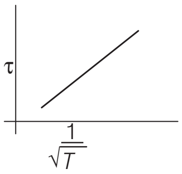
A. The mean free path of the molecules decreases.
B. The mean collision time between the molecules decreases.
C. The mean free path remains unchanged.
D. The mean collision time remains unchanged.

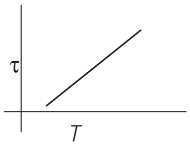
(a) B and C (b) A and B
(c) C and D (d) A and D

**31.** The plot that depicts the behaviour of the mean free time $\tau$ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature ($T$), qualitatively is (graphs are schematic and not drawn to scale)

(a) 

(b) 

(c) 

(d) 

**32.** Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently, the mean collision time between the gas molecule changes from $\tau_1$ to $\tau_2$. If $\frac{C_p}{C_V} = \gamma$ for this gas, then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by **[JEE Main 2020]**

(a) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$ (b) $\frac{1}{2}$
(c) 2 (d) $\left(\frac{1}{2}\right)^{\gamma}$

**33.** Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to **[JEE Main 2020]**

(a) 4.67 (b) 2.3
(c) 3.67 (d) 1.09

**34.** Two moles of an ideal gas with $\frac{C_p}{C_V} = \frac{5}{3}$ are mixed with 3 mol of another ideal gas with $\frac{C_p}{C_V} = \frac{4}{3}$. The value of $\frac{C_p}{C_V}$ for the mixture is **[JEE Main 2020]**

(a) 1.42 (b) 1.47
(c) 1.50 (d) 1.45

**35.** Number of molecules in a volume of $4 \text{ cm}^3$ of a perfect monoatomic gas at some temperature $T$ and at a pressure of 2 cm of mercury is close to
(Given, mean kinetic energy of a molecule at $T$ is $4 \times 10^{-14}$ erg, $g = 980 \text{ cm/s}^2$, density of mercury $= 13.6 \text{ g/cm}^3$) **[JEE Main 2020]**

(a) $5.8 \times 10^{16}$ (b) $4.0 \times 10^{16}$
(c) $4.0 \times 10^{18}$ (d) $5.8 \times 10^{18}$

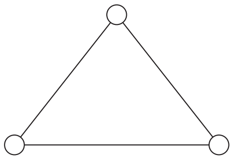
**36.** Consider a mixture of $n$ moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its $C_p / C_V$ value will be **[JEE Main 2020]**

(a) 40/27 (b) 23/15
(c) 19/13 (d) 67/45

**37.** An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, $m$ is its mass and $k_B$ is Boltzmann constant, then its temperature will be **[JEE Main 2019]**

(a) $\frac{m\bar{v}^2}{3k_B}$ (b) $\frac{m\bar{v}^2}{7k_B}$ (c) $\frac{m\bar{v}^2}{5k_B}$ (d) $\frac{m\bar{v}^2}{6k_B}$

**38.** Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature $T$ is **[JEE Main 2020]**



(a) $\frac{5}{2}RT$ (b) $\frac{3}{2}RT$

(c) $\frac{9}{2}RT$ (d) $3\,RT$

## ROUND II Mixed Bag

### Only One Correct Option

**1.** Three perfect gases at absolute temperatures $T_1$, $T_2$ and $T_3$ are mixed. The masses of molecules are $m_1$, $m_2$ and $m_3$ and the number of molecules are $n_1$, $n_2$ and $n_3$ respectively. Assuming no loss of energy, the final temperature of the mixture is **[AIEEE 2011]**

(a) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$ (b) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(c) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$ (d) $\frac{(T_1 + T_2 + T_3)}{3}$

**2.** The temperature of an open room of volume 30 $m^3$ increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains $1 \times 10^5$ Pa. If $n_i$ and $n_f$ are the number of molecules in the room before and after heating, then $n_f - n_i$ will be **[JEE Main 2017]**

(a) $1.38 \times 10^{23}$ (b) $2.5 \times 10^{25}$

(c) $-2.5 \times 10^{25}$ (d) $-1.61 \times 10^{23}$

**3.** An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume $V_1$ and contains ideal gas at pressure $p_1$ and temperature $T_1$. The other chamber has volume $V_2$ and contains ideal gas at pressure $p_2$ and temperature $T_2$. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be **[AIEEE 2008, 04]**

(a) $\frac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_2 + p_2V_2T_1}$

(b) $\frac{p_1V_1T_1 + p_2V_2T_2}{p_1V_1 + p_2V_2}$

(c) $\frac{p_1V_1T_2 + p_2V_2T_1}{p_1V_1 + p_2V_2}$

(d) $\frac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_1 + p_2V_2T_2}$

**4.** 22 g of carbon dioxide at 27°C is mixed in a closed container with 16 g of oxygen at 37°C. If both gases are considered as ideal gases, then the temperature of the mixture is

(a) 24.2°C (b) 28.5°C

(c) 31.5°C (d) 33.5°C

**5.** A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is $TV^x$ = constant, then $x$ is **[JEE Main 2019]**

(a) $\frac{2}{5}$ (b) $\frac{2}{3}$

(c) $\frac{5}{3}$ (d) $\frac{3}{5}$

**6.** A cylinder is containing nitrogen at 2 atm and temperature 17°C. The radius of a nitrogen molecule is roughly 1.0A. Molecular mass of nitrogen = 28.0 u, Boltzmann constant, $k = 1.38 \times 10^{-23}\ \text{JK}^{-1}$. The mean free path of nitrogen molecule is

(a) $1.1 \times 10^{-7}$ m (b) $2.1 \times 10^{-7}$ m

(c) $3.1 \times 10^{-7}$ m (d) $0.8 \times 10^{-7}$ m

**7.** A polyatomic ideal gas has 24 vibrational modes. What is the value of $\gamma$? **[JEE main 2021]**

(a) 1.03 (b) 1.30

(c) 1.37 (d) 10.3

**8.** Inside a cylinder closed at both ends is a movable piston. On one side of the piston is a mass $m$ of a gas, and on the other side a mass 2 $m$ of the same gas.

What fraction of the volume of the cylinder will be occupied by the larger mass of the gas when the piston is in equilibrium? The temperature is the same throughout.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

**9.** Half-mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from $20^\circ$ C to $90^\circ$ C . Work done by gas is close to
(Take, gas constant, $R = 8.31$ J/mol-K)
(a) 291 J (b) 581 J **[JEE Main 2019]**
(c) 146 J (d) 73 J

**10.** One kg of a diatomic gas is at a pressure of $8 \times 10^4$ Nm$^{-2}$. The density of the gas is 4 kgm$^{-3}$. What is the energy of the gas due to its thermal motion? **[AIEEE 2009]**
(a) $3 \times 10^4$ J (b) $5 \times 10^4$ J
(c) $6 \times 10^4$ J (d) $7 \times 10^4$ J

**11.** The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where $a$ and $b$ are constants and $x$ is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, $D$ is **[AIEEE 2010]**
(a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12\,a}$
(c) $\frac{b^2}{4\,a}$ (d) $\frac{b^2}{6\,a}$

**12.** $A$ 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled is about (Take, $R = 8.3$ J/mol K) **[JEE Main 2019]**
(a) 10 kJ (b) 0.9 kJ
(c) 14 kJ (d) 6 kJ

**13.** Two chambers containing $m_1$ and $m_2$ gram of a gas at pressures $p_1$ and $p_2$ respectively are put in communication with each other, temperature remaining constant. The common pressure reached will be
(a) $\frac{p_1p_2\,(m_1 + m_2)}{p_2m_1 + p_1m_2}$ (b) $\frac{p_1p_2m_1}{p_2m_1 + p_1m_2}$
(c) $\frac{m_1m_2\,(p_1 + p_2)}{p_2m_1 + p_1m_2}$ (d) $\frac{m_1m_2p_2}{p_2m_1 + p_1m_2}$

**14.** An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is $6 \times 10^{-8}$ s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to **[JEE Main 2019]**
(a) $4 \times 10^{-8}$ s (b) $3 \times 10^{-6}$ s
(c) $2 \times 10^{-7}$ s (d) $0.5 \times 10^{-8}$ s

**15.** Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? (Take, $R = 8.3$ J/mol-K) **[JEE Main 2019]**
(a) 19.7 J/mol-K (b) 15.7 J/mol-K
(c) 17.4 J/mol-K (d) 21.6 J/mol-K

**16.** Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as $V^q$, where $V$ is the volume of the gas. The value of $q$ is $\left(\gamma = \frac{C_p}{C_V}\right)$ **[JEE Main 2015]**
(a) $\frac{3\gamma + 5}{6}$ (b) $\frac{3\gamma - 5}{6}$
(c) $\frac{\gamma + 1}{2}$ (d) $\frac{\gamma - 1}{2}$

