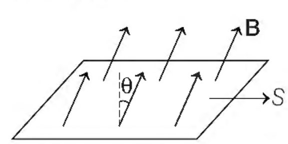
CHAPTER 20

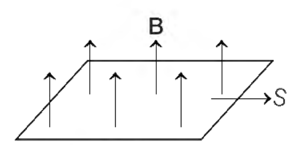
Electromagnetic Induction and Alternating Current

Magnetic Flux

- Magnetic flux $\phi = \int \mathbf{B} \cdot d\mathbf{S}$
- SI unit of magnetic flux is weber.
- If $\mathbf{B}$ is uniform, then



$\phi = BS \cos\theta$



$\phi = BS$

Faraday's Laws

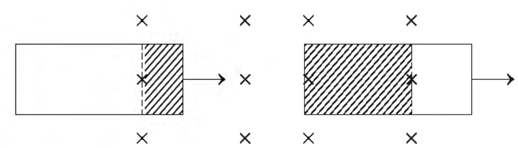
- Induced emf, $e = -\dfrac{d\phi}{dt}$
- Induced current, $i = e/R = \dfrac{(-d\phi/dt)}{R}$
- Induced flow of charge, $\Delta q = (i\Delta t) = -\dfrac{\Delta\phi}{R}$
- According to Lenz law, any induced event always opposes the cause, due to which it is happening.
- As we have seen, induced emf is produced only when there is a change in magnetic flux passing through a loop. The flux passing through the loop is given by $\phi = BS \cos\theta$.

  This flux can be changed in several ways

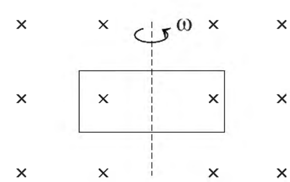
  (i) The magnitude of $\mathbf{B}$ can change with time. Thus, $B = B(t)$.

(ii) The current which is producing the magnetic field can change with time. Hence, $i = i(t)$.

(iii) The area of the loop inside the magnetic field can change with time. This can be done by pulling a loop inside (or outside) a magnetic field.



(iv) The angle θ between **B** and the normal to the loop (or **S**) can change with time.



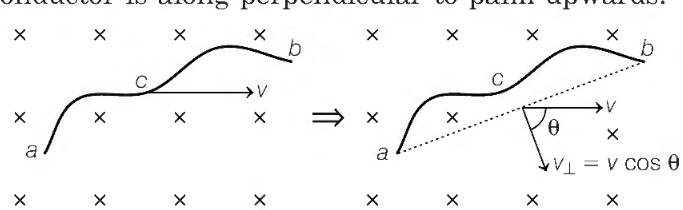
This can be done by rotating a loop in a magnetic field.

Motional EMF

- On a straight conducting wire $e = Bvl$
- On a rotating conducting wire about one end,

$$e = \frac{B\omega l^2}{2}$$

- The direction of motional emf or current can be given by right hand rule. The stretched fingers point in the direction of magnetic field. Thumb is along the velocity of conductor. The upper side of the palm is at higher potential and lower side at lower potential. If the circuit is closed, the induced current within the conductor is along perpendicular to palm upwards.



In the above figure, from right hand rule we can see that b is at higher potential and a at lower potential.

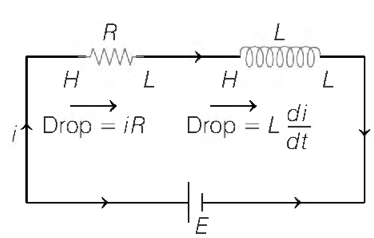
$$V_{ba} = V_b - V_a = (ab)(v\cos\theta)(B)$$

Self and Mutual Induction

- Coefficient of self induction, $L = \dfrac{N\phi}{i}$ or $\left(\dfrac{-e}{di/dt}\right)$

