Chapter: Twelve

Linear Programming

Competency Based Questions

✦ Multiple Choice Questions (MCQs)

1. Z = $20x_1 + 20x_2$, subject to $x_1 \geq 0$, $x_2 \geq 0$, $x_1 + 2x_2 \geq 8$, $3x_1 + 2x_2 \geq 15$, $5x_1 + 2x_2 \geq 20$. The minimum value of Z occurs at

(*a*) (8, 0) (*b*) $\left(\frac{5}{2}, \frac{15}{4}\right)$ (*c*) $\left(\frac{7}{2}, \frac{9}{4}\right)$ (*d*) (0, 10)

[Ans. (c)

2. Z = $7x + y$, subject to $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at

(*a*) (3, 0) (*b*) $\left(\frac{1}{2}, \frac{5}{2}\right)$ (*c*) (7, 0) (*d*) (0, 5)

[Ans. (d)

3. Minimize Z = $20x_1 + 9x_2$, subject to $x_1 \geq 0$, $x_2 \geq 0$, $2x_1 + 2x_2 \geq 36$, $6x_1 + x_2 \geq 60$.

(*a*) 360 at (18, 0) (*b*) 336 at (6, 4)
(*c*) 540 at (0, 60) (*d*) 0 at (0, 0) *[Ans. (b)*

4. Z = $8x + 10y$, subject to $2x + y \geq 1$, $2x + 3y \geq 15$, $y \geq 2$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at

(*a*) (4.5, 2) (*b*) (1.5, 4) (*c*) (0, 7) (*d*) (7, 0)

[Ans. (b)

5. Z = $4x_1 + 5x_2$, subject to $2x_1 + x_2 \geq 7$, $2x_1 + 3x_2 \leq 15$, $x_2 \leq 3$, $x_1, x_2 \geq 0$. The minimum value of Z occurs at

(*a*) (3.5, 0) (*b*) (3, 3) (*c*) (7.5, 0) (*d*) (2, 3)

[Ans. (a)

6. The maximum value of $f = 4x + 3y$ subject to constraints $x \geq 0$, $y \geq 0$, $2x + 3y \leq 18$; $x + y \geq 10$ is

(*a*) 35 (*b*) 36
(*c*) 34 (*d*) none of these

[Ans. (d)

7. Objective function of a L.P.P. is

(*a*) a constant
(*b*) a function to be optimised
(*c*) a relation between the variables
(*d*) none of these *[Ans. (b)*

8. The optimal value of the objective function is attained at the points

(*a*) on *x*-axis
(*b*) on *y*-axis
(*c*) which are corner points of the feasible region
(*d*) none of these *[Ans. (c)*

9. In solving the LPP: "minimize $f = 6x + 10y$ subect to constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$" redundant constraints are

(*a*) $x \geq 6$, $y \geq 2$
(*b*) $2x + y \geq 10$, $x \geq 0$, $y \geq 0$
(*c*) $x \geq 6$
(*d*) none of these *[Ans. (b)*

10. Region represented by $x \geq 0$, $y \geq 0$ is

(*a*) first quadrant
(*b*) second quadrant
(*c*) third quadrant
(*d*) fourth quadrant *[Ans. (a)*

11. The region represented by the inequalities $x \geq 6$, $y \geq 2$, $2x + y \leq 0$, $x \geq 0$, $y \geq 0$ is

(*a*) unbounded
(*b*) a polygon
(*c*) exterior of a triangle
(*d*) None of these *[Ans. (d)*

12. The minimum value of Z = $4x + 3y$ subjected to the constraints $3x + 2y \geq 160$, $5 + 2y \geq 200$, $2y \geq 80$; $x, y \geq 0$ is

(*a*) 220 (*b*) 300
(*c*) 230 (*d*) None of these

[Ans. (a)

13. The maximum value of Z = $3x + 2y$, subjected to $x + 2y \le 2$, $x + 2y \ge 8$; $x, y \ge 0$ is

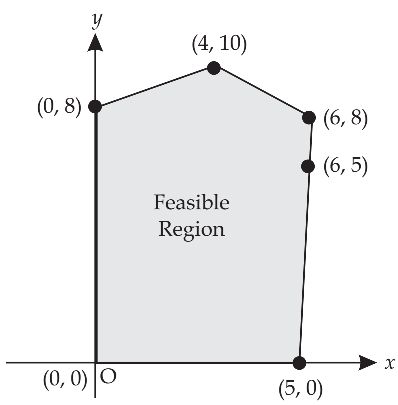
(*a*) 32 (*b*) 24
(*c*) 40 (*d*) None of these

