01

Electric Charges and Fields

1.2	Electric Charges	1.9	Electric Field Lines
1.3	Conductors and Insulators	1.10	Electric Flux
1.4	Charging by Induction	1.11	Electric Dipole
1.5	Basic Properties of Electric Charge	1.12	Dipole in a Uniform External Field
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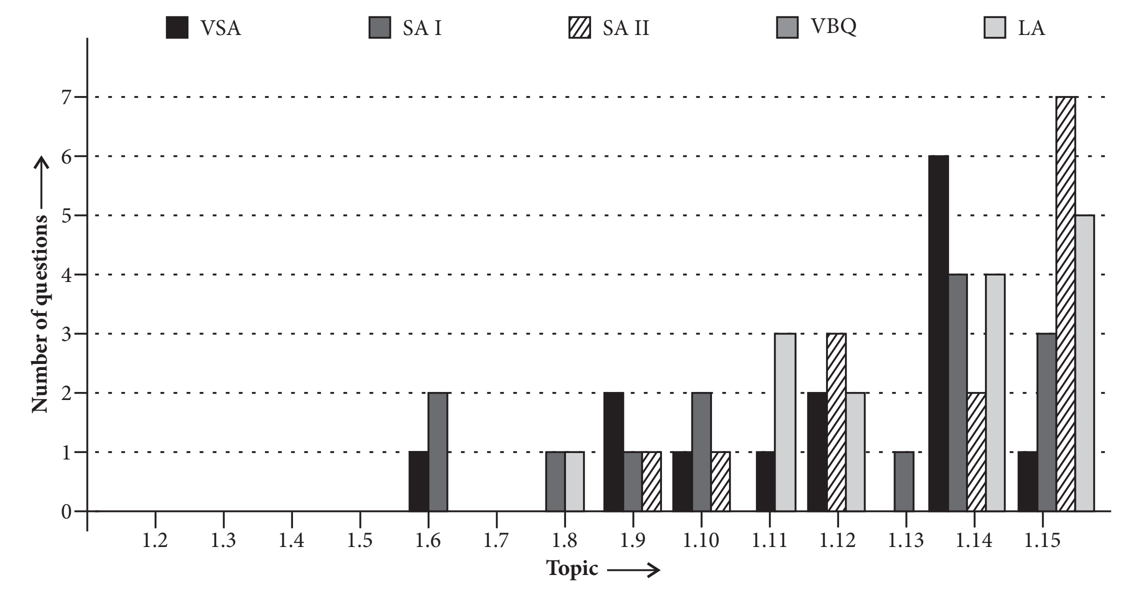
1.7	Forces between Multiple Charges	1.14	Gauss's Law
1.8	Electric Field	1.15	Applications of Gauss's Law

Topicwise Analysis of Last 10 Years' CBSE Board Questions



- Maximum weightage is of *Applications of Gauss's Law.*
- Maximum VSA type questions were asked from *Gauss's Law.*
- Maximum SA II and LA type questions were asked from *Applications of Gauss's Law.*
- No VBQ type questions were asked till now.

QUICK RECAP

- **Charge :** Electric charge is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects.
- **Quantization :** Charge is always in the form of an integral multiple of electronic charge and never its fraction.
  $q = \pm ne$
  where n is an integer and $e = 1.6 \times 10^{-19}$ C.
- Millikan's oil drop experiment showed the discrete nature of charge. Charge cannot be fractional multiple of e.
- **Conservation of charge :** Net charge of an isolated physical system always remains constant. Charge can neither be created nor destroyed. It can be transferred from one body to another.
- Electric charge is additive, *i.e.*, total charge is the algebraic sum of the individual charges.
- Electric charge is invariant as it does not depend upon the motion of the charged body or the observer.

- **Coulomb's inverse square law :** It states that the electrostatic force of attraction or repulsion acting between two stationary point charges is given by

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

where F denotes the force between two charges q_1 and q_2 separated by a distance r in free space. ε_0 is a constant known as permittivity of free space. Free space is vacuum and may be taken to be air practically.

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ \frac{\text{N m}^2}{\text{C}^2}$$

- If free space is replaced by a medium, then ε_0 is replaced by $(\varepsilon_0 K)$ or $(\varepsilon_0 \varepsilon_r)$ where K is known as dielectric constant or relative permittivity.

$$\vec{F} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 \varepsilon_r}\frac{q_1 q_2}{r^2}$$

$K = \dfrac{\varepsilon}{\varepsilon_0}$ or $\varepsilon_r = \dfrac{\varepsilon}{\varepsilon_0}$

$K = 1$ for vacuum (or air) and $K = \infty$ for conductor/metal.

$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$ and its dimensional formula is $[\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]$.

Vector form of the law (q_1 and q_2 are like charges)

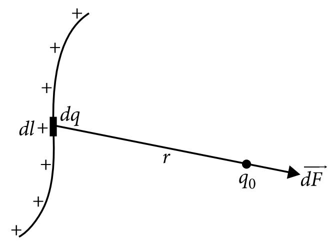
(i) $$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^3}\vec{r}_{21}$$

(ii) $$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r_{12}^3}\vec{r}_{12}$$

- **Electrostatic force due to continuous charge distribution**

The region in which charges are closely spaced is said to have continuous distribution of charge. It is of three types :

- **Linear charge distribution**

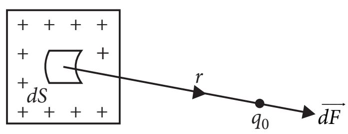


$dq = \lambda\, dl$
where, λ = linear charge density

$$\overrightarrow{dF} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0(dq)}{|r|^2}\hat{r} \Rightarrow \overrightarrow{dF} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0(\lambda\, dl)}{|r|^2}\hat{r}$$

Net force on charge q_0, $\vec{F} = \dfrac{q_0}{4\pi\varepsilon_0}\displaystyle\int_l \frac{\lambda\, dl}{|r|^2}\hat{r}$

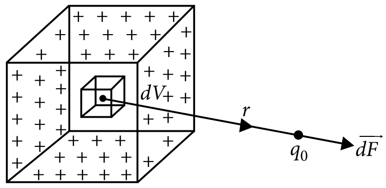
- **Surface charge distribution**



$dq = \sigma\, dS$
where, σ = surface charge density

Net force on charge q_0, $\vec{F} = \dfrac{q_0}{4\pi\varepsilon_0}\displaystyle\int_S \frac{\sigma\, dS}{|r|^2}\hat{r}$

- **Volume charge distribution**



$dq = \rho\, dV$

where, ρ = volume charge density

Net force on charge q_0, $\vec{F} = \dfrac{q_0}{4\pi\varepsilon_0}\displaystyle\int_V \dfrac{\rho dV}{|r|^2}\hat{r}$

Electric field intensity : The electric field intensity at any point due to source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge. It is expressed as

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

Here, $q_0 \to 0$, *i.e.*, the test charge q_0 must be small, so that it does not produce its own electric field.

SI unit of electric field intensity (E) is N/C and it is a vector quantity.

- **Electric field intensity due to a point charge**

Electric field intensity at P is,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{|r|^2}\hat{r}$$

The magnitude of the electric field at a point P is given by

$$|E| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

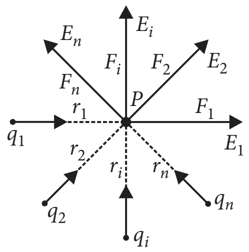
If $q > 0$, *i.e.*, positive charge, then E is directed away from source charge. On the other hand if $q < 0$, *i.e.*, negative charge, then E is directed towards the source charge.

$$E \propto \frac{1}{r^2}$$

- **Electric field due to a system of charges**

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots\ldots\ldots + \vec{E}_n$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{|r|^2}\hat{r}_i$$



A system of charges

Electric dipole : Two equal and opposite charges (q) each, separated by a small distance ($2l$) constitute an electric dipole. Many of the atoms/molecules are dipoles.

(i) Electric dipole moment, $\vec{p} = 2q\vec{l}$.

