CHAPTER – 9

DIFFERENTIAL EQUATIONS

Exercise 9.6

Question 1: For each of the differential equations given in question, find the general solution:

$\frac{dy}{dx} + 2y = \sin x$

Answer:

It is given that $\frac{dy}{dx} + 2y = \sin x$

This is equation in the form of $\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{py} = \mathrm{Q}$ (where, p = 2 and Q = sin x)

Now, I.F. = $\mathrm{e}^{\int \mathrm{pdx}} = \mathrm{e}^{\int 2\mathrm{dx}} = \mathrm{e}^{2\mathrm{x}}$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int(\mathrm{Q} \times \mathrm{I.F.})\mathrm{dx} + \mathrm{C}$

$\Rightarrow ye^{2x} = \int \mathrm{sinx}.e^{2x}\, dx + C$ --------(1)

Let I = $\int \mathrm{sinx}.e^{2x}\, dx$

$\Rightarrow$ I = sinx. $\int e^{2x}\, dx - \int \left(\frac{d}{dx}(\sin x).\mathrm{e}^{\int 2\mathrm{dx}}\right) dx$

= sinx. $\frac{e^{2x}}{2} - \int \left(\cos x.\frac{e^{2x}}{2}\right) dx$

= $\frac{e^{2x}\sin x}{2} - \frac{1}{2}[\cos \int e^{2x} - \int \left(\frac{d}{dx}(\cos x)\ .\int \int e^{2x}\, dx)\right)]$

= $\frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{2} - \frac{1}{4}\int(\mathrm{sinx}.e^{2x})\, dx$

$\Rightarrow \frac{e^{2x}}{2}(2\sin x - \cos x) - \frac{1}{4}\mathrm{I}$

$\Rightarrow \frac{5}{4}\mathrm{I} = \frac{e^{2x}}{4}(2\sin x - \cos x)$

$\Rightarrow \mathrm{I} = \frac{e^{2x}}{5}(2\sin x - \cos x)$

Now, putting the value of I in (1), we get,

$\Rightarrow ye^{2x} = \frac{e^{2x}}{5}(2\sin x - \cos x) + C$

$\Rightarrow y = \frac{1}{5}(2\sin x - \cos x) + Ce^{-2x}$

Therefore, the required general solution of the given differential equation is

$y = \frac{1}{5}(2\sin x - \cos x) + Ce^{-2x}$

Question 2: For each of the differential equations given in question, find the general solution:

$\frac{dy}{dx} + 3y = e^{-2x}$

Answer:

It is given that $\frac{dy}{dx} + 3y = e^{-2x}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = 3 and Q $= e^{-2x}$)

Now, I.F. = $e^{\int p dx} = e^{\int 3dx} = e^{3x}$

Thus, the solution of the given differential equation is given by the relation:

$\mathrm{y(I.F.)} = \int(\mathrm{Q} \times \mathrm{I.F.})\,\mathrm{dx} + \mathrm{C}$

$\Rightarrow \mathrm{ye^{3x}} = \int(\mathrm{e^{-2x}} \times \mathrm{e^{2x}})\,\mathrm{dx} + \mathrm{C}$

$\Rightarrow ye^{3x} = e^{x} + C$

$\Rightarrow y = e^{-2x} + Ce^{-3x}$

Therefore, the required general solution of the given differential equation is $y = e^{-2x} + Ce^{-3x}$

Question 3: For each of the differential equations given in question, find the general solution:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Answer:

It is given that $\frac{dy}{dx} + \frac{y}{x} = x^2$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $\frac{1}{x}$ and Q =x^2)

Now, I.F. = $\int (\mathrm{Q} \times \mathrm{I.F.})\, \mathrm{dx} + \mathrm{C}$

$\Rightarrow \mathrm{y(x)} = \int (\mathrm{x}^2.\mathrm{x})\mathrm{dx} + \mathrm{C}$

$\Rightarrow \mathrm{xy} = \int (\mathrm{x}^3)\mathrm{dx} + \mathrm{C}$

$\Rightarrow \mathrm{xy} = \frac{\mathrm{x}^4}{4} + \mathrm{C}$

Therefore, the required general solution of the given differential equation is $\mathrm{xy} = \frac{\mathrm{x}^4}{4} + \mathrm{C}$.

