Oscillations

14
Chapter

1 SPECIAL TYPES OF MOTIONS

- **Periodic Motion**

  A motion which repeats itself after regular intervals of time, (T) is periodic. Examples:

  - Motion a particle in circle with constant speed
  - Skipping
  - Spring block system
  - Simple pendulum
  - Motion of Earth around sun
  - Motion of needle of sewing machine
  - A boat tossing up and down in a lake
  - Piston of engine going back and forth can be periodic

- **Oscillatory Motion**

  Special type of periodic motion in which a particle moves to and fro about a fixed point. The force acting on the particle in a direction directed towards equilibrium position is called **restoring force**.

  - Every oscillatory motion is periodic but every periodic motion may not be oscillatory.
  - Back and Forth motion can be oscillatory or vibratory. When oscillations frequency is small we call it oscillatory, at high frequency we call it vibratory.

  Oscillations can be

  A. **Free oscillations**

  - When a system oscillates with its natural frequency the oscillations are called free oscillations.

  B. **Damped oscillations**

  - If some external resisting force appears opposing restoring force, oscillatory amplitude starts decreasing with time.

  C. **Forced oscillations**

  - Forced oscillations are those in which damping is not allowed by applying an external time varying force, which compensates the effect of damping force acting on it.

2 SIMPLE HARMONIC MOTION

- Simple harmonic motion is an example of periodic oscillatory motion.
- Special type of oscillatory motion which satisfies following conditions.
  A. Oscillatory amplitude of particle is small.
  B. During oscillation, acceleration towards mean position, due to net restoring force, is directly proportion to displacement from mean position.
- Force displacement relation in S.H.M.
  $F = -ky$, where K is force constant (Force law in S.H.M.), y is displacement from mean position.
- Acceleration of particle

  $a = \frac{F}{m} = -\left(\frac{K}{m}\right)y = -\omega^2 y$

  ∴ Acceleration and displacement are antiparallel

  $\frac{d^2y}{dt^2} + \omega^2 y = 0$, here $\omega = \sqrt{\frac{K}{m}}$ (Angular frequency)

  m is mass oscillating, K is force constant.

- **General equation for displacement in S.H.M.**

  $y = A\sin(\omega t + \phi)$ or $y = A\cos(\omega t \pm \phi)$

  $\omega = \frac{2\pi}{T} = 2\pi n$ is angular frequency and $(\omega t + \phi)$ is called phase, a time varying quantity.

  Here ϕ is called **epoch** or initial phase.

  A. If particle at $t = 0$ is at equilibrium position. $(y = 0)$

  $y = A\sin\omega t$

  B. If particle at $t = 0$ is at extreme right position $(y = A)$

  $y = A\cos\omega t$

- **Velocity of particle in SHM.**

  $v_P = \frac{dy}{dt} = \omega A\cos(\omega t \pm \phi)$

  If at $t = 0$ particle is at origin.

  $v_P = \omega A\cos\omega t = \omega\sqrt{A^2 - y^2}$

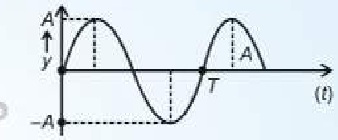
- **Acceleration of particle in SHM**

  $a_P = -\omega^2 A\sin\omega t$, at $t = 0$ particle is at mean position.

- Velocity displacement graph will be an ellipse $(\omega \neq 1)$ or a circle $(\omega = 1\ \text{rad s}^{-1})$.
- The maximum velocity of particle executing SHM will be at mean position and at extremes speed becomes minimum (zero).
- Different graphs for a particle executing SHM

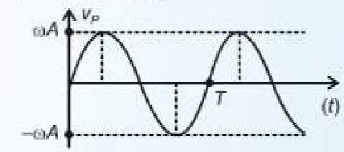
  (A) **Displacement - time graph**

  If at $t = 0$ particle is at mean position



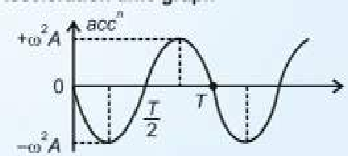
  $y = A\sin(\omega t)$

  (B) **Velocity - time graph**



  $v = A\omega\cos(\omega t)$

  (C) **Acceleration time graph**

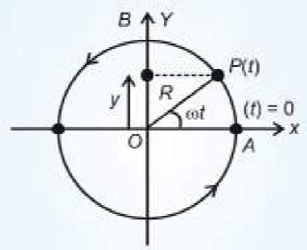


  $a = -\omega^2 A\sin(\omega t)$

  - Velocity leads displacement by a phase of $(\pi/2)$ rad.
  - Acceleration leads velocity a phase of $\pi/2$ rad.

③ SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION

- Projection of uniform circular motion on a diameter of the circle follows simple harmonic motion.



Particle position
$y = R \sin \omega t$ is SHM.

This is an equation of S.H.M. for particle displacement at any time.

④ SPECIAL PARAMETERS IN SHM

- Since particles speed is not constant ; from mean position to half of amplitude it takes half of time than to move from half of amplitude to extreme position.
- Minimum velocity in S.H.M. is $v_{min} = 0$ at extremes and maximum velocity at equilibrium position. $v_{max} = \omega A$
- Maximum acceleration of particle is at extreme positions $a_{max} = \omega^2 A$ and minimum (zero) is at equilibrium.
- Maximum force on particle is at extreme positions and zero at mean, in between it varies linearly always directed towards equilibrium.

  $F_{max} = m\omega^2 A$ and F_{min} = zero

⑤ MECHANICAL ENERGY IN SIMPLE HARMONIC OSCILLATOR

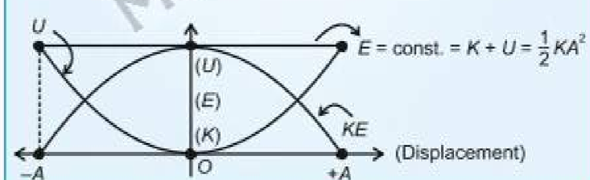
- Potential energy in SHM $U = \frac{1}{2}ky^2 + U_0$

  U_0 is generally taken zero at equilibrium.

  $F_{ist} = -\frac{dU}{dy}$; instantaneous force on particle.
- Maximum potential energy occurs at extreme positions and minimum at mean position.
- Graph of potential energy versus displacement of particle will be parabolic, symmetric about y-axis.
- Kinetic energy of particle in S.H.M. varies directly as square of its velocity at any location.

  $KE = \frac{1}{2}(m\omega^2)(A^2 - y^2) = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$
- Kinetic energy can not be negative. Potential energy increases at expense of KE and vice versa.
- Kinetic energy will be maximum at mean position and zero at extreme position.
- Total mechanical energy is independent of time.
- Potential energy and kinetic energy peaks twice during every period. Element of springiness stores potential energy and element of inertia stores its kinetic energy.
- Graph of kinetic energy versus position of particle will be an inverted parabola.
- In absence of damping ; total mechanical energy of harmonic oscillator will remain constant.

  $E = k_{max} = U_{max} = \frac{1}{2}m\omega^2 A^2$
- Potential energy and kinetic energy is periodic with period $\frac{T}{2}$.
- The graphs for energy versus position are



⑥ OSCILLATIONS DUE TO A SPRING

(1) **Oscillations of a spring block system**

(Linear S.H.M.)

Force law, $F = -kx$

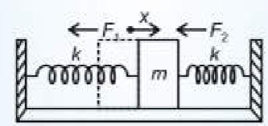
$F = -ma = -m\omega^2 x$

$\therefore\ k = m\omega^2$ or $\omega = \sqrt{\frac{k}{m}}$

Where k spring constant of spring and m is mass of block executing SHM.

(2) **For two Identical Springs**

This is also linear harmonic oscillator



When displaced right, restoring forces towards left

$F_1 = -kx, F_2 = -kx, F = F_1 + F_2$

$F = -2kx$

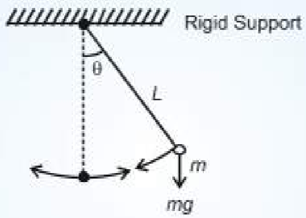
Since force acting on mass is proportional to displacement and directed towards mean position. It is SHM. The period of oscillation is

$T = 2\pi\sqrt{\frac{m}{2k}}$

7 SIMPLE PENDULUM

Simple Pendulum

- By attaching a small mass to an inextensible string, a simple pendulum can be made.
- The mass executes SHM for small displacements only.



$T = 2\pi\sqrt{\frac{I}{mgL}}$ Also here $I = mL^2$, about rigid support point.

$T = 2\pi\sqrt{\frac{L}{g}}$

- The time period of a simple pendulum depends on its length and acceleration due to gravity but is independent of its mass and amplitude.
- Time period of a clock pendulum which ticks every second is 2s and its length is approximately 1 metre.

