EXERCISE

1. A simple harmonic oscillator has an amplitude A and time period T. The time required by it to travel from $x = A$ to $x = A/2$ is
(a) $T/6$ (b) $T/4$ (c) $T/3$ (d) $T/2$

2. A mass $m = 100$ g is attached at the end of a light spring which oscillates on a friction less horizontal table with an amplitude equal to 0.16 m and the time period equal to 2 s. Initially the mass is released from rest at $t = 0$ and displacement $x = -0.16$ m. The expression for the displacement of the mass at any time (t) is
(a) $x = 0.16 \cos(\pi t)$
(b) $x = -0.16 \cos(\pi t)$
(c) $x = 0.16 \cos(\pi t + \pi)$
(d) $x = -0.16 \cos(\pi t + \pi)$

3. The motion of a particle executing SHM is given by $x = 0.10 \sin 100\pi(t + 0.05)$, where x is in meters and time t is in seconds. The time period is
(a) 0.01 s (b) 0.02 s (c) 0.1 s (d) 0.2 s

4. Two equations of two SHM are $x = a \sin(\omega t - \alpha)$ and $y = b \cos(\omega t - \alpha)$. The phase difference between the two is
(a) 0° (b) $\alpha°$ (c) 90° (d) 180°

5. A particle starts SHM from the mean position. Its amplitude is A and time period is T. At the time when its speed is half of the maximum speed. Its displacement y is
(a) $A/2$ (b) $A/\sqrt{2}$ (c) $A\sqrt{3}/2$ (d) $2A/\sqrt{3}$

6. A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude (A) and time period (T). The speed of the pendulum at $x = A/2$ will be
(a) $\frac{\pi A\sqrt{3}}{T}$ (b) $\frac{\pi A}{T}$ (c) $\frac{\pi A\sqrt{3}}{2T}$ (d) $\frac{3\pi^2 A}{T}$

7. When the potential energy of a particle executing simple harmonic motion is one-fourth of the maximum value during the oscillation, its displacement from the equilibrium position in terms of amplitude a is
(a) $a/4$ (b) $a/3$ (c) $a/2$ (d) $2a/3$

8. A particle of mass 10 g is executing SHM with an amplitude of 0.5 m and circular frequency of 10 rad/s. The maximum value of the force acting on the particle during the course of oscillation is
(a) 25 N (b) 5 N (c) 2.5 N (d) 0.5 N

9. Two particles executes SHM of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions. Each time their displacement is half of their amplitude. The phase difference between them is
(a) 30° (b) 60° (c) 90° (d) 120°

10. The total energy of the body executing SHM is E. Then the kinetic energy when the displacement is half of the amplitude is
(a) $E/2$ (b) $E/4$ (c) $3E/4$ (d) $\sqrt{3}E/4$

11. A body executing simple harmonic motion has a maximum acceleration equal to 24 m/s^2 and maximum velocity equal to 16 m/s. The amplitude of simple harmonic motion is
(a) $\frac{32}{3}$m (b) $\frac{3}{32}$m
(c) $\frac{1024}{9}$m (d) $\frac{64}{9}$m

12. The displacement of an oscillating particle varies with time (in seconds) according to the equation, $y(\text{cm}) = \sin\frac{\pi}{2}\left(\frac{t}{2}+\frac{1}{3}\right)$. The maximum acceleration of the particle is, approximately,
(a) 5.21 cm/s^2 (b) 3.62 cm/s^2
(c) 1.81 cm/s^2 (d) 0.62 cm/s^2

13. The kinetic energy and potential energy of a particle executing SHM will be equal, when displacement is (amplitude = a)

(a) $a/2$ (b) $a\sqrt{2}$ (c) $\frac{a}{\sqrt{2}}$ (d) $\frac{a\sqrt{2}}{3}$

14. The phase of a particle executing SHM is $\frac{\pi}{2}$ when it has
(a) Maximum velocity
(b) Maximum acceleration
(c) Maximum energy
(d) Maximum displacement

15. A particle moves such that its acceleration a is given by $a = -bx$, where x is the displacement from equilibrium position and b is a constant. The period of oscillation is
(a) $2\pi\sqrt{b}$ (b) $\frac{2\pi}{\sqrt{b}}$ (c) $\frac{2\pi}{b}$ (d) $2\sqrt{\frac{\pi}{b}}$

16. The equation of motion of a particle is $\frac{d^2y}{dt^2} + ky = 0$, where k is a positive constant.
The time period of the motion is given by
(a) $\frac{2\pi}{k}$ (b) $3\pi k$ (c) $\frac{2\pi}{\sqrt{k}}$ (d) $2\pi\sqrt{k}$

