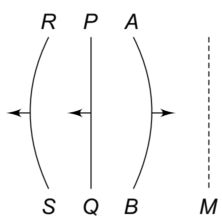
CHAPTER 14

Wave Optics

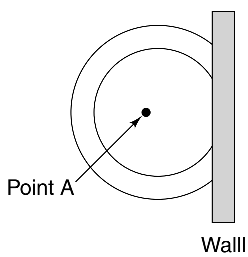
Level 1

Q. 1: In the figure shown M represents a mirror. AB is an incident wavefront. Which type of mirror is M if the reflected wave front is



(a) PQ (b) RS

Q. 2: On the surface of a calm lake a source at A is causing disturbance. The circular ripples formed get reflected at a wall in the lake. The reflected ripples can be thought to be generated from a virtual source. Indicate the position of virtual source in the diagram.



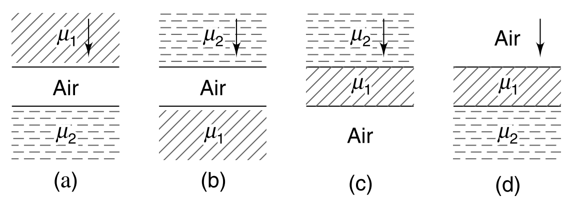
Q. 3: A wavefront expressed by $x + y - z = 4$ is incident on a plane mirror which is lying parallel to xy plane. Write a unit vector in the direction of reflected ray.

Q. 4: A parallel beam of light is travelling along a direction making an angle of 30° with the positive X direction and 60° with positive Y direction. Wavelength is λ. Find the phase difference between points having co-ordinates $(1, \sqrt{3}, 2)$ and $(0, 0, 0)$.

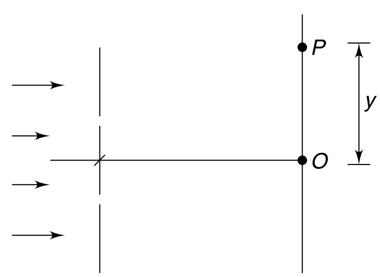
Q. 5: A point source of light is being moved closer to a thin concave lens from a large distance on its principal axis. Is the radius of curvature of the refracted wave front, close to the lens, increasing or decreasing?

Q. 6: Three coherent sources S_1, S_2 and S_3 can throw light on a screen. With S_1 switched on intensity at a point P on the screen was observed to be I. With only S_2 on, intensity at P was $2I$ and when all three are switched on the intensity at P becomes zero. Intensity at P is I when S_1 and S_2 are kept on. Find the phase difference between the waves reaching at P from sources S_1 and S_3.

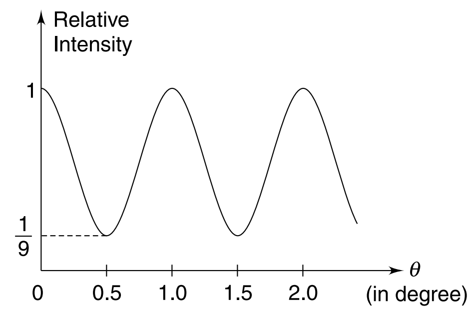
Q. 7: Three media are arranged in various ways as shown in figures a, b, c and d. Light of wavelength λ is incident perpendicularly on the boundary of the middle layer and interference between waves reflected at the boundaries of the middle layer is studied. In which of the four cases the reflected light is eliminated by destructive interference when the thickness of middle layer approaches zero. $\mu_1 = 1.5$ and $\mu_2 = 1.8$



Q. 8: In young's double slit experiment, when the slit plane is illuminated with light of wavelength λ_1, it was observed that point P is closest point from central maximum O, where intensity was 75% the intensity at O. When the light of wavelength λ_2 is used, point P happens to be the nearest point from O where intensity is 50% of that at O. Find the ratio $\frac{\lambda_1}{\lambda_2}$.



Q. 9: In young's double slit experiment relative intensity at a point on the screen may be defined as ratio of intensity at that point to the maximum intensity on the screen. Light of wavelength 7500 Å passing through a double slit, produces interference pattern of relative intensity variation as shown in Fig. θ on horizontal axis represents the angular position of a point on the screen.

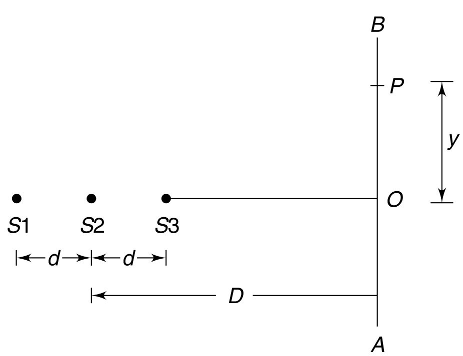


(a) Find separation d between the slits.

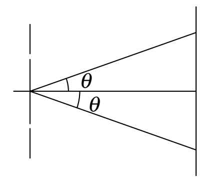
(b) Find the ratio of amplitudes of the two waves producing interference pattern on the screen.

Q. 10(a): A monochromatic point source (S_1) is at a distance d from a screen. Another identical source (S_2) is at a large distance from the screen. The two sources are on a line which is perpendicular to the screen. Sources are coherent. What is the shape of interference fringes on the screen?

(b): Three identical coherent sources are placed on a straight line with two neighbouring ones separated by a distance $d = 0.06$ mm. The sources produce monochromatic light of wavelength $\lambda = 550.4$ nm. A line AB is located at a distance $D = 2.50$ m from the sources and is perpendicular to line joining the three sources. Intensity of light at P with any one of the sources switched on is I_o. Find the intensity when all three sources are switched on. Distance of point P from O is $y = 1.72$ cm (see figure)

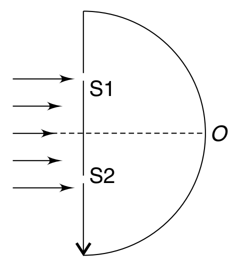


Q. 11: In Young's double-slit experiment, the separation between two slits is $d = 0.32$ mm and the wavelength of light used is $\lambda = 5000$ Å. Find the number of maxima in the angular range $-\sin^{-1}(0.6) \le \theta \le \sin^{-1}(0.6)$.



Q. 12: In young's double slit experiment (with identical slits) the intensity of a maxima is I. P is a point on the screen where 10^{th} maxima is formed with light of wavelength $\lambda = 6000$ Å. Find the intensity at point P if the entire experimental set up is submerged in water of refractive index $\mu = \frac{4}{3}$. Assume that intensity due to individual slits remains unchanged after the system is dipped in water.

Q. 13: Coherent light of wavelength $\lambda = 500$ nm is sent through two narrow parallel slits in a large vertical wall. The two slits are 5 μm apart. In front of the wall there is a semi cylindrical screen with its horizontal axis at the line running on the wall parallel to the slits and midway between them. Radius of the cylindrical screen is $R = 2.0$ m. Find the vertical height of the second order interference maxima from the centre (O) of the screen.



Q. 14: In a Young double slit experiment, the two slits are named as A and B. Two transparent films of thickness t_1 and t_2 having refractive indices μ_1 and μ_2 placed in front of the slits A and B respectively. It is given that $\mu_1 t_1 = \mu_2 t_2$ and $t_1 < t_2$

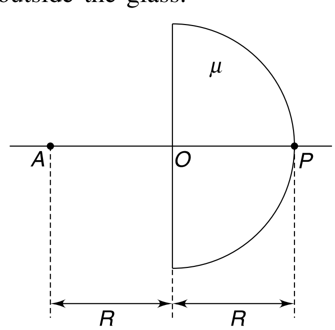
In which direction will the central maximum shift after the two films are placed?

Level 2

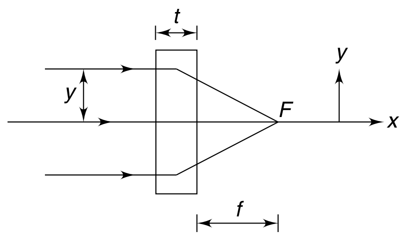
Q. 15: A point source (A) is kept on the axis of a hemispherical paperweight made of glass of refractive index $\mu = \frac{3}{2}$. The distance of the point source from the centre (O) of the sphere is R where R is radius of the hemisphere. Use paraxial approximations for answering following questions

(a) Find the change in radius of curvature of the wavefronts just after they enter the glass at O.