9 FORCED OSCILLATIONS AND RESONANCE

- An external agency can maintain motion by resisting damping forces. These are called driven or forced oscillations. An external force which is periodic is applied to damped oscillator. Equation of oscillations of mass is $\frac{md^2y}{dt^2} + b\frac{dy}{dt} + ky = F_0 \cos\omega_d t$, and after natural oscillation, die out eqn. is

  $y = A\cos(\omega_d t + \phi)$ and A depends on ω_d and ω.
- If ω_d is close to ω then $A = \frac{F_0}{\omega_d b}$
- The phenomenon of increase of amplitude when driving frequency is close to natural frequency of the oscillators is called resonance.

8 DAMPED SIMPLE HARMONIC MOTION

- A viscous surroundings will apply force on simple pendulum or a spring pendulum and system will ultimately come to rest.
- The damping force depends on nature of surrounding medium. When damping is high, energy is quickly dissipated. This force is generally proportion to velocity of oscillator.

$\vec{F}_d \propto \vec{v} \Rightarrow \vec{F} = -b\vec{v}$

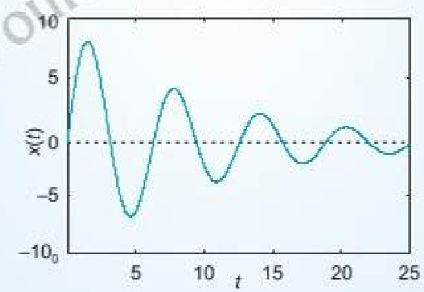
- Net force $F = -ky - bv$ ($b \rightarrow$ damping factor)

$ma = -ky - bv$

$\frac{md^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$ is damped equation, whose solution is given by

$y = Ae^{-\frac{bt}{2m}}\cos(\omega' t + \phi)$ for displacement of oscillator.

Where $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$



- Small damping means $\frac{b}{\sqrt{km}} << 1$
- $\left[E = \frac{1}{2}kA^2 e^{-bt/m}\right]$ energy eqn.

Sharpen Your Understanding

NCERT Based MCQs

1. The equation of motion is represented by $y = \sin \omega t + \cos \omega t$. The time period of periodic motion is [NCERT Pg. 339]

(1) $\frac{\pi}{\omega}$ (2) $\frac{2\pi}{\omega}$

(3) $\frac{2\pi}{\omega}$ (4) $\frac{4\pi}{\omega}$

2. The equation of motion of particle executing SHM is given as $y = \sin^2 \omega t$. The position of equilibrium is [NCERT Pg. 341]

(1) $y = 0$ (2) $y = 1$

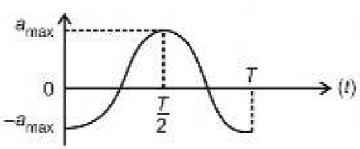
(3) $y = \frac{1}{2}$ (4) $y = -1$

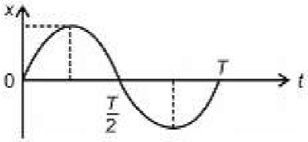
3. A body execute SHM according to equation $x = 10 \cos(2\pi t + \pi/4)$. At $t = 3/2$ s, what is speed of the particle? [NCERT Pg. 339]

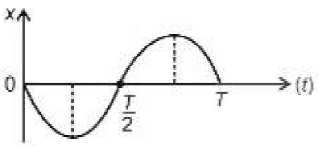
(1) 10 ms^{-1} (2) 20 ms^{-1}

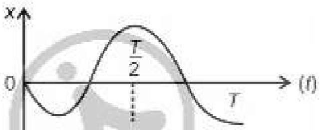
(3) 22 ms^{-1} (4) 44 ms^{-1}

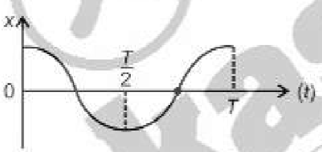
4. Acceleration versus time graph for a particle executing SHM is shown in figure below. Corresponding position time graph will be [NCERT Pg. 344]



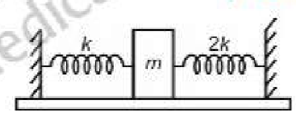
(1)


(2)


(3)


(4)


5. Two springs with spring constants K and 2K are attached to a block of mass m and with fixed supports as shown. When mass is displaced from equilibrium position on either side, it executes SHM. The frequency of oscillation is [NCERT Pg. 345]



(1) $\frac{1}{2\pi}\sqrt{\frac{3m}{K}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{m}{2K}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{3m}{2K}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{3K}{m}}$