17. A clock which keeps correct time at 20°C, is subjected to 40°C. If coefficient of linear expansion of the pendulum is 12×10^{-6}/°C. How much will it gain or loose in time?
(a) 10.3 s/day (b) 20.6 s/day
(c) 5 s/day (d) 20 min/day

18. The metallic bob of simple pendulum has the relative density ρ. The time period of this pendulum is T. If the metallic *bob* is immersed in water, then the new time period is given by
(a) $T\left(\frac{\rho-1}{\rho}\right)$ (b) $T\left(\frac{\rho}{\rho-1}\right)$
(c) $T\sqrt{\frac{\rho-1}{\rho}}$ (d) $T\sqrt{\frac{\rho}{\rho-1}}$

19. A simple pendulum is executing SHM with a time period T. If the length of the pendulum is increased by 21% the percentage increase in the time period of the pendulum is
(a) 10% (b) 21% (c) 30% (d) 50%

20. A spring of force constant k is cut into two piece such that one pieces is double the length of the other. Then the long piece will have a force constant of
(a) $\frac{2}{3}k$ (b) $\frac{3}{2}k$ (c) $3k$ (d) $6k$

21. Two bodies M and N of equal masses are suspended from two separate mass less springs of force constants $k_1$and k_2, respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of M to that of N is
(a) k_1/k_2 (b) $\sqrt{k_1/k_2}$
(c) k_2/k_1 (d) $\sqrt{k_2/k_1}$

22. Two identical springs of constant k are connected in series and parallel as shown in Fig. 12. A mass m is suspended from them. The ratio of their frequencies of vertical oscillation will be

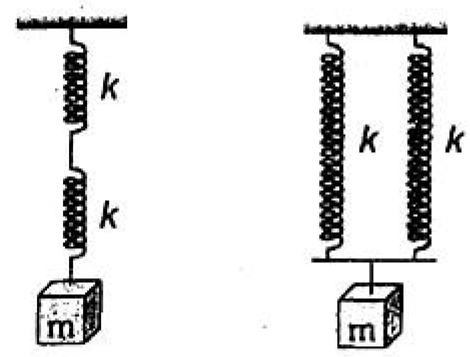

Fig. 12

(a) 2 : 1 (b) 1 : 1 (c) 1 : 2 (d) 4 : 1

23. A block of mass m attached to a spring of spring constant k oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. The block has a speed v when the spring is at its natural length. Before coming to an instantaneous rest, if the block moves a distance x from the mean position, then
(a) $x = \sqrt{\frac{m}{k}}$ (b) $x = \frac{1}{v}\sqrt{\frac{m}{k}}$
(c) $x = v\sqrt{\frac{m}{k}}$ (d) $x = \sqrt{\frac{mv}{k}}$

24. A block is placed on a frictionless horizontal table. The mass of the block is m and springs of force constant k_1, k_2 are attached on either side with if the block is displaced a little and left to oscillate, then the angular frequency of oscillation will be
(a) $\left(\frac{k_1+k_2}{m}\right)^{1/2}$ (b) $\left[\frac{k_1k_2}{m(k_1+k_2)}\right]^{1/2}$
(c) $\left[\frac{k_1k_2}{(k_1-k_2)m}\right]^{1/2}$ (d) $\left[\frac{k_1^2+k_2^2}{(k_1+k_2)m}\right]^{1/2}$

25. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with platform from the first time
(a) at the mean position of platform.
(b) for an amplitude g/ω^2.
(c) for an amplitude of g^2/ω^2.
(d) at the highest position of the platform.

26. Two springs of force constants K_1 and K_2 are connected to a mass m as shown in Fig. 13. The frequency of oscillation of the mass is f. If both K_1 and K_1 are made four times their original values, the frequency of oscillation becomes

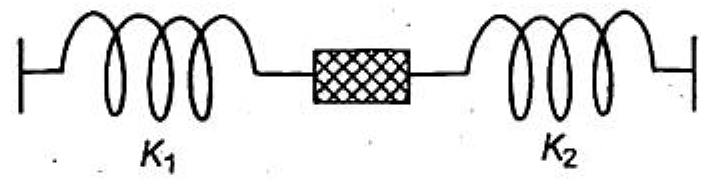

Fig. 13

(a) $f/4$ (b) $4f$ (c) $2f$ (d) $f/2$

27. A particle of mass m executes simple harmonic motion with amplitude a and frequency v. The average kinetic energy during its motion form the position of equilibrium to the end is