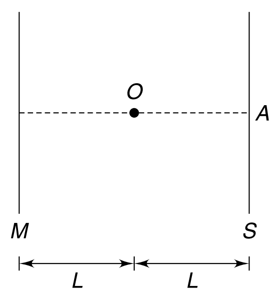
(b) Find the radius of curvature of the wavefronts at point P just outside the glass.



Q. 16: A parallel beam of light travelling in x direction is incident on a glass slab of thickness t. The refractive index of the slab changes with y as $\mu = \mu_0\left(1 - \frac{y^2}{y_0^2}\right)$ where μ_0 is the refractive index along x axis and y_0 is a constant. The light beam gets focused at a point F on the x axis. By using the concept of optical path length calculate the focal length f. Assume $f >> t$ and consider y to be small.



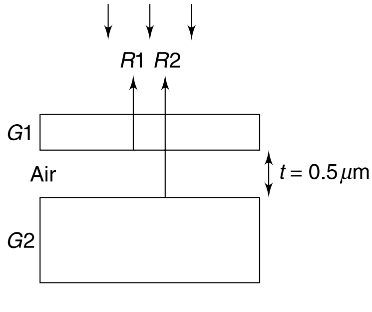
Q. 17: In the figure shown, O is a point source of light and S is a screen placed at a distance L from the source. Intensity of light at point A on the screen due to the source is $81I$ where I is some unit. Now a large mirror (M) is placed behind the source at a distance L from it. The mirror reflects 100% of the light energy incident on it. Calculate the intensity at point A.



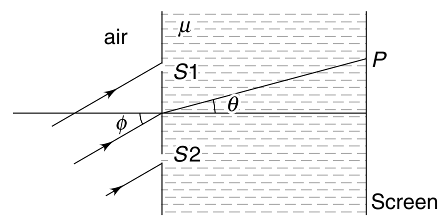
Q. 18: In a routine Young's double slit experiment the parallel beam of light incident on the slit plane is a mixture of wavelength distributed between $\lambda + \Delta\lambda$ and $\lambda - \Delta\lambda$. Because of this the interference fringes are poorly defined compared to the ideal case of monochromatic light. It is understood that the central maxima and the first order maxima will be well resolved if there is no overlapping in the first order minima and the first order maxima. Write the values of $\Delta\lambda$ so that the first order maxima is well resolved from the central fringe.

Q. 19: A thin glass slab $G1$ is held over a large glass slab $G2$, creating an air gap of uniform thickness $t = 0.5\ \mu$m between them. Electromagnetic wave having wavelengths ranging from 0.4 μm to 1.15 μm is incident normally on the slab $G1$. When interference between waves reflected from boundaries of air gap (the two reflected waves are shown in fig as $R1$ and $R2$) was studied, it was found that only two wavelengths interfered constructively. One of these two wavelengths is $\lambda_1 = 0.04\ \mu$m.

Find the other wavelength (λ_2) that interferes constructively.



Q. 20: Light is incident at an angle ϕ with the normal to a vertical plane containing two narrow slits ($S1$ and $S2$) at separation d. The medium to the left of slit plane is air and wavelength of the incident light is λ. The medium to the right of the slit plane has refractive index μ. Find all values of angular position (θ) of a point P where we will observe constructive interference. Wavelength of incident light is λ.

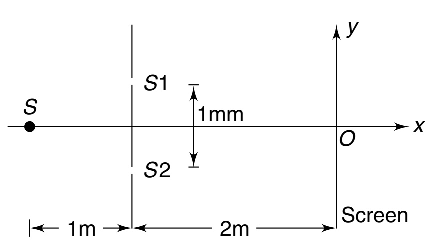


Q. 21: In a Young's double slit experiment set up source S of wavelength λ = 500 nm illuminates two symmetrically located slits $S1$ and $S2$. The source S oscillates about its shown position parallel to the screen according to the equation y = (0.5 mm) $\sin(\pi t)$.

Where t is time in second. Distances are as marked in the figure.

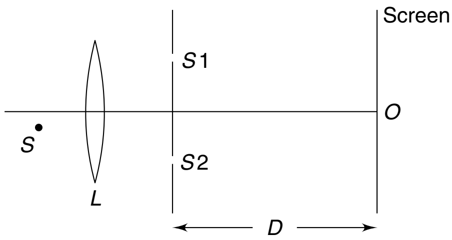
(a) Write the y co-ordinate (y') of central maximum as a function of time

(b) Find least value of time (t) at which the intensity becomes maximum at a point on the screen that is exactly in front of slit $S1$.



Q. 22: In a young's double slit experiment the distance between slits $S1$ and $S2$ is d and distance of slit plane from the screen is D (>> d). The point source of light (S) is placed a distance $\frac{d}{2}$ below the principal axis in the focal plane of the convex lens (L). The slits $S1$ and $S2$ are located symmetrically with respect to the principal axis of the lens. Focal length of the lens is f (>> d).

Find the distance of the central maxima of the fringe pattern from the centre (O) of the screen.



Q. 23: A Soap bubble has a thickness of 90 nm and its refractive index is $\mu = 1.4$. What colour does the bubble appear to be at a point on its surface closest to an observer when it is illuminated by white light?

Q. 24: A parallel beam of white light falls from air on a thin film whose refractive index is $\sqrt{3}$. The medium on both sides of the film is air. The angle of incdidence is 60°. Find the minimum film thickness if reflected light is most intense for $\lambda = 600$ nm.

Q. 25: A thin film having refractive index $\mu = 1.5$ has air on both sides. It is illuminated by white light falling normally on it. Analysis of the reflected light shows that the wavelengths 450 nm and 540 nm are the only missing wavelengths in the visible portion of the spectrum. Assume that visible range is 400 nm to 780 nm.

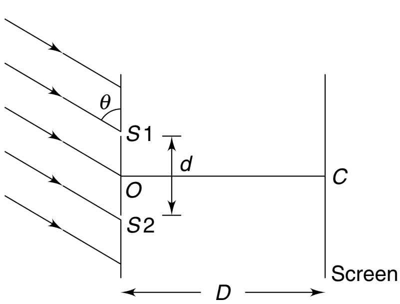
(a) Find thickness of the film.

(b) Which wavelengths are brightest in the interference pattern of the reflected light?

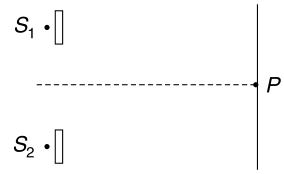
Q. 26: In a double slit experiment a parallel beam of light strikes the slit plane at an angle θ as shown in the figure. The two slits are covered with transparent plastic sheets of equal thickness t but of different refractive indices 1.2 and 1.5. The central maxima is formed at the centre of the screen at C.

(a) Which sheet was used to cover the slit $S1$?

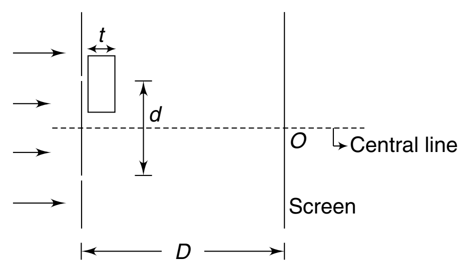
(b) Find θ.



Q. 27: In Young's double slit experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate having same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum fell before the glass plates were inserted, now has $\frac{3}{4}$ the original intensity. It is also observed that what used to be 5[th] maximum earlier, lies below the point P while the 6[th] minimum lies above P. Calculate the thickness of glass plates. Absorption of light by glass plate may be neglected.



Q. 28: In Young's double slit experiment a transparent sheet of thickness t and refractive index μ is placed in front of one of the slits and the central fringe moves away from the central line. It was found that when temperature was raised by $\Delta\theta$ the central fringe was back on the central line (at C). It is known that temperature coefficient of linear expansion of the material of the transparent sheet is α. A young scientist modeled that the refractive index of the material changes with temperature as $\Delta\mu = -\gamma\Delta\theta$. Find $\Delta\theta$ in terms of other given quantities. D and d are given and have usual meaning.