6. A particle executes SHM. Its time period is T. The kinetic energy of the particle is also periodic with time period of [NCERT Pg. 346]

(1) T

(2) $2T$

(3) $\frac{T}{2}$

(4) Infinity

7. A block whose mass is 500 g is fastened to a spring. The spring has spring constant of 100 N/m. The block is pulled to a distance of x = 10 cm from its equilibrium position state of x = 0 from rest at t = 0. What is kinetic energy of block at x = 5 cm? [NCERT Pg. 347]

(1) 0.375 J (2) 0.19 J

(3) 0.56 J (4) 0.76 J

8. A block of mass 2 kg is attached to a spring of spring constant 200 Nm^{-1} oscillates without friction over a smooth horizontal surface. The block is displaced by 10 cm from equilibrium position and released. What is maximum acceleration of block? [NCERT Pg. 348]

(1) 1 ms^{-2} (2) 2 ms^{-2}

(3) 0.5 ms^{-2} (4) 1.5 ms^{-2}

9. Length of a simple pendulum whose time period is 2 second on earth surface will be nearly [NCERT Pg. 351]

(1) 0.5 m

(2) 1 m

(3) 1.5 m

(4) 2 m

10. A block of mass 500 g and attached to one end of a spring of spring constant K = 450 Nm^{-1}. The friction is also present which dissipate energy and damping constant of system is 25 g/s. What is time taken for its amplitude of oscillation to drop to half of its initial value. [NCERT Pg. 352]

(1) 18.73 s

(2) 27.72 s

(3) 32.2 s

(4) 6.52 s

11. Which of the following example does not represents SHM? [NCERT Pg. 358]

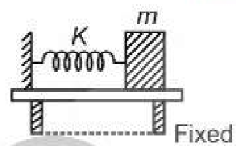
(1) Oscillations of a spring block system

(2) Motion of ball bearing inside smooth curved bowl, when released slightly away from equilibrium

(3) Motion of oscillating mercury column in vertical U-tube

(4) Rotation of earth about its axis

12. A spring having spring constant of 800 Nm^{-1} is mounted on a horizontal table as shown. A mass of 2 kg is attached to free end of the spring. The mass is pulled sideways to distance of 2.5 cm and released. How much time the mass takes from one extreme to other [NCERT Pg. 359]



(1) 0.157 s (2) 0.2 s

(3) 0.314 s (4) 0.782 s

13. The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What will be period of oscillation of a simple pendulum on the surface of moon if its time period on the surface of earth is 2s? [NCERT Pg. 361]

(1) 4.8 s (2) 2.8 s

(3) 1.8 s (4) 3.5 s

14. A particle executes SHM has maximum speed of 20 cm s^{-1} and maximum acceleration of 40 cm s^{-2}. The period of oscillation is [NCERT Pg. 361]

(1) π s (2) $\frac{\pi}{2}$s

(3) $\frac{\pi}{3}$s (4) 2π s

15. A spring balance has a scale that reads from 0 to 100 kg. The length of scale is 25 cm. A block suspended from this balance when displaced and released oscillates with time period of 0.2 s. What is mass of block approximately? [NCERT Pg. 359]

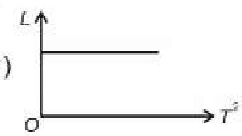
(1) 2 kg

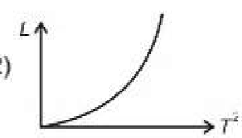
(2) 4 kg

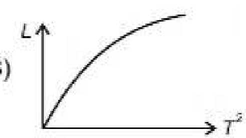
(3) 5 kg

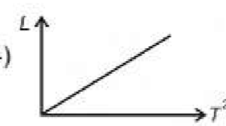
(4) 6 kg

16. The graph between length of pendulum and square of its time period is shown below. The best graph is [NCERT Pg. 351]

(1)


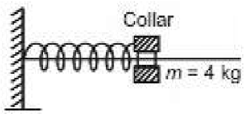
(2)


(3)


(4)


17. A collar of mass 4 kg is attached to a spring of spring constant 500 Nm^{-1}. If collar is displaced from equilibrium position by a distance of 2 cm and released, what is frequency of oscillation?