**17.** A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio $\frac{C_p}{C_V}$ of the mixture is **[AIEEE 2005]**
(a) 1.59 (b) 1.62 (c) 1.4 (d) 1.54

**18.** A vertical closed cylinder is separated into two parts by a frictionless piston of mass $m$ and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is $l_1$ and that below the piston is $l_2$, such that $l_1 > l_2$. Each part of the cylinder contains $n$ moles of an ideal gas at equal temperature $T$. If the piston is stationary, its mass $m$, will be given by (where, $R$ is universal gas constant and $g$ is the acceleration due to gravity) **[JEE Main 2019]**
(a) $\frac{nRT}{g}\left[\frac{l_1 - l_2}{l_1l_2}\right]$ (b) $\frac{nRT}{g}\left[\frac{1}{l_2} + \frac{1}{l_1}\right]$
(c) $\frac{RT}{g}\left[\frac{2l_1 + l_2}{l_1\,l_2}\right]$ (d) $\frac{RT}{ng}\left[\frac{l_1 - 3l_2}{l_1\,l_2}\right]$

**19.** One mole of ideal monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of diatomic gas $\left(\gamma = \frac{7}{5}\right)$. What is $\gamma$ for the mixture ? $\gamma$ denotes the ratio of specific heat at constant pressure, to that at constant volume. **[AIEEE 2004]**
(a) $\frac{3}{2}$ (b) $\frac{23}{15}$ (c) $\frac{35}{23}$ (d) $\frac{4}{3}$

**20.** Two rigid boxes $\frac{1}{2}$ containing different ideal gases are placed on a table. Box $A$ contains one mole of nitrogen at temperature $T_0$, while box $B$ contains one mole of helium at temperature (7/3) $T_0$. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature (ignore the heat capacity of boxes). Then, the final temperature of the gases, $T_f$, in terms of $T_0$ is **[AIEEE 2006]**
(a) $T_f = \frac{3}{7}T_0$ (b) $T_f = \frac{7}{3}T_0$
(c) $T_f = \frac{3}{2}T_0$ (d) $T_f = \frac{5}{2}T_0$

**21.** The number density of molecules of a gas depends on their distance $r$ from the origin as, $n(r) = n_0 e^{-ar^4}$.Then, the total number of molecules is proportional to **[JEE Main 2019]**

(a) $n_0\alpha^{-3/4}$ (b) $\sqrt{n_0}\ \alpha^{1/2}$

(c) $n_0\alpha^{1/4}$ (d) $n_0\alpha^{-3}$

## Numerical Value Questions

**22.** The total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 $m^3$ at a temperature of 27°C and 1 atm pressure, is found to be $6.1 \times 10^{13x}$, then the value of $x$ is ...... . (Take, Boltzmann constant = $1.38 \times 10^{-23} JK^{-1}$)

**23.** A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to …… . **[JEE Main 2020]**

**24.** If four molecules of a gas have speed 2, 4, 6, 8 $kms^{-1}$ respectively, then their average speed (in $kms^{-1}$) and root mean square speed (in $kms^{-1}$) will be ...... and ......, respectively.

**25.** Initially a gas of diatomic molecules is contained in a cylinder of volume $V_1$ at a pressure $p_1$ and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by $p_2$. The ratio $p_2/p_1$ is ......... . **[JEE Main 2020]**

**26.** From the following data, find the magnitude of Joule's mechanical equivalent of heat. If $C_p$ for hydrogen $= 3.409$ cal $g^{-1}{}^\circ C^{-1}$, $C_V$ for hydrogen = 2.409 cal $g^{-1}{}^\circ C^{-1}$ and molecular weight of hydrogen = 2, then the magnitude of joule's mechanical equivalent of heat (in $Jcal^{-1}$) will be ...... .

**27.** Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a $H_2$ molecule would be equal to the rms speed of a $N_2$ molecule is ………

(Molar mass of $N_2$ gas is 28 g.) **[JEE Main 2020]**

**28.** In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cc of water at 27°C. The final temperature is 40ºC. If heat losses to the surroundings is not negligible, then the value of specific heat (in J/g°C) of the metal will be ...... .

**29.** A geyser heats water glowing at the rate of 3.0 $Lmin^{-1}$ from 27°C to 77°C. If the geyser operates on a gas burner, then the rate of combustion (in $gmin^{-1}$) of fuel, when its heat of combustion is $4.0 \times 10^4 Jg^{-1}$, will be ...... .

**30.** The molecules of a given mass of a gas have root mean square speeds of 100 $ms^{-1}$ at 27°C and 1.00 atm pressure. If the root mean square speed of the molecules of the gas at 127°C and 2 atm pressure is found to be $\frac{200}{\sqrt{n}}$ m/s, then find the value of $n$. **[JEE Main 2021]**

**31.** An air bubble of volume 1.0 $cm^3$ rises from the bottom of a lake 40 m deep at a temperature of 12°C. The volume of bubble when it reaches the surface, which is at temperature of 3°C, is found to be $5.275 \times 10^{-x} m^3$, then the value of $x$ is ……… . (Given, 1 atm = $1.01 \times 10^5$ Pa)

# Answers

## *Round I*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (c) | **2.** (d) | **3.** (d) | **4.** (c) | **5.** (c) | **6.** (c) | **7.** (c) | **8.** (b) | **9.** (c) | **10.** (c) |
| **11.** (d) | **12.** (d) | **13.** (c) | **14.** (b) | **15.** (c) | **16.** (c) | **17.** (a) | **18.** (d) | **19.** (d) | **20.** (b) |
| **21.** (c) | **22.** (b) | **23.** (d) | **24.** (b) | **25.** (c) | **26.** (d) | **27.** (d) | **28.** (a) | **29.** (a) | **30.** (a) |
| **31.** (c) | **32.** (a) | **33.** (d) | **34.** (a) | **35.** (c) | **36.** (c) | **37.** (b) | **38.** (d) | | |

## *Round II*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (a) | **2.** (c) | **3.** (a) | **4.** (c) | **5.** (a) | **6.** (a) | **7.** (a) | **8.** (a) | **9.** (a) | **10.** (b) |
| **11.** (c) | **12.** (a) | **13.** (a) | **14.** (a) | **15.** (c) | **16.** (c) | **17.** (b) | **18.** (a) | **19.** (a) | **20.** (c) |
| **21.** (a) | **22.** 2 | **23.** 266 | **24.** 5, 5.48 | | **25.** 5 | **26.** 4.11 | **27.** 41 | **28.** 0.1 | |
| **29.** 15.75 | **30.** 3 | **31.** 6 | | | | | | | |

# Solutions

## Round I

**1.** Absolute pressure, $p_1 = (15 + 1)$ atm

$$[\because \text{Absolute pressure } = \text{Gauge pressure} + 1 \text{ atm}]$$

$$= 16 \times 1.013 \times 10^5 \text{ Pa}$$

$$V_1 = 30 \text{ L} = 30 \times 10^{-3} \text{ m}^3$$

$$T_1 = 273.15 + 27 = 300.15 \text{ K}$$

Using ideal gas equation, $pV = nRT$

or $$n = \frac{pV}{RT}$$

$$= \frac{p_1V_1}{RT_1} = \frac{16 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 300.15}$$

$$= 19.48$$

Final pressure, $p_2 = (11 + 1) = 12$ atm Pa

$$V_2 = 30 \text{ L} = 30 \times 10^{-3} \text{m}^3$$

$$T_2 = 273.15 + 17 = 290.15 \text{ K}$$

$$\text{Number of moles} = \frac{p_2V_2}{RT_2} = \frac{12 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 290.15}$$

$$= 15.12$$

Hence, moles removed $= 19.48 - 15.12 = 4.36$

Mass removed $= 4.36 \times 32 \text{ g} = 0.1396 \text{ kg} \approx 0.14 \text{ kg}$

**2.** As, $c_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

or $$M = \frac{3RT}{c_{\text{rms}}^2}$$

$$= \frac{3 \times 8.31 \times 300}{(1920)^2}$$

$$= 2 \times 10^{-3} \text{ kg} = 2 \text{ g}$$

Since, $M = 2$ for the hydrogen molecule. Hence, the gas is hydrogen.