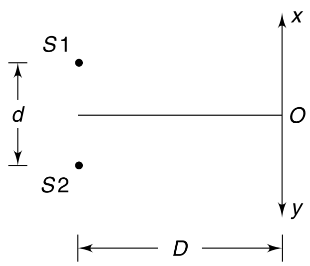


Q. 29: In Young's double slit experiment a monochromatic light of wavelength λ from a distant point source is incident upon the two identical slits.The interference pattern is viewed on a distant screen. Intensity at a point P is equal to the intensity due to individual slits (equal to I_0). A thin piece of glass of thickness t and refractive index μ is placed in front of the slit which is at larger distance from point P; perpendicular to the light path. Assume no absorption of light energy by the glass.

(a) Write intensity at point P as a function of t.

(b) Write all values of t for which the intensity at P is minimum.

Q. 30: In the shown fig. $S1$ and $S2$ are two identical coherent sources of sound, separated by a distance d. A receiver moves along the line XY (which is parallel to line $S1\,S2$) to detect the intensity of sound at various points on the line. Distance of line XY from line $S1S2$ is $D(>> d)$. Point O is the foot of perpendicular bisector of the line $S1S2$ on XY. Distance of first intensity maxima (on line XY) measured from O is y. Find percentage change in value of y if the temperature of air increases by 1%.

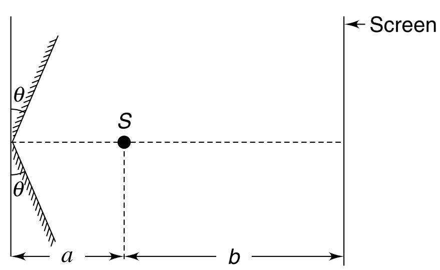


Q. 31: Two plane mirrors, a source S of light, emitting wavelengths of $\lambda_1 = 4000$ Å and $\lambda_2 = 5600$ Å and a screen are arranged as shown in figure. The angle θ shown is 0.05 radian and distance a and b are 1 cm and 38 cm respectively.

(a) Find the fringe width of interference pattern formed on screen by the blue light.

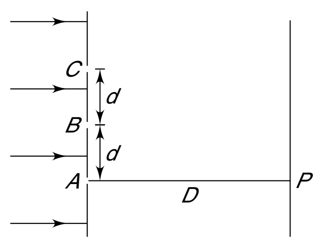
(b) Calculate the distance of first black line from central bright fringe.

(c) Find the distance between two black lines which are nearest to the central bright fringe.



Q. 32: Three narrow slits A, B and C are illuminated by a parallel beam of light of wavelength λ, P is a point on the screen exactly in front of point A. Slit plane is at a distance D from the screen ($D >> \lambda$). It is know that

$$BP - AP = \frac{\lambda}{3}$$



(a) Find d in terms of D and λ

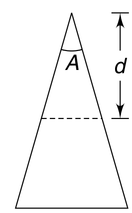
(b) Write the phase difference between waves reaching at P from C and A.

(c) If intensity of P due to any of the three slits individually is I_0, find the resultant intensity at P.

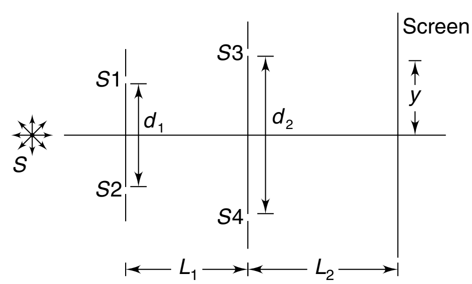
Q. 33: A very thin prism has an apex angle A and its material has refractive index $\mu = 1.48$. Light is made to fall on one of the refracting faces at near normal incidence. Interference results from light reflected from the outer surface and that emerging after reflection at the inner surface. When violet light of wavelength $\lambda = 400$ nm is used, the first constructive interference band is observed at a distance $d = 3.0$ cm from the apex of the prism.

(a) Find the apex angle A.

(b) If red light ($\lambda = 800$ nm) is used, at what distance from the apex will we observe the first constructive interference band.

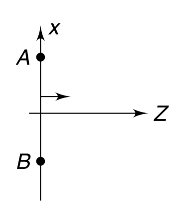


Q. 34: In the arrangement shown, S is a point source of monochromatic light. $S1$ and $S2$ are two slits located symmetrically with respect to the source with separation between them d_1. Parallel to this slit plane there are two more slits ($S3$ and $S4$) at separation d_2. These slits are also symmetrically located with respect to S. A screen is at a distance L_2 from this slit plane. How does the intensity on the screen change with y and d_1?

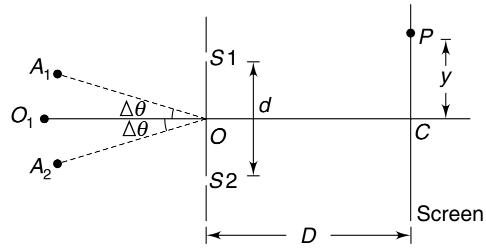


Level 3

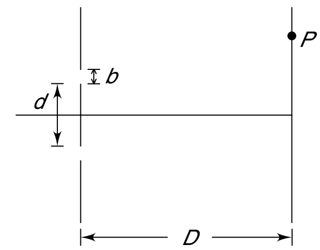
Q. 35: The refractive index of a medium changes as $\mu = \mu_0\left[1 - \frac{x^2 + y^2}{d^2}\right]^{1/2}$ where μ_0 is the refractive index on the z axis. A plane wavefront (AB) is incident along z axis as shown in the figure. Draw the wavefront at a later time Δt. Is the wavefront getting focused?



Q. 36: In the arrangement shown in the figure, $S1$ and $S2$ are two parallel slits at a separation d. There is a screen at a distance D (>> d) from the slits. Two point sources $A1$ and $A2$ have been placed symmetrically with respect to the slits at a large distance with $\langle A_1OO_1 = \langle A_2OO_1 = \Delta\theta$. The sources are monochromatic giving out light of wavelength λ but they are incoherent. Write the intensity of light at a point P on the screen as a function of y. Take I_0 to be maximum intensity on the screen when one of the two sources ($A1$ and $A2$) is switched on. Assume y to be small.



Q. 37 The figure represents two identical slits in a Young's double slit experiment. The width of each slit is b and distance between the centres of the two slits is d. Consider a point P on the screen that is close to centre of the screen. Δx_t represents the optical path difference to point P from the top edges of the two slits and Δx_b represents the optical path difference to P from the bottom edges of the two slits. Find $\Delta x_b - \Delta x_t$.

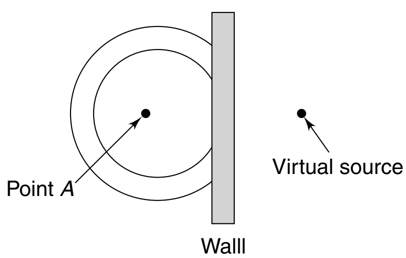


It is given that $D >> d$.

ANSWERS

1. (a) Concave (b) Plane

2.



3. $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

4. $\frac{2\sqrt{3}\pi}{\lambda}$

5. Decreasing.

6. $\pi/2$

7. *a*, *b*.

8. $\frac{3}{2}$.

9. (a) 0.04 mm (b) 2 : 1

10. (a) Circular
(b) I_0

11. 769.

12. $\frac{I}{4}$.

13. 0.4 m

14. shift towards A

15. (a) $\frac{R}{2}$ (b) $10R$.

16. $\frac{y_0^2}{2\mu_o t}$

17. 90 I

18. $\Delta\lambda < \frac{\lambda}{3}$.

19. 0.67 μm

20. $\theta = \sin^{-1}\left[\frac{1}{\mu d}(n\lambda - d\sin\phi)\right]$; $[n = 0, \pm 1, \pm 2 ...]$

21. (a) $y' = -\sin \pi t$ (b) $\frac{1}{6}$ s

22. $\frac{Dd}{2f}$

23. Green

24. 100 nm.