(ii) Dipole moment is a vector quantity and is directed from negative to positive charge.

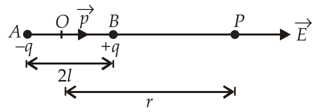
(iii) Unit of dipole moment is coulomb metre (Cm).

(iv) Dimension of dipole moment = [ATL]

- **Intensity of electric field due to a dipole**

– Along axis at distance r from centre of dipole

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

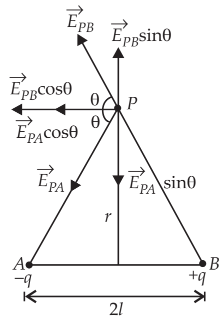


Direction of E is along the direction of dipole moment.

– Along equator of dipole at distance r from centre

$$\vec{E} = \frac{-1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$$

Direction of E is antiparallel to direction of p.



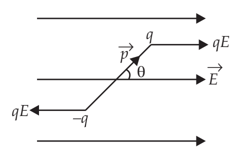
– At any point along direction θ

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1 + 3\cos^2\theta}$$

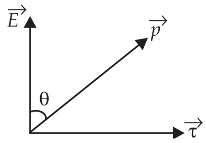
The direction of E makes an angle β with the line joining the point with centre of dipole where $\tan\beta = \dfrac{1}{2}\tan\theta$.

Electric dipole in a uniform electric field

- The resultant electric force on dipole $= qE - qE = 0$

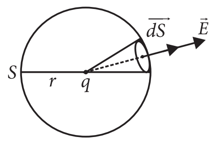


- Two forces [qE and ($-qE$)] equal and opposite, separated by a distance constitute a couple (torque).
- Torque on a dipole $= pE \sin\theta$ numerically.
  Vectorially,
  Torque $(\vec{\tau}) = \vec{p} \times \vec{E}$

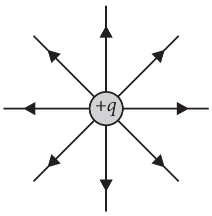


- The direction of τ is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$.
- The torque tends to align the dipole in the direction of field.
- Torque is maximum when $\theta = 90°$ *i.e.*, dipole is perpendicular to E.
  $\therefore$ Maximum torque $= pE$.
  When $\theta = 0°$ or $180°$ then $\tau_{min} = 0$.
- When dipole is parallel to electric field, it is in stable equilibrium. When it is antiparallel to electric field, it is in unstable equilibrium.

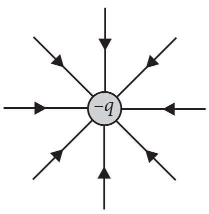
Gauss's law : For a closed surface enclosing a net charge q, the net electric flux ϕ emerging out is given by $\phi = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$



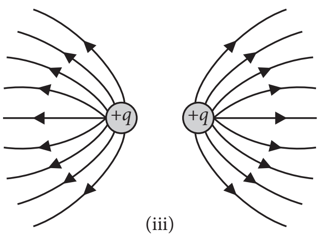
- If a dipole is enclosed by a closed surface, flux ϕ is equal to zero.
  Here the algebraic sum of charges $(+q - q = 0)$ is zero.
- The electric field lines due to positive and negative charges and their combinations are :



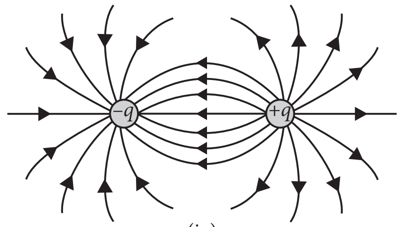
(i)



(ii)



(iii)



(iv)

- Flux from a cube
  (i) If q is at the centre of cube, total flux $(\phi) = \frac{q}{\varepsilon_0}$
  (ii) From each face of cube, flux $= \frac{q}{6\varepsilon_0}$
- Electric field due to a thin, infinitely long straight wire of uniform linear charge density λ,
  $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}$, where r is the perpendicular distance of the observation point from the wire.
- Electric field due to uniformly charged thin spherical shell of uniform surface charge density σ and radius R at a point distant r from the centre of the shell is given as follows :
  At a point outside the shell *i.e.*, $r > R$,
  $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
  At a point on the shell *i.e.*, $r = R$, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$
  At a point inside the shell *i.e.*, $r < R$, $\vec{E} = 0$
  Here, $q = 4\pi R^2\sigma$
- Electric field due to a thin non conducting infinite sheet of charge with uniform surface charge density σ is $\vec{E} = \frac{\sigma}{2\varepsilon_0}$
- Electric field between two infinite thin plane parallel sheets of uniform surface charge density σ and $-\sigma$ is $\vec{E} = \sigma/\varepsilon_0$.
- Gaussian surface
  – For a sphere or spherical shell : A concentric sphere.
  – For cylinder or an infinite rod : A coaxial cylinder.
  – For a plate : A cube or a cuboid.

Previous Years' CBSE Board Questions

1.6 Coulomb's Law

VSA (1 mark)

1. Two equal balls having equal positive charge 'q' coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two?

(AI 2014)

SA I (2 marks)

2. Plot a graph showing the variation of coulomb force (F) versus $\left(\frac{1}{r^2}\right)$, where r is the distance between the two charges of each pair of charges : (1μC, 2μC) and (2μC, – 3μC), interpret the graphs obtained.

(AI 2011)

3. An infinite number of charges, each of q coulomb, are placed along x-axis at x = 1m, 3m, 9m and so on. Calculate the electric field at the point x = 0, due to these charges if all the charges are of the same sign.

(Delhi 2009)

1.8 Electric Field

SA I (2 marks)

4. A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge. *(Delhi 2007)*

LA (5 marks)

5. Consider a system of n charges q_1, q_2 .... q_n with position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, ... \vec{r}_n$ relative to some origin 'O'. Deduce the expression for the net electric field $\vec{E}$ at a point P with position vector $\vec{r}_p$ due to this system of charges. *(3/5, Foreign 2015)*

1.9 Electric Field Lines

VSA (1 mark)

6. Why do the electrostatic field lines not form closed loops? *(AI 2014, AI 2012C)*

7. Why do the electric field lines never cross each other? *(AI 2014)*

SA I (2 marks)

8. The electric field $\vec{E}$ due to a point charge at any point near it is defined as $\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$, where q is the test charge and $\vec{F}$ is the force acting on it. What is the physical significance of $\lim_{q \to 0}$ in this expression? Draw the electric field lines of a point charge Q when (i) $Q > 0$ and (ii) $Q < 0$. *(Delhi 2007)*

SA II (3 marks)

9. A point charge (+Q) is kept in the vicinity of an uncharged conducting plate. Sketch the electric field lines between the charge and the plate. *(1/3, Foreign 2014)*

1.10 Electric Flux

VSA (1 mark)

10. Write an expression for the flux $\Delta\phi$, of the electric field $\vec{E}$ through an area element $\Delta\vec{S}$. *(Delhi 2010C)*

SA I (2 marks)

11. (i) Define the term 'electric flux'. Write its SI unit.