- Coefficient of self induction of a circular coil,
$$L \propto N^2$$
- Potential difference across an inductor,
$$PD = -L\left(\frac{di}{dt}\right)$$
- Energy stored in the magnetic field of inductor,
$$U = \frac{1}{2}Li^2$$
- Coefficient of mutual inductance,
$$M = \frac{N_2\,\phi_2}{i_1} \quad \text{or} \quad \left(\frac{-e_2}{di_1/dt}\right)$$
- Coefficient of mutual inductance of two closely wound circular coils,
$$M \propto N_1 N_2$$
- Inductors in series, $L = L_1 + L_2$
- Inductors in parallel, $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ or $L = \frac{L_1 L_2}{L_1 + L_2}$
- Energy density or energy per unit volume in magnetic field,
$$u = \frac{1}{2}\frac{B^2}{\mu_0}$$
- **Kirchhoff's potential law with an inductor**

In Kirchhoff's potential equation when we jump an inductor in the direction of current, we encounter a voltage drop equal to $L\,di/dt$, where di/dt is to be substituted with sign.



For example, in the loop shown in figure, Kirchhoff's second law gives the equation.
$$E - iR - L\frac{di}{dt} = 0$$
- Current growth in an L-R circuit,
$$i = i_0\,(1 - e^{-t/\tau_L})$$
- Current decay in L-R circuit, $i = i_0\, e^{-t/\tau_L}$
- τ_L = time constant = $\frac{L}{R}$
- Unit of τ_L or τ_C is second.

Oscillations in L-C circuit

In L - C circuit, charge, current and rate of change of current oscillate simple harmonically. These oscillations are similar to oscillations of spring-block system in the chapter of SHM. Table below shows a comparison of oscillations of a mass-spring system and L - C circuit.

A comparison of oscillations of a mass-spring system and *L*-*C* circuit

Mass-spring system	Inductor-capacitor circuit
Displacement (x)	Charge (q)
Velocity (v)	Current (i)
Acceleration (a)	Rate of change of current $\left(\frac{di}{dt}\right)$
$\frac{d^2x}{dt^2} - \omega^2 x$, where $\omega = \sqrt{\frac{k}{m}}$	$\frac{d^2q}{dt^2} = -\omega^2 q$, where $\omega = \frac{1}{\sqrt{LC}}$
$x = A\sin(\omega t \pm \phi)$ or $x = A\cos(\omega t \pm \phi)$	$q = q_0 \sin(\omega t \pm \phi)$ or $q = q_0 \cos(\omega t \pm \phi)$
$v = \frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$	$i = \frac{dq}{dt} = \omega\sqrt{q_0^2 - q^2}$
$a = \frac{dv}{dt} = -\omega^2 x$	Rate of change of current $= \frac{di}{dt} = -\omega^2 q$
Kinetic energy $= \frac{1}{2}mv^2$	Magnetic energy $= \frac{1}{2}Li^2$
Potential energy $= \frac{1}{2}kx^2$	Potential energy $= \frac{1}{2}\frac{q^2}{C}$
$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant} = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$	$\frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C} = \text{constant} = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}Li_{max}^2$
$\lvert v_{max} \rvert = A\omega$	$i_{max} = q_0\omega$
$\lvert a_{max} \rvert = \omega^2 A$	$\left\lvert \left(\frac{di}{dt}\right)_{max} \right\rvert = \omega^2 q_0$
$\frac{1}{k}$	C
m	L

Induced Electric Field

- The line integral of $\mathbf{E}$ around a closed path is not zero. This line integral is given by

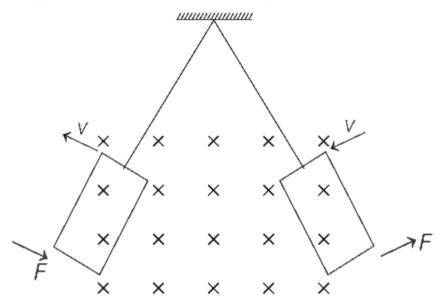
$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi_B}{dt}$$

This electric field is produced by change in magnetic field.