**3.** As, $c = \sqrt{\dfrac{3pV}{M}} = \sqrt{\dfrac{3RT}{M}}$

and the new rms speed,

$$c_1 = \sqrt{\frac{3R(T/2)}{(2M)}} = \frac{1}{2}\sqrt{\frac{3RT}{M}}$$

$$= \frac{c}{2} = \frac{300}{2} = 150 \text{ ms}^{-1}$$

**4.** As, $p = \dfrac{n_1RT + n_2RT + n_3RT}{V} = (n_1 + n_2 + n_3)\dfrac{RT}{V}$

$$= \left(\frac{8}{16} + \frac{14}{28} + \frac{22}{44}\right) \times \frac{0.082 \times 300}{10} = 3.69 \text{ atm} \approx 3.7 \text{atm}$$

**5.** As, $c = \sqrt{\dfrac{3RT}{M}}$

So, $\dfrac{c_{\text{H}}}{c_{\text{O}}} = \sqrt{\dfrac{M_{\text{O}}}{M_{\text{H}}}} = \sqrt{\dfrac{16}{1}} = 4$

**6.** For a closed system, the total number of moles remains constant.

So, $p_1V = n_1RT_1 \Rightarrow \dfrac{n_1R}{V} = \dfrac{p_1}{T_1}$

and $p_2V = n_2RT_2 \Rightarrow \dfrac{n_2R}{V} = \dfrac{p_2}{T_2}$

$$\therefore\ p(2V) = (n_1 + n_2)RT$$

where, $V$ is the volume of each vessel. When the vessels are joined.

$$\therefore \quad \frac{p}{T} = \frac{(n_1 + n_2)}{2V}R = \frac{1}{2}\left(\frac{p_1}{T_1} + \frac{p_2}{T_2}\right) = \frac{1}{2}\left(\frac{p_1T_2 + p_2T_1}{T_1T_2}\right)$$

**7.** $pV = (n_1 + n_2 + n_3)RT$

$$= \left(\frac{16}{32} + \frac{28}{28} + \frac{44}{44}\right)RT$$

$$= \left(\frac{1}{2} + 1 + 1\right)RT$$

$$\Rightarrow \quad pV = \frac{5}{2}RT$$

$$\Rightarrow \quad p = \frac{5}{2} \cdot \frac{RT}{V}$$

**8.** rms velocity does not change with pressure, till temperature remains constant.

**9.** As, $c = \sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3 \times 1.38 \times 10^{-23} \times 273}{5 \times 10^{-17}}}$

$$= 15 \times 10^{-3} \text{ ms}^{-1} = 1.5 \text{ cms}^{-1}$$

**10.** As, $c_s = \sqrt{\dfrac{\gamma p}{\rho}}$

and $c = \sqrt{\dfrac{3pV}{M}} = \sqrt{\dfrac{3p}{\rho}}$

$$\frac{c_s}{c} = \sqrt{\frac{\gamma}{3}} \quad \text{or} \quad c_s = c\sqrt{\frac{\gamma}{3}}$$

**11.** In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's energy distribution law. As each such atom has three translational degrees of freedom and two rotational degrees of freedom, therefore, at a given temperature, rotational energy is $\frac{2}{3}$rd the translational KE of each molecule. Energy associated with each molecule per degree of freedom is fixed $\left(= \frac{1}{2}k_B T\right)$.

**12.** As face *ABCD* has positive charge on it and the gas consists of ionized hydrogen, therefore, isotropy is lost, The usual expression for pressure on the basis of kinetic theory will not be valid as ions would also experience forces, other than the forces due to collisions with the walls of the container.

**13.**
$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$
$$v_{\text{avg}} = \sqrt{\frac{8}{\pi} \cdot \frac{RT}{M}}$$
$$\therefore \quad \frac{v_{\text{rms}}}{v_{\text{avg}}} = \sqrt{\frac{3\pi}{8}}$$

**14.** As, $p = \frac{1}{3}\frac{M}{V}c^2$

or $$c^2 = \frac{3\,pV}{M} = \frac{3\,RT}{M}$$

For gas $A$, $$V_1^2 = \frac{3 \times RT}{M} = \frac{3\,RT}{M}$$

For gas $B$, $$V_2^2 = \frac{3\,RT}{M};$$

So, $$\frac{V_1^2}{V_2^2} = 1 \Rightarrow \frac{V_1}{V_2} = 1$$

**15.** Momentum imparted to the surface in one collision, $\Delta p = (p_i - p_f) = mv - (-mv) = 2mv$ ...(i)

Force on the surface due to $n$ collision per second,
$$F = \frac{n}{t}(\Delta p) = n\Delta p \qquad (\because t = 1\,\text{s})$$
$$= 2\,mnv \qquad \text{[from Eq. (i)]}$$

So, pressure on the surface, $p = \frac{F}{A} = \frac{2mnv}{A}$

Here, $m = 10^{-26}$ kg, $n = 10^{22}\ \text{s}^{-1}$,
$v = 10^4\ \text{ms}^{-1}$, $A = 1\ \text{m}^2$

$\therefore$ Pressure, $p = \frac{2 \times 10^{-26} \times 10^{22} \times 10^4}{1} = 2\ \text{N/m}^2$

So, pressure exerted is of order of $10^0$.

**16.** Given, in constant pressure process,
$$\Delta Q = 160\ \text{cal}$$
$$\Delta T = 50°\text{C}$$

So, $$\Delta Q = nC_p\Delta T$$
$$\Rightarrow \quad 160 = nC_p(50) \qquad \text{...(i)}$$

Also, in constant volume process,
$$\Delta Q = 240\ \text{cal}$$
$$\Delta T = 100°\,\text{C} \Rightarrow \Delta Q = nC_V\Delta T$$
$$\Rightarrow \quad 240 = nC_V(100) \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we have
$$\frac{160}{240} = \frac{nC_p(50)}{nC_V(100)}$$
$$\Rightarrow \quad \frac{C_p}{C_V} = \frac{2 \times 160}{240} = \frac{4}{3}$$

Now, $\frac{C_p}{C_V} = \gamma$ and degrees of freedom of a gas,
$$f = \frac{2}{\gamma - 1}$$
$$\Rightarrow \quad f = \frac{2}{\gamma - 1} = \frac{2}{\frac{4}{3} - 1} \Rightarrow f = 6$$

**17.** A diatomic gas molecule has 5 degrees of freedom, *i.e.* 3 translational and 2 rotational, at low temperature ranges (~ 250 K to 750 K). At temperatures above 750 K, molecular vibrations occurs and this causes two extra degrees of freedom.

Now, in given case,
For gas $A$, $C_p = 29$, $C_V = 22$
For gas $B$, $C_p = 30$, $C_V = 21$

By using $\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$, we have

For gas $A$,
$$1 + \frac{2}{f} = \frac{29}{22} \approx 1.3 \Rightarrow f = 6.67 \approx 7$$

So, gas $A$ has vibrational mode of degree of freedom.

For gas $B$,
$$1 + \frac{2}{f} = \frac{30}{21} \approx 1.4 \Rightarrow f = 5$$

Hence, gas $B$ does not have any vibrational mode of degree of freedom.

**18.** Given, volume, $V = 25 \times 10^{-3}\ \text{m}^3$
$$N = 1 \text{ mole of } O_2 = 6.023 \times 10^{23} \text{ atoms of } O_2,$$
$$T = 300\ \text{K}$$

Root mean square velocity of a gas molecule of $O_2$, $v_{\text{rms}} = 200$ m/s

Radius, $r = \frac{0.3}{2}\ \text{nm} = \frac{0.3}{2} \times 10^{-9}$ m

Now, average time, $\frac{1}{\tau} = \frac{v_{\text{av}}}{\lambda}$

where, $$\lambda = \frac{RT}{\sqrt{2}\,N\pi r^2 p}$$

As, $$p = \frac{RT}{V}$$

$$\Rightarrow \quad \lambda = \frac{V}{\sqrt{2}\,N\pi r^2}$$

$\therefore$ Average collision per second,
$$\frac{1}{\tau} = \frac{v_{\text{av}}}{\lambda} = \frac{\sqrt{\frac{8}{3\pi}} \times v_{\text{rms}}}{\lambda}$$
$$\frac{200 \times \sqrt{2} \times 6.023 \times 10^{23} \times \pi \times \frac{0.09}{4} \times 10^{-18}}{25 \times 10^{-3}}$$
$$\Rightarrow \quad \frac{1}{\tau} = 4.4 \times 10^8 \text{ per second} \approx 10^8$$

**19.** For a gas, value of specific heat at constant volume,
$$C_V = \frac{1}{2}fR$$

where, $f$ = degrees of freedom

and $R$ = gas constant.

For a diatomic gas molecule, degrees of freedom is in general 5 but when vibration occurs, then degrees of freedom increases to 7.

So, for gas $A$, specific heat is $C_V^A = \frac{1}{2} 5R$

and for gas $B$, specific heat is $C_V^B = \frac{1}{2} 7R$

$\therefore$ Ratio of specific heats of gas $A$ and $B$ is

$$\frac{C_V^A}{C_V^B} = \frac{5}{7}$$

**20.** Given, $T_1 = 27° \text{ C} = 300 \text{ K}$,

$$k_1 = 6.21 \times 10^{-21} \text{ J}$$

and $T_2 = 127° \text{ C} = 400 \text{ K}$.