[Ans. (d)

14. Maximize Z = $11x + 8y$, subject to $x \le 4$, $y \le 6$, $x \ge 0$, $y \ge 0$.

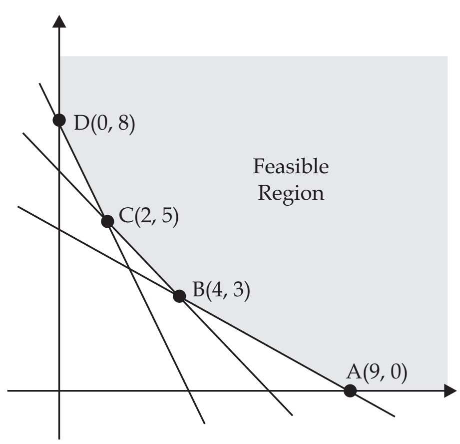
(*a*) 44 at (4, 2) (*b*) 60 at (4, 2)
(*c*) 62 at (4, 0) (*d*) 48 at (4, 2) *[Ans. (b)*

15. The feasible, region for an LPP is shown shaded in the figure. Let Z = $3x - 4y$ be the objective function. A minimum of Z occurs at



(*a*) (0, 0) (*b*) (0, 8)
(*c*) (5, 0) (*d*) (4, 10) *[Ans. (b)*

16. The feasible region for an LPP is shown shaded in the following figure. Minimum of Z = $4x + 3y$ occurs at the point



(*a*) (0, 8) (*b*) (2, 5)
(*c*) (4, 3) (*d*) (9, 0) *[Ans. (b)*

17. Maximize Z = $3x + 5y$, subject to $x + 4y \le 24$, $3x + 4y \le 21$, $x + y \le 9$, $x \ge 0$, $y \ge 0$.

(*a*) 20 at (1, 0) (*b*) 30 at (0, 6)
(*c*) 37 at (4, 5) (*d*) 33 at (6, 3) *[Ans. (c)*

18. Maximize Z = $4x + 6y$, subject to $3x + 2y \le 12$, $x + y \ge 4$, $x, y \ge 0$.

(*a*) 16 at (4, 0) (*b*) 24 at (0, 4)
(*c*) 24 at (6, 0) (*d*) 36 at (0, 6) *[Ans. (d)*

19. Maximize Z = $6x + 4y$, subject to $x \le 2$, $x + y \le 3$, $-2x + y \le 1$, $x \ge 0$, $y \ge 0$.

(*a*) 12 at (2, 0)
(*b*) 16 at (2, 1)
(*c*) $\frac{140}{3}$ at $\left(\frac{2}{3}, \frac{1}{3}\right)$
(*d*) 4 at (0, 1) *[Ans. (c)*

✦ Case Based Questions

1. Linear programming is a method for finding the optimal values (maximum or minimum) of quantities subject to the constraints when relationship is expressed as linear equations or inequations.

Based on the above information, answer the following questions:

(*i*) The optimal value of the objective function is attained at the points

(*a*) on X-axis
(*b*) on Y-axis
(*c*) which are corner points of the feasible region
(*d*) none of these

Sol. (*c*) which are corner points of the feasible region

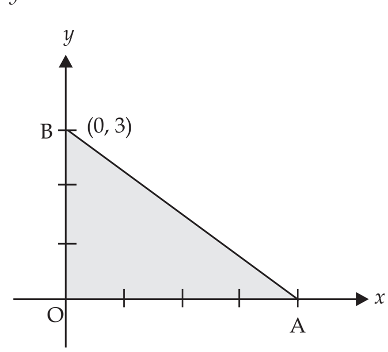
Explanation. When we solve an L.P.P. graphically, the optimal (or optimum) value of the objective function is attained at corner points of the feasible region.