Question 4: For each of the differential equations given in question, find the general solution:

$$\frac{dy}{dx} + (\sec x)y = \tan x \left(0 \leq x < \frac{\pi}{2}\right)$$

Answer:

It is given that $\frac{dy}{dx} + (sec\ x)y = tan\ x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = sec x and Q = tan x)

Now, I.F. = $\int(\text{Q} \times \text{I. F.})\ \text{dx} + \text{C}$

$\Rightarrow \text{y}(\sec x + \tan x) = \int \tan x\,(\sec x + \tan x)\text{dx} + \text{C}$

$\Rightarrow \text{y}(\sec x + \tan x) = \int \sec x \tan x\, dx + \int \tan^2 x\, dx + \text{C}$

$\Rightarrow \text{y}(\sec x + \tan x) = \sec x + \int(\sec^2 x - 1)dx + C$

⇒ y (sec x + tan x) = sec x + tan x – x+ C

Therefore, the required general solution of the given differential equation is

y (sec x + tan x) = sec x + tan x – x+ C.

Question 5: For each of the differential equations given in question, find the general solution:

$\cos^2 x \frac{dy}{dx} + y = \tan x \left(0 \leq x < \frac{\pi}{2}\right)$

Answer:

It is given that $\cos^2 x \frac{dy}{dx} + y = \tan x$

$\Rightarrow \frac{dy}{dx} + \sec^2 x . y = \sec^2 x \tan x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $\sec^2 x$ and Q =$\sec^2 x \tan x$)

Now, I.F. =$e^{\int pdx} = e^{\int \sec^2 x dx} = e^{\tan x}$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int (Q \times I.F.)\, dx + C$

$\Rightarrow y.e^{\tan x} = \int e^{\tan x}\, dx + C$ ……… (1)

Now, Let t = tan x

$\Rightarrow \frac{d}{dx}(\tan x) = \frac{dt}{dx}$

$\Rightarrow \sec^2 x = \frac{dt}{dx}$

$\Rightarrow \sec^2 x dx = dt$

Thus, the equation (1) becomes,

$\Rightarrow y.e^{\tan x} = \int (e^{t} \cdot t)\, dt + C$

$\Rightarrow y.e^{\tan x} = \int (t.e^{t})\, dt + C$

$\Rightarrow y.e^{\tan x} = t \int e^{t}\, dt - \int \left(\frac{d}{dt}(t). \int e^{t} dt\right) + C$

$\Rightarrow y.e^{\tan x} = t.e^{t} - \int e^{t}\, dt + C$

$\Rightarrow t\, e^{\tan x} = (t - 1)\, e^{t} + C$

$\Rightarrow t\, e^{\tan x} = (\tan x - 1)\, e^{\tan x} + C$

$\Rightarrow y = (\tan x - 1) + C\, e^{-\tan x}$

Therefore, the required general solution of the given differential equation is

$y = (\tan x - 1) + C\, e^{-\tan x}$.

Question 6: For each of the differential equations given in question, find the general solution:

$x \frac{dy}{dx} + 2y = x^2 \log x$

Answer:

It is given that $x\frac{dy}{dx} + 2y = x^2 \log x$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = x^2 \log x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $\frac{2}{x}$ and Q = $x \log x$)

Now, I.F. =$e^{\int p dx} = e^{\int \frac{2}{x} dx} = e^{2(\log x)} = e^{\log x^2} = x^2$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int(\mathrm{Q} \times \mathrm{I.F.})\,\mathrm{dx} + \mathrm{C}$