- Being a non-conservative field, the concept of potential has no meaning for such a field.
- This field is different from the electrostatic field produced by stationary charges (which is conservative in nature).
- The relation $\mathbf{F} = q\mathbf{E}$ is valid for this field.

Eddy Currents

- When a changing magnetic flux is applied to a piece of conducting material, circulating currents called eddy currents are induced in the material. These eddy currents often have large magnitudes and heat up the conductor.
- When a metal plate is allowed to swing through a strong magnetic field, then in entering or leaving the field, the eddy currents are set up in the plate which opposes the motion as shown in figure.



- The kinetic energy dissipates in the form of heat. The slowing down of the plate is called the **electromagnetic damping.**
- The electromagnetic damping is used to damp the oscillations of a galvanometer coil or chemical balance and in braking electric trains. Otherwise, the eddy currents are often undesirable.
- To reduce the eddy currents some slots are cut into moving metallic parts of machinery. These slots intercept the conducting paths and decreases the magnitudes of the induced currents.

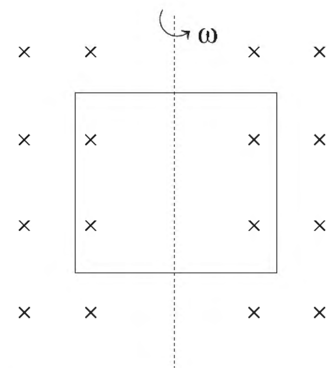
Back EMF of Motors

- An electric motor converts electrical energy into mechanical energy and is based on the fact that a current carrying coil in a uniform magnetic field experiences a torque.
- As the coil rotates in the magnetic field, the flux linked with the rotating coil will change and hence, an emf called back emf is produced in the coil.
- When the motor is first turned ON, the coil is at rest and so there is no back emf. The 'start up' current can be quite large. To reduce 'start up' current a resistance called 'starter' is put in series with the motor for a short period when the motor is started.
- As the rotation rate increases the back emf increases and hence, the current reduces.

Electric Generator or Dynamo

- A dynamo converts mechanical energy (rotational kinetic energy) into electrical energy. It consists of a coil rotating in a magnetic field.

- Due to rotation of the coil, magnetic flux linked with it changes, so an emf is induced in the coil.



Suppose at time $t = 0$, plane of coil is perpendicular to the magnetic field.
The flux linked with it at any time t will be given by

$$\phi = NBA \cos \omega t \qquad \text{(where, } N = \text{number of turns in the coil)}$$

$$\therefore \qquad e = -\frac{d\phi}{dt} = NBA\omega \sin \omega t \text{ or } e = e_0 \sin \omega t$$

where, $\quad e_0 = NBA\,\omega$

Alternating Current

- Frequency of AC in India is 50 Hz.
- The AC is converted into DC with the help of rectifier while DC is converted into AC with the help of inverter.
- An AC cannot produce electroplating or electrolysis.
- The AC is measured by hot wire ammeter.
- An AC can be stepped up or down with the help of a transformer.
- An AC can be transmitted over long distances without much power loss.
- An AC can be regulated by using choke coil without any significant waste of energy.
- In an AC (sinusoidal) current or voltage can have following four values :
  (i) instantaneous value
  (ii) peak value (i_0 or V_0)
  (iii) irms value (i_{rms} or V_{rms})
  (iv) average value; (In full cycle, average value is zero while in half cycle it is non-zero.)
- $i_{rms} = \dfrac{i_0}{\sqrt{2}}, V_{rms} = \dfrac{V_0}{\sqrt{2}}$
- $\langle i \rangle_{\text{positive half cycle}} = \dfrac{2}{\pi} i_0 \approx 0.636\, i_0$

  Similarly, $\langle V \rangle_{\text{positive half cycle}} = \dfrac{2}{\pi} V_0 \approx 0.636\, V_0$

Note *In sinusoidal AC, even the average value in half cycle can also be zero. It depends on the time interval over which half, average value is desired.*