(*ii*) The graph of the inequality $3x + 4y < 12$ is

(*a*) half plane that contains the origin
(*b*) half plane that neither contains the origin nor the points of the line $3x + 4y = 12$.
(*c*) whole *XOY*-plane excluding the points on line $3x + 4y = 12$
(*d*) None of these

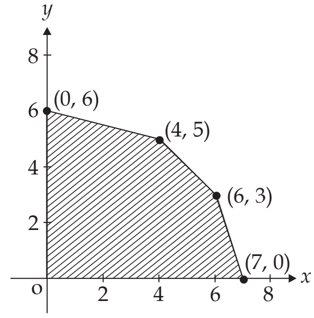
Sol. (*d*) None of these

Explanation. From the graph of $3x + 4y < 12$, it is clear that if contains the origin but not the points on the line $3x + 4y = 12$.



(*iii*) The feasible region for an LPP is shown in the figure. Let Z = $2x + 5y$ be the objective function.

Maximum of Z occurs at



(*a*) (7, 0) (*b*) (6, 3)
(*c*) (0, 6) (*d*) (4, 5)

Sol. (*d*) (4, 5)

Explanation. Maximum of objective function occurs at corner points.

Corner Points	*Value of Z = 2x + 5y*
(0, 0)	0
(7, 0)	14
(6, 3)	27
(4, 5)	**33 ← Maximum**
(0, 6)	30

(*iv*) The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = $px + qy$, where p, q > 0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

(*a*) $p = q$ (*b*) $p = 2q$
(*c*) $q = 2p$ (*d*) $q = 3p$

Sol. (*d*) $q = 3p$

Explanation. Value of Z = $px + qy$ at (15, 15) = $15p + 15q$ and that at (0, 20) = $20q$. According to given condition, we have

$15p + 15q = 20q \Rightarrow 15p = 5q \Rightarrow q = 3p$

(*v*) The corner points of the feasible determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is Z = $4x + 3y$.

Compare the quantity in Column A & Column B.

Column A	*Column B*
Maximum of Z	325

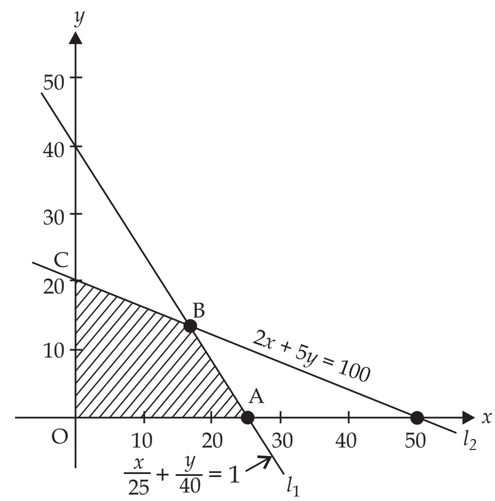
(*a*) The quantity in column A is greater
(*b*) The quantity in column B is greater
(*c*) The two quantities are equal
(*d*) The relationship cannot be determined on the basis of the information supplied.

Sol. (*b*) The quantity in column B is greater

Explanation. Construct the following table of values of the objective function:

Corner Points	Value of Z = $4x + 3y$
(0, 0)	$4 \times 0 + 3 \times 0 = 0$
(0, 40)	$4 \times 0 + 3 \times 40 = 120$
(20, 40)	$4 \times 20 + 3 \times 40 = 200$
(60, 20)	$4 \times 60 + 3 \times 20 =$ **300 (max.)**
(60, 0)	$4 \times 60 + 3 \times 0 = 240$

2. Deepa rides her car at 25 km/hr. She has to spend ₹2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to ₹5 per km. She has ₹100 to spend on diesel. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr. The feasible region for the LPP is shown below:



Based on the above information, answer the following questions:

(*i*) What is the point of intersection of line l_1 and l_2?

(*a*) $\left(\frac{40}{3}, \frac{50}{3}\right)$ (*b*) $\left(\frac{50}{3}, \frac{40}{3}\right)$

(*c*) $\left(\frac{-50}{3}, \frac{40}{3}\right)$ (*d*) $\left(\frac{-50}{3}, \frac{-40}{3}\right)$

Sol. (*b*) $\left(\frac{50}{3}, \frac{40}{3}\right)$

Explanation. Let B(x, y) be the point of intersection of the given lines.