(a) $\frac{1}{4}ma^2v^2$ (b) $4\pi^2ma^2v^2$

(c) $2\pi^2 ma^2v^2$ (d) πma^2v^2

28. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x/2 \times 10^{-2} \cos \pi t$ meters. The time at which the maximum speed first occurs is
(a) 0.75 s (b) 0.125 s
(c) 0.25 s (d) 0.5 s

29. A point mass oscillates along the x-axis according to the law $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$. If acceleration of the particle is written as $a = A \cos (\omega t + \delta)$, then

(a) $A = x_0\omega^2, \delta = \frac{\pi}{4}$ (b) $A = x_0\omega^2, \delta = -\frac{\pi}{4}$

(c) $A = x_0\omega^2, \delta = \frac{3\pi}{4}$ (d) $A = x_0, \delta = -\frac{\pi}{4}$

30. If x, v, and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?

(a) $a^2T^2 + 4\pi^2v^2$ (b) $\frac{aT}{x}$

(c) $aT + 2\pi v$ (d) $\frac{aT}{v}$

31. A mass M, attached to a horizontal spring, executes SHM with an amplitude A_1. When the mass M passes through its mean position, then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of (A_1/A_2) is

(a) $\frac{M}{M+m}$ (b) $\frac{M+m}{M}$

(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left(\frac{M+m}{M}\right)^{1/2}$

32. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along x-axis. Their mean position is separated by distance x_0 $(x_0 > A)$. If the maximum separation between them is $(x_0 + A)$, the phase difference between their motion is
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/6$

SOLUTIONS

1.(a) Because the SHM starts from extreme position, so $y = a \cos \omega t$ form of SHM should be used.

$$\frac{A}{2} = A\cos\frac{2\pi}{T}t \Rightarrow \cos\frac{\pi}{3} = \cos\frac{2\pi}{T}t \Rightarrow t = T/6$$

2.(b) Standard equation for given condition,

$$x = a\cos\frac{2\pi}{T}t \Rightarrow x = -0.16 \cos (\pi t)$$

[as $a = -0.16$ m, $T = 2$ s]

3.(b) By comparing the given equation with standard equation,

$$y = a \sin (\omega t + \phi)$$

$$\omega = 100\pi \text{ so } T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = 0.02 \text{ sec}$$

4.(c) $x = a \sin (\omega t - \alpha)$ and
$y = b \cos (\omega t - \alpha) = b \sin (\omega t - \alpha + \pi/2)$

Now, phase difference

$$= \left(\omega t - \alpha + \frac{\pi}{2}\right) - (\omega t - \alpha) = \pi/2 = 90°$$

5.(c) $v = \omega\sqrt{a^2 - y^2}$

$$\Rightarrow \quad \frac{a\omega}{2} = \omega\sqrt{a^2 - y^2}$$

$$\Rightarrow \quad \frac{a^2}{4} = a^2 - y^2 \Rightarrow y = \frac{\sqrt{3}A}{2}$$

$$\left[\text{as } v = \frac{v_{max}}{2} = \frac{a\omega}{2}\right]$$

6.(a) $v = \omega\sqrt{a^2 - y^2} \Rightarrow v = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{\pi A\sqrt{3}}{T}$

[as $y = A/2$]

7.(c) According to potential energy $= \frac{1}{4}$ (Maximum energy)

$$\Rightarrow \quad \frac{1}{2}m\omega^2 y^2 = \frac{1}{4}\left(\frac{1}{2}m\omega^2 a^2\right) \Rightarrow y^2 = \frac{a^2}{4}$$

$$\Rightarrow \quad y = a/2$$

8.(d) Maximum force = Mass × Maximum acceleration

$= m\omega^2 a = 10 \times 10^{-3}(10)^2(0.5)$

$= 0.5$ N

9.(d) Let two simple harmonic motions are $y = a \sin \omega t$ and $y = a \sin (\omega t + \phi)$.