We know that, $\frac{k_1}{k_2} = \frac{T_1}{T_2}$

$$k_2 = \frac{T_2}{T_1} \times k_1$$
$$= 6.21 \times 10^{-21} \times 400/300$$
$$= 8.28 \times 10^{-21} \text{ J}$$

**21.** As, $\left(\frac{c_{\text{He}}}{c_{\text{H}}}\right) = \sqrt{\frac{\rho_{\text{H}}}{\rho_{\text{He}}}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$$(c_{\text{He}})_t = (c_{\text{He}})_0 \sqrt{\frac{T}{T_0}}$$

$\therefore$ $\frac{(c_{\text{He}})_t}{(c_{\text{H}})_0} = \frac{(c_{\text{He}})_0}{(c_{\text{H}})_0} \sqrt{\frac{T}{T_0}} = \frac{5}{7}$

or $T \approx 2\,T_0 = 2 \times 273 = 546° \text{ K}$

$$= 273° \text{ C}$$

**22.** According to law of equipartition of energy, average kinetic energy per molecule per degree of freedom at temperature $T$ is $\frac{1}{2} kT$. The average kinetic energy per molecule of polyatomic gas molecule $= \frac{n}{2} kT$

($n$ = number of mole)

The average kinetic energy per mol of polyatomic gas

$$E = \frac{n}{2} kT \times N = \frac{n}{2} RT$$

$$C_V = \frac{d}{dT}\left(\frac{n}{2} RT\right) = \frac{n}{2} R$$

**23.** As, $\gamma = 1 + \frac{2}{n}$ or $\frac{2}{n} = \gamma - 1$ or $n = \frac{2}{\gamma - 1}$

**24.** Molar specific heat of the mixture at constant volume is

$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{(n_1 + n_2)}$$

$$= \frac{2\left(\frac{3}{2} R\right) + 3\left(\frac{5}{2} R\right)}{2 + 3} = 2.1\, R$$

**25.** Internal energy of '$n$' moles of a gas with degree of freedom $f$ ($= 3$ for an ideal gas), at temperature $T$ is

$$E = \frac{f}{2} \cdot n\,RT = \frac{3}{2} nRT$$

For an ideal gas, internal energy, $E = \frac{3}{2} nRT$

$$= \frac{3}{2} p \cdot V \qquad (\because \text{ from } pV = nRT)$$

Substituting the given values, we get

$$= \frac{3}{2} \times 3 \times 10^6 \times 2 = 9 \times 10^6 \text{ J}$$

**26.** $$\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$$

$$\lambda = 102 \text{ nm}$$

**27.** Here, $n = 6$,

$$C_p = \left(1 + \frac{n}{2}\right) R = \left(1 + \frac{6}{2}\right) R = 4\,R$$

**28.** At NTP, $T = 273$ K, $p = 1.01 \times 10^5 \text{ Nm}^{-2}$

Here, $d = 2.4 \times 10^{-10}$ m

$$\lambda = \frac{kT}{\sqrt{2}\,\pi d^2 p}$$

$$= \frac{(1.38 \times 10^{-23}) \times 273}{1.414 \times 3.14 \times (2.4 \times 10^{-10})^2 \times 1.01 \times 10^5}$$

$$= 1.46 \times 10^{-7} \text{ m}$$

**29.** $\because$ $\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$

where, $f$ = degree of freedom.

Now,

(A) for monatomic gas, $f = 3$

so, $\gamma = 1 + \frac{2}{3} = \frac{5}{3}$

(B) for diatomic rigid gas, $f = 5$

so, $\gamma = 1 + \frac{2}{5} = \frac{7}{5}$

(C) for diatomic non-rigid gas, $f = 7$

so, $\gamma = 1 + \frac{2}{7} = \frac{9}{7}$

(D) for triatomic rigid gas, $f = 6$

so, $\gamma = 1 + \frac{2}{6} = \frac{8}{6} = \frac{4}{3}$

Therefore,

A-IV, B-I, C-II, D-III.

Hence, correct option is (a).

**30.** Mean free path, $\lambda = \frac{V}{\sqrt{2}\pi d^2 N} = \frac{kT}{\sqrt{2}\pi d^2 p}$

where, $V$ = volume of conductor,
$N$ = number of molecules,
$d$ = diameter of molecule,
$k$ = Boltzmann's constant,
$T$ = temperature
and $p$ = pressure.

But from gas equation, $\frac{p}{T} = \frac{nR}{V}$

For a constant $n$ and $V$, $\dfrac{p}{T}$ = constant.

So, mean free path does not depends on $\dfrac{p}{T}$ or $p$ and $T$.

So, in given condition (closed container), mean free path remains constant or unchanged.

$\therefore$ Statement $A$ is incorrect, but $C$ is correct.

Also, mean time between two collisions is

$$\tau = \frac{\text{mean free path}}{v_{\text{avg}}} = \frac{\lambda}{v_{\text{avg}}}$$

As, $\quad v_{\text{avg}} \propto \sqrt{T}$

$\Rightarrow$ Average time, $\tau \propto \dfrac{1}{\sqrt{T}}$

As, the temperature is increasing, so mean collision time ($\tau$) decreases.

$\therefore$ Statement $B$ is correct.

Hence, option (a) is correct.

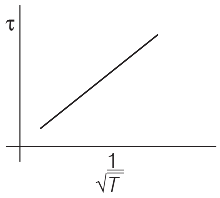
**31.** Mean free time ($\tau$) for a gas molecule is time elapsed between two successive collisions.

$$\therefore \quad \tau = \frac{\text{Mean free path length } (\lambda)}{\text{Average speed } (v_{\text{av}})}$$

$$= \frac{\left(\dfrac{1}{\sqrt{2}\cdot \pi d^2 \cdot n}\right)}{\left(\sqrt{\dfrac{8}{\pi}\dfrac{R}{M}\cdot T}\right)}$$

So, for an ideal gas, $\tau \propto \dfrac{1}{\sqrt{T}}$

Thus, graph of $\tau$ *versus* $\dfrac{1}{\sqrt{T}}$ is a straight line as shown below



**32.** Mean free path ($\lambda$) of a gas molecule and mean relaxation time (or collision time) $\tau$ are related as

$$\tau = \frac{\lambda}{v_{\text{mean}}} = \frac{\dfrac{1}{\sqrt{2}}\dfrac{k_BT}{\pi d^2 p}}{\sqrt{\dfrac{8}{\pi}\cdot\dfrac{k_B}{m}\cdot T}} = \frac{1}{4d^2}\left(\frac{k_Bm}{\pi}\right)^{1/2}\cdot\left(\frac{T^{1/2}}{p}\right)$$

Using $pV = nRT$, we have

$$\tau = \frac{1}{4d^2}\cdot\left(\frac{k_Bm}{\pi}\right)^{1/2}\cdot\frac{VT^{1/2}}{nRT}$$

$$= \left\{\frac{1}{4d^2}\cdot\left(\frac{k_Bm}{\pi}\right)^{1/2}\cdot\frac{1}{nR}\right\}\cdot\frac{V}{\sqrt{T}}$$

$\Rightarrow \tau \propto \dfrac{V}{\sqrt{T}}$, as other quantities remains constant during the process.

So, $$\frac{\tau_1}{\tau_2} = \frac{V_1}{V_2}\cdot\left(\frac{T_2}{T_1}\right)^{1/2} \quad \ldots \text{(i)}$$

As the process is adiabatic,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

So, Eq. (i) becomes

$$\frac{\tau_1}{\tau_2} = \frac{V_1}{V_2}\cdot\left(\frac{V_1}{V_2}\right)^{\frac{\gamma-1}{2}}$$

$$\Rightarrow \quad \frac{\tau_1}{\tau_2} = \left(\frac{V_1}{V_2}\right)^{\frac{\gamma+1}{2}}$$

Now, given $V_2 = 2V_1$, so

$$\frac{\tau_1}{\tau_2} = \left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$$

**33.** Mean free path for a gas,

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$$

where, $\quad n$ = number density

and $\quad d$ = diameter of gas atom.

Mean speed of a gas atom,

$$v = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$$

where, $\quad k$ = Boltzmann's constant,

$\quad T$ = temperature of sample

and $\quad m$ = atomic mass.

Hence, mean relaxation time,

$$T = \frac{\lambda}{v} = \frac{\left(\dfrac{1}{\sqrt{2}\pi n d^2}\right)}{\sqrt{\dfrac{8}{\pi}\dfrac{kT}{m}}}$$

$\tau \propto \dfrac{\sqrt{m}}{d^2}$, when $T$ and $n$ are same.