$2x + 5y = 100$ ...(*i*)

and $\frac{x}{25} + \frac{y}{40} = 1 \Rightarrow 8x + 5y = 200$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = \frac{50}{3}$, $y = \frac{40}{3}$.

∴ The point of intersection B(x, y) = $\left(\frac{50}{3}, \frac{40}{3}\right)$

(ii) The corner points of the feasible region shown in the above graph are

(a) (0, 25), (20, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$

(b) (0, 0), (25, 0), (0, 20)

(c) (0, 0), $\left(\frac{40}{3}, \frac{50}{3}\right)$, (0, 20)

(d) (0, 0), (25, 0), $\left(\frac{50}{3}, \frac{40}{3}\right)$, (0, 20)

Sol. **(d)** (0, 0), (25, 0), $\left(\frac{50}{3}, \frac{40}{3}\right)$, (0, 20)

Explanation. The corner points of the feasible region shown in the given graph are:

(0, 0), A(25, 0), B $\left(\frac{50}{3}, \frac{40}{3}\right)$, C (0, 20)

(iii) If Z = $x + y$ be the objective function and max Z = 30. The maximum value occurs at point

(a) $\left(\frac{50}{3}, \frac{40}{3}\right)$ **(b)** (0, 0)

(c) (25, 0) **(d)** (0, 20)

Sol. **(a)** $\left(\frac{50}{3}, \frac{40}{3}\right)$

Explanation. Here Z = $x + y$

Corner Points	*Value of Z = $x + y$*
$\left(\frac{50}{3}, \frac{40}{3}\right)$	**30 ← Maximum**
(0, 0)	0
(25, 0)	25
(0, 20)	20

Thus, max Z = 30 occurs at point $\left(\frac{50}{3}, \frac{40}{3}\right)$.

(iv) If Z = $6x - 9y$ be the objective function, then maximum value of Z is

(a) –20 **(b)** 150

(c) 180 **(d)** 20

Sol. **(b)** 150

Corner Points	*Value of Z = $6x - 9y$*
(0, 0)	0
(25, 0)	**150 ← Maximum**
$\left(\frac{50}{3}, \frac{40}{3}\right)$	–20
(0, 20)	–180

(v) If Z = $6x + 3y$ be the objective function, then what is the minimum value of Z?

(a) 120 **(b)** 130

(c) 0 **(d)** 150

Sol. **(c)** 0

Corner Points	*Value of Z = $6x + 3y$*
(0, 0)	**0 ← Minimum**
(25, 0)	150
$\left(\frac{50}{3}, \frac{40}{3}\right)$	140
(0, 20)	60

3. Corner points of the feasible region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8). Let Z = $4x - 6y$ be the objective function.

Based on the above information, answer the following questions:

(i) The minimum value of Z occurs at

(a) (6, 8) **(b)** (5, 0)

(c) (0, 3) **(d)** (0, 8)

Sol. Construct the following table of values of objective function.

Corner points	*Value of Z = $4x - 6y$*
(0, 3)	–18
(5, 0)	**20 (Max.)**
(6, 8)	–24
(0, 8)	**–48 (Min.)**

(d) (0, 8)

Explanation. Minimum value of Z is –48 which occurs at (0, 8).

(ii) Maximum value of Z occurs at

(a) (5, 0) **(b)** (0, 8)

(c) (0, 3) **(d)** (6, 8)

Sol. **(a)** (5, 0)

Explanation. Maximum value of Z is 20, which occurs at **(5, 0).**

(iii) Maximum of Z - Minimum of Z =

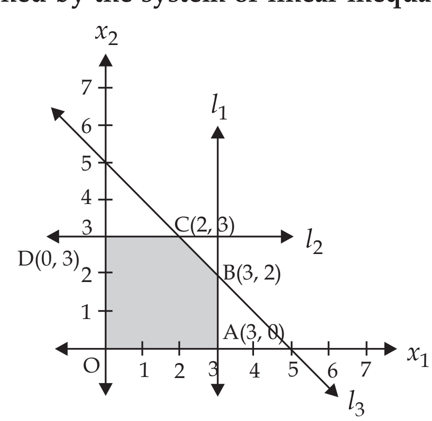
(a) 58 **(b)** 68

(c) 78 **(d)** 88

Sol. **(b)** 68

Explanation. Maximum of Z – Minimum of Z = 20 – (–48) = 20 + 48 = 68

(iv) The corner points of the feasible region determined by the system of linear inequalities are



(*a*) (0, 0), (–3, 0), (3, 2), (2, 3)
(*b*) (3, 0), (3, 2), (2, 3), (0, –3)
(*c*) (0, 0), (3, 0), (3, 2), (2, 3), (0, 3)
(*d*) None of these

Sol. (*c*) (0, 0), (3, 0), (3, 2), (2, 3), (0, 3)

Explanation. The corner points of the feasible region are O(0, 0), A(3, 0), B(3, 2), C(2, 3), D(0, 3).