25. (a) 900 nm (b) 415 nm, 491 nm, 600 nm, 771 nm

26. (a) Sheet of $\mu = 1.5$ (b) $\cos^{-1}\left(\frac{3t}{10d}\right)$

27. 9.3 μm

28. $\Delta\theta = \left[\frac{\mu - 1}{(\mu - 1)\alpha - \gamma}\right]$

29. (a) $4I_0\cos^2\left(\frac{\pi}{3} + \frac{\pi}{\lambda}(\mu - 1)t\right)$

(b) $t = \frac{(2n-1)\lambda}{2(\mu - 1)} - \frac{\lambda}{3(\mu - 1)}$ where $n = 1, 2, 3...$

30. 0.5%

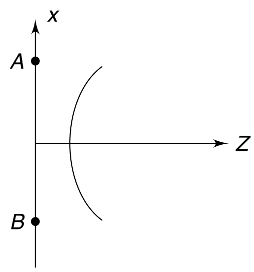
31. (a) 80 μm (b) 280 μm (c) 560 μm

32. (a) $\sqrt{\dfrac{2D\lambda}{3}}$ (b) $\dfrac{8\pi}{3}$ (c) $3I_0$

33. (a) 1.3×10^{-4} degree (b) 6 cm.

34. Intensity $\propto \cos^2\left(\dfrac{\pi d_1 d_2}{2\lambda L_1}\right)\cos^2\left(\dfrac{\pi d_2 y}{\lambda L_2}\right)$

35.



Yes the wavefront is getting focussed.

36. $I_0\cos^2\left[\dfrac{2\pi}{\lambda}d\left(\dfrac{yd}{D} + \sin\Delta\theta\right)\right]$

$$+ I_0\cos^2\left[\frac{2\pi}{\lambda}d\left(\frac{yd}{D} - \sin\Delta\theta\right)\right]$$

37. $\dfrac{db}{D}$

SOLUTIONS

3. Light ray is normal to the wavefront.

Incident ray is parallel to $\vec{a} = \hat{i} + \hat{j} - \hat{k}$.

The normal to the surface is parallel to z axis. The component of $\vec{a}$ parallel to xy plane does not change during reflection and the z component gets reversed.

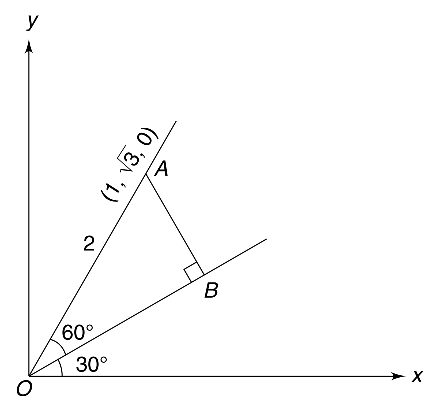
Therefore a vector in the direction of reflected ray is

$$\vec{b} = \hat{i} + \hat{j} + \hat{k}$$

unit vector along $\vec{b}$ is

$$\hat{b} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$$

One more answer is possible if the light is assumed to be travelling in opposite direction.



4. Wave front is a plane perpendicular to line OB. Hence phase at point $(1, \sqrt{3}, 2)$ is same as that at point A $(1, \sqrt{3}, 0)$.

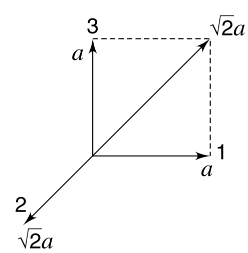
$$OB = OA \cos 30° = \sqrt{3}$$

$$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda}\sqrt{3}.$$

5. When object is at large distance its image is at focus. The diverging wave fronts have their centre of curvature at the focus of the lens. As the object gets closer the image also gets closer to the lens. It means that the radius of diverging wavefronts will be decreasing.

6. Resultant intensity due to $S1$ and $S2$ is I and adding third to this makes the resultant zero. This implies that intensity due to $S3$ alone is I.

Amplitudes of waves reaching P from the three sources $S1$, $S2$ and $S3$ can be written as a, $\sqrt{2}a$ and a respectively. These three waves can produce zero resultant if phase difference between the first and third wave is $\pi/2$. The figure explains the situation.



7. **Hint:** Light suffers a phase change of π when it gets reflected while travelling from a rarer to a denser medium.

8. $\left(\dfrac{I_P}{I_{\max}}\right)_{\lambda_1} = \cos^2\dfrac{\phi}{2} = 0.75$

[ϕ = phase difference between two waves arriving at P]

$$\therefore \quad \cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\phi = \pi/3$$

$$\Rightarrow \quad \frac{2\pi}{\lambda_1}(\Delta x)_1 = \pi/3$$

$$\Rightarrow \quad (\Delta x)_1 = \lambda_1/6 \qquad ...(1)$$

Similarly, $\left(\frac{I_P}{I_{max}}\right)_{\lambda 2} = \cos^2\frac{\phi'}{2} = 0.5$

$$\therefore \quad \cos\frac{\phi'}{2} = \frac{1}{\sqrt{2}}$$

$$\phi' = \pi/2$$

$$\therefore \quad (\Delta x)_2 = \frac{\lambda_2}{4}$$

Because $(\Delta x)_1 = (\Delta x)_2$ ...(2)

$$\frac{\lambda_1}{6} = \frac{\lambda_2}{4} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{3}{2}$$

9. (a) At $\theta = 0.5°$, we have first minima

$$\therefore \quad d\sin\theta = \frac{\lambda}{2}$$

$$d\theta \simeq \frac{\lambda}{2}$$

$$d = \frac{7500 \times 10^{-7}\text{mm}}{2 \times \left(0.5 \times \frac{3.14}{180}\text{rad}\right)} = 0.04 \text{ mm}$$

(b) $$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

$$9 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

$$\Rightarrow \frac{a_1 + a_2}{a_1 - a_2} = 3$$

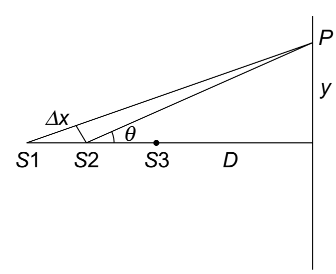
$$\Rightarrow \frac{a_1}{a_2} = \frac{2}{1}$$

10. (a) **Hint:** The wave from S_2 can be treated as a plane wave. All points on the screen illuminated due to S_2 will have same phase. Due to S_1 alone the locus of points of constant phase are circles.

(b) Angular position of P is

$$\tan\theta = \frac{y}{D}$$

$$\Rightarrow \quad \theta \simeq \frac{y}{D} \quad [\because y << D]$$



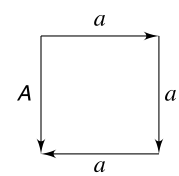
Path difference between waves reaching P from $S1$ and $S2$ is $\Delta x = d\sin\theta = d\cdot\theta$

$$\therefore \text{ Phase difference } \delta = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}d\cdot\theta$$

$$= \frac{2\pi}{\lambda}\frac{d\cdot y}{D} = \frac{2\pi \times 0.06 \times 17.2}{550.4 \times 10^{-6} \times 250}$$

$$= 1.5\pi = \frac{3\pi}{2}$$

Similarly phase difference between waves from $S2$ and $S3$ will be $\frac{3\pi}{2}$.



Using pharor method of amplitude addition, the resultant amplitude at P is

$$A = a \text{ [see fig]}$$

Where a = amplitude due to individual waves

$\therefore$ Resultant intensity at P is I_0

11. Position of n^{th} order maxima is given by

$$d \sin\theta = n\lambda; \quad n = 0, \pm 1, \pm 2....$$

At $\sin\theta = 0.6$; $d = 0.32$ mm; $\lambda = 5000$ Å we have

$$n = \frac{d\sin\theta}{\lambda} = \frac{0.32 \times 0.6}{5000 \times 10^{-7}} = 384$$

It means there are 384 maxima in the range $0 < \theta \leq \sin^{-1}(0.6)$. By symmetry we have same number of maxima on the other side and there is one central maxima (corresponding to $n = 0$)

Therefore, total number of maxima = 384 + 384 + 1

= 769

12. Path difference of two interfering waves at P is $\Delta x = 10\lambda = 60000$ Å.

In water the wavelength of light changes to $\lambda' = \frac{\lambda}{\mu} = \frac{6000}{4/3} = 4500$ Å.