[NCERT Pg. 348]



(1) 5.4 Hz

(2) 1.78 Hz

(3) 9.36 Hz

(4) 3.26 Hz

18. Two identical springs of spring constant K each are attached to block of mass m and fixed supports as shown in figure (a). The period of oscillation was observed to be T. If one more identical spring is attached as shown in figure (b) then new period will be

[NCERT Pg. 345]

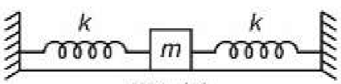


Fig. (a)

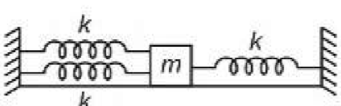


Fig. (b)

(1) $\sqrt{\frac{2}{3}}T$

(2) $\sqrt{\frac{3}{2}}T$

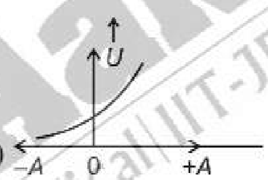
(3) $2T$

(4) $\sqrt{\frac{1}{3}} \times T$

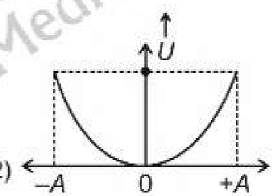
19. A particle executes SHM, has potential energy which changes with position. If potential energy at equilibrium position is assumed to be zero, then potential energy versus position graph is best represented by

[NCERT Pg. 347]

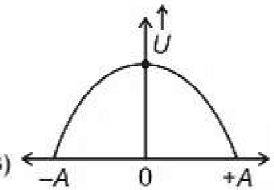
(1)



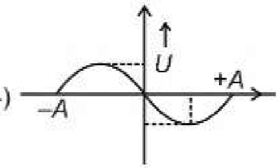
(2)



(3)

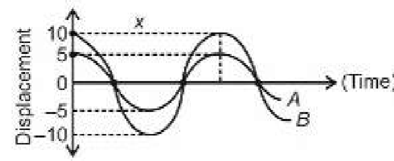


(4)



20. The graph of a particle executing SHM is shown for two particles A and B. The ratio of maximum accelerations of A to B is

[NCERT Pg. 341]



(1) 1 : 1

(2) 1 : 2

(3) 2 : 1

(4) 1 : 4

Thinking in Context

1. The direction of acceleration in case of SHM is always towards mean position and is ________ to displacement. [NCERT Pg. 344]
2. The ratio of distance travelled by an oscillator to its amplitude in one time period is ________. [NCERT Pg. 344]
3. In case of a particle executing SHM, phase difference between velocity and acceleration is ________. [NCERT Pg. 343]
4. The length of second's pendulum on surface of Earth is 1 m. The length of second's pendulum on surface of moon will be ________. [NCERT Pg. 351]
5. The displacement of the particle executing SHM where PE and KE are equal is ________ times amplitude of motion. [NCERT Pg. 346]
6. A spring pendulum and simple pendulum have equal time period on earth surface. On the surface of moon simple pendulum has time period ________ than spring pendulum. [NCERT Pg. 348]
7. A body is performing SHM, then its average velocity over a complete cycle is ________. [NCERT Pg. 343]
8. Motion of a ball bearing inside a smooth hemispherical bowl, when released from a point slightly above equilibrium point is ________. [NCERT Pg. 358]
9. The phenomenon of increase in amplitude when driving force is closed to natural frequency of the oscillation is called ________. [NCERT Pg. 354]
10. The mechanical energy in a real oscillating system decreases during oscillations. The real oscillator and its motion are then said to be ________. [NCERT Pg. 351]
11. Every periodic motion is not simple harmonic motion, only that periodic motion governed by the force law in which force is linear proportion to ________ is simple harmonic motion. [NCERT Pg. 345]
12. The kinetic energy of a particle executing simple harmonic motion is maximum at ________. [NCERT Pg. 346]
13. The graph between acceleration and displacement of a particle executing S.H.M. is ________. [NCERT Pg. 344]
14. When amplitude of a particle executing S.H.M. increases its time period ________. [NCERT Pg. 348]
15. The motion of a sewing needle when handle rotated at constant speed is ________. [NCERT Pg. 337]
16. Maximum possible time period of a simple pendulum on the surface of earth will be ________. [NCERT Pg. 344]
17. The projection of uniform circular motion on a diameter of circle follows ________. [NCERT Pg. 342]
18. A particle executing SHM of amplitude 10 cm. At extreme position the force acting is 6 N. At a point midway between mean and extreme position the force is ________. [NCERT Pg. 345]
19. A particle executes SHM with amplitude A and period T. The time taken by the particle from extreme position to half of amplitude is ________. [NCERT Pg. 344]
20. In simple harmonic motion at mean position ________ energy is minimum and ________ energy is maximum. [NCERT Pg. 346]

❑ ❑ ❑