(*v*) **The feasible solution of LPP belongs to**
(*a*) first and second quadrant
(*b*) first and third quadrant
(*c*) only second quadrant
(*d*) only first quadrant

Sol. (*d*) only first quadrant

4. Suppose a dealer in rural area wishes to purpose a number of sewing machines. He has only ₹5,760 to invest and has space for at most 20 items for storage. An electronic sewing machine costs him ₹360 and a manually operated sewing machine ₹240. He can sell an electronic sewing machine at a profit of ₹22 and a manually operated sewing machine at a profit of ₹18.

Based on the above information, answer the following questions:

(*i*) **Let x and y denotes the number of electronic sewing machines and manually operated sewing machines purchased by the dealer. If it is assumed that the dealer purchased atleast one of the given machines, then**

(*a*) $x + y \geq 0$ (*b*) $x + y < 0$
(*c*) $x + y > 0$ (*d*) $x + y \leq 0$

Sol. (*c*) $x + y > 0$

(*ii*) **Let the constraints in the given problem is represented by the following inequalities:**
$x + y \leq 20$
$360x + 240y \leq 5760$
$x, y \geq 0$
Then which of the following point lie in its feasible region.

(*a*) (0, 24) (*b*) (8, 12)
(*c*) (20, 2) (*d*) None of these

Sol. (*b*) (8, 12)

Explanation. Since (8, 12) satisfy all the inequalities therefore (8, 12) is the point in its feasible region.

(*iii*) **If the objective function of the given problem is maximise $z = 22x + 18y$, then its optimal value occur at**

(*a*) (0, 0) (*b*) (16, 0)
(*c*) (8, 12) (*d*) (0, 20)

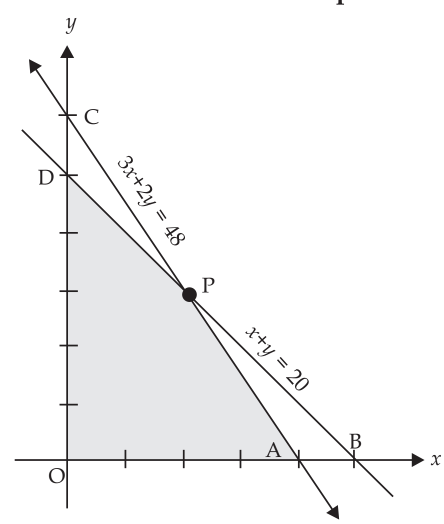
Sol. (*c*) (8, 12)

Explanation. At (0, 0), z = 0
At (16, 0), z = 352
At (8, 12), z = 392
At (0, 20), z = 360

It can be observed that max z occur at (8, 12). Thus, z will attain its optimal value at (8, 12).

(*iv*) **Suppose the following shaded region APDO, represent the feasible region corresponding to mathematical formulation of given problem.**

Then which of the following represent the coordinates of one of its corner points.



(*a*) (0, 24) (*b*) (12, 8)
(*c*) (8, 12) (*d*) (6, 14)

Sol. (*c*) (8, 12)

Explanation. We have, $x + y = 20$ ...(i)
and $3x + 2y = 48$ ...(ii)
On solving (*i*) and (*ii*), we get $x = 8$, $y = 12$
Thus, the coordinates of P are (8, 12) and hence (8, 12) is one of its corner points.

(*v*) **If an LPP admits optimal solution at two consecutive vertices of a feasible region, then**
(*a*) the required optimal solution is at the midpoint of the line joining two points.
(*b*) the optimal solution occurs at every point on the line joining these two points.
(*c*) the LPP under consideration is not solvable.
(*d*) the LPP under consideration must be reconstructed.