    (ii) What is the flux due to electric field $\vec{E} = 3 \times 10^3 \hat{i}$ N/C through a square of side 10 cm, when it is held normal to $\vec{E}$? *(AI 2015C)*

12. Given a uniform electric field $\vec{E} = 5 \times 10^3 \hat{i}$ N/C. Find the flux of this field through a square of 10 cm on a side whose plane is parallel to the

y-z plane. What would be the flux through the same square if the plane makes a 30° angle with the x-axis? *(Delhi 2014)*

SA II (3 marks)

13. Consider a uniform electric field $\vec{E} = 3\times10^3\,\hat{i}$ N/C. Calculate the flux of this field through a square surface of area 10 cm^2 when
(i) its plane is parallel to the y-z plane
(ii) the normal to its plane makes a 60° angle with the x-axis. *(Delhi 2013C)*

1.11 Electric Dipole

VSA (1 mark)

14. Define the term electric dipole moment of a dipole. State its S.I. unit. *(Foreign 2013, AI 2011)*

LA (5 marks)

15. An electric dipole of dipole moment $\vec{p}$ consists of point charges $+q$ and $-q$ separated by a distance $2a$ apart. Deduce the expression for the electric field $\vec{E}$ due to the dipole at a distance x from the centre of the dipole on its axial line in terms of the dipole moment $\vec{p}$. Hence show that in the limit $x >> a$, $\vec{E} \longrightarrow 2\,\vec{p}/(4\pi\varepsilon_0 x^3)$. *(3/5, Delhi 2015)*

16. Find the resultant electric field due to an electric dipole of dipole moment $2aq$ ($2a$ being the separation between the charges $\pm\, q$) at a point distance x on its equator. *(2/5, Foreign 2015)*

17. Define electric dipole moment. Is it a scalar or a vector quantity? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole. *(3/5, AI 2013)*

1.12 Dipole in a Uniform External Field

VSA (1 mark)

18. Write the expression for the torque $\vec{\tau}$ acting on a dipole of dipole moment $\vec{p}$ placed in an electric field $\vec{E}$. *(Foreign 2015)*

19. At what position is the electric dipole in uniform electric field in its most stable equilibrium position? *(AI 2008)*

SA II (3 marks)

20. An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. Obtain the expression for the torque $\vec{\tau}$ experienced by the dipole. Identify two pairs of perpendicular vectors in the expression. *(Delhi 2015C)*

21. An electric dipole is kept in a uniform electric field. Derive an expression for the net torque acting on it and write its direction. State the conditions under which the dipole is in (i) stable equilibrium and (ii) unstable equilibrium. *(Delhi 2012C)*

22. Derive an expression for the torque experienced by an electric dipole kept in a uniform electric field. *(Delhi 2008)*

LA (5 marks)

23. (a) Define torque acting on a dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Express it in the vector form and point out the direction along which it acts.
(b) What happens if the field is non-uniform?
(c) What would happen if the external field $\vec{E}$ is increasing (i) parallel to $\vec{p}$ and (ii) anti-parallel to $\vec{p}$? *(Foreign 2016)*

24. Deduce the expression for the torque acting on a dipole of dipole moment $\vec{p}$ in the presence of a uniform electric field $\vec{E}$. *(3/5, AI 2014)*

1.13 Continuous Charge Distribution

SA I (2 marks)

25. Deduce the expression for the electric field $\vec{E}$ due to a system of two charges q_1 and q_2 with position vectors $\vec{r}_1$ and $\vec{r}_2$ at a point $\vec{r}$ with respect to the common origin O. *(Delhi 2010C)*

1.14 Gauss's Law

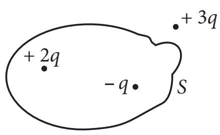
VSA (1 mark)

26. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased ? *(Delhi 2016)*

27. What is the electric flux through a cube of side 1 cm which encloses an electric dipole? *(Delhi 2015)*

28. A charge 'q' is placed at the centre of a cube of side l. What is the electric flux passing through each face of the cube? *(AI 2012)*

29. Figure shows three point charges, $+2q$, $-q$, $+3q$. Two charges $+2q$ and $-q$ are enclosed within a surface 'S'. What is the electric flux due to this configuration through the surface 'S' ? *(Delhi 2010)*



30. A charge Q μC is placed at the centre of a cube. What is the electric flux coming out from any one surface ? *(AI 2010)*

31. If the radius of the Gaussian surface enclosing a charge is halved, how does the electric flux through the Gaussian surface change? *(AI 2008)*

SA I (2 marks)

32. Show that the electric field at the surface of a charged conductor is given by $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$, where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction. *(AI 2010)*

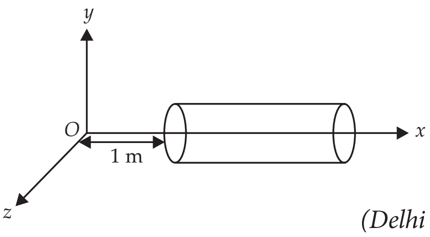
33. Define electric flux. Write its S.I. unit. A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change? *(AI 2009)*

34. A sphere S_1 of radius r_1 encloses a charge Q, if there is another concentric sphere S_2 of radius $r_2 (r_2 > r_1)$ and there are no additional charges between S_1 and S_2. Find the ratio of electric flux through S_1 and S_2. *(AI 2009)*

35. Define electric flux. Write its S.I. unit. A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blown up and increases in size, how does the total electric flux coming out of the surface change? Give reason. *(Delhi 2007)*

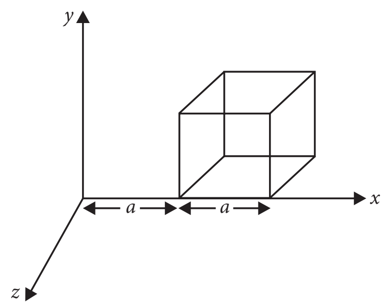
SA II (3 marks)

36. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50x\hat{i}$, where E is in N C^{-1} and x is in metres. Find
(i) net flux through the cylinder.
(ii) charge enclosed by the cylinder.



(Delhi 2013)

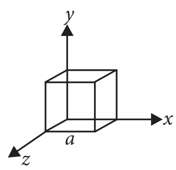
37. State Gauss's law in electrostatic. A cube with each side 'a' is kept in an electric field given by $\vec{E} = Cx\hat{i}$, (as is shown in the figure) where C is a positive dimensional constant. Find out



(i) the electric flux through the cube
(ii) the net charge inside the cube. *(Foreign 2012)*

LA (5 marks)

38. Given the electric field in the region $\vec{E} = 2x\hat{i}$, find the electric flux through the cube and the charge enclosed by it.



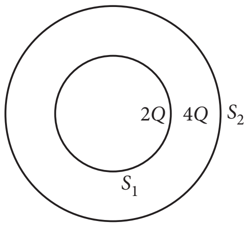
(2/5, Delhi 2015)

39. Define electric flux. Write its S.I. unit.
"Gauss's law in electrostatics is true for any closed surface, no matter what its shape or size is". Justify this statement with the help of a suitable example. *(AI 2015)*

40. Consider two hollow concentric spheres S_1 and S_2, enclosing charges $2Q$ and $4Q$ respectively as shown in figure.

(i) Find out the ratio of the electric flux through them.

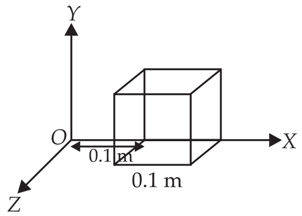
(ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r' is introduced in the space inside S_1 in place of air? Deduce the necessary expression.



(AI 2014)

41. (a) Define electric flux. Write its SI units.

(b) The electric field components due to a charge inside the cube of side 0.1m are as shown: $E_x = \alpha\, x$, where $\alpha = 500$ N/C-m
$E_y = 0$, $E_z = 0$.
Calculate (i) the flux through the cube, and (ii) the charge inside the cube.



(AI 2008)

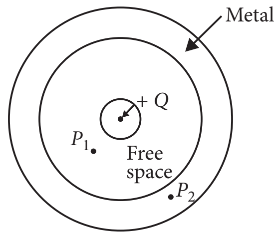
1.15 Applications of Gauss's Law

VSA (1 mark)

42. Two charges of magnitudes $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius '$3a$' with its centre at the origin? *(AI 2013)*

SA I (2 marks)

43. A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure. Use Gauss's law to find the expressions for the electric field at points P_1 and P_2.



(AI 2014)

44. Two concentric metallic spherical shells of radii R and $2R$ are given charges Q_1 and Q_2 respectively. The surface charge densities on the outer surfaces of the shells are equal. Determine the ratio $Q_1 : Q_2$. *(Foreign 2013)*

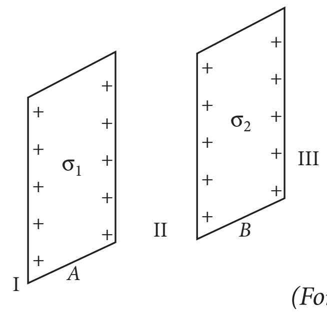
45. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q. A charge q is placed at the centre of the shell.

