

# COORDINATE GEOMETRY

## DISTANCE FORMULA

The distance between any two points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or  $AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$

### Distance of a point from origin

The distance of a point  $P(x, y)$  from origin  $O$  is given by  $OP = \sqrt{x^2 + y^2}$

### Problems based on geometrical figure

To show that a given figure is a

- ☞ Parallelogram – prove that the opposite sides are equal
- ☞ Rectangle – prove that the opposite sides are equal and the diagonals are equal.
- ☞ Parallelogram but not rectangle – prove that the opposite sides are equal and the diagonals are not equal.
- ☞ Rhombus – prove that the four sides are equal
- ☞ Square – prove that the four sides are equal and the diagonals are equal.
- ☞ Rhombus but not square – prove that the four sides are equal and the diagonals are not equal.
- ☞ Isosceles triangle – prove any two sides are equal.
- ☞ Equilateral triangle – prove that all three sides are equal.
- ☞ Right triangle – prove that sides of triangle satisfies Pythagoras theorem.

## IMPORTANT QUESTIONS

**Show that the points (1, 7), (4, 2), (-1, -1) and (-4, 4) are the vertices of a square.**

**Solution :** Let  $A(1, 7)$ ,  $B(4, 2)$ ,  $C(-1, -1)$  and  $D(-4, 4)$  be the given points.

$$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$$

$$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$$

$$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$$

$$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$$

$$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$$

$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$$

Since,  $AB = BC = CD = DA$  and  $AC = BD$ , all the four sides of the quadrilateral  $ABCD$  are equal and its diagonals  $AC$  and  $BD$  are also equal. Therefore,  $ABCD$  is a square.

**Find a point on the y-axis which is equidistant from the points  $A(6, 5)$  and  $B(-4, 3)$ .**

**Solution :** We know that a point on the y-axis is of the form  $(0, y)$ . So, let the point  $P(0, y)$  be equidistant from  $A$  and  $B$ . Then  $AP^2 = BP^2$

$$\Rightarrow (6-0)^2 + (5-y)^2 = (-4-0)^2 + (3-y)^2$$

$$\Rightarrow 36 + 25 + y^2 - 10y = 16 + 9 + y^2 - 6y \Rightarrow 4y = 36 \Rightarrow y = 9$$

So, the required point is  $(0, 9)$ .

### Questions for practice

1. Show that the points A(1, 2), B(5, 4), C(3, 8) and D(-1, 6) are vertices of a square.
2. Show that the points A(5, 6), B(1, 5), C(2, 1) and D(6, 2) are vertices of a square.
3. Show that the points A(1, -3), B(13, 9), C(10, 12) and D(-2, 0) are vertices of a rectangle.
4. Show that the points A(1, 0), B(5, 3), C(2, 7) and D(-2, 4) are vertices of a rhombus.
5. Prove that the points A(-2, -1), B(1, 0), C(4, 3) and D(1, 2) are vertices of a parallelogram.
6. Find the point on x-axis which is equidistant from (7, 6) and (-3, 4).
7. Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).
8. Find a point on the y-axis which is equidistant from the points A(5, 2) and B(-4, 3).
9. Find a point on the y-axis which is equidistant from the points A(5, -2) and B(-3, 2).
10. Find the values of y for which the distance between the points P(2, -3) and Q(10, y) is 10 units.
11. Find the value of a, if the distance between the points A(-3, -14) and B(a, -5) is 9 units.
12. If the point A(2, -4) is equidistant from P(3, 8) and Q(-10, y), find the values of y. Also find distance PQ.

### Section formula

The coordinates of the point P(x, y) which divides the line segment joining the points A(x<sub>1</sub>, y<sub>1</sub>) and B(x<sub>2</sub>, y<sub>2</sub>), internally, in the ratio m<sub>1</sub> : m<sub>2</sub> are

$$\left( \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

### Mid-point formula

The coordinates of the point P(x, y) which is the midpoint of the line segment joining the points

A(x<sub>1</sub>, y<sub>1</sub>) and B(x<sub>2</sub>, y<sub>2</sub>), are  $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

### IMPORTANT QUESTIONS

**Find the coordinates of the point which divides the line segment joining the points (4, -3) and (8, 5) in the ratio 3 : 1 internally.**

**Solution :** Let P(x, y) be the required point.

Using the section formula,  $x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}$ ,  $y = \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}$  we get

$$x = \frac{3(8) + 1(4)}{3 + 1} = 7, y = \frac{3(5) + 1(-3)}{3 + 1} = 3$$

Therefore, (7, 3) is the required point.

**In what ratio does the point (-4, 6) divide the line segment joining the points A(-6, 10) and B(3, -8)?**

**Solution :** Let (-4, 6) divide AB internally in the ratio k : 1.

Using the section formula,  $x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}$ ,  $y = \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}$  we get

$$y = \frac{k(-8) + 1(10)}{k + 1} = 6$$

$$\Rightarrow -8k + 10 = 6k + 6 \Rightarrow -8k - 6k = 6 - 10$$

$$\Rightarrow -14k = -4 \Rightarrow k = \frac{4}{14} = \frac{2}{7}$$

Therefore, the point (-4, 6) divides the line segment joining the points A(-6, 10) and B(3, -8) in the ratio 2 : 7.