Series *L*-*C*-*R* AC circuit

- Capacitive reactance, $X_C = \dfrac{1}{\omega C}$
- Inductive reactance, $X_L = \omega L$
- Impedance, $Z = \sqrt{R^2 + (X_C - X_L)^2}$
- If $X_C > X_L$, current leads and if $X_L > X_C$, voltage leads by an angle ϕ given by

$$\cos\phi = \frac{R}{Z}$$

or

$$\tan\phi = \frac{X_C \sim X_L}{R}$$

- Instantaneous power = instantaneous current × instantaneous voltage
- Average power = $V_{\text{rms}}\, i_{\text{rms}} \cos\phi$, where $\cos\phi = \dfrac{R}{Z}$ = power factor.
- $i_0 = \dfrac{V_0}{Z}$ or $i_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}$
- $(V_C)_{\text{rms}} = (i_{\text{rms}})\, X_C$, $(V_L)_{\text{rms}} = (i_{\text{rms}})\, X_L$ and $(V_R)_{\text{rms}} = (i_{\text{rms}})\, R$
- $V = \sqrt{V_R^2 + (V_C - V_L)^2}$

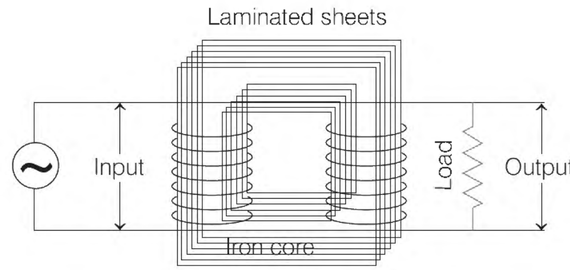
  Here, V is the rms value of applied voltage,
  V_R is the rms value of voltage across resistance,
  V_C across capacitor and V_L across inductor, etc.
- At $\omega = \omega_r = \dfrac{1}{\sqrt{LC}}$, $X_C = X_L$ and Z has the minimum value equal to R. Power factor in this case is 1.
- At $\omega > \omega_r$, $X_L > X_C$, voltage leads the current function and circuit is inductive.
- At $\omega < \omega_r$, $X_C > X_L$, current leads the voltage function and circuit is capacitive.
- **Quality factor** $Q = \dfrac{\omega_r L}{R}$

  It is a measure of sharpness of resonance. If value of Q is large, sharpness is more and it is more selective.

Transformer

- It is a device which is either used to increase or decrease the voltage in AC circuits through mutual induction. A transformer consists of two coils wound on the same core.
- The coil connected to input is called primary while the other connected to output is called secondary coil. An alternating current passing through the primary creates a continuously changing flux through the core.

This changing flux induces an alternating emf in the secondary.



- As magnetic lines of force are closed curves, the flux per turn of primary must be equal to flux per turn of the secondary. Therefore,

$$\frac{\phi_P}{N_P} = \frac{\phi_S}{N_S}$$

or $$\frac{1}{N_P} \cdot \frac{d\phi_P}{dt} = \frac{1}{N_S} \cdot \frac{d\phi_S}{dt} \qquad \left(\text{As } e \propto \frac{d\phi}{dt}\right)$$

$\therefore$ $$\frac{e_S}{e_P} = \frac{N_S}{N_P}$$

- In an ideal transformer, there is no loss of power.

Hence, $ei = \text{constant}$

$\Rightarrow$ $$\frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{i_P}{i_S}$$

- Regarding a transformer, given below are few important points
  - (i) In step-up transformer, $N_S > N_P$. It increases voltage and reduces current.
  - (ii) In step-down transformer, $N_P > N_S$. It increases current and reduces voltage.
  - (iii) It works only on AC.
  - (iv) A transformer cannot increase (or decrease) voltage and current simultaneously. As, $ei = \text{constant}$
  - (v) Some power is always lost due to eddy currents, hysteresis, etc.