(a) What is the surface charge density on the (i) inner surface, (ii) outer surface of the shell?

(b) Write the expression for the electric field at a point $x > r_2$ from the centre of the shell. *(AI 2010)*

SA II (3 marks)

46. Two infinitely large plane thin parallel sheets having surface charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are shown in the figure. Write the magnitudes and directions of the net fields in the regions marked II and III.



(Foreign 2014)

47. (i) State Gauss's law.

(ii) A thin straight infinitely long conducting wire of linear charge density 'λ' is enclosed by a cylindrical surface of radius 'r' and length 'l'. Its axis coinciding with the length of the wire. Obtain the expression for the electric field, indicating its direction, at a point on the surface of the cylinder. *(Delhi 2012C)*

48. Using Gauss's law obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for $r > R$ and $r < R$. *(Delhi 2011, 2009)*

49. State Gauss's law in electrostatics. Using this law derive an expression for the electric field due to a uniformly charged infinite plane sheet. *(Delhi 2009)*

50. State Gauss's law in electrostatics. Use this law to derive an expression for the electric field due to an infinitely long straight wire of linear charge density λ Cm^{-1}. *(Delhi 2009)*

51. A positive point charge ($+q$) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point on to the surface of the plate. Derive the expression for the electric field at the surface of a charged conductor. *(AI 2009)*

52. Use Gauss's law to derive the expression for the electric field between two uniformly charged large parallel sheets with surface charge densities $+\sigma$ and $-\sigma$ respectively. *(AI 2009)*

LA (5 marks)

53. Use Gauss's law to find the electric field due to a uniformly charged infinite plane sheet. What is the direction of field for positive and negative charge densities? *(AI 2016)*

54. Use Gauss's law to prove that the electric field inside a uniformly charged spherical shell is zero. *(AI 2015)*

55. (a) A small conducting sphere of radius 'r' carrying a charge $+q$ is surrounded by a large concentric conducting shell of radius R on which a charge $+Q$ is placed. Using Gauss's law derive the expressions for the electric field at a point 'x'

(i) between the sphere and the shell ($r < x < R$).

(ii) outside the spherical shell.

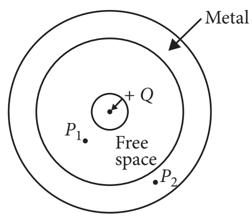
(3/5, Foreign 2015)

56. Using Gauss' law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point (i) outside and (ii) inside the shell. Plot a graph showing variation of electric field as a function of $r > R$ and $r < R$. (r being the distance from the centre of the shell). *(AI 2013)*

57. Using Gauss's law, derive the expression for the electric field at a point (i) outside and (ii) inside a uniformly charged thin spherical shell. Draw a graph showing electric field $\vec{E}$ as a function of distance from the centre. *(AI 2013C)*

58. (i) Define electric flux. Write its S.I. unit.

(ii) A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure. Use Gauss's law to find the expressions for the electric field at points P_1 and P_2.



(iii) Draw the pattern of electric field lines in this arrangement. *(AI 2012C)*

59. (a) Using Gauss's law, derive an expression for the electric field intensity at any point outside a uniformly charged thin spherical shell of radius R and charge density σ C/m^2. Draw the field lines when the charge density of the sphere is (i) positive, (ii) negative.

(b) A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100 μC/m^2. Calculate the

(i) charge on the sphere.

(ii) total electric flux passing through the sphere. *(Delhi 2008)*

Detailed Solutions

1. As in air, $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

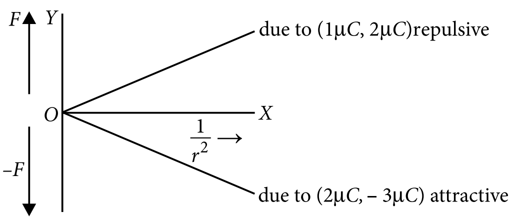
In medium, $F' = \frac{1}{4\pi\varepsilon_0 K}\frac{q^2}{r^2}$

$\therefore \quad F' = \frac{F}{K}$

where K is dielectric constant of material and $K > 1$ for insulators.

Hence, the force is reduced, when a plastic sheet is inserted.

2.



(i) Pair (1mC, 2mC) : From upper graph it is clear that the force of repulsion increases with the reducing distance between two charges.

(ii) Pair (2μC, –3μC) : From lower graph it is clear that the force of attraction increases as the distance between two charges reduces.

3. Here, r_1 = 1m, r_2 = 3m, r_3 = 9m

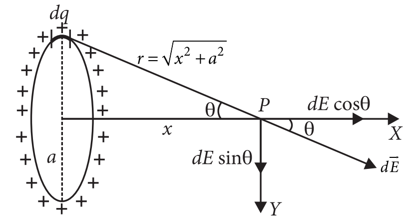
Electric .field,

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(1)^2}+\frac{1}{(3)^2}+\frac{1}{(9)^2}+......+\infty\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-\frac{1}{9}}\right] = \frac{q}{4\pi\varepsilon_0}\times\frac{9}{8} \qquad \left\{\text{using } S_\infty = \frac{a}{1-r}\right\}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{9q}{8}\text{NC}^{-1}$$

4. Suppose total charge on ring of radius a is q. Charge q is uniformly distributed. We want to find electric field at point P on the axis of the charged ring. Consider a small element of the ring carrying charge dq. Electric field due to this small element is $d\vec{E}$.



$d\vec{E}$ can be resolved into two components as (i) dE cosθ along PX and (ii) dE sinθ along PY.

Due to symmetry of ring all components of electric fields of small elements along y-axis cancel out.

Resultant electric field at point P,

$$E = \int dE\cos\theta$$

Here, $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(x^2+a^2)}$

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+a^2}}$$

$$\therefore \quad E = \int\frac{1}{4\pi\varepsilon_0}\times\frac{dq}{(x^2+a^2)}\times\frac{x}{\sqrt{(x^2+a^2)}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{x}{(x^2+a^2)^{3/2}}\int dq$$

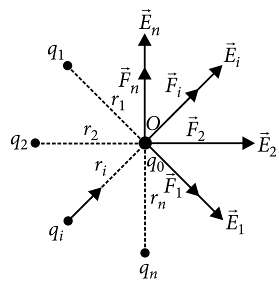
$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2+a^2)^{3/2}}$$

For large x as $x >> a$, so a^2 can be neglected,

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{x^3} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

which is the electric intensity due to a point charge at a distance x. Hence charged ring behaves as a point charge for points at large distances from it.

5. Let us consider a system of n charges q_1, q_2, q_3,... q_n with position vectors r_1, r_2, r_3, ... r_n relative to origin O.



Let $\vec{F}_i$ be the force due to i^{th} charge q_i on q_0

Then,

$$\vec{F}_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_0}{r_i^2}\hat{r}_i$$

Here, r_i is the distance of the test charge q_0 from q_i

The electric field at the observation point P is given by

$$\vec{E}_i = \lim_{q_0 \to 0}\frac{\vec{F}_i}{q_0} = \lim_{q_0 \to 0}\frac{1}{q_0}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i q_0}{r_i^2}\hat{r}_i\right)$$

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\hat{r}_i \qquad \text{...(i)}$$

If $\vec{E}$ is the electric field at point P due to the system of charges, then by the principle of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots.. + \vec{E}_n = \sum_{i=1}^{n}\vec{E}_i$$

Using (i), we get

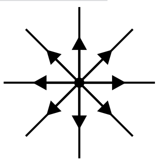
$$\vec{E} = \sum_{i=1}^{n}\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\hat{r}_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

6. Electrostatic field lines do not form closed loops due to conservative nature of electric field.

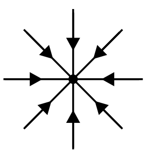
7. At the point of intersection of two field lines, there will be two directions for the resultant electric field. This is not acceptable.