Hence, $$\frac{\tau_{\text{Ar}}}{\tau_{\text{Xe}}} = \frac{\sqrt{m_{\text{Ar}}}}{\sqrt{m_{\text{Xe}}}}\cdot\frac{d_{\text{Xe}}^2}{d_{\text{Ar}}^2} = \frac{\sqrt{m_{\text{Ar}}}}{\sqrt{m_{\text{Xe}}}}\cdot\frac{r_{\text{Xe}}^2}{r_{\text{Ar}}^2}$$

Here, $m_{\text{Ar}} = 40$, $m_{\text{Xe}} = 140$,

$r_{\text{Ar}} = 0.07$ nm, $r_{\text{Xe}} = 0.1$ nm

$$\therefore \quad \frac{\tau_{\text{Ar}}}{\tau_{\text{Xe}}} = \sqrt{\frac{40}{140}}\times\left(\frac{0.1}{0.07}\right)^2 = 1.09$$

**34.** When $n_1$ moles of an ideal gas having specific heats $C_{p_1}$ and $C_{V_1}$ is mixed with $n_2$ moles of another ideal gas with specific heats $C_{p_2}$ and $C_{V_2}$, specific heats of mixture are given by

$$C_{p(\text{mix})} = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1 + n_2}$$

and $$C_{V(\text{mix})} = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$$

So, ratio of specific heats of mixture will be

$$\frac{C_{p(\text{mix})}}{C_{V(\text{mix})}} = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1C_{V_1} + n_2C_{V_2}} \qquad \ldots\text{(i)}$$

Now, using $C_p = \left(\frac{\gamma}{\gamma-1}\right)R$ and $C_V = \frac{R}{\gamma-1}$

**Note** As, $\frac{C_p}{C_V} = \gamma$ and $C_p - C_V = R$

So, $C_p - \frac{C_p}{\gamma} = R$ or $C_p = \frac{\gamma}{\gamma-1}$ and $C_V = \frac{R}{\gamma-1}$ ]

We can write Eq. (i) as

$$\frac{C_{p(\text{mix})}}{C_{V(\text{mix})}} = \frac{n_1\left(\frac{\gamma_1}{\gamma_1-1}\right)R + n_2\left(\frac{\gamma_2}{\gamma_2-1}\right)R}{n_1\left(\frac{R}{\gamma_1-1}\right) + n_2\left(\frac{R}{\gamma_2-1}\right)}$$

$$= \frac{n_1\gamma_1(\gamma_2-1) + n_2\gamma_2(\gamma_1-1)}{n_1(\gamma_2-1) + n_2(\gamma_1-1)} \qquad \ldots\text{(ii)}$$

Here, we are given

$$n_1 = 2,\ \gamma_1 = \frac{5}{3},\ n_2 = 3 \text{ and } \gamma_2 = \frac{4}{3}$$

Substituting the above values in Eq.(ii), we have

$$\frac{C_{p(\text{mix})}}{C_{V(\text{mix})}} = \frac{2\times\frac{5}{3}\left(\frac{4}{3}-1\right) + 3\times\frac{4}{3}\left(\frac{5}{3}-1\right)}{2\times\left(\frac{4}{3}-1\right) + 3\times\left(\frac{5}{3}-1\right)}$$

$$= \frac{\frac{10}{9}+\frac{24}{9}}{\frac{2}{3}+\frac{6}{3}}$$

$$= \frac{34\times3}{9\times8} = \frac{17}{12} = 1.42$$

**35.** Given, height, $h = 2$ cm

Kinetic energy, $U = 4\times10^{-14}$ erg

Density of mercury, $\rho = 13.6\ \text{g/cm}^3$

Volume, $V = 4\ \text{cm}^3$ $g = 980\ \text{cm/s}^2$

According to gas equation,

$$pV = NKT \;\Rightarrow\; N = \frac{pV}{KT}$$

For monoatomic gas,

Energy, $U = \frac{3}{2}KT \Rightarrow KT = \frac{2}{3}U$

Now, $N = \frac{3pV}{2U}$

So, $N = \frac{3g\rho hV}{2U}$ $\qquad (\because \text{pressure, } p = \rho gh)$ …(i)

Substituting the given values in eq. (i), we get

$$N = \frac{3\times980\times13.6\times2\times4}{2\times4\times10^{-14}}$$

$$= 3.99\times10^{18}$$

$$\approx 4\times10^{18}$$

**36.** For a mixture of two gases, ratio of specific heats is given by

$$\gamma_{\text{mixture}} = \frac{C_{p_{\text{mixture}}}}{C_{V_{\text{mixture}}}}$$

$$= \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1C_{V_1} + n_2C_{V_2}} \qquad \ldots\text{(i)}$$

Here, gas 1 is helium which is monoatomic, so

$$n_1 = n,\ C_{p_1} = \frac{5}{2}R,\ C_{V_1} = \frac{3}{2}R$$

and gas 2 is oxygen which is diatomic, so

$$n_2 = 2n,\ C_{p_2} = \frac{7}{2}R,\ C_{V_2} = \frac{5}{2}R$$

Hence, from Eq. (i), we get

$$\gamma_{\text{mix}} = \frac{n\times\frac{5R}{2} + 2n\times\frac{7R}{2}}{n\times\frac{3R}{2} + 2n\times\frac{5R}{2}} = \frac{19}{13}$$

**37.** According to the law of equipartition of energy, $\frac{1}{2}mv_{\text{rms}}^2 = \frac{n}{2}k_BT$ where, $n$ is the degree of freedom.

Since, HCl is a diatomic molecule that has rotational, translational and vibrational motion.

So, $n = 7$

$$\Rightarrow \quad \frac{1}{2}mv_{\text{rms}}^2 = \frac{7}{2}k_BT$$

Here, $v_{\text{rms}}^2 = \bar{v}$

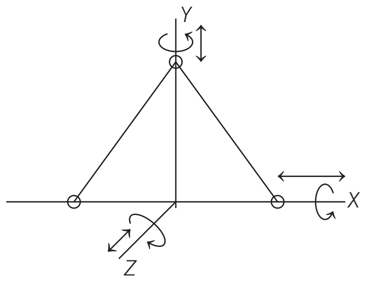
$$\Rightarrow \quad T = \frac{m\bar{v}^2}{7k_B}$$

**38.** Internal energy $U$ of a mole of the gas at temperature $T$ is given by

$$U = \frac{f}{2}RT$$

where, $f$ = degree of freedom of gas molecule

and $R$ = gas constant.



A triatomic triangular gas molecules can have three-translational motions (translation along $X, Y$ or $Z$-axis) and three-rotational motions (rotation along $X, Y$ or $Z$-axis). So, it has total 6 degrees of freedom.

So, internal energy, $U = \frac{f}{2}RT = \frac{6}{2}RT = 3RT$

## *Round II*

**1.** $$\frac{F}{2}n_1kT_1+\frac{F}{2}n_2kT_2+\frac{F}{2}n_3kT_3=\frac{F}{2}(n_1+n_2+n_3)\,kT$$

[$\because$ $F$ is degree of freedom]

Temperature of mixture, $T=\dfrac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$

**2.** From $pV=nRT=\dfrac{N}{N_A}RT$, we have

$$n_f-n_i=\frac{pVN_A}{RT_f}-\frac{pVN_A}{RT_i}$$

$$\Rightarrow\quad n_f-n_i=\frac{10^5\times30}{8.3}\times6.02\times10^{23}\cdot\left(\frac{1}{300}-\frac{1}{290}\right)$$

$$=-2.5\times10^{25}$$

$$\therefore\quad \Delta n=-2.5\times10^{25}$$

**3.** As no work is done and system is thermally insulated from surrounding, it means sum of internal energy of gas in two partitions is constant, *i.e.* $U=U_1+U_2$.

Assuming both gases have same degree of freedom, then

Internal energy, $U=\dfrac{f(n_1+n_2)RT}{2}$

and $\quad U_1=\dfrac{fn_1RT_1}{2},\; U_2=\dfrac{fn_2RT_2}{2}$

Solving, we get

Common temperature, $T=\dfrac{(p_1V_1+p_2V_2)\,T_1T_2}{p_1V_1T_2+p_2V_2T_1}$

**4.** For carbon dioxide, number of mole $(n_1)=\dfrac{22}{44}=\dfrac{1}{2}$;

Molar specific heat of $CO_2$ at constant volume, $C_{V_1}=3R$

For oxygen, number of moles $(n_2)=\dfrac{16}{32}=\dfrac{1}{2}$;

molar specific heat of $O_2$ at constant volume, $C_{V_2}=\dfrac{5R}{2}$.

Let $T$ K be the temperature of mixture.