Phase difference between waves at P is

$$\lambda = \frac{2\pi}{\lambda'}\Delta x = \frac{2\pi}{4500} \times 60000 = \frac{80\pi}{3} = 26\pi + \frac{2\pi}{3}$$

Intensity at P will be

$$I' = 4I_0 \cos^2\left(\frac{\delta}{2}\right) = 4I_0 \cos^2\left[13\pi + \frac{\pi}{3}\right]$$

$\Rightarrow$ $$I' = 4I_0 \cdot \frac{1}{4}$$

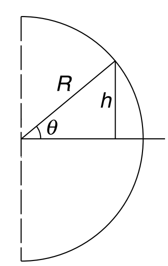
It is given that $4I_o = I$

$\therefore$ $$I' = \frac{I}{4}.$$

13. The second order maxima occurs in direction given by

$$d\sin\theta = 2\lambda$$

$\Rightarrow$ $$\sin\theta = \frac{2\lambda}{d} = \frac{2 \times 500 \times 10^{-9}}{5 \times 10^{-6}} = 0.2$$



The required height is $h = R\sin\theta = 2 \times 0.2 = 0.4\,\text{m}$

14. Extra optical path length for wave from A; $\Delta x_1 = t_1(\mu_1 - 1)$

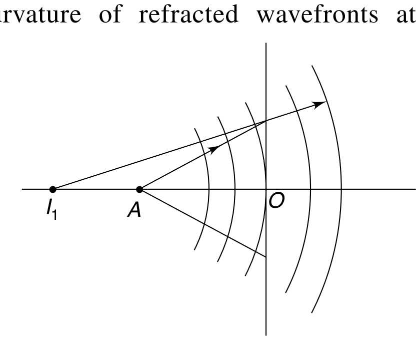
Extra optical path length for wave from B; $\Delta x_2 = t_2(\mu_2 - 1)$

Under given conditions $\Delta x_1 > \Delta x_2$ Hence the central fringe will shift towards A

15. (a) Due to refraction at plane surface the image (I_1) is formed at a distance $\mu R = \frac{3}{2}R$ from O (to left of O).

Refracted rays appear to be diverging from I_1. Hence radius of curvature of refracted wavefronts at $O = OI_1 = \frac{3}{2}R$.

Change in radius of wavefronts $= \frac{3}{2}R - R = \frac{R}{2}$



(b) I_1 acts as object for refraction at curved surface. Image position (I_2) can be calculated as

$$\frac{1}{V} - \frac{\mu}{-\left(\frac{3}{2}R + R\right)} = \frac{1-\mu}{-R}$$

$$\frac{1}{V} = \frac{1}{2R} - \frac{3}{5R} = -\frac{1}{10R}$$

$$\therefore \qquad V = -10R.$$

Image is at a distance of $10R$ from P (to left).

All emergent wave fronts appear to be diverging from I_2. Hence radius of curvature is $10R$.

16. Optical path length of ray along x axis and a ray at a height y will be same.

$$\therefore \qquad \mu_0 t + f \simeq \mu t + (y^2 + f^2)^{\frac{1}{2}}$$

$$\left[\mu_0 - \mu_0\left(1 - \frac{y^2}{y_0^2}\right)\right]t = (y^2 + f^2)^{\frac{1}{2}} - f$$

$$\mu_0 \frac{y^2}{y_0^2} t = f\left[\left(1 + \frac{y^2}{f^2}\right)^{1/2} - 1\right]$$

$$\mu_0 \frac{y^2}{y_0^2} t \simeq f \frac{y^2}{2f^2}$$

$$\Rightarrow \qquad f = \frac{y_0^2}{2\mu_0 t}$$

17. Imagine the image of O to be another point source. The mirror reflects the light as if it is diverging from the image.

The wavefronts around a point source are spherical in shape and the intensity falls off with distance (x) from the source as

$$I \propto \frac{1}{x^2}$$

Image, as a source, is at a distance 3L from the point A.

$\therefore$ Intensity due to it will be $\frac{1}{9}$th of that caused by the original source.

$$\therefore \quad \text{Intensity at } A = 81I + \frac{81I}{9}$$

$$= 90\,I$$

18. The given condition implies the angular position (θ_1) of first minima for $\lambda + \Delta\lambda$ should be less than the angular position (θ_2) for first maxima of $\lambda - \Delta\lambda$. This means

$$\theta_1 < \theta_2$$

$$\sin\theta_1 < \sin\theta_2$$

$$d\sin\theta_1 < d\sin\theta_2$$

$$\frac{1}{2}(\lambda + \Delta\lambda) < (\lambda - \Delta\lambda)$$

$$3\Delta\lambda < \lambda$$

$$\Delta\lambda < \frac{\lambda}{3}$$

19. Let thickness of air gap be t.

On reflection $R2$ suffers a phase change of π.

$\therefore$Condition for constructive interference is

$$2t = (2n_1 + 1)\frac{\lambda_1}{2} \quad \text{where } \lambda_1 = 0.4\ \mu\text{m}$$

$$\therefore \qquad 2t = (2n_1 + 1)\,0.2 \qquad \text{...(1)}$$

For other wavelength

$$2t = (2n_2 + 1)\frac{\lambda_2}{2} \quad ...(2)$$

From (1), $2 \times 0.5\ \mu m = (2n_1 + 1)\frac{0.4}{2}\ \mu m$

$\Rightarrow \quad n_1 = 2$

According to the question; $\lambda_2 > \lambda_1$

(Since $\lambda_1 = 0.4\ \mu m$ is the smallest incident wavelength)

$\therefore \quad n_2 < n_1$

$\therefore \quad n_2 = 1$

$\therefore \quad \lambda_2 = \dfrac{4t}{2n_2 + 1} = \dfrac{4 \times 0.5\ \mu m}{3} = 0.67\ \mu m$

20. Path difference between light incident on two slits is

$$\Delta x_1 = d \sin \phi$$

Geometrical path difference between S_2P and S_1P is $\Delta x_2 = d \sin \theta$

The equivalent length in air will be

$$(\Delta x_2)_{\text{air}} = \mu d \sin \theta.$$

Therefore, total path difference between two waves reaching at P will be

$$\Delta x = \Delta x_1 + (\Delta x_2)_{\text{air}}$$
$$= d \sin \phi + \mu d \sin \theta$$

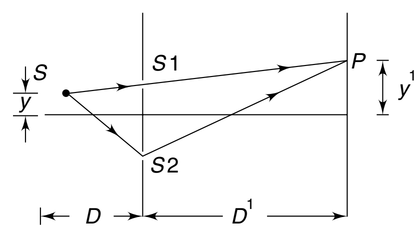
For a maxima at P, we must have

$$\Delta x = n\lambda; \quad n = 0, 1, 2, 3 \ldots$$
$$d \sin \phi + \mu d \sin \theta = n\lambda$$

$\Rightarrow \quad \mu d \sin \theta = n\lambda - d \sin \phi$

$\Rightarrow \quad \theta = \sin^{-1}\left[\dfrac{1}{\mu d}(n\lambda - d \sin \phi)\right]$

21. (a) Displacement of the source at time t is



$$Y = (0.5 \text{ mm}) \sin(\pi t)$$

Path difference of the two waves reaching at P is

$$\Delta x = \frac{yd}{D} + \frac{y'd}{D'}$$

For central maximum $\Delta x = 0$

$\Rightarrow \quad y' = -\dfrac{D'}{D}y = -\left(\dfrac{2}{1}\right) 0.5 \sin \pi t$