8. $\lim\limits_{q \to 0}$ represents that the charge q in $\vec{E} = \lim\limits_{q \to 0}\frac{\vec{F}}{q}$, is a test charge of infinitely small is magnitude, so that it may not alter the electric field of the source charge.

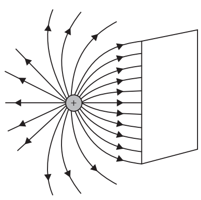
(i) $Q > 0$



(ii) $Q < 0$



9.



10. Electric flux $\Delta\phi = \vec{E}\cdot\Delta\vec{S} = E\,\Delta S\cos\theta$.

11. (i) Electric flux : Total number of electric field lines crossing a surface normally is called electric flux. SI unit of electric flux is N m^2 C^{-1}.

(ii) The area of a surface can be represented as a vector along normal to the surface.

Here $\vec{E} = 3\times10^3\hat{i}$ NC^{-1}

Area of the square $\Delta S = 10 \times 10$ cm^2

$\Delta S = 100$ cm^2 = 10^{-2} m^2

Since normal to the square is along x-axis, we have

$\Delta S = 10^{-2}\,\hat{i}$ m^2

Electric flux through the square,

$\phi = \vec{E}\cdot\Delta\vec{S} = (3\times10^3\hat{i})\cdot(10^{-2}\hat{i})$

$\phi = 30$ N m^2 C^{-1}

12. Here, $\vec{E} = 5\times10^3\hat{i}$ N/C

Side of square = a = 10 cm = 0.1 m

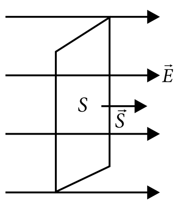
Area of square,

$S = a^2 = (0.1)^2 = 0.01$ m^2

Case I : Area vector is along x-axis,

$\vec{S} = 0.01\hat{i}$ m^2

Required flux, $\phi = \vec{E}\cdot\vec{S}$



$\Rightarrow \phi = (5\times10^3\hat{i})\cdot(0.01\hat{i}) \Rightarrow \phi = 50$ N m^2/C

Case II : Plane of the square makes a 30° angle with the x-axis.

Here, angle between area vector and the electric field is 60°.

So, required flux $\phi' = E\cdot S\cos\theta$

= $(5 \times 10^3)(10^{-2})\cos60° = 25$ N m^2/C

13. Given electric field $\vec{E} = 3 \times 10^3\,\hat{i}$ NC^{-1}

Magnitude of area, $S = 10$ cm^2

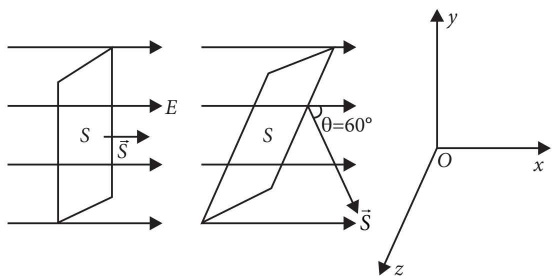
= 1×10^{-3} m^2

(i) When the surface is parallel to y-z plane, the normal to plane is along x-axis.

normal to plane is along x-axis.

In this case $\theta = 0$; so electric flux,

$\phi = \vec{E}\cdot\vec{S} = (3\times10^3\hat{i})\cdot(1\times10^{-3}\hat{i})$ = 3 N m^2 C^{-1}



(ii) In this case $\theta = 60°$, so electric flux,

$\phi = ES\cos\theta$

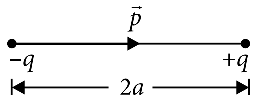
$= 3 \times 10^3 \times 1 \times 10^{-3} \cos 60° = 3 \times \dfrac{1}{2}$

$= 1.5\ \text{Nm}^2\ \text{C}^{-1}$.

14. Strength of an electric dipole is measured by its electric dipole moment, whose magnitude is equal to product of magnitude of either charge and separation between the two charges *i.e.*,

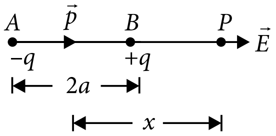
$\vec{p} = q \cdot \overrightarrow{2a}$

and is directed from negative to positive charge, along the line joining the two charges. Its SI unit is Cm.



15. Electric field intensity at any point on axis of electric dipole at distance x from its centre is directed parallel to dipole moment and is given by

$\vec{E} = \vec{E}_{PB} - \vec{E}_{PA}$ $\quad [\because E_{PB} > E_{PA}]$



or $\quad E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(x-a)^2} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(x+a)^2}$

$= \dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{1}{(x-a)^2} - \dfrac{1}{(x+a)^2}\right]$

or $\quad E = \dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{x^2+a^2+2ax-x^2-a^2+2ax}{(x^2-a^2)^2}\right]$

$= \dfrac{1}{4\pi\varepsilon_0}\dfrac{q\cdot 2a \cdot 2x}{(x^2-a^2)^2}$

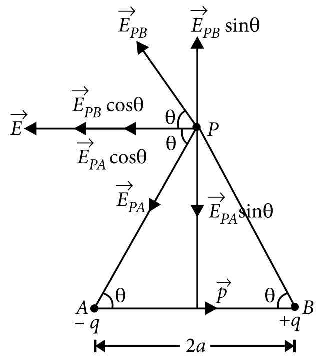
or $\quad \vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\vec{p}x}{(x^2-a^2)^2}$

directed parallel to dipole moment $\vec{p}$. For short dipole, when $x >>> a$, then electric field at point P is

$\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\vec{p}}{x^3}$

16. Consider a dipole consisting of two electric charges $+q$ and $-q$ between a small distance $AB = 2a$ with centre O.

Now, let us find the electric field intensity at point P.



The magnitudes of the electric field at point P due to the two charges $+q$ and $-q$ are given by

$E_{+q} = \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{x^2+a^2}$ $\quad$ ...(i)

and $E_{-q} = \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{x^2+a^2}$ $\quad$ ...(ii)

$\therefore \quad E_{+q} = E_{-q}$

The directions of E_{+q} and E_{-q} are as shown in the figure. The components normal to the dipole axis cancel away. The components along the dipole axis add up.

Therefore, the resultant electric field at point P is given as; $E = -(E_{+q} + E_{-q})\cos\theta$

(The negative sign shows that the field is opposite to the dipole moment of the dipole).

From figure $\cos\theta = \dfrac{a}{(x^2+a^2)^{1/2}}$

$\therefore \quad E = \dfrac{-2qa}{4\pi\varepsilon_0(x^2+a^2)^{3/2}}$ $\quad$ ...(iii)

When point P lies at a large distance ($x >> a$) from the dipole, the above expression reduces to

$E = \dfrac{-2qa}{4\pi\varepsilon_0 x^3}$ $\quad$ ...(iv)

$\because \quad p = q \times 2a$

$\therefore \quad E = \dfrac{-p}{4\pi\varepsilon_0 x^3} (x >> a)$

17. *Refer to answers 14 and 16.*

18. The torque $\vec{\tau}$ acting on a dipole of dipole moment $\vec{p}$ placed in an electric field $\vec{E}$ is given by

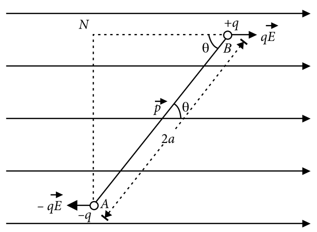
$\vec{\tau} = \vec{p} \times \vec{E}$

or $\quad \tau = pE\sin\theta$

where θ = Angle between dipole moment and electric field.

19. When $\theta = 0°$ between $\vec{p}$ and $\vec{E}$, the dipole is in the most stable equilibrium state in uniform external field.

20. Dipole in a uniform external field



Consider an electric dipole consisting of charges $-q$ and $+q$ and of length $2a$ placed in a uniform electric field $\vec{E}$ making an angle θ with electric field.