$\Rightarrow y.x^2 = \int(x \log x . x^2)dx + C$

$\Rightarrow x^2 y = \int(x^3 \log x)dx + C$

$\Rightarrow x^2 y = \log x . \int x^3 dx - \int \left[\frac{d}{dx}(\log x) . \int x^3 dx\right] dx + C$

$\Rightarrow x^2 y = \log x . \frac{x^4}{4} - \int \left(\frac{1}{x} . \frac{x^4}{4}\right) dx + C$

$\Rightarrow x^2 y = \frac{x^4 \log x}{4} - \frac{1}{x} . \frac{x^4}{4} + C$

$\Rightarrow x^2 y = \frac{1}{16} x^4 - (4 \log x - 1) + C$

$\Rightarrow y = \frac{1}{16} x^2 - (4 \log x - 1) + Cx^{-2}$

Therefore, the required general solution of the given differential equation

$y = \frac{1}{16} x^2 - (4 \log x - 1) + Cx^{-2}$

Question 7: For each of the differential equations given in question, find the general solution:

$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$

Answer:

It is given that $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x^2}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $\frac{1}{x \log x}$ and Q = $\frac{2}{x^2}$)

Now, I.F. =$e^{\int p dx} = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int (\mathrm{Q} \times \mathrm{I.F.})\, \mathrm{dx} + \mathrm{C}$

$\Rightarrow y. \log x = \int \left[\frac{2}{x^2}. \log x\right] dx + C$ …… (1)

Now, $\int \left[\frac{2}{x^2}. \log x\right] dx = 2 \int \left(\log \frac{1}{x^2}\right) dx$

= $2 \left[\log x. \int \frac{1}{x^2} dx - \int \left\{\frac{d}{dx}(\log x). \int \frac{1}{x^2} dx\right\} dx\right]$

= $2 \left[\log x \left(-\frac{1}{x}\right) - \int \left(\frac{1}{x}. \left(-\frac{1}{x}\right)\right)\right]$

= $2 \left[-\frac{\log x}{x} + \int \frac{1}{x^2} dx\right]$

= $-\frac{2}{x}(1 + \log x)$

Now, substituting the value in (1), we get,

$\Rightarrow y. \log x = -\frac{2}{x}(1 + \log x) + C$

Therefore, the required general solution of the given differential equation is

$y.\log x = -\frac{2}{x}(1 + \log x) + C$

Question 8: For each of the differential equations given in question, find the general solution:
$(1 + x^2)dy + 2xy\ dx = \cot x\ dx\ (x \neq 0)$

Answer:

It is given that $(1 + x^2)\ dy + 2xy\ dx = \cot x\ dx$

$\Rightarrow \frac{dy}{dx} + \frac{2xy}{(1+x^2)} = \frac{\cot x}{1+x^2}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $\frac{2xy}{(1+x^2)}$ and Q = $\frac{\cot x}{1+x^2}$)

Now, I.F. $= e^{\int p dx} = e^{\int \frac{2xy}{(1+x^2)} dx} = e^{\log(1+x^2)} = 1 + x^2$

Thus, the solution of the given differential equation is given by the relation:

$y(\text{I.F.}) = \int (\text{Q} \times \text{I.F.})\ \text{dx} + \text{C}$

$\Rightarrow y.(1 + x^2) = \int \left[\frac{\cot x}{1+x^2}.(1 + x^2)\right] \text{dx} + \text{C}$

$\Rightarrow y.(1 + x^2) = \int \cot x\ \text{dx} + \text{C}$

$\Rightarrow y(1 + x^2) = \log|\sin x| + \text{C}$

Therefore, the required general solution of the given differential equation is

$y(1 + x^2) = \log|\sin x| + \text{C}$

Question 9: For each of the differential equations given in question, find the general solution:

$$x\frac{dy}{dx}+y-x+xy\cot x=0(x\neq 0)$$

Answer:

