TANGENT AND NORMALS

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. If a variable tangent to the curve $x^2y = c^3$ makes intercepts a, b on x and y axis respectively, then the value of a^2b is -

(A) $27c^3$ (B) $\frac{4}{27}c^3$ (C) $\frac{27}{4}c^3$ (D) $\frac{4}{9}c^3$

2. The number of values of c such that the straight line $3x + 4y = c$ touches the curve $\frac{x^4}{2} = x + y$ is -

(A) 0 (B) 1 (C) 2 (D) 4

3. Let $f(x) = x^3 + ax + b$ with $a \neq b$ and suppose the tangent lines to the graph of f at $x = a$ and $x = b$ have the same gradient. Then the value of $f(1)$ is equal to -

(A) 0 (B) 1 (C) $-\frac{1}{3}$ (D) $\frac{2}{3}$

4. The tangent to the curve $3xy^2 - 2x^2y = 1$ at (1,1) meets the curve again at the point -

(A) $\left(\frac{16}{5}, \frac{1}{20}\right)$ (B) $\left(-\frac{16}{5}, -\frac{1}{20}\right)$ (C) $\left(\frac{1}{20}, \frac{16}{5}\right)$ (D) $\left(-\frac{1}{20}, \frac{16}{5}\right)$

5. The curve $y - e^{xy} + x = 0$ has a vertical tangent at -

(A) (1,1) (B) (0,1) (C) (1,0) (D) no point

6. Suppose f and g both are linear function with $f(x) = -2x + 1$ and $f(g(x)) = 6x - 7$, then slope of line $y = g(x)$ is -

(A) 3 (B) –3 (C) 6 (D) –2

7. A curve with equation of the form $y = ax^4 + bx^3 + cx + d$ has zero gradient at the point (0,1) and also touches the x-axis at the point (–1,0) then the values of x for which the curve has a negative gradient are -

(A) $x > -1$ (B) $x < 1$ (C) $x < -1$ (D) $-1 \leq x \leq 1$

8. The line which is parallel to x-axis and crosses the curve $y = \sqrt{x}$ at an angle of $\frac{\pi}{4}$ is -

(A) $y = -\frac{1}{2}$ (B) $x = \frac{1}{2}$ (C) $y = \frac{1}{4}$ (D) $y = \frac{1}{2}$

9. The lines tangent to the curves $y^3 - x^2y + 5y - 2x = 0$ and $x^4 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

10. The angle of intersection of the curves $2y = x^3$ and $y^2 = 32x$ at origin is -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

11. The angle of intersection of $x = \sqrt{y}$ and $x^3 + 6y = 7$ at (1, 1) is -

(A) $\frac{\pi}{5}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

12. At any point of a curve $\sqrt{\frac{\text{subnormal}}{\text{sub tan gent}}}$ is equal to -

(A) the abscissa of that point (B) the ordinate of that point

(C) slope of the tangent at that point (D) slope of the normal at that point

13. The length of the tangent to the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ at θ points is -

(A) $2a\sin\frac{\theta}{2}$ (B) $a\sin\theta$ (C) $2a\sin\theta$ (D) $a\cos\theta$

14. The length of the subnormal of the curve $y^2 = 8ax$ $(a > 0)$ is -

(A) 2a (B) 4a (C) 6a (D) 8a

15. A 13 ft. ladder is leaning against a wall when its base starts to slide away. At the instant when the base is 12 ft. away from the wall, the base is moving away from the wall at the rate of 5 ft/sec. The rate at which the angle θ between the ladder and the ground is changing is -

(A) $-\frac{12}{13}$ rad/sec. (B) -1 rad/sec. (C) $-\frac{13}{12}$ rad/sec. (D) $-\frac{10}{13}$ rad/sec.

16. Water is poured into an inverted conical vessel of which the radius of the base is 2m and height 4m, at the rate of 77 litre/minute. The rate at which the water level is rising at the instant when the depth is 70 cm is - (use $\pi = 22/7$)

(A) 10 cm/min (B) 20 cm/min (C) 40 cm /min (D) none

17. A point is moving along the curve $y^3 = 27x$. The interval in which the abscissa changes at slower rate than ordinate, is -

(A) $(-3, 3)$ (B) $(-\infty,\infty)$ (C) $(-1, 1)$ (D) $(-\infty, -3)\cup(3, \infty)$

18. A particle moves along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units per second. The value of $\frac{dx}{dt}$ when $x = 3$ is -

(A) 4 (B) $\frac{9}{2}$ (C) $\frac{3\sqrt{3}}{2}$ (D) none of these

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

19. Which of the following pair(s) of curves is/are orthogonal.

(A) $y^2 = 4ax$; $y = e^{-x/2a}$ (B) $y^2 = 4ax$; $x^2 = 4ay$

(C) $xy = a^2$; $x^2 - y^2 = b^2$ (D) $y = ax$; $x^2 + y^2 = c^2$

20. If $\frac{x}{a}+\frac{y}{b}=1$ is a tangent to the curve $x = Kt$, $y=\frac{K}{t}$, $K>0$ then :

(A) $a > 0, b > 0$ (B) $a > 0, b < 0$ (C) $a < 0, b > 0$ (D) $a < 0, b < 0$

21. The coordinates of the point(s) on the graph of the function, $f(x)=\frac{x^3}{3}-\frac{5x^2}{2}+7x-4$ where the tangent drawn cut off intercepts from the coordinate axes which are equal in magnitude but opposite in sign, is -

(A) (2,8/3) (B) (3,7/2) (C) (1,5/6) (D) none

22. For the curve represented parametrically by the equations, $x = 2\,\ell n \cot t + 1$ and $y = \tan t + \cot t$

(A) tangent at $t = \pi/4$ is parallel to x-axis

(B) normal at $t = \pi/4$ is parallel to y-axis

(C) tangent at $t = \pi/4$ is parallel to the line $y = x$

(D) normal at $t = \pi/4$ is parallel to the line $y = x$

23. Consider the curve $f(x) = x^{1/3}$, then -

(A) the equation of tangent at (0, 0) is $x = 0$ (B) the equation of normal at (0, 0) is $y = 0$

(C) normal to the curve does not exist at (0, 0) (D) f(x) and its inverse meet at exactly 3 points.