Force on charge $-q$ at $A = -q\vec{E}$ (opposite to $\vec{E}$)
Force on charge $+q$ at $B = q\vec{E}$ (along $\vec{E}$)
Electric dipole is under the action of two equal and unlike parallel forces, which give rise to a torque on the dipole.
τ = Force × Perpendicular distance between the two forces
$\tau = qE(AN) = qE(2a\sin\theta)$
$\tau = q(2a)E\sin\theta$
$\tau = pE\sin\theta$

$\vec{\tau} = \vec{p} \times \vec{E}$
Pairs of perpendicular vectors
(a) $(\vec{\tau}, \vec{p})$ (b) $(\vec{\tau}, \vec{E})$

21. *Refer to answer 20.*

(i) When $\theta = 0$; $\tau = 0$ and $\vec{p}$ and $\vec{E}$ are parallel and the dipole is in a position of stable equilibrium.

(ii) When $\theta = 180°$, $\tau = 0$ and $\vec{p}$ and $\vec{E}$ are antiparallel and the dipole is in a position of unstable equilibrium.

22. *Refer to answer 20.*

23. (a) Suppose an electric dipole of dipole moment $\vec{p}$ is placed along a direction, making an angle θ with the direction of an external uniform electric field $\vec{E}$. Then, the torque acting on the dipole is defined as $pE\sin\theta$ or $\vec{\tau} = \vec{p} \times \vec{E}$.
Its direction will be perpendicular to both $\vec{p}$ and $\vec{E}$.

(b) If the field is non-uniform there would be a net force on the dipole in addition to the torque and the resulting motion would be a combination of translation and rotation.

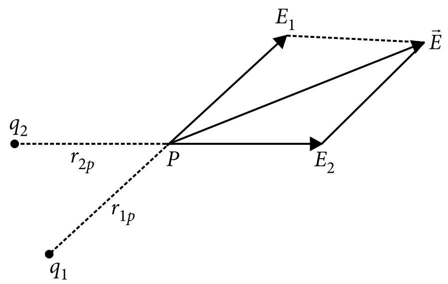
(b) If the field is non-uniform there would be a net force on the dipole in addition to the torque and the resulting motion would be a combination of translation and rotation.

(c) (i) $\vec{E}$ is increasing parallel to $\vec{p}$ then $\theta = 0°$. So torque becomes zero but the net force on the dipole will be in the direction of increasing electric field and hence it will have linear motion along the dipole moment.

(ii) $\vec{E}$ is increasing anti-parallel to $\vec{p}$. So, the torque still remains zero but the net force on the dipole will be in the direction of increasing electric field which is opposite to the dipole moment, hence it will have linear motion opposite to the dipole moment.

24. *Refer to answer 20.*

25. Electric field due to a system of charges :
Consider a system of charges q_1 and q_2 with position vectors $\vec{r}_1$ and $\vec{r}_2$ relative to common origin O. Let P be any point with position vector $\vec{r}$ at which electric field is to be determined.



Electric field $\vec{E}_1$ due to q_1 is given by

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{1p}^2} \hat{r}_{1p}$$

where $\hat{r}_{1p}$ is a unit vector in the direction from q_1 to P and r_{1p} is the distance between q_1 and P.

Similarly, electric field $\vec{E}_2$ due to q_2 is

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_{2p}^2} \cdot \hat{r}_{2p}$$

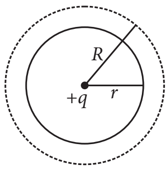
where $\hat{r}_{2p}$ is a unit vector in the direction from q_2 to P and r_{2p} is the distance between q_2 and P.

By the superposition principle, the electric field $\vec{E}$ at $\vec{r}$ due to the system of charges is $\vec{E}(r) = \vec{E}_1(r) + \vec{E}_2(r)$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{1p}^2} \cdot \hat{r}_{1p} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_{2p}^2} \cdot \hat{r}_{2p}$$

$$\therefore \quad \vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{1p}^2}\hat{r}_{1p} + \frac{q_2}{r_{2p}^2}\hat{r}_{2p}\right]$$

26. According to Gauss's law, the electric flux passing through a closed surface is given by



$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

When radius of spherical Gaussian surface is increased, its surface area will be increased but point charge enclosed in the sphere remains same. Hence there will be no change in the electric flux.

27. According to Gauss's law, net flux through a closed surface, $\phi_E = \oint \vec{E}\cdot d\vec{s} = \frac{q_{en}}{\varepsilon_0}$

Total charge enclosed, $q_{en} = 0$

as net charge on dipole is zero.

$\therefore \quad \phi_E = \frac{0}{\varepsilon_0} = 0$

28. By Gauss's theorem, total flux through whole of the cube, $\phi = \frac{q}{\varepsilon_0}$

where, q is the total charge enclosed by the cube. As, charge is at centre, therefore, electric flux is symmetrically distributed on all 6 faces.

Therefore,

Flux through each face of the cube, $\phi' = \frac{\phi}{6} = \frac{q}{6\varepsilon_0}$

29. Total charge within a surface S

$= +2q + (-q) = +q$

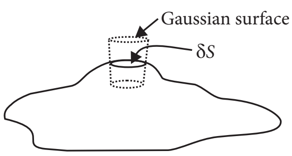
$\therefore \quad$ Electric flux $\phi = \frac{q}{\varepsilon_0}$

30. *Refer to answer 28.*

31. The electric flux remains the same, as the charge enclosed remains the same.

32. Consider an elementary area δS on the surface of the charged conductor.

Enclose this area element with a cylindrical gaussian surface as shown in figure.



Now electric field inside a charged conductor is zero. Therefore, direction of field, just outside δS will be normally outward *i.e.* in direction of $\hat{n}$.

According to Gauss's theorem, total electric flux coming out is

$$\vec{E}\cdot\delta\vec{S} = \frac{\sigma\,\delta S}{\varepsilon_0} \qquad [\vec{E} \text{ is electric field at the surface}]$$

$$\Rightarrow \quad E\,\delta S\cos 0° = \frac{\sigma\,\delta S}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

33. Electric flux linked with a surface is the number of electric lines of force cutting through the surface normally.

It's SI unit is N m^2 C^{-1} or V m. On decreasing the radius of spherical surface to half there will be no effect on the electric flux.

34. By Gauss's theorem, total flux passing through a closed surface, $\phi = \frac{q}{\varepsilon_0}$

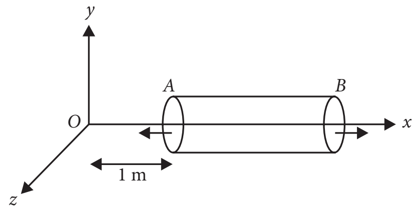
$\therefore \quad \frac{\phi_1}{\phi_2} = \frac{q_1}{\varepsilon_0}\times\frac{\varepsilon_0}{q_2} = \frac{q_1}{q_2} = \frac{Q}{Q} = 1$

$\therefore \quad \phi_1 : \phi_2 = 1 : 1.$

35. *Refer to answer 33.*

The total electric flux linked with surface of balloon remains the same because the charge on its surface remains the same on blowing it up.

36. (i)



Given, $\vec{E} = 50x\hat{i}$ and $A = 25$ cm^2 $= 25 \times 10^{-4}$ m^2

As the electric field is only along the x-axis, so, flux will pass only through the cross-section of cylinder.