It is given that $x\frac{dy}{dx}+y-x+xy\cot x=0$

$\Rightarrow x\frac{dy}{dx}+y\,(1+x\cot x)=x$

$\Rightarrow x\frac{dy}{dx}+\left(\frac{1}{x}+\cot x\right)y=1$

This is equation in the form of $\frac{dy}{dx}+py=Q$ (where, p = $\frac{1}{x}$ and Q = 1)

Now, I.F. $=e^{\int pdx}=e^{\int\left(\frac{1}{x}+\cot x\right)dy}=e^{\log x+\log(\sin x)}=e^{\log(x\sin x)}=x\sin x$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int(\mathrm{Q}\times\mathrm{I.F.})\,\mathrm{dx}+\mathrm{C}$

$\Rightarrow$ y.$(x\sin x)=\int[1\times x\sin x]\,\mathrm{dx}+\mathrm{C}$

$\Rightarrow$ y.$(x\sin x)=\int[x\sin x]\,\mathrm{dx}+\mathrm{C}$

$\Rightarrow$ y$(x\sin x)=x\int\sin x\,dx-\int\left[\frac{d}{dx}(x).\int\sin x\,dx\right]+C$

$\Rightarrow$ y$(x\sin x)=x(-\cos x)-\int 1.(-\cos x)dx+C$

$\Rightarrow$ y (x sin x) = - x cos x + sin x + C

$\Rightarrow y=\frac{-x\cos x}{x\sin x}+\frac{\sin x}{x\sin x}+\frac{C}{x\sin x}$

$\Rightarrow y=\cot x+\frac{1}{x}+\frac{C}{x\sin x}$

Therefore, the required general solution of the given differential equation is

$y = \cot x + \frac{1}{x} + \frac{C}{x \sin x}$

Question 10: For each of the differential equations given in question, find the general solution:

$(x + y)\frac{dy}{dx} = 1$

Answer:

It is given that $(x + y)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x+y}$

$\Rightarrow \frac{dx}{dy} = x + y$

$\Rightarrow \frac{dx}{dy} - x = y$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = -1 and Q = y)

Now, I.F. $= e^{\int p dy} = e^{\int -dy} = e^{-y}$

Thus, the solution of the given differential equation is given by the relation:

$x(\text{I.F.}) = \int (\text{Q} \times \text{I. F.})\, \text{dy} + \text{C}$

$\Rightarrow xe^{-y} = \int [y. e^{-y}]\, \text{dy} + \text{C}$

$\Rightarrow xe^{-y} = y \int e^{-dy} - \int \left[\frac{d}{dx}(y). \int e^{-y} dy\right] + C$

$\Rightarrow xe^{-y} = y(-e^{-y}) - \int (-e^{-y}) dy + C$

$\Rightarrow xe^{-y} = -ye^{-y} - \int e^{-y} dy + C$

$\Rightarrow xe^{-y} = -ye^{-y} - e^{-y} + C$

$\Rightarrow x = - y - 1 + Ce^{y}$

$\Rightarrow x + y + 1 = Ce^{y}$

Therefore, the required general solution of the given differential equation is

$x + y + 1 = Ce^{y}$.

Question 11: For each of the differential equations given in question, find the general solution:

$y\ dx + (x - y^2)\ dy = 0$

Answer:

It is given that $y\ dx + (x - y^2)\ dy = 0$

$\Rightarrow y\,dx = (y^2 - x)dy$

$\Rightarrow \frac{dx}{dy} = \frac{(y^2 - x)}{y} = y - \frac{x}{y}$

$\Rightarrow \frac{dx}{dy} + \frac{x}{y} = y$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, $p = \frac{1}{y}$ and $Q = y$)

Now, I.F. $= e^{\int p dy} = e^{\int \frac{dy}{y}} = e^{\log y} = y$

Thus, the solution of the given differential equation is given by the relation:

$x(\text{I.F.}) = \int (\text{Q} \times \text{I. F.})\ \text{dy} + \text{C}$

$\Rightarrow x.y = \int [y.y]\ \text{dy} + \text{C}$

$\Rightarrow x.y = \int y^2\ \text{dy} + \text{C}$

$\Rightarrow x.y = \frac{y^3}{3} + C$

$\Rightarrow xy = \frac{y^3}{3} + \frac{C}{y}$

Therefore, the required general solution of the given differential equation is $xy = \frac{y^3}{3} + \frac{C}{y}$

Question 12: For each of the differential equations given in question, find the general solution:

$(x + 3y^2)\frac{dy}{dx} = y(y > 0)$.