Sol. (*b*) The optimal solution occurs at every point on the line joining these two points.

5. Let R be the feasible region (convex polygon) for a linear programming problem and let Z = $ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Based on the given information, answer the following questions:

(*i*) Objective function of a L.P.P. is

(*a*) a constant

(*b*) a function to be optimised

(*c*) a relation between the variables

(*d*) none of these

Sol. (*b*) a function to be optimised

Explanation. Objective function is a linear function (involve variable) whose maximum or minimum value is to be found.

(*ii*) Which of the following statement is correct?

(*a*) Every LPP has at least one optimal solution

(*b*) Every LPP has a unique optimal solution.

(*c*) If an LPP has two optimal solutions, then it has infinitely many solutions.

(*d*) None of these

Sol. (*c*) If an LPP has two optimal solutions, then it has infinitely many solutions.

Explanation. If optimal solution is obtained at two distinct points A and B (corner of the feasible region), then optimal solution is obtained at every point of segment [AB].

(*iii*) In solving the LPP : "minimize $f = 6x + 10y$ subject to constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$" redundant constraints are

(*a*) $x \geq 6$, $y \geq 2$

(*b*) $2x + y \geq 10$, $x \geq 0$, $y \geq 0$

(*c*) $x \geq 6$

(*d*) none of these

Sol. (*b*) $2x + y \geq 10$, $x \geq 0$, $y \geq 0$

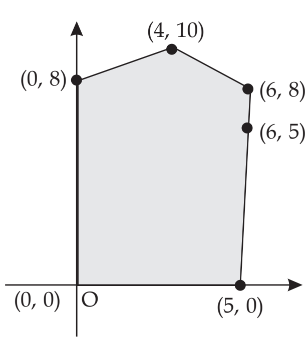
Explanation. When $x \geq 6$ and $y \geq 2$, then

$2x + y \geq 2 \times 6 + 2$, *i.e.*, $2x + y \geq 14$

Hence, $x \geq 0$, $y \geq 0$ and $2x + y \geq 10$ are automatically satisfied by every point of the region.

$[(x, y) : x \geq 6\} \cap \{(x, y) : y \geq 2\}$

(*iv*) The feasible region for a LPP is shown shaded in the figure. Let Z = $3x - 4y$ be the objective function. Minimum of Z occurs at



(*a*) (0, 0)
(*b*) (0, 8)
(*c*) (5, 0)
(*d*) (4, 10)

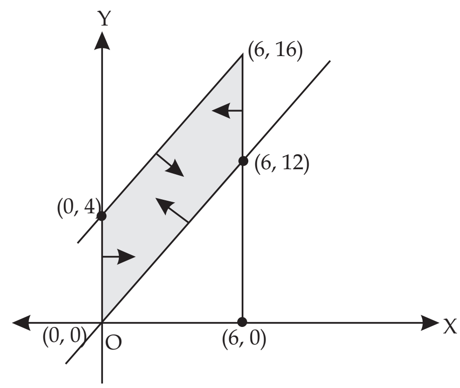
Sol. (*b*) (0, 8)

Explanation. Construct the following tabhle of values of the objective function:

Corner Points	*Value of Z = 3x – 4y*
(0, 0)	3 × 0 – 4 × 0 = 0
(5, 0)	3 × 5 – 4 × 0 = 15
(6, 5)	3 × 6 – 4 × 5 = –2
(4, 10)	3 × 4 – 4 × 10 = –28
(0, 8)	3 × 0 – 4 × 8 = **–32** ← **Minimum**

∴ Minimum of Z = –32 at (0, 8)

(*v*) The feasible region for a LPP is shown shaded in the figure. Let F = $3x - 4y$ be the objective function. Maximum value of F is



(*a*) 0
(*b*) 8
(*c*) 12
(*d*) –18

Sol. (*a*) 0

Explanation. Construct the following table of values of the objective function F:

Corner Points	*Value of F = 3x – 4y*
(0, 0)	3 × 0 – 4 × 0 = 0 ← **Minimum**
(6, 12)	3 × 6 – 4 × 12 = –30
(6, 16)	3 × 6 – 4 × 16 = –46
(0, 4)	3 × 0 – 4 × 4 = –16

Hence, maximum of F = 0