10
Hydrostatics

Fluids

Fluids are those substances which can flow when an external force is applied on them.

Liquids and gases are fluids.

The key property of fluids is that they offer very little resistance to shear stress. Hence, fluids do not have finite shap but takes the shape of the containing vessel.

In fluid mechanics, the following properties of fluid would be considered

(i) When the fluid is at rest– **hydrostatics**

(ii) When the fluid is in motion– **hydrodynamics**

Thrust

The total normal force exerted by liquid at rest on a given surface is called **thrust** of liquid.

The SI unit of thrust is newton.

Pressure

Pressure of liquid at a point is $p = \frac{\text{Thrust}}{\text{Area}} = \frac{F}{A}$.

Pressure is a scalar quantity, SI unit is Nm^{-2} and its dimensional formula $[\text{ML}^{-1}\text{T}^{-2}]$.

Pressure Exerted by the Liquid

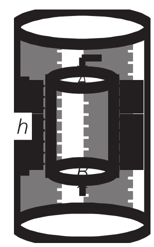
The normal force exerted by a liquid per unit area of the surface in contact is called **pressure of liquid** or **hydrostatic pressure.**

Pressure exerted by a liquid column, $p = h\rho g$

where, h = height of liquid column, ρ = density of liquid
and g = acceleration due to gravity.

Mean pressure on the walls of a vessel containing liquid upto height h is $\left(\frac{h\rho g}{2}\right)$.

Variation of Pressure with Depth



Consider a fluid at rest having density ρ (roh) contained in a cylindrical vessel as shown in figure. Let the two points A and B separated by a vertical distance h.

The pressure p at depth below the surface of a liquid open is given by

Pressure, $$p = p_a + h\rho g$$

where, ρ = density of liquid and g = acceleration due to gravity.

Atmospheric Pressure

The pressure exerted by the atmosphere on earth is called **atmospheric pressure.**

It is equivalent to a weight of 10 tones on 1 m^2.

At sea level, atmospheric pressure is equal to 76 cm of mercury column. Then, atmospheric pressure

$$= hdg = 76 \times 13.6 \times 980 \text{ dyne/cm}^2$$

$$= 0.76 \times 13.6 \times 10^3 \times 9.8 \text{ N/m}^2$$

Thus, 1 atm $= 1.013 \times 10^5$ Nm^{-2} (or Pa)

The atmospheric pressure does not crush our body because the pressure of the blood flowing through our circulatory system is balanced by this pressure.

Atmospheric pressure is also measured in torr and bar.

1 torr = 1 mm of mercury column

1 bar $= 10^5$ Pa

Aneroid barometer is used to measure atmospheric pressure.

Pressure measuring devices are open tube manometer, tyre pressure gauge, sphygmomanometer etc.

Gauge Pressure

Gauge pressure at a point in a fluid is the difference of total pressure at that point and atmospheric pressure.

Hydrostatic Paradox

The liquid pressure at a point is independent of the quantity of liquid but depends upon the depth of point below the liquid surface. This is known as hydrostatic paradox.

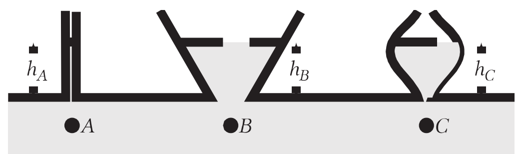
Important Points Related with Fluid Pressure

Important points related with fluid pressure are given below

(i) At a point in the liquid column, the pressure applied on it is same in all directions.

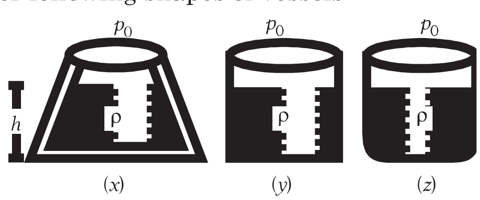
(ii) In a liquid, pressure will be same at all points at the same level.

(iii) The pressure exerted by a liquid depends only on the height of fluid column and is independent of the shape of the containing vessel.



If $h_A = h_B = h_C$, then $p_A = p_B = p_C$

(iv) Consider following shapes of vessels



Pressure at the base of each vessel

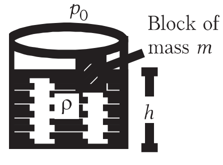
$$p_x = p_y = p_z = p_0 + \rho g h \text{ but } w_x \neq w_y \neq w_z$$

where, ρ = density of liquid in each vessel,

h = height of liquid in each vessel

and p_0 = atmospheric pressure.

(v) In the figure, a block of mass 'm' floats over a fluid surface



If ρ = density of the liquid
A = area of the block

Pressure at the base of the vessel in $p = p_0 + \rho gh + \dfrac{mg}{A}