### Questions for practice

1. Find the coordinates of the point which divides the join of  $(-1, 7)$  and  $(4, -3)$  in the ratio  $2 : 3$ .
2. Find the coordinates of the points of trisection of the line segment joining  $(4, -1)$  and  $(-2, -3)$ .
3. Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points  $A(2, -2)$  and  $B(-7, 4)$ .
4. Find the ratio in which the y-axis divides the line segment joining the points  $(5, -6)$  and  $(-1, -4)$ . Also find the point of intersection.
5. Find the ratio in which the line segment joining the points  $(-3, 10)$  and  $(6, -8)$  is divided by  $(-1, 6)$ .
6. Find the ratio in which the line segment joining  $A(1, -5)$  and  $B(-4, 5)$  is divided by the x-axis. Also find the coordinates of the point of division.
7. Find the coordinates of the points which divide the line segment joining  $A(-2, 2)$  and  $B(2, 8)$  into four equal parts.
8. If the points  $A(6, 1)$ ,  $B(8, 2)$ ,  $C(9, 4)$  and  $D(p, 3)$  are the vertices of a parallelogram, taken in order, find the value of  $p$ .
9. If  $(1, 2)$ ,  $(4, y)$ ,  $(x, 6)$  and  $(3, 5)$  are the vertices of a parallelogram taken in order, find  $x$  and  $y$ .
10. In what ratio does the x-axis divide the line segment joining the points  $(-4, -6)$  and  $(-1, 7)$ ? Find the coordinates of the point of division.
11. If  $P(9a - 2, -b)$  divides line segment joining  $A(3a + 1, -3)$  and  $B(8a, 5)$  in the ratio  $3 : 1$ , find the values of  $a$  and  $b$ .
12. If  $(a, b)$  is the mid-point of the line segment joining the points  $A(10, -6)$  and  $B(k, 4)$  and  $a - 2b = 18$ , find the value of  $k$  and the distance  $AB$ .
13. The centre of a circle is  $(2a, a - 7)$ . Find the values of  $a$  if the circle passes through the point  $(11, -9)$  and has diameter  $10\sqrt{2}$  units.
14. The line segment joining the points  $A(3, 2)$  and  $B(5, 1)$  is divided at the point  $P$  in the ratio  $1 : 2$  and it lies on the line  $3x - 18y + k = 0$ . Find the value of  $k$ .
15. Find the coordinates of the point  $R$  on the line segment joining the points  $P(-1, 3)$  and  $Q(2, 5)$  such that  $PR = \frac{3}{5}PQ$ .
16. Find the values of  $k$  if the points  $A(k + 1, 2k)$ ,  $B(3k, 2k + 3)$  and  $C(5k - 1, 5k)$  are collinear.
17. Find the ratio in which the line  $2x + 3y - 5 = 0$  divides the line segment joining the points  $(8, -9)$  and  $(2, 1)$ . Also find the coordinates of the point of division.
18. The mid-points  $D$ ,  $E$ ,  $F$  of the sides of a triangle  $ABC$  are  $(3, 4)$ ,  $(8, 9)$  and  $(6, 7)$ . Find the coordinates of the vertices of the triangle.

### MCQ (1 MARK)

1. The distance of the point  $P(4, -3)$  from the origin is  
(a) 1 unit                      (b) 7 units                      (c) 5 units                      (d) 3 units
2. The distance between the points  $A(2, -3)$  and  $B(2, 2)$  is  
(a) 2 units                      (b) 4 units                      (c) 5 units                      (d) 3 units
3. What is the midpoint of a line with endpoints  $(-3, 4)$  and  $(10, -5)$ ?  
(a)  $(-13, -9)$                       (b)  $(-6.5, -4.5)$                       (c)  $(3.5, -0.5)$                       (d) none of these
4. If the distance between the points  $(8, p)$  and  $(4, 3)$  is 5 then value of  $p$  is  
(a) 6                      (b) 0                      (c) both (a) and (b)                      (d) none of these
5. If the origin is the mid-point of the line segment joined by the points  $(2, 3)$  and  $(x, y)$ , then the value of  $(x, y)$  is  
(a)  $(2, -3)$                       (b)  $(2, 3)$                       (c)  $(-2, 3)$                       (d)  $(-2, -3)$

6. The distance of the point  $P(2, 3)$  from the x-axis is:  
(a) 2 (b) 3 (c) 1 (d) 5
7. The distance between the points  $A(0, 6)$  and  $B(0, -2)$  is:  
(a) 2 (b) 6 (c) 4 (d) 8
8. The distance of the point  $P(-6, 8)$  from the origin is:  
(a) 8 (b) 27 (c) 10 (d) 6
9. The distance between the points  $(0, 5)$  and  $(-5, 0)$  is:  
(a) 5 (b) 52 (c) 25 (d) 10
10. AOBC is a rectangle whose three vertices are  $A(0, 3)$ ,  $O(0, 0)$  and  $B(5, 0)$ . The length of its diagonal is:  
(a) 5 (b) 3 (c) 34 (d) 4
11. The perimeter of a triangle with vertices  $(0, 4)$ ,  $(0, 0)$  and  $(3, 0)$  is:  
(a) 5 (b) 12 (c) 11 (d)  $7 + 5$
12. The points  $(-4, 0)$ ,  $(4, 0)$ ,  $(0, 3)$  are the vertices of a :  
(a) Right triangle (b) Isosceles triangle (c) Equilateral triangle (d) Scalene triangle
13. Point on x – axis has coordinates:  
(a)  $(a, 0)$  (b)  $(0, a)$  (c)  $(-a, a)$  (d)  $(a, -a)$
14. Point on y – axis has coordinates:  
(a)  $(-a, b)$  (b)  $(a, 0)$  (c)  $(0, b)$  (d)  $(-a, -b)$