In the first case, $\frac{a}{2} = a \sin \omega t \Rightarrow \sin \omega t = 1/2$

$$\therefore \quad \cos \omega t = \frac{\sqrt{3}}{2}$$

In the second case, $\frac{a}{2} = a \sin(\omega t + \phi)$

$$\Rightarrow \quad \frac{1}{2} = [\sin \omega t . \cos \phi + \cos \omega t \sin \phi]$$

$$\Rightarrow \quad \frac{1}{2} = \left[\frac{1}{2}\cos\phi + \frac{\sqrt{3}}{2}\sin\phi\right]$$

$$\Rightarrow \quad 1 - \cos\phi = \sqrt{3}\sin\phi$$

$$\Rightarrow \quad (1 - \cos \phi)^2 = 3 \sin^2 \phi$$

$$\Rightarrow \quad (1 - \cos \phi)^2 = 3(1 - \cos^2 \phi)$$

By solving, we get $\cos \phi = +1$ or $\cos \phi = -1/2$

i.e., $\phi = 0$ or $\phi = 120°$

10.(c) Kinetic energy $= \frac{1}{2}m\omega^2(a^2 - y^2)$

$$= \frac{1}{2}m\omega^2\left(a^2 - \frac{a^2}{4}\right)$$

$$= \frac{3}{4}\left(\frac{1}{2}m\omega^2 a^2\right) = \frac{3E}{4}$$

$$[\text{as } y = a/2]$$

11.(a) Maximum acceleration, $\omega^2 a = 24$ (i)

and maximum velocity, $a\omega = 16$ (ii)

Dividing (i) by (ii), we get

$$\omega = \frac{3}{2}$$

Substituting this value in (ii), we get $a = 32/3$ m.

12.(d) By comparing the given equation with standard equation,

$$y = a \sin (\omega t + \phi)$$

We find that $a = 1$ and $\omega = \pi/4$

Now maximum acceleration

$$= \omega^2 a = \left(\frac{\pi^2}{4}\right) = \left(\frac{3.14}{4}\right)^2 = 0.62 \text{ cm/s}^2$$

13.(c) According to the problem, Kinetic energy = Potential energy

$$\Rightarrow \quad \frac{1}{2}m\omega^2(a^2 - y^2) = \frac{1}{2}m\omega^2 y^2$$

$$\Rightarrow \quad a^2 - y^2 = y^2$$

$$\therefore \quad y = a/\sqrt{2}$$

14.(b, d) Phase $\pi/2$ means extreme position. At extreme position, acceleration and displacement will be maximum.

15.(b) We know that Acceleration = $-\omega^2$ (displacement) and $a = -bx$ (given in the problem)

Comparing above two equation $\omega^2 = b \Rightarrow \omega = \sqrt{b}$

$\therefore$ Time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{b}}$

16.(c) Standard equation $m\frac{d^2y}{dt^2} + ky = 0$ and in a given equation $m = 1$ and $k = k$

So, $T = 2\pi\sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{k}}$

17.(a) $\frac{\Delta T}{T}=\frac{1}{2}\alpha\Delta\theta=\frac{1}{2}\times 12\times 10^{-6}\times(40-20)$

$\Delta t = 12 \times 10^{-5} \times 86400$ s/day = 10.3 s/day.

18.(d) Formula $\frac{T'}{T}=\sqrt{\frac{\rho}{\rho-\sigma}}$ Here $\sigma = 1$ for water so

$T'=T\sqrt{\frac{\rho}{\rho-1}}$.

19.(a) As $T \propto \sqrt{l}$

$\therefore \quad \frac{T_2}{T_1}=\sqrt{\frac{l_2}{l_1}}=\sqrt{1.21}$

$\Rightarrow \quad T_2 = 1.1\, T = T + 10\%\, (T)$

20.(b) If $l_1 = nl_2$, then $k_1=\frac{(n+1)k}{n}=\frac{3}{2}k$ [as $n = 2$]

21.(d) Given that maximum velocities are equal $a_1\omega_1 = a_1\omega_2$.

$\Rightarrow \quad a_1\sqrt{\frac{k_1}{m}}=a_2\sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2}=\sqrt{\frac{k_2}{k_1}}$

22.(c) For series combination, $n_1 \propto \sqrt{k/2}$

For parallel combination, $n_2 \propto \sqrt{2k}$

so $\frac{n_1}{n_2}=\sqrt{\frac{k/2}{2k}}=\frac{1}{2}$

23.(c) Kinetic energy of block $\left(\frac{1}{2}mv^2\right)$ = Elastic potential energy of spring $\left(\frac{1}{2}kx^2\right)$

By solving, we get $x=v\sqrt{\frac{m}{k}}$

24.(a) Given condition match with parallel combination, so $k_{eff} = k_1 + k_2$

$\therefore \quad \omega=\sqrt{\frac{k_{eff}}{m}}=\sqrt{\frac{k_1+k_2}{m}}$

25.(d) At extreme position acceleration is maximum $a_{max} = r\omega^2$. So the coin will leave the platform for first time at extreme position only when amplitude is increases.