Answer:

It is given that $(x + 3y^2)\frac{dy}{dx} = y$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x+3y^2}$

$\Rightarrow \frac{dx}{dy} = \frac{x+3y^2}{y} = \frac{x}{y} + 3y$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $-\frac{1}{y}$ and Q = $3y$)

Now, I.F. $= e^{\int p dy} = e^{\int \frac{dy}{y}} = e^{-\log y} = e^{\log\left(\frac{1}{y}\right)} = \frac{1}{y}$

Thus, the solution of the given differential equation is given by the relation:

x(I.F.) = $\int (\mathrm{Q} \times \mathrm{I.F.})\, \mathrm{dy} + \mathrm{C}$

$\Rightarrow$ x.$\frac{1}{y}$ = $\int \left[3y.\frac{1}{y}\right] \mathrm{dy} + \mathrm{C}$

$\Rightarrow \frac{x}{y} = 3y + C$

$\Rightarrow$ x = $3y^2$ + Cy

Therefore, the required general solution of the given differential equation is x = $3y^2$ + Cy.

Question 13: For each of the differential equations given in question, find a particular solution satisfying the given condition:

$\frac{dy}{dx} + 2\,y\tan x = \sin x; y = 0$ when x = $\frac{\pi}{3}$

Answer:

It is given that $\frac{dy}{dx} + 2\,y\tan x = \sin x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = 2 $\tan x$ and Q =$\sin x$)

Now, I.F. = $e^{\int p dx} = e^{\int 2\tan x dx} = e^{2\log(\sec x)} = e^{\log(\sec^2 x)} = \sec^2 x$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int(\text{Q} \times \text{I. F.})\,\text{dx} + \text{C}$

$\Rightarrow$ y.($\sec^2 x$) = $\int[\sin x\,.\sec^2 x]\,\text{dx} + \text{C}$

$\Rightarrow$ y.($\sec^2 x$) = $\int[\sec x\,.\tan x]\,\text{dx} + \text{C}$

$\Rightarrow y.(\sec^2 x) = \sec x + C$ .... (1)

Now, it is given that y = 0 at x = $\frac{\pi}{3}$

$0 \times \sec^2\frac{\pi}{3} = \sec\frac{\pi}{3} + C$

$\Rightarrow$ 0 = 2 + C

$\Rightarrow$ C = -2

Now, Substituting the value of C = -2 in (1), we get,

$\Rightarrow y.(\sec^2 x) = \sec x - 2$

$\Rightarrow$ y = cos x – 2cos²x

Therefore, the required general solution of the given differential equation is

y = cos x – 2cos²x.

Question 14: For each of the differential equations given in question, find a particular solution satisfying the given condition:

$(1+x^2)\frac{dy}{dx} + 2\,xy = \frac{1}{1+x^2}; y = 0$ when x = 1

Answer:

It is given that $(1+x^2)\frac{dy}{dx} + 2\,xy = \frac{1}{1+x^2}$

$\Rightarrow \frac{dy}{dx} + \frac{2\,xy}{(1+x^2)} = \frac{1}{(1+x^2)^2}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $\frac{2\,x}{(1+x^2)}$ and Q = $\frac{1}{(1+x^2)^2}$)

Now, I.F. = $e^{\int pdx} = e^{\int \frac{2\,x}{(1+x^2)}dx} = e^{\log(1+x^2)} = 1 + x^2$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int (\mathrm{Q} \times \mathrm{I.F.})\,\mathrm{dx} + \mathrm{C}$