$\therefore \quad y' = -\sin(\pi t)$ in mm

(b) $y' = \dfrac{d}{2}$ for a point exactly in front of $S1$

$\therefore \quad \Delta x = \dfrac{yd}{D} + \dfrac{\frac{d}{2}d}{D'}$

For maximum intensity $\Delta x = n\lambda$

$$\frac{yd}{D} + \frac{\frac{d^2}{2}}{D'} = n\lambda$$

$\Rightarrow \quad 0.5 \sin \pi t + \dfrac{(1)^2}{4} = n \times 500 \times 10^{-6} \times 10^3$

$\Rightarrow \quad 0.5 \sin \pi t + 0.25 = 0.5n$

$$\Rightarrow \qquad \sin(\pi t) = \frac{0.5n - 0.25}{0.5}$$

For minimum 't' we have $n = 1$

$$\therefore \qquad \sin(\pi t) = \frac{0.5 - 0.25}{0.5} = 0.5$$

$$\Rightarrow \qquad \pi t = \frac{\pi}{6}$$

$$\Rightarrow \qquad t = \frac{1}{6}\, s$$

22. All rays transmitted through the lens will be parallel since the source (s) is in focal plane.

$$\tan\phi = \frac{d/2}{f}$$

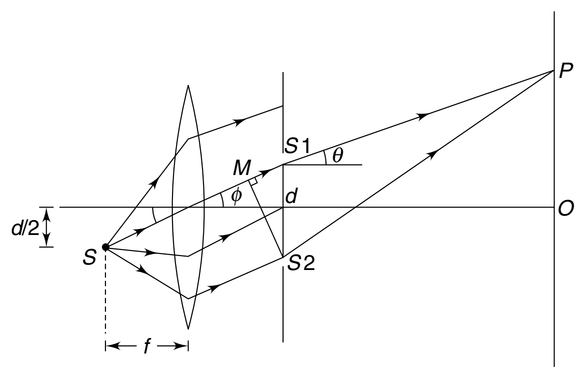
Path difference in waves reaching the slits $S1$ and $S2$ is

$$\Delta x_1 = S_1M = d\sin\phi \simeq d\tan\phi = \frac{d^2}{2f}$$

Central maxima will be formed at a point P where

$$S_2P - S_1P = \Delta X_1$$

If angular position of point P is θ then,



$$S_2P - S_1P = d\sin\theta$$

$$\therefore \qquad d\sin\theta = \frac{d^2}{2f}$$

$$\Rightarrow \qquad \sin\theta = \frac{d}{2f}$$

$$\therefore \qquad \tan\theta \simeq \frac{d}{2f}$$

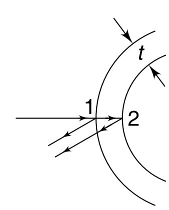
$$\frac{\text{OP}}{\text{D}} = \frac{d}{2f}$$

$$\text{OP} = \frac{Dd}{2f}.$$

23. Colours are seen due to interference between the light waves reflected from the outer and inner surfaces of the liquid film. Light wave, reflected at 1 undergoes a phase change of π and for near normal incidence (as said in the question. Read carefully) the path difference between two interfering waves will be

$$\Delta x = \mu(2t)$$

If constructive interference occurs for wavelength λ then



$$2\mu t = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots$$

$$\Rightarrow \qquad \lambda = 4\mu t, \frac{4\mu t}{3}, \frac{4\mu t}{5} \ldots$$

$$= (4 \times 1.4 \times 90), \frac{4 \times 1.4 \times 90}{3}, \frac{4 \times 1.4 \times 90}{5} \ldots$$

$$= 510 \text{ nm}, 168 \text{ nm}, 101 \text{ nm} \ldots$$

510 nm is visible green light and all other wavelength, for which constructive interference takes place, are invisible.

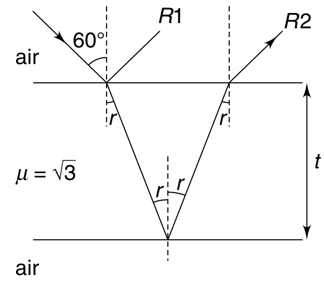
$\therefore$ The point will appear green.

24. 1. $\sin 60° = \sqrt{3}\,\sin r \quad \Rightarrow \quad r = 30°$.

The optical path difference between $R1$ and $R2$ is given by

$$\Delta x = 2\mu t \sec r - 2t \tan r \sin i$$

$$= 2\sqrt{3}\,(t \sec 30°) - 2t \tan 30° \cdot \sin 60° = 3t$$



$\therefore$ For constructive interference, we have

$$3t = \frac{\lambda}{2}; \frac{3\lambda}{2} \ldots$$

For minimum t

$$3t = \frac{\lambda}{2}$$

$$\Rightarrow \quad t = \frac{\lambda}{6} = 100 \text{ nm}.$$

25. (a) Condition for destructive interference in the thin film will be

$$2\mu t = \lambda, 2\lambda, 3\lambda \ldots$$

$$\Rightarrow \quad \lambda = \frac{2\mu t}{n} \quad \text{where} \quad n = 1, 2, 3 \ldots$$

For $\lambda_1 = 450$ nm

$$450 \text{ nm} = \frac{2\mu t}{n} \qquad \ldots(1)$$

For $\lambda_2 = 540$ nm

$$540 \text{ nm} = \frac{2\mu t}{m} \qquad \ldots(2)$$

Where n and m are integers. $[n = m + 1]$

$$(1) \div (2) \quad \frac{450}{540} = \frac{m}{n}$$

$$\Rightarrow \quad \frac{5}{6} = \frac{m}{n}$$

$$\therefore \quad m = 5, n = 6$$

Putting in (1) $\quad t = \dfrac{450 \times 6}{2 \times 1.5} = 900$ nm.

(b) Condition for constructive interference is

$$2\mu t = \frac{\lambda}{2}, \frac{3\lambda}{2} \ldots$$

$$\Rightarrow \quad 2\mu t = \left(m + \frac{1}{2}\right)\lambda \qquad [m = 0, 1, 2, \ldots]$$

$$\Rightarrow \quad \lambda = \frac{2\mu t}{m + \frac{1}{2}} = \frac{2 \times 1.5 \times 900 \text{ nm}}{m + \frac{1}{2}} = \frac{2700}{m + \frac{1}{2}} \text{ nm}.$$

For $m = 0$; $\lambda = 5400$ nm
For $m = 1$; $\lambda = 1800$ nm
For $m = 2$; $\lambda = 1080$ nm
For $m = 3$; $\lambda = 771$ nm
For $m = 4$; $\lambda = 600$ nm
For $m = 5$; $\lambda = 491$ nm
For $m = 6$; $\lambda = 415$ nm
For $m = 7$; $\lambda = 360$ nm

Out of all these 415 nm, 491 nm, 600 nm, and 771 nm fall in visible range.

26. Path difference between light wave reaching at $S2$ and $S1$ is.

$$\Delta x = d\cos\theta$$

To the left of the slit plane the optical path from $S1$ to C must be large by the same amount.

For this the upper slit must be covered using the sheet of refractive index 1.5.

The geometrical path length from $S1$ to C and from $S2$ to C is same. The difference in optical path arises due to different refractive indices of the two sheets.

$$\therefore \quad \Delta x = (1.5 - 1.2)t$$

$$\therefore \quad d\cos\theta = \frac{3}{10}t$$

$$\therefore \quad \cos\theta = \frac{3t}{10d}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{3t}{10d}\right)$$

27. After plates are inserted

$$\Delta x = (\mu_2 - \mu_1)t = (1.7 - 1.4)t = 0.3t$$

Since point P lies between original 5th maximum and 6th minimum

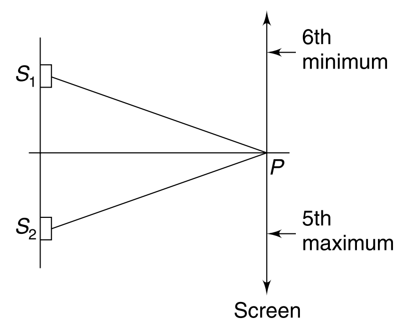
$\therefore$ path difference of waves reaching P can be written as

$$\Delta x = 5\lambda + \Delta$$

$$\therefore \quad 0.3t = 5\lambda + \Delta \qquad \text{...(a)}$$

It is given that

$$\frac{I}{I_0} = \frac{3}{4} = \cos^2\left[\frac{\pi\Delta}{\lambda}\right]$$



$$\Rightarrow \quad \frac{\cos\pi\Delta}{\lambda} = \frac{\sqrt{3}}{2}$$

$$\text{so,} \quad \frac{\pi\Delta}{\lambda} = \frac{\pi}{6}; \qquad \therefore \Delta = \frac{\lambda}{6}$$

$$\therefore \text{ from (a)} \quad 0.3t = 5\lambda + \frac{\lambda}{6}$$

$$\Rightarrow \quad t = \frac{31 \times 5400 \times 10^{-10}}{6 \times 0.3} = 9.3 \times 10^{-6} \text{ m}$$