Magnitude of electric field at cross-section A,

$E_A = 50 \times 1 = 50$ N C^{-1}

Magnitude of electric field at cross-section B,

$E_B = 50 \times 2 = 100$ N C^{-1}

The corresponding electric fluxes are

$\phi_A = \vec{E}_A\cdot\vec{A} = 50 \times 25 \times 10^{-4} \cos 180°$

$= -0.125$ N m^2 C^{-1}

$\phi_B = \vec{E}_B\cdot\vec{A} = 100 \times 25 \times 10^{-4} \cos 0°$

$= 0.25$ N m^2 C^{-1}

So, the net flux through the cylinder,

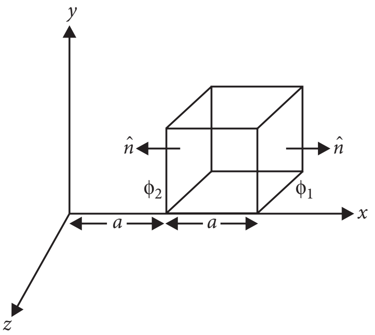
$\phi = \phi_A + \phi_B = -0.125 + 0.25 = 0.125$ N m^2 C^{-1}

(ii) Using Gauss's law

$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0} \Rightarrow 0.125 = \frac{q}{8.85\times10^{-12}}$$

$\Rightarrow \quad q = 8.85 \times 0.125 \times 10^{-12} = 1.1 \times 10^{-12}$ C

37. (i) Gauss's law in electrostatics states that the total electric flux through a closed surface enclosing a charge is equal to $\frac{1}{\varepsilon_0}$ times the magnitude of that charge.



$$\phi = \oint \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

(ii) Net flux, $\phi = \phi_1 + \phi_2$

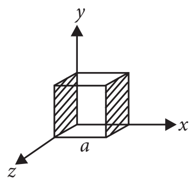
where $\phi_1 = 2aC\, dS \cos 0° = 2aC \times a^2 = 2a^3C$

$\phi_2 = aC \times a^2 \cos 180° = -a^3C$

$\phi = 2a^3C + (-a^3C) = a^3C$ Nm^2 C^{-1}

(iii) Net charge $(q) = \varepsilon_0 \times \phi = a^3C\, \varepsilon_0$ coulomb

38. $\vec{E} = 2x\hat{i}$



So, flux passes through faces of cube which are perpendicular to x-axis.

The magnitude of electric field at the left face $(x = 0)$, $E_L = 0$

The magnitude of electric field at the right face, $(x = a)$, $E_R = 2a$

So, net flux, $\phi = \vec{E} \cdot \Delta\vec{s}$

$= E_L\, \Delta s \cos 180° + E_R \Delta s \cos 0°$

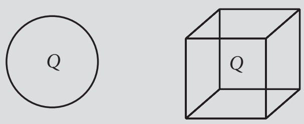
$= 0 + 2a \times a^2 = 2a^3$

Assume enclosed charge is q.

Use Gauss's law, $\phi = \frac{q}{\varepsilon_0}$; $q = \varepsilon_0 \phi$ $\therefore$ $q = 2a^3\varepsilon_0$

39. *Refer to answer 33.*

Let us take a charge Q inside a cube or a sphere.



The flux through both the closed surfaces will be same. *i.e.*, $\phi_{net} = \frac{Q}{\varepsilon_0}$

40. (i) Charge enclosed by sphere $S_1 = 2Q$

By Gauss law, electric flux through sphere S_1 is

$\phi_1 = 2Q\varepsilon_0$

Charge enclosed by sphere,

$S_2 = 2Q + 4Q = 6Q$

$\phi_2 = 6Q\varepsilon_0$

The ratio of the electric flux is

$\phi_1 : \phi_2 = 2Q\varepsilon_0 : 6Q\varepsilon_0 = 2 : 6 = 1 : 3$

(ii) When a medium of dielectric constant ε_r is introduced in sphere S_1, the flux through S_1 would be $\phi_1' = \frac{2Q}{\varepsilon_r}$

41. (a) *Refer to answer 11(i).*

(b) (i) Electric flux linked with cubical surface is

$$\phi = \int E_1 ds \cos 180° + \int E_2 ds \cos 0°$$

or $\phi = -E_1A + E_2A = (-E_1 + E_2)\, A$

or $\phi = [-\alpha x_1 + \alpha x_2]\, A = [-x_1 + x_2]\alpha a^2$

or $\phi = [-0.1 + 0.2] \times 500 \times 0.1^2$

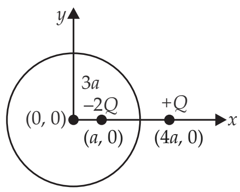
or $\phi = 0.5$ V m

(ii) Net charge inside the cube is

$q = \phi\varepsilon_0 = 0.5 \times 8.85 \times 10^{-12}$

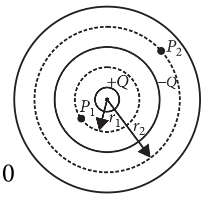
or $q = 4.425 \times 10^{-12}$ C

42.



Electric flux $\phi = \frac{q_{\text{inside}}}{\varepsilon_0} = \frac{-2Q}{\varepsilon_0}$

43. Using Gauss's theorem, electric field at P_1



$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_1^2}$$

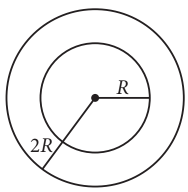
Again field at P_2, $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_2^2} = 0$

Because electric field inside a conductor is zero.

44. Surface charge density, σ = constant

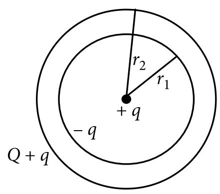
Charge $Q_1 = 4\pi R^2\sigma$

Charge $Q_2 = 4\pi(2R)^2\sigma$



$$\therefore \quad \frac{Q_1}{Q_2} = \frac{4\pi R^2\sigma}{4\pi(2R)^2\sigma} = \frac{1}{4}$$

45.

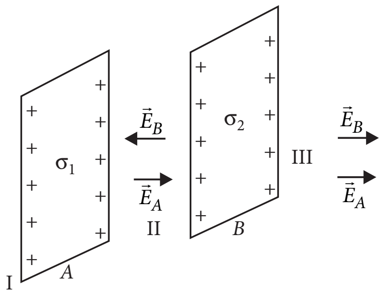


(a) (i) Surface charge density on the inner surface of the shell is $\sigma_{in} = \dfrac{-q}{4\pi r_1^2}$

(ii) Surface charge density on the outer surface of shell is $\sigma_{out} = \dfrac{Q+q}{4\pi r_2^2}$

(b) Using, Gauss's law, $E(x) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{x^2}$

46.



In region II :

The electric field due to the sheet of charge *A* will be from left to right (along the positive direction) and that due to the sheet of charge *B* will be from right to left (along the negative direction). Therefore, in region II, we have

$$E = \frac{\sigma_1}{\varepsilon_0} + \left(-\frac{\sigma_2}{\varepsilon_0}\right)$$

$\Rightarrow \quad \vec{E} = \dfrac{1}{\varepsilon_0}(\sigma_1 - \sigma_2)$ along positive direction

In region III :

The electric fields due to both the charged sheets will be from left to right, *i.e.*, along the positive direction. Therefore, in region III, we have

$$E = \frac{\sigma_1}{\varepsilon_0} + \frac{\sigma_2}{\varepsilon_0}$$

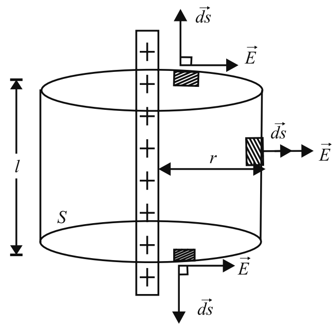
$\Rightarrow \quad \vec{E} = \dfrac{1}{\varepsilon_0}(\sigma_1 + \sigma_2)$ along positive direction

47. (i) According to Gauss's law, total flux over a closed surface *S* in vacuum is $\dfrac{1}{\varepsilon_0}$ times the total charge enclosed by closed surface *S*

$$\phi = \oint_s \vec{E}\cdot\vec{ds} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

(ii) Electric field intensity due to line charge or infinite long uniformly charged wire at point *P* at distance *r* from it is obtained as :

Assume a cylindrical Gaussian surface *S* with charged wire on its axis and point *P* on its surface, then net electric flux through surface *S* is



$$\phi = \oint_s \vec{E}\cdot\vec{ds} = \int_{\substack{\text{upper}\\\text{plane}\\\text{face}}} Eds\cos 90° + \int_{\substack{\text{curved}\\\text{surface}}} Eds\cos 0° + \int_{\substack{\text{lower}\\\text{plane}\\\text{face}}} Eds\cos 90°$$

or $\quad \phi = 0 + EA + 0$ or $\phi = E\cdot 2\pi rl$

But by Gauss's theorem, $\phi = \dfrac{q}{\varepsilon_0} = \dfrac{\lambda l}{\varepsilon_0}$

where q is the charge on length l of wire enclosed by cylindrical surface *S*, and λ is uniform linear charge density of wire.