$\Rightarrow$ y.$(1+x^2) = \int \left[\frac{1}{(1+x^2)^2}.(1+x^2)\right] \mathrm{dx} + \mathrm{C}$

$\Rightarrow$ y.$(1 + x^2) = \int \frac{1}{(1+x^2)} \mathrm{dx} + \mathrm{C}$

$\Rightarrow y.(1 + x^2) = \tan^{-1} x + C$ ………. (1)

Now, it is given that y = 0 at x = 1

$0 \times \sec^2 \frac{\pi}{3} = \sec \frac{\pi}{3} + C$

$\Rightarrow$ 0 = $\tan^{-1} 1$ + C

$\Rightarrow$ C = $-\frac{\pi}{4}$

Now, Substituting the value of C = $-\frac{\pi}{4}$ in (1), we get,

$\Rightarrow y.(\sec^2 x) = \sec x - 2$

$\Rightarrow y.(1 + x^2) = \tan^{-1} x - \frac{\pi}{4}$

Therefore, the required general solution of the given differential equation is

$y.(1 + x^2) = \tan^{-1} x - \frac{\pi}{4}$

Question 15: For each of the differential equations given in question, find a particular solution satisfying the given condition:

$\frac{dy}{dx} - 3\, y \cot x = \sin 2x\, ; y = 2$ when x = $\frac{\pi}{2}$

Answer:

It is given that $\frac{dy}{dx} - 3\, y \cot x = \sin 2x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $-3 \cot x$ and Q = $\sin 2x$)

Now, I.F. = $e^{\int p dx} = e^{-3 \int \cot x dx} = e^{-3 \log|\sin x|} = e^{\log\left|\frac{1}{\sin^3 x}\right|} = \frac{1}{\sin^3 x}$

Thus, the solution of the given differential equation is given by the relation:

$y(\text{I.F.}) = \int (Q \times \text{I.F.})\, dx + C$

$\Rightarrow y.\frac{1}{\sin^3 x} = \int \left[\sin 2x \,.\, \frac{1}{\sin^3 x}\right] dx + C$

$\Rightarrow y.\text{cosec}^3 x = 2 = \int (\cot x \,\text{cosec}\, x)\, dx + C$

$\Rightarrow y \,\text{cosec}^3 x = 2\text{cosec}x + C$

$\Rightarrow y = -\frac{2}{\text{cosec}^2 x} + \frac{3}{\text{cosec}^3 x}$

$\Rightarrow y = -2 \sin^2 x + C \sin^3 x$ ------------(1)

Now, it is given that y = 2 when $x = \frac{\pi}{2}$

Thus, we get,

2 = -2 + C

⇒ C = 4

Now, Substituting the value of C = 4 in (1), we get,

$y = -2\sin^2 x + 4\sin^3 x$

$\Rightarrow y = 4\sin^3 x - 2\sin^2 x$

Therefore, the required general solution of the given differential equation is

$y = 4\sin^3 x - 2\sin^2 x$.

Question 16: Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

Answer:

Let F (x, y) be the curve passing through origin and let (x, y) be a point on the curve.

We know the slope of the tangent to the curve at (x, y) is $\frac{dy}{dx}$.

According to the given conditions, we get,

$\frac{dy}{dx} = x + y$

$\Rightarrow \frac{dy}{dx} - y = x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = -1 and Q = x)

Now, I.F. =$e^{\int pdx} = e^{\int(-1)dx} = e^{-x}$

Thus, the solution of the given differential equation is given by the relation:

y(I.F.) = $\int(\mathrm{Q} \times \mathrm{I.F.})\,\mathrm{dy} + \mathrm{C}$

$\Rightarrow$ ye^{-x} = xe^{-x}dx + C .... (1)

Now, $\int xe^{-x}dx = x\int e^{-x}dx - \int\left[\frac{d}{dx}(x).\int e^{-x}dx\right]dx$

= $x(e^{-x}) - \int(-e^{-x})dx$

= $x(e^{-x}) - (-e^{-x})$

= $-e^{-x}(x+1)$

Thus, from equation (1), we get,

$\Rightarrow$ ye^{-x} = $-e^{-x}(x+1) + \mathrm{C}$

$\Rightarrow$ y = -(x+1) + Ce^{x}

$\Rightarrow$ x + y + 1 = Ce^{x} .... (2)

Now, it is given that curve passes through origin.