Heat lost by $O_2$ = Heat gained by $CO_2$.

$$n_2C_{V_2}\Delta T_2=n_1C_{V_1}\Delta T_1$$

$$\frac{1}{2}\left(\frac{5}{2}R\right)(310-T)=\frac{1}{2}\times(3R)\,(T-300)$$

or $\quad 1550-5T=6T-1800$ or $T=304.54\text{ K}=31.5^\circ\text{ C}$

**5.** For an ideal gas undergoing an adiabatic process at room temperature,

$pV^\gamma=$ constant or $TV^{\gamma-1}=$ constant

For a diatomic gas, degree of freedom, $f=5$

$$\therefore\quad \gamma=1+2/f=1+\frac{2}{5}=\frac{7}{5}$$

As for adiabatic process, $TV^{\gamma-1}=$ constant ...(i)

and it is given that, here $TV^x=$ constant ...(ii)

Comparing Eqs. (i) and (ii), we get

$$\gamma-1=x\Rightarrow\frac{7}{5}-1=x\quad\text{or}\quad x=2/5$$

**6.** Here, $p=2\text{ atm}=2\times1.013\times105\text{ Nm}^{-2}$

$$T=17+273=290\text{ K}$$

$$\sigma=2r=2\times1\text{ Å}=2\times10^{-10}\text{ m}$$

$$\lambda=\frac{kT}{\sqrt{2}\,m\sigma^2p}$$

$$=\frac{(1.38\times10^{-23})\times290}{1.414\times3.14\times(2\times10^{-10})^2\times2.026\times10^5}$$

$$=1.11\times10^{-7}\text{ m}$$

**7.** Since, each vibrational mode has 2 degrees of freedom, hence total vibrational degrees of freedom $=24\times2=48$

$\therefore$ Total number of degree of freedom

$$=3+3+48=54$$

$$\therefore\quad \gamma=1+\frac{2}{f}$$

$$=1+\frac{2}{54}$$

$$=\frac{28}{27}=1.03$$

**8.** When the piston is in equilibrium, the pressure is same on both the sides of the piston. It is given that temperature and weight of gas on the two sides of piston does not change. From ideal gas equation, $pV=n\,RT$, we have $V\propto$ mass of the gas.

So, $$\frac{V_1}{V_2}=\frac{m_1}{m_2}$$

or $$\frac{V_1}{V_2}+1=\frac{m_1}{m_2}+1$$

or $$\frac{V_1+V_2}{V_2}=\frac{m_1+m_2}{m_2}$$

or $$\frac{V_2}{V_1+V_2}=\frac{m_2}{m_1+m_2}$$

$$=\frac{2m}{m+2m}=\frac{2}{3}$$

**9.** Work done by gas during heating process at constant pressure is given by

$$\Delta W=p\Delta V$$

Using ideal gas equation,

$$pV=nRT$$

$$\Rightarrow\quad p\Delta V=nR\Delta T$$

So, $$\Delta W=nR\Delta T\quad\text{... (i)}$$

Now, it is given that, $n=\dfrac{1}{2}$ and

$$\Delta T=90^\circ\text{ C}-20^\circ\text{ C}$$

$$=363\text{ K}-293\text{ K}=70\text{ K}$$

and $R$ (gas constant) = 8.31 J/mol-K

Substituting these values in Eq. (i), we get

$$\Delta W=\frac{1}{2}\times8.31\times70=290.85\text{ J}$$

$$\Delta W\simeq291\text{ J}$$

**10.** Thermal energy corresponds to internal energy

$$\text{Mass} = 1 \text{ kg}$$

$$\text{Density} = 4 \text{ kgm}^{-3}$$

$$\Rightarrow \qquad \text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{1}{4} \text{ m}^3$$

$$\text{Pressure} = 8 \times 10^4 \text{ Nm}^{-2}$$

As internal energy $= \dfrac{f}{2} nRT = \dfrac{f}{2} pV$

Degree of freedom for diatomic = 5

$\therefore$ Internal energy $= \dfrac{5}{2} p \times V$

Putting the values of $p = 8 \times 10^4 \text{ Nm}^{-2}$ and $V = \dfrac{1}{4} \text{m}^3$,

we get

Internal energy $= \dfrac{5}{2} \times 8 \times 10^4 \times \dfrac{1}{4} = 5 \times 10^4$ J

**11.** $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

$U(x = \infty) = 0$

As, $F = -\dfrac{dU}{dx} = -\left[\dfrac{12a}{x^{13}} + \dfrac{6\,b}{x^7}\right]$

At equilibrium, $F = 0$

$$\therefore \qquad x^6 = \frac{2a}{b}$$

$$\therefore \qquad U_{\text{at equilibrium}} = \frac{a}{\left(\dfrac{2a}{b}\right)^2} - \frac{b}{\left(\dfrac{2a}{b}\right)} = \frac{-b^2}{4\,a}$$

$$\therefore \qquad D = [U\,(x = \infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

**12.** We know that, from kinetic theory of gases,

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\text{or} \qquad v_{\text{rms}} \propto \sqrt{T}$$

where, $R$ is gas constant, $T$ is temperature and $M$ is molecular mass of a gas.

Here, to double the $v_{\text{rms}}$, temperature must be 4 times of the initial temperature.

$\therefore$ New temperature, $T_2 = 4 \times T_1$

As, $\qquad T_1 = 27°\text{ C} = 300 \text{ K}$

$\Rightarrow \qquad T_2 = 4 \times 300 = 1200 \text{ K}$

As, gas is kept inside a closed vessel.

$$\therefore \qquad Q = nC_V \cdot dT$$

$$= \frac{15}{28} \times \frac{5R}{2} (1200 - 300)$$

$$\left[\text{As, } n = \frac{m}{M} = \frac{15}{28} \text{ for nitrogen}\right]$$

$$= \frac{15}{28} \times \frac{5}{2} \times 8.3 \times 900$$

[Given, $R = 8.3$ J/K-mol]

$$\text{or} \qquad Q = 10004.4 \text{ J} \simeq 10 \text{ kJ}$$

**13.** According to Boyle's law, $pV = k$ (a constant)

$$\text{or} \qquad p\frac{m}{\rho} = k \text{ or } \rho = \frac{pm}{k}$$

$$\text{or} \qquad \rho = \frac{p}{k} \qquad \left(\text{where, } \frac{k}{m} = k = \text{constant}\right)$$

$$\text{So,} \qquad \rho_1 = \frac{p_1}{k}$$

$$\text{and} \qquad V_1 \frac{p_1}{k} = \frac{m_1}{\rho_1} = \frac{m_1}{p_1/k} = \frac{km_1}{p_1}$$

$$\text{Similarly,} \qquad V_2 = \frac{km_2}{p_2}$$

Total volume $= V_1 + V_2 = k\left(\dfrac{m_1}{p_1} + \dfrac{m_2}{p_1}\right)$

Let $p$ be the common pressure and $\rho$ be the common density of mixture. Then

$$\rho = \frac{m_1 + m_2}{V_1 + V_2} = \frac{m_1 + m_2}{k\left(\dfrac{m_1}{p_1} + \dfrac{m_2}{p_2}\right)}$$

$$\therefore \qquad p = k\rho = \frac{m_1 + m_2}{\dfrac{m_1}{p_1} + \dfrac{m_2}{p_2}} = \frac{p_1 p_2\,(m_1 + m_2)}{(m_1 p_2 + m_2 p_1)}$$

**14.** Mean time elapsed between two successive, collisions is $t = \dfrac{\lambda}{v}$

where, $\lambda$ = mean free path length and

$v$ = mean speed of gas molecule

$$\therefore \qquad t = \frac{\left(\dfrac{k_B t}{\sqrt{2}\,\pi\, d^2 p}\right)}{\sqrt{\dfrac{8}{\pi} \cdot \dfrac{k_B T}{M}}} \Rightarrow t = \frac{C\sqrt{T}}{p}$$

where, $C = \dfrac{1}{4d^2}\sqrt{\dfrac{k_B M}{\pi}}$ = a constant for a gas.

$$\text{So,} \qquad \frac{t_2}{t_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}} \cdot \left(\frac{p_1}{p_2}\right) \qquad \dots \text{(i)}$$

Here given, $\dfrac{p_1}{p_2} = \dfrac{1}{2}$, $\sqrt{\dfrac{T_2}{T_1}} = \sqrt{\dfrac{500}{300}} = \sqrt{\dfrac{5}{3}}$

and $\qquad t_1 = 6 \times 10^{-8}$ s

Substituting there values in Eq. (i), we get

$$t_2 = 6 \times 10^{-8} \times \sqrt{\frac{5}{3}} \times \frac{1}{2}$$

$$= 3.86 \times 10^{-8} \text{ s} \approx 4 \times 10^{-8} \text{ s}$$

**15.** Let molar specific heat of the mixture is $C_V$.

Total number of molecules in the mixture = 3 + 2 = 5

$\therefore$ $C_V$ can be determined using

$$n\, C_V\; dT = n_1\, C_{V_1}\; dT + n_2\, C_{V_2} dT$$

or $(n_1 + n_2)\;(C_V)_{\text{mix}} = n_1 C_{V_1} + n_2\, C_{V_2}$

[here, $n = n_1 + n_2$]