28. Shift in fringe pattern caused by the transparent sheet is

$$S = \frac{D}{d}(\mu - 1)t$$

$$\therefore \quad \Delta S = \frac{D}{d}[\Delta(\mu - 1)t + (\mu - 1)\Delta t]$$

$$= \frac{D}{d}[\Delta\mu t + (\mu - 1)\Delta t]$$

$$= \frac{D}{d}[-t\cdot\gamma\Delta\theta + (\mu - 1)t\alpha\Delta\theta]$$

For the central fringe to be back at its original location.

$$S + \Delta s = 0$$

$$\Rightarrow \quad \frac{D}{d}(\mu - 1)t + \frac{D}{d}[-t\gamma\Delta\theta + (\mu - 1)t\alpha\Delta\theta] = 0$$

$$[-\gamma + (\mu - 1)\alpha]\Delta\theta = -(\mu - 1)$$

$$\Rightarrow \quad \Delta\theta = \frac{(\mu - 1)}{(\mu - 1)\alpha - \gamma}$$

29. (a) If δ is phase difference between the two waves arriving at P then

$$I_0 = 4I_0 \cos^2\left(\frac{\delta}{2}\right)$$

$$\Rightarrow \quad \delta = \frac{2\pi}{3}$$

After placing the glass piece the new phase difference will be –

$$\delta' = \frac{2\pi}{3} + \frac{2\pi(\mu - 1)t}{\lambda}$$

(b) Intensity at P will be- $4I_0\cos^2\left(\frac{\pi}{3} + \frac{\pi}{\lambda}(\mu - 1)t\right)$

Intensity at P will be minimum (= 0) if

$$\frac{\pi}{3} + \frac{\pi}{\lambda}(\mu - 1) = (2n - 1)\frac{\pi}{2} \text{ where } n = 1, 2, 3...$$

$$t = \frac{(2n - 1)\lambda}{2(\mu - 1)} - \frac{\lambda}{3(\mu - 1)} \text{ where } n = 1, 2, 3...$$

30. From the theory of interference, the position of first maxima (from O) is given as

$$y = \frac{D\lambda}{d}$$

or, $$\Delta y = \frac{D}{d}\Delta\lambda$$

$\therefore$ $$\frac{\Delta y}{y} = \frac{\Delta\lambda}{\lambda} \quad ...(i)$$

Now, velocity of sound is proportional to square root of its temperature

$\therefore$ $$V = K'\sqrt{T}; \quad \text{where } K' \text{ is a constant.}$$

The frequency of source remains unaltered.

$\therefore$ $$\lambda v = K'\sqrt{T}$$

or, $$\lambda = \frac{K'}{v}\sqrt{T} = K\sqrt{T} \quad \left[\text{where } K = \frac{K'}{v} = \text{constant}\right]$$

$\therefore$ $$\Delta\lambda = \frac{K\Delta T}{2\sqrt{T}}$$

or, $$\frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{2T} = \frac{0.01}{2} \quad \left[\because \frac{\Delta T}{T} = 0.01\right]$$

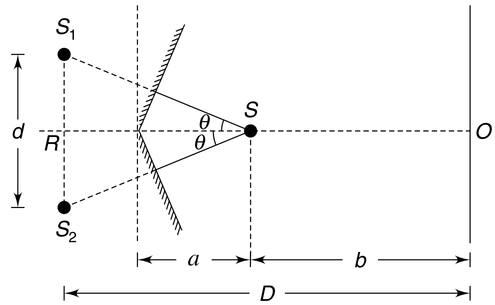
$$= 0.005 \quad ...(ii)$$

Form (i) and (ii)

$$\frac{\Delta y}{y} = 0.005$$

$\therefore$ $$\% \text{ change} = \frac{\Delta y}{y} \times 100 = 0.5\% \text{ (increment).}$$

31. Interference is due to reflected waves, therefore, images S_1 and S_2 of source S behave like two coherent sources.



Distance of S from each mirror $= a\cos\theta$.

$$\therefore \quad SS_1 = SS_2 = 2a\cos\theta$$

$$\therefore \quad S_1S_2 = d = SS_1\sin\theta + SS_2\sin\theta$$
$$= 4a\cos\theta\sin\theta$$

Since θ is small

$$\cos\theta \simeq 1 \quad \text{and} \quad \sin\theta \simeq \theta$$
$$d = 4a\theta$$

Distance $RS = SS_1\cos\theta = 2a\cos^2\theta$

$\simeq 2a.$

$$\therefore \quad D = RO = RS + SO = (2a + b)$$

$$\therefore \text{ Fringe width} \quad \beta = \frac{D\lambda}{d} = \frac{(2a+b)\lambda}{4a\theta}$$

(a) For blue light $\lambda = 4000$ Å, $a = 1$ cm, $b = 38$ cm

$$\therefore \quad \beta_1 = \frac{2 \times 1 + 38}{4 \times 1 \times 0.05} \times 4000 \times 10^{-10} \text{ m}$$
$$= \frac{40}{20} \times 4 \times 10^{-5} \text{ m} = 80\ \mu\text{m}$$

(b) Fringe width for light of wavelength 5600 Å is

$$\beta_2 = \frac{2 \times 1 + 38}{4 \times 1 \times 0.05} \times 5600 \times 10^{-10} \text{ m} = 112\ \mu\text{m}.$$

A black line is formed at the position where dark fringes are formed for both the wavelengths.

Let the first black line be at a distance y from the central fringe. Let this be the position of m^{th} dark fringe of 4000 Å and n^{th} dark fringe of 5600 Å light.

$$\therefore \quad y = \left(m - \frac{1}{2}\right)\beta_1 = \left(n - \frac{1}{2}\right)\beta_2$$

$$\Rightarrow \quad \frac{2m-1}{2n-1} = \frac{\beta_2}{\beta_1}$$

For first black line, y should be minimum possible which corresponds to least possible integral values of m and n.

$$\therefore \quad \frac{2m-1}{2n-1} = \frac{7}{5}$$

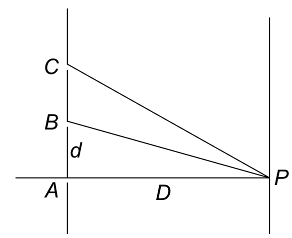
or, $\quad m = 4, n = 3$

$\therefore$ position of first black line $y = \left(m - \frac{1}{2}\right)\beta_1 = 280\ \mu$m.

(c) Since the interference pattern is symmetric about the central fringe, there will be one dark line at a distance y above it and the other will be y below it.

$\therefore$ Relevant answer is $2y = 560\ \mu$m.