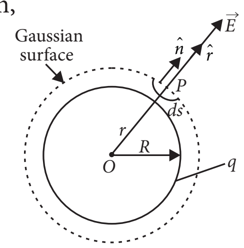
$\therefore \quad E \times 2\pi rl = \dfrac{\lambda l}{\varepsilon_0}$ or $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

directed normal to the surface of charged wire.

48. Consider a thin spherical shell of radius *R* carrying charge *q*. To find the electric field outside the shell, we consider a spherical Gaussian surface of radius *r* (> *R*), concentric with given shell.

The electric field $\vec{E}$ is same at every point of Gaussian surface and directed radially outwards (as is unit vector $\hat{n}$ so that $\theta = 0°$)

According to Gauss's theorem,



$$\oint_s \vec{E}\cdot\vec{ds} = \oint_s \vec{E}\cdot\hat{n}\,ds = \frac{q}{\varepsilon_0}$$

or $\quad E\oint_s ds = \dfrac{q}{\varepsilon_0}$

$\therefore \quad E(4\pi r^2) = \dfrac{q}{\varepsilon_0}$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Vectorially, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

Special cases

(i) At the point on the surface of the shell, $r = R$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

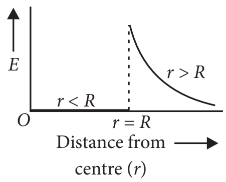
(ii) If σ is the surface charge density on the shell then $q = 4\pi R^2\sigma$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4\pi R^2\sigma}{R^2} = \frac{\sigma}{\varepsilon_0}$$

(iii) If the point P lies inside the spherical shell then the Gaussian surface encloses no charge

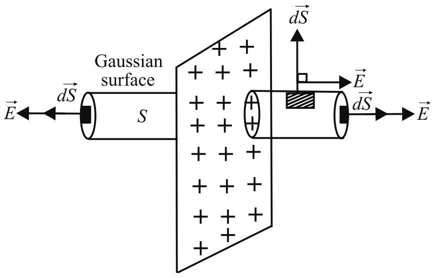
i.e., $r < R$

$\because \quad q = 0$, hence $E = 0$



49. (i) *Refer to answer 47 (i).*

(ii) Assume a cylindrical Gaussian surface S cutting through plane sheet of charge, such that point P lies on its plane face, then net electric flux through surface S is



$$\phi = \oint_S \vec{E}\cdot\vec{ds} = \int_{\substack{\text{left}\\\text{plane}\\\text{face}}} \vec{E}\cdot\vec{ds} + \int_{\substack{\text{curved}\\\text{surface}}} \vec{E}\cdot\vec{ds} + \int_{\substack{\text{right}\\\text{plane}\\\text{face}}} \vec{E}\cdot\vec{ds}$$

$$\text{or} \quad \phi = \int_{\substack{\text{left}\\\text{plane face}}} Eds\cos 0° + \int_{\substack{\text{curved}\\\text{surface}}} Eds\cos 90° + \int_{\substack{\text{right}\\\text{plane face}}} Eds\cos 0°$$

or $\quad \phi = EA + 0 + EA = 2EA$

But by Gauss's theorem $\phi = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$

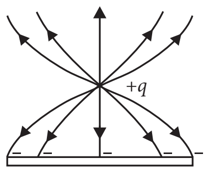
where q is the charge in area A of sheet enclosed by cylindrical surface S and σ is uniform surface charge density of sheet.

$$\therefore \quad 2EA = \frac{\sigma A}{\varepsilon_0} \text{ or } E = \frac{\sigma}{2\varepsilon_0}$$

directed normal to surface of charged sheet (i) away from it, if it is positively charged and (ii) towards it, if it is negatively charged.

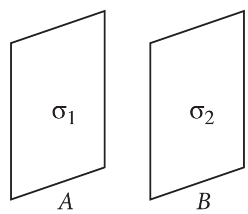
50. *Refer to answer 47.*

51.



Refer to answer 49(ii).

52. Consider two infinite plane parallel sheets of charge A and B. Let $\sigma_1 = +\sigma$ and $\sigma_2 = -\sigma$ be the uniform surface densities of charge on A and B respectively.



The electric field between two plates is given by

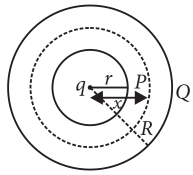
$$E = E_1 - E_2 = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} - \left(\frac{-\sigma}{2\varepsilon_0}\right) = \frac{\sigma+\sigma}{2\varepsilon_0}$$

$$\therefore \quad E = \frac{2\sigma}{2\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

53. *Refer to answer 49.*

54. *Refer to answer 48.*

55. Consider a sphere of radius r with centre O surrounded by a large concentric conducting shell of radius R.



To calculate the electric field intensity at any point P, where $OP = x$, imagine a Gaussian surface with

centre O and radius x, as shown in the figure.
The total electric flux through the Gaussian surface is given by

$$\phi = \oint E ds = E \oint ds$$

Now, $\oint ds = 4\pi x^2$

$$\therefore \quad \phi = E \times 4\pi x^2 \qquad \text{...(i)}$$

Since the charge enclosed by the Gaussian surface is q, according to Gauss's theorem,

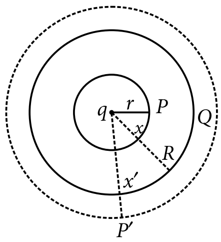
$$\phi = \frac{q}{\varepsilon_0} \qquad \text{...(ii)}$$

From (i) and (ii), we get

$$E \times 4\pi x^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0 x^2}$$

To calculate the electric field intensity at any point P', where point P' lies outside the spherical shell, imagine a Gaussian surface with centre O and radius x', as shown in the figure



According to Gauss's theorem,

$$E'(4\pi x'^2) = \frac{q+Q}{\varepsilon_0}$$

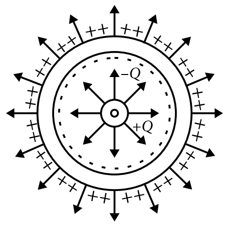
$$\Rightarrow \quad E' = \frac{q+Q}{4\pi\varepsilon_0 x'^2}$$

As the charge always resides only on the outer surface of a conduction shell, the charge flows essentially from the sphere to the shell when they are connected by a wire. It does not depend on the magnitude and sign of charge Q.

56. *Refer to answer 48.*

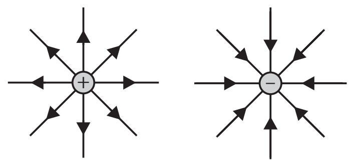
57. *Refer to answer 48.*

58. (i) *Refer to answer 11 (i).*
(ii) *Refer to answer 43.*
(iii) The electric field lines due to the arrangement is as shown



Charges will be uniformly distributed on the whole surfaces hence, all field lines will be uniformly separated.

59. (a) *Refer to answer 48.*
Electric field lines due to positively and negatively charged spherical shells are as given in figures (a) and (b) respectively



(a) (b)

(b) Given $D = 2.5$ m;

$$R = \frac{2.5}{2} = 1.25 \text{ m}, \ \sigma = 100 \ \mu\text{C/m}^2.$$

(i) Charge on the sphere is

$$q = \sigma 4\pi R^2 = 100 \times 10^{-6} \times 4 \times 3.14 \times (1.25)^2$$

or $\quad q = 1.96 \times 10^{-3}$C

(ii) $\phi = \dfrac{q}{\varepsilon_0} = \dfrac{1.96 \times 10^{-3}}{8.85 \times 10^{-12}} = 2.2 \times 10^8$ V-m