26.(c) Here K_1 and K_2 are in parallel, $K = K_1 + K_2$ The frequency of oscillation is

$f=\frac{1}{2\pi}\sqrt{\frac{K}{m}}=\frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$

when K_1 and K_2 are made four times their original values, then

$K' = 4K_1 + 4K_2 = 4(K_1 + K_2)$

So, new frequency of oscillation is

$f'=\frac{1}{2\pi}\sqrt{\frac{4(K_1+K_2)}{m}}$

$\frac{f'}{f}=\sqrt{4}=2 \Rightarrow f' = 2f$

27.(d) The average value of kinetic energy during motion from mean to extreme position

$=\frac{1}{2}\left(\frac{1}{2}m\omega^2a^2+-0\right)$

$=\frac{1}{4}m(2\pi v)^2a^2=\pi ma^2v^2$

28.(d) $x = 2 \times 10^{-2}\cos \pi t$

Comparing it with standard form $x = \cos \omega t$. We have $\omega = \pi$

$\Rightarrow \quad \frac{2\pi}{T}=\pi$

$\Rightarrow \quad T = 2$ s

Now in time $t = T/4$, the particle goes form extreme position to mean position where KE becomes maximum.

$t=\frac{2}{4}=0.5$

29.(c) $x=x_0\cos\left(\omega t-\frac{\pi}{4}\right)$

$\Rightarrow \quad v=\frac{dx}{dt}=-x_0\omega\sin\left(\omega t-\frac{\pi}{4}\right)$

$\therefore$ Acceleration, $a=\frac{dv}{dt}=-x_0\omega^2\cos\left(\omega t-\frac{\pi}{4}\right)$

$=x_0\omega^2\cos\left(\pi+\omega t-\frac{\pi}{4}\right)$

Now comparing it with $a = A\cos(\omega t + \delta)$, we have

$A = x_0\omega^2$ and $\delta=\frac{3\pi}{4}$

30.(b) $\frac{aT}{x}=\frac{\omega^2xT}{x}=\frac{4\pi^2}{T^2}\times T=\frac{4\pi^2}{T}$ = constant.

31.(d) COLM: $MV_{max} = (m + M)V_{new}$, $V_{max} = A_1\omega$

$$\Rightarrow \quad V_{new} = \frac{MV_{max}}{(m+M)}$$

Now, $V_{new} = A_2 \cdot \omega_2$

$$\Rightarrow \quad \frac{M.A_1}{(m+M)}\sqrt{\frac{K}{M}} = A_2\sqrt{\frac{K}{(m+M)}}$$

$$\Rightarrow \quad A_2 = A_1\sqrt{\frac{M}{(m+M)}}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \left(\frac{m+M}{M}\right)^{1/2}$$

32.(b) $x_1 = A \sin(\omega t + \phi_1)$, $x_2 = A \sin(\omega t + \phi_2)$

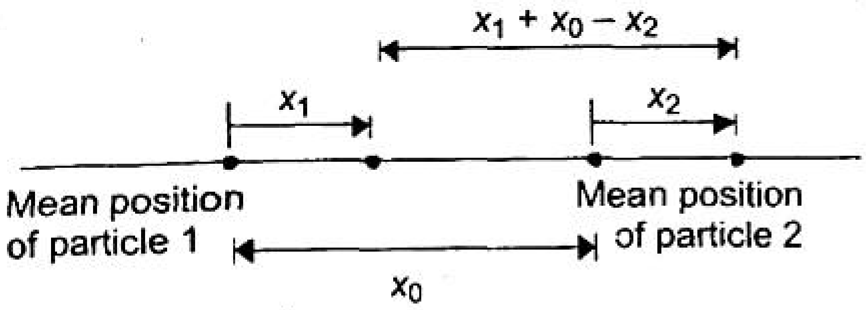

Fig. 14

$$x_1 - x_2 = A\left[2\sin\left(\omega t + \frac{\phi_1 + \phi_2}{2}\right)\sin\left(\frac{\phi_1 - \phi_2}{2}\right)\right]$$

$$(x_1 - x_2 + x_0)_{max} = x_0 + A$$

$$\Rightarrow \quad (x_1 - x_2)_{max} = A$$

$$\Rightarrow \quad 2A\sin\left(\frac{\phi_1 - \phi_2}{2}\right) = A$$

$$\Rightarrow \quad \frac{\phi_1 - \phi_2}{2} = \frac{\pi}{6}$$

$$\Rightarrow \quad \phi_1 - \phi_2 = \frac{\pi}{3}$$