Here, $C_{V_1} = \dfrac{3R}{2}$ (for helium); $n_1 = 2$

$C_{V_2} = \dfrac{5R}{2}$ (for hydrogen); $n_2 = 3$

[Because for monoatomic gases, $C_V = \dfrac{3}{2}R$ and for diatomic gases, $C_V = \dfrac{5}{2}R$]

$$\therefore \quad 5 \times C_V = \left(2 \times \frac{3R}{2}\right) + \left(3 \times \frac{5R}{2}\right) \Rightarrow 5C_V = \frac{21R}{2}$$

or $$C_V = \frac{21R}{10} = \frac{21 \times 8.3}{10} = \frac{174.3}{10}$$

or $$C_V = 17.4 \text{ J/mol-K}$$

**16.** For an adiabatic process $TV^{\gamma - 1} = \text{constant}$.

We know that average time of collision between molecules

$$\tau = \frac{1}{n\pi\sqrt{2}\, v_{\text{rms}}\, d^2}$$

where, $n$ = number of molecules per unit volume

$v_{\text{rms}}$ = rms velocity of molecules

As $$n \propto \frac{1}{V} \text{ and } v_{\text{rms}} \propto \sqrt{T}$$

$$\tau \propto \frac{V}{\sqrt{T}}$$

Thus, we can write $n = K_1 V^{-1}$ and $v_{\text{rms}} = K_2 T^{1/2}$

where, $K_1$ and $K_2$ are constants.

For adiabatic process, $TV^{\gamma - 1}$ = constant. Thus, we can write

$$\tau \propto VT^{-1/2} \propto V\,(V^{1-\gamma})^{-1/2}$$

or $$\tau \propto V^{\frac{\gamma + 1}{2}}$$

**17.** For mixture, $C_V = \dfrac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

For helium,

Number of moles, $n_1 = \dfrac{16}{4} = 4$ and $\gamma_1 = \dfrac{5}{3}$

For oxygen,

Number of moles, $n_2 = \dfrac{16}{32} = \dfrac{1}{2}$ and $\gamma_2 = \dfrac{7}{5}$

Then, $C_{V_1} = \dfrac{R}{\gamma_1 - 1} = \dfrac{R}{\frac{5}{3} - 1} = \dfrac{3}{2}R$

and $C_{V_2} = \dfrac{R}{\gamma_2 - 1} = \dfrac{R}{\frac{7}{5} - 1} = \dfrac{5}{2}R$

∴ For mixture,

$$C_V = \frac{4 \times \frac{3}{2}R + \frac{1}{2} \times \frac{5}{2}R}{4 + \frac{1}{2}} = \frac{6R + \frac{5}{4}R}{\frac{9}{2}}$$

$$= \frac{29R \times 2}{9 \times 4} = \frac{29R}{18}$$

Now, $$C_V = \frac{R}{\gamma - 1}$$

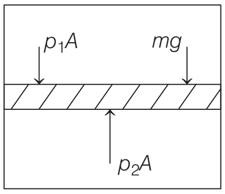
$$\Rightarrow \quad \gamma - 1 = \frac{R}{C_V}$$

or $$\gamma = \frac{R}{C_V} + 1 = \frac{R}{\frac{29}{18}R} + 1$$

$$\Rightarrow \quad \frac{C_p}{C_V} = \frac{18}{29} + 1 = \frac{18 + 29}{29} = 1.62$$

**18.** As piston is in equilibrium, so net force on piston is zero.

When the piston is stationary, *i.e.* on equilibrium as shown in the figure below



then $p_1A + mg = p_2A$

$$\Rightarrow \quad mg = p_2A - p_1A$$

or $$mg = \left(\frac{nRTA}{V_2} - \frac{nRTA}{V_1}\right)$$

[$\because pV = nRT$ (ideal gas equation)]

$$= nRT\left(\frac{A}{Al_2} - \frac{A}{Al_1}\right) = nRT\left(\frac{l_1 - l_2}{l_1 l_2}\right)$$

or $$m = \frac{nRT}{g}\left(\frac{l_1 - l_2}{l_1 l_2}\right)$$

**19.** Mayer's formula is given by, $C_p - C_V = R$ and $\gamma = \dfrac{C_p}{C_V}$

Therefore, using above two relations, we find

$$C_V = \frac{R}{\gamma - 1}$$

For a mole of monatomic gas, $\gamma = \dfrac{5}{3}$

$$\therefore \quad C_V = \frac{R}{\left(\frac{5}{3}\right) - 1} = \frac{3}{2}R$$

For a mole of diatomic gas, $\gamma = \dfrac{7}{5}$

$$\therefore \quad C_V = \frac{R}{\left(\frac{7}{5}\right) - 1} = \frac{5}{2}R$$

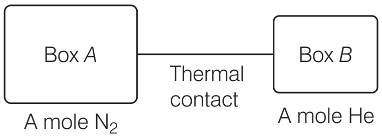
When these two moles are mixed, then heat required to raise the temperature to 1°C is $C_V = \dfrac{3}{2}R + \dfrac{5}{2}R = 4R$

Hence, for one mole, heat required $= \dfrac{4R}{2} = 2R$

$$\therefore \quad C_V = 2R$$

$$\Rightarrow \quad \frac{R}{\gamma - 1} = 2R \quad \text{or} \quad \gamma = \frac{3}{2}$$

**20.** Here, change in internal energy of the system is zero, *i.e.* increase in internal energy of one is equal to decrease in internal energy of other.



Change in internal energy in box *A*,

$$\Delta U_A = 1 \times \frac{5}{2} R\,(T_f - T_0)$$

Change in internal energy in box *B*

$$\Delta U_B = 1 \times \frac{3R}{2}\left(T_f - \frac{7}{3}T_0\right)$$

Now, $\Delta U_A + \Delta U_B = 0$

$$\Rightarrow \quad \frac{5R}{2}(T_f - T_0) + \frac{3R}{2}\left(T_f - \frac{7T_0}{3}\right) = 0$$

$$\text{or} \quad 5T_f - 5T_0 + 3T_f - 7T_0 = 0$$

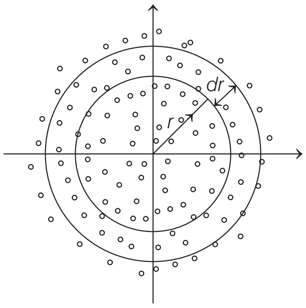
$$\text{or} \quad 8T_f = 12T_0$$

$$\text{or} \quad T_f = \frac{12}{8}T_0 = \frac{3}{2}T_0$$

**21.** Number density of gas molecules,

$$n = \frac{\text{Number of molecules}}{\text{Volume of gas}}$$

$\Rightarrow$ Number of molecules, $N = n \times$ Volume of gas



Now, consider a shell of radius *r* and thickness *dr* with centre at $r = 0$.

Volume of shell of differentiable thickness ($dr$),
$dV =$ surface area $\times$ thickness $= 4\pi r^2\,dr$

Now, number of molecules in this shell is

$$dN = n(r)\cdot dV = n_0 e^{-\alpha r^4}\cdot 4\pi r^2\,dr$$

So, total number of molecules present in given volume (extending from $r = 0$ to $r = \infty$) is

$$N = \int_0^\infty n(r)\cdot dV = \int_0^\infty n_0\, e^{-\alpha r^4}\cdot 4\pi r^2\,dr$$

$$= \int_0^\infty 4\pi n_0\, e^{-\alpha r^4}\cdot r^2\,dr \qquad \ldots\text{(i)}$$

Here, we take $\alpha r^4 = t$

$$\Rightarrow \quad r = t^{\frac{1}{4}}\cdot \alpha^{\frac{-1}{4}} \Rightarrow 4\alpha r^3 dr = dt$$

$$\Rightarrow \quad r^2\,dr = \frac{dt}{4\alpha r} = \frac{dt}{4\alpha t^{\frac{1}{4}}\cdot \alpha^{-\frac{1}{4}}} = \frac{dt}{4\alpha^{3/4}\,t^{\frac{1}{4}}}$$

Also, when $r = 0$, $t = 0$ and when $r = \infty$, $t = \infty$ substituting in Eq. (i), we get

$$N = \int_0^\infty 4\pi\, n_0\, e^{-t}\cdot \frac{dt}{4\,\alpha^{\frac{3}{4}}\,t^{\frac{1}{4}}}$$

$$N = \pi\alpha^{-3/4}\cdot n_0 \cdot \int_0^\infty e^{-t}\cdot t^{-\frac{1}{4}}\,dt$$