24. Equation of common tangent(s) of $x^2 - y^2 = 12$ and $xy = 8$ is (are) -

(A) $y = 3x + 4\sqrt{6}$ (B) $y = -3x + 4\sqrt{6}$ (C) $3y = x + 4\sqrt{6}$ (D) $y = -3x - 4\sqrt{6}$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	B	B	B	C	B	C	D	D	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	D	C	A	B	B	B	C	A	A,C,D	A,D
Que.	21	22	23	24						
Ans.	A,B	A,B	A,B,D	B,D						

EXTRA PRACTICE QUESTIONS ON TANGENT AND NORMALS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. The angle at which the curve $y = Ke^{Kx}$ intersects the y-axis is -

(A) $\tan^{-1}k^2$ (B) $\cot^{-1}(k^2)$ (C) $\sin^{-1}\left(\frac{1}{\sqrt{1+k^4}}\right)$ (D) $\sec^{-1}\left(\sqrt{1+k^4}\right)$

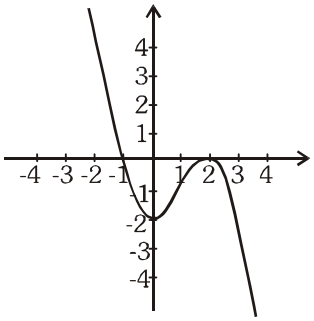
2. The coordinates of points(s) at each of which the tangents to the curve $y = x^3 - 3x^2 - 7x + 6$ cut off on the positive semi axis OX a line segment half that on the negative semi axis OY is/are given by :

(A) (–1,9) (B) (3,–15) (C) (1,–3) (D) none

3. The abscissa of the point on the curve $\sqrt{xy} = a + x$, the tangent at which cuts off equal intercepts from the co-ordinate axes is (a > 0)

(A) $\frac{a}{\sqrt{2}}$ (B) $-\frac{a}{\sqrt{2}}$ (C) $a\sqrt{2}$ (D) $-a\sqrt{2}$

4. A cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ has a graph which is tangent to the x-axis at 2, has another x-intercept at –1, and has y-intercept at –2 as shown. The values of, a + b + c + d equals-



(A) –2

(B) –1

(C) 0

(D) 1

5. Equation of a tangent to the curve $y\cot x = y^3 \tan x$ at the point where the abscissa is $\frac{\pi}{4}$ is -

(A) $4x + 2y = \pi + 2$ (B) $4x - 2y = \pi + 2$ (C) $x = 0$ (D) $y = 0$

6. Consider the curve represented parametrically by the equation

$x = t^3 - 4t^2 - 3t$ and $y = 2t^2 + 3t - 5$ where $t \in R$

If H denotes the number of point on the curve where the tangent is horizontal and V the number of point where the tangent is vertical then-

(A) H = 2 and V = 1 (B) H = 1 and V = 2 (C) H = 2 and V = 2 (D) H = 1 and V = 1

7. If y = f(x) be the equation of a parabola which is touched by the line y = x at the point where x = 1. Then -

(A) f'(1) = 1 (B) f'(0) = f'(1) (C) 2f(0) = 1 – f'(0) (D) f(0) + f'(0) + f''(0) = 1

8. At the point $P(a, a^n)$ on the graph of $y = x^n (n \in N)$ in the first quadrant a normal is drawn. The normal intersects the y-axis at the point (0, b), If $\lim_{a\to 0} b = \frac{1}{2}$, then n equals -

(A) 1 (B) 3 (C) 2 (D) 4

9. A horse runs along a circle with a speed of 20 km/hr. A lantern is at the centre of the circle. A fence is along the tangent to the circle at the point at which the horse starts. The speed with which the shadow of the horse move along the fence at the moment when it covers 1/8 of the circle in km/hr is -

(A) 20 (B) 60 (C) 30 (D) 40

10. Equation of the line through the point (1/2,2) and tangent to the parabola $y = \frac{-x^2}{2} + 2$ and secant to the curve $y = \sqrt{4 - x^2}$ is -

(A) $2x + 2y - 5 = 0$ (B) $2x + 2y - 9 = 0$

(C) $y - 2 = 0$ (D) none

11. If the tangent at P of the curve $y^2 = x^3$ intersects the curve again at Q and the straight lines OP,OQ make angles α, β with the x-axis where 'O' is the origin then $\tan\alpha/\tan\beta$ has the value equal to -

(A) -1 (B) -2 (C) 2 (D) $\sqrt{2}$

12. Let f (x) be a nonzero function whose all successive derivative exist and are nonzero. If f (x), f '(x) and f ''(x) are in G.P. and f (0) = 1 , f '(0) = 1, then -

(A) f '(x) < 0 $\forall$ x $\in$ R (B) f ''(x) < 1 $\forall$ x $\in$ R

(C) f ''(0) $\neq$ f '''(0) (D) f ''(x) > 0 $\forall$ x $\in$ R

13. If the line ax + by + c = 0 is a normal to the curve xy = 1, then - **[JEE 1986]**

(A) a > 0, b > 0 (B) a > 0, b < 0 (C) a < 0, b > 0 (D) a < 0, b < 0

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B,C	B	A,B	B	A,B,D	B	A,C	C	D	A
Que.	11	12	13							
Ans.	B	D	B,C							