32. (a)



$$BP - AP = \frac{\lambda}{3}$$

$$\sqrt{D^2 + d^2} - D = \frac{\lambda}{3}$$

$$\Rightarrow \quad D\left[1 + \frac{d^2}{D^2}\right]^{1/2} - D = \frac{\lambda}{3}$$

$$\Rightarrow \quad D\left[1 + \frac{d^2}{2D^2}\right] - D = \frac{\lambda}{3}$$

$$\left[d << D \text{ as } BP - AP = \frac{\lambda}{3} << D\right]$$

$$\therefore \quad d = \sqrt{\frac{2D\lambda}{3}} \qquad \text{...(a)}$$

(b) Path difference for two waves

$$\Delta x_{CA} = CP - AP$$

$$= \sqrt{D^2 + (2d)^2} - D$$

$$= D\left(1 + \frac{4d^2}{D^2}\right)^{1/2} - D$$

$$\simeq D\left(1 + \frac{2d^2}{D^2}\right) - D = \frac{2d^2}{D}$$

$$= \frac{4\lambda}{3}\left[\because \quad d = \sqrt{\frac{2D\lambda}{3}}\right]$$

$\therefore$ phase difference

$$\Delta\phi_{CA} = \frac{2\pi}{\lambda}\cdot\Delta x_{CA} = \frac{2\pi}{\lambda}\cdot\frac{4\lambda}{3} = \frac{8\pi}{3}$$

(c) Path difference at P for waves from B and A

$$\Delta x_{BA} = \frac{\lambda}{3}$$

$\therefore$ phase difference

$$\Delta\phi_{CA} = \frac{2\pi}{\lambda}\cdot\Delta x_{BA} = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{3} = \frac{2\pi}{3}$$

The three waves at P can be represented as

$$y_1 = a\sin\omega t$$

$$y_2 = a\sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$y_3 = a\sin\left(\omega t + \frac{8\pi}{3}\right) = a\sin\left(\omega t + \frac{2\pi}{3}\right)$$

$\therefore$ Resultant wave is

$$y = y_1 + y_2 + y_3$$

$$= a\sin\omega t + 2a\sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$= A\sin(\omega t + \delta)$$

Where $$A = \sqrt{a^2 + (2a)^2 + 2\cdot a\cdot 2a\cdot\cos\frac{2\pi}{3}} = \sqrt{3}a.$$

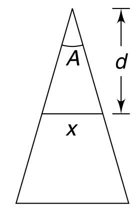
Intensity is proportional to square of amplitude.

$\therefore \qquad I = 3I_0.$

33. (a) $\qquad x \simeq A \cdot d \qquad \ldots(1)$

Path difference between the two interfering waves is

$$\Delta x = 2\mu x$$



For constructive interference the condition is

$$2\mu x = n\lambda + \frac{\lambda}{2}$$

$$n = 0, 1, 2, \ldots$$

Since reflection at the first surfaces causes an additional phase change of π.

For first maxima–

$$2\mu x = \frac{\lambda}{2} \Rightarrow 2 \times 1.48x = \frac{400 \times 10^{-9}}{2}$$

$\Rightarrow \qquad x = 6.8 \times 10^{-8}$ m = 68 nm

Form (1)

$$A = \frac{x}{d}$$

$$= \frac{68 \times 10^{-9}}{3 \times 10^{-2}} = 22.7 \times 10^{-7} \text{ rad}$$

$$= 2.3 \times 10^{-6} \text{ rad} \simeq 1.3 \times 10^{-4} \text{ degree.}$$

(b) For red light

$$2\mu x = \frac{\lambda_R}{2} \Rightarrow x = 136 \text{ nm}$$

$\therefore \qquad d = \dfrac{x}{A} = 6$ cm

34. Let I_0 = intensity at S_3 (and S_4) due to either of S_1 or S_2.

Path difference between light waves reaching S_3 (or S_4) = $d_1\left(\dfrac{d_2}{2L_1}\right)$

Phase difference $\qquad \Delta\phi = \dfrac{2\pi}{\lambda} \cdot \dfrac{d_1 d_2}{2L_1} = \dfrac{\pi d_1 d_2}{\lambda L_1}$

Intensity at slits S_3 and S_4 will be

$$I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right) = 4I_0 \cos^2\left(\frac{\pi d_1 d_2}{2\lambda L_1}\right)$$

Now, S_3 and S_4 are two coherent sources producing interference fringes on the screen. At distance y from the centre

$$\Delta x' = d_2 \frac{y}{L_2}$$

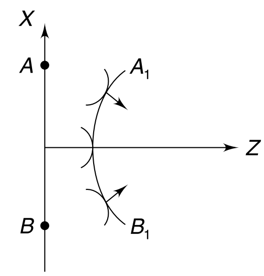
$$\Delta\phi' = \frac{2\pi}{\lambda} \frac{d_2 y}{L_2}$$

$\therefore$ Intensity at distance y will be proportional to

$$I' \propto I \cdot \cos^2 \frac{\Delta\phi'}{2}$$

$\Rightarrow \qquad I' \propto \cos^2\left(\dfrac{\pi d_1 d_2}{2\lambda L_1}\right) \cdot \cos^2\left(\dfrac{\pi d_2 y}{\lambda L_2}\right)$

35. The refractive index decrease as x and y increase. The speed of wave increases as we move away from the z axis. When we draw spherical wavefronts originating from different points on AB, we get spheres of increasing radius as we move away from the z axis. When we draw common tangent to all these spheres, the resulting wavefront is as shown in the figure. Draw one more wavefront and you will realize that it is converging.



For the above construction to be valid, Δt should be very small so that we can treat the secondary wavelets to be spherical.

36. Assume that only A_1 is on.

Path difference $\quad A_1S_2 - A_1S_1 = d\sin\Delta\theta$

Path difference $\quad S_2P - S_1P = \dfrac{yd}{D}$

$\therefore$ Total path difference

$$\Delta x = A_1S_2P - A_1S_1P$$

$$= d\sin\Delta\theta + \frac{yd}{D}$$

$$\therefore \qquad \delta = \frac{2\pi}{\lambda}d\left[\sin\Delta\theta + \frac{y}{D}\right]$$

$\therefore$ Intensity at P when only A_1 is on is

$$I_1 = I_0\cos^2\delta = I_0\cos^2\left[\frac{2\pi}{\lambda}d\left(\sin\Delta\theta + \frac{y}{D}\right)\right]$$

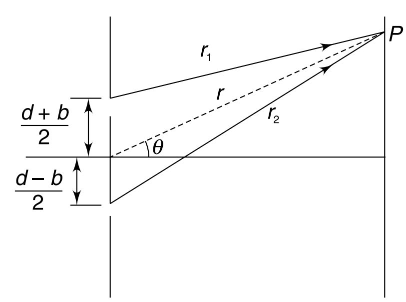
Similarly, intensity at P when only A_2 is on is

$$I_2 = I_0\cos^2\left[\frac{2\pi}{\lambda}d\left(\frac{y}{d} - \sin\Delta\theta\right)\right]$$

Since sources are incoherent; the resultant intensity when both are on is

$$I = I_1 + I_2 = I_0\cos^2\left[\frac{2\pi}{\lambda}d\left(\sin\Delta\theta + \frac{y}{D}\right)\right] + I_0\cos^2\left[\frac{2\pi}{\lambda}d\left(\frac{y}{D} - \sin\Delta\theta\right)\right]$$

37. The path length from top edges of the slits to P is r_1 and r_2. Using law of cosines:



$$r_1^2 = r^2 + \left(\frac{d+b}{2}\right)^2 - 2\cdot\left(\frac{d+b}{2}\right)\cdot r\cdot\cos\left(\frac{\pi}{2} - \theta\right)$$

$$= r^2 + \left(\frac{d+b}{2}\right)^2 - (d+b)r\cdot\sin\theta$$

And
$$r_2^2 = r^2 + \left(\frac{d-b}{2}\right)^2 - 2\cdot\left(\frac{d-b}{2}\right)r\cdot\cos\left(\frac{\pi}{2} + \theta\right)$$

$$= r^2 + \left(\frac{d-b}{2}\right)^2 + (d-b)r\cdot\sin\theta$$

$$r_2^2 - r_1^2 = -db + 2d\cdot r\cdot\sin\theta$$

$$\Rightarrow \qquad (r_2 + r_1)(r_2 - r_1) = 2d\, r\sin\theta - d\cdot b$$

$$\Rightarrow \qquad (r_2 - r_1) = \Delta x_t = d\sin\theta - \frac{d\cdot b}{2r} \qquad [\because \; r_1 + r_2 = 2r]$$

Similarly, consider r_3 and r_4 as path length from the bottom edge of the two slits and prove that

$$r_4 - r_3 = \Delta x_b = d\sin\theta + \frac{d\cdot b}{2r}$$

$$\therefore \qquad \Delta x_b - \Delta x_t = \frac{db}{r} \simeq \frac{db}{D}.$$