Thus, equation (2) becomes:

1 = C

⇒ C = 1

Substituting C = 1 in equation (2), we get,

$x + y - 1 = e^x$

Therefore, the required general solution of the given differential equation is

$x + y - 1 = e^x$

Question 17: Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

Answer:

Let F (x, y) be the curve and let (x, y) be a point on the curve.

We know the slope of the tangent to the curve at (x, y) is $\frac{dy}{dx}$.

According to the given conditions, we get,

$\frac{dy}{dx} + 5 = x + y$

$\Rightarrow \frac{dy}{dx} - y = x - 5$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = -1 and Q =-5)

Now, I.F. $= e^{\int p dx} = e^{\int (-1) dx} = e^{-x}$

Thus, the solution of the given differential equation is given by the relation:

$y(\text{I.F.}) = \int (\text{Q} \times \text{I.F.})\, \text{dy} + \text{C}$

$\Rightarrow ye^{-x} = (x - 5)e^{-x} dx + C$ ………. (1)

Now, $\int (x-5)e^{-x}dx = (x-5)\int e^{-x}dx - \int\left[\frac{d}{dx}(x-5).\int e^{-x}dx\right]dx$

$= (x-5)(e^{-x}) - \int(-e^{-x})dx$

$= (x-5)(e^{-x}) - (-e^{-x})$

$= (4-x) - e^{-x}$

Thus, from equation (1), we get,

$\Rightarrow ye^{-x} = (4-x)e^{-x} + C$

$\Rightarrow y = 4 - x + Ce^{x}$

$\Rightarrow x + y - 4 = Ce^{x}$

Now, it is given that curve passes through (0,2).

Thus, equation (2) becomes:

$0 + 2 - 4 = Ce^{0}$

$\Rightarrow -2 = C$

$\Rightarrow C = -2$

Substituting C = -2 in equation (2), we get,

$x + y - 4 = -2e^{x}$

$\Rightarrow y = 4 - x - 2e^{x}$

Therefore, the required general solution of the given differential equation is

$y = 4 - x - 2e^{x}$

Question 18: The Integrating Factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

A. e^{-x} B. e^{-y}

C. 1/x D. x

Answer:

It is given that $\frac{dy}{dx} - y = 2x^2$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $-\frac{1}{x}$ and Q = 2x)

Now, I.F. = $e^{\int p dx} = e^{\int -\frac{1}{x} dx} = e^{\log(x^{-1})} = x^{-1} = \frac{1}{x}$

Question 19: The Integrating Factor of the differential equation $(1 - y^2)\frac{dy}{dx} + yx = ay(-1 < y < 1)$is

A. $\frac{1}{y^2-1}$ B. $\frac{1}{\sqrt{y^2-1}}$

C. $\frac{1}{1-y^2}$ D. $\frac{1}{\sqrt{1-y^2}}$

Answer:

It is given that $(1 - y^2)\frac{dy}{dx} + yx = ay$

$\Rightarrow \frac{dy}{dx} - \frac{yx}{1-y^2} = \frac{ay}{1-y^2}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = $\frac{y}{1-y^2}$ and Q = $\frac{a}{1-y^2}$)

Now, I.F. = $e^{\int p dy} = e^{\int \frac{y}{1-y^2} dy} = e^{\frac{1}{2}\log(1-y^2)} = e^{\log\left[\frac{1}{\sqrt{1-y^2}}\right]}$

$= \frac{1}{\sqrt{1-y^2}}$