As value of definite integral $\int_0^\infty e^{-t}\cdot t^{-\frac{1}{4}}\,dt$ is a constant ($= k$ let), we have

$$N = \pi\, k n_0 \alpha^{-3/4}$$

$$\Rightarrow \quad N \propto n_0 \alpha^{-3/4}$$

**22.** Here, $V = 25.0\ \text{m}^3$

$$T = 27 + 273 = 300\ \text{K}$$

$$k = 1.38 \times 10^{-23}\ \text{JK}^{-1}$$

Now, $pV = nRT = n\,(Nk)T$

$$= (nN)kT = N'\,kT$$

where, $nN = N' =$ total number of air molcules in the given gas

$$N' = \frac{pV}{kT} = \frac{(1.01 \times 10^5) \times 25}{(1.38 \times 10^{-23}) \times 300}$$

$$= 6.10 \times 10^{26} = 6.10 \times 10^{13 \times 2}$$

$$\therefore \quad x = 2$$

**23.** If *T* be the final equilibrium temperature, then applying conservation of internal energy,

$$U_1 + U_2 = U_1' + U_2'$$

$$n_1 C_V T_1 + n_2 C_V T_2 = (n_1 + n_2) C_V T$$

$$n_1 T_1 + n_2 T_2 = (n_1 + n_2)\,T$$

$$(0.1)\,(200) + (0.05)\,(400) = (0.1 + 0.05)\,T$$

$$20 + 20 = (0.15)\,T$$

$$T = \frac{40}{0.15} = 266.66\ \text{K} \simeq 266\ \text{K}$$

**24.** Here, $c_1 = 2\ \text{kms}^{-1}$; $c_2 = 4\ \text{kms}^{-1}$

$c_3 = 6\ \text{kms}^{-1}$ and $c_4 = 8\ \text{kms}^{-1}$

(i) Average speed,

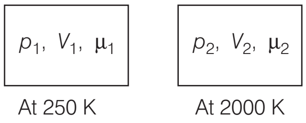
$$c_{\text{av}} = \frac{c_1 + c_2 + c_3 + c_4}{4} = \frac{2 + 4 + 6 + 8}{4} = 5\ \text{kms}^{-1}$$

(ii) Root mean square speed,

$$c = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 + c_4^2}{4}} = \sqrt{\frac{2^2 + 4^2 + 6^2 + 8^2}{4}}$$

$$= 5.48\ \text{kms}^{-1}$$

**25.**



Let $\qquad \mu_1 = \mu_0,\ V_2 = 2V_1 \qquad$ (given)

Total number of moles at 2000 K = Number of moles of diatomic molecules + Number of moles of monoatomic

$$\mu_2 = 0.75\mu_0 + 2 \times 0.25\mu_0 = 1.25\,\mu_0$$

Ideal gas equation for situation 1,

$$p_1V_1 = \mu_0 RT_1$$

$$p_1V_1 = \mu_0 R(250) \qquad \text{... (i)}$$

For situation 2,

$$p_2V_2 = (1.25\mu_0)R(2000)$$

$$p_2(2V_1) = (1.25\mu_0)R(2000) \qquad \text{... (ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{p_1V_1}{p_2(2V_1)} = \frac{\mu_0R(250)}{(1.25\mu_0)R(2000)}$$

$$\frac{p_1}{p_2} = \frac{1}{5} \text{ or } \frac{p_2}{p_1} = 5$$

**26.** Here, $J = ?$

$$C_p = 3.409 \text{ calg}^{-1\circ}\text{C}^{-1}$$

$$C_V = 2.409 \text{ calg}^{-1\circ}\text{C}^{-1}, M = 2$$

$$R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$$

As, $C_p - C_V = \frac{r}{J} = \frac{R}{MJ}$

$$\therefore \quad 3.409 - 2.409 = \frac{8.31}{2J}$$

$$1 = \frac{4.155}{J}$$

$$J = 4.11 \text{ J cal}^{-1}$$

**27.** The rms speed is given by $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

For nitrogen molecule, $(v_{\text{rms}})_{N_2} = \sqrt{\frac{3RT_1}{M_1}}$

and for hydrogen molecule, $(v_{\text{rms}})_{H_2} = \sqrt{\frac{3RT_2}{M_2}}$

According to question, $(v_{\text{rms}})_{N_2} = (v_{\text{rms}})_{H_2}$

$$\Rightarrow \quad \sqrt{\frac{3RT_1}{M_1}} = \sqrt{\frac{3RT_2}{M_2}} \Rightarrow \frac{T_1}{M_1} = \frac{T_2}{M_2}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{M_1}{M_2} \Rightarrow T_2 = \frac{T_1M_2}{M_1} \qquad \text{...(i)}$$

Given, $T_1 = 300^\circ \text{C} = 300 + 273 = 573 \text{ K}$

$M_1 = 28 \text{ g and } M_2 = 2 \text{ g}$

Substituting all these values in Eq. (i), we get

$$T_2 = 573 \times \frac{2}{28} = 40.93 \text{ K} \approx 41 \text{ K}$$

Hence, the required temperature is 41 K.

**28.** Here, mass of metal, $m = 0.20 \text{ kg} = 200 \text{ g}$

Fall in temperature of metal

$$\Delta T = 150^\circ \text{C} - 40^\circ \text{C} = 110^\circ\text{C}$$

If $c$ is specific heat of the metal, then heat lost by the metal,

$$\Delta Q = mc\Delta T = 200\, s \times 110 \qquad \text{...(i)}$$

Volume of water = 150 cc

Mass of water, $m' = 150$ g

Water equivalent of calorimeter

$$w = 0.025 \text{ kg} = 25\text{g}$$

Rise in temperature of water in calorimeter

$$\Delta T' = 40^\circ \text{C} - 27^\circ \text{C} = 13^\circ \text{ C}$$

Heat gained by water and calorimeter

$$\Delta Q' = (m' + w)\,\Delta T' = (150 + 25) \times 13$$

$$\Delta Q' = 175 \times 13 \qquad \text{...(ii)}$$

As $\quad \Delta Q = \Delta Q'$

$\therefore$ From Eqs. (i) and (ii), we get

$$200 \times s \times 100 = 175 \times 13$$

$$s = \frac{175 \times 13}{200 \times 110} \approx 0.1 \text{ J/g°C}$$

**29.** Here, volume of water heated = 3.0 $\text{Lmin}^{-1}$

Mass of water heated, $m = 3000 \text{ gmin}^{-1}$

Rise of temperature, $\Delta T = 77^\circ \text{C} - 27^\circ \text{C} = 50^\circ \text{C}$

Specific heat of water, $s = 4.2 \text{ Jg}^{-1}\,{}^\circ\text{C}^{-1}$

Amount of heat used, $\Delta Q = ms\Delta T = 3000 \times 4.2 \times 50$

$= 63 \times 10^4 \text{ Jmin}^{-1}$

Heat of combustion $= 4 \times 10^4 \text{ Jg}^{-1}$

Rate of combustion of fuel $= \frac{63 \times 10^4}{4 \times 10^4} = 15.75 \text{ gmin}^{-1}$

**30.** Given, at 27°C, root mean square speed,

$$(v_{\text{rms}})_1 = 100 \text{ ms}^{-1}$$

$$(v_{\text{rms}})_1 = \sqrt{\frac{3p_1}{d_1}} = \sqrt{\frac{3p_1V_1}{M}} \qquad \text{...(i)}$$

According to ideal gas equation,

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{p_2T_1}{p_1T_2} = \frac{2 \times 300}{400} = \frac{3}{2}$$

$\therefore$ At 127°C, root mean square speed

$$(v_{\text{rms}})_2 = \sqrt{\frac{3p_2}{d_2}} = \sqrt{\frac{3p_2V_2}{M}} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\therefore \quad (v_{\text{rms}})_2^2 = (v_{\text{rms}})_1^2 \times \frac{V_2}{V_1} \times \frac{p_2}{p_1} = (100)^2 \times \frac{2}{3} \times 2$$

$$\text{or} \quad (v_{\text{rms}})_2 = \frac{200}{\sqrt{3}} \text{ ms}^{-1}$$

$$\therefore \quad n = 3$$

**31.** When the air bubble is at 40 m depth, then

$$V_1 = 1 \text{ cm}^3 = 1.0 \times 10^{-6} \text{ m}^3$$

$$T_1 = 12\,^\circ\text{C} = 12 + 273 = 285 \text{ K}$$

$$P_1 = 1\text{atm} + h_1\rho g = 1.01 \times 10^5 + 40 \times 10^3 \times 9.8$$

$$= 493000 \text{ Pa}$$

When the air bubble reaches at the surface of lake, then, $\quad V_2 = ?, T_2 = 35^\circ \text{C} = 35 + 273 = 308 \text{ K}$

$$P_2 = 1\text{atm} = 1.01 \times 10^5 \text{ Pa}$$

Now, $\quad \frac{p_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ or $V_2 = \frac{p_1V_1T_2}{T_2p_2}$

$$\therefore \quad V_2 = \frac{(493000) \times 1.0 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5}$$

$$= 5.275 \times 10^{-6} \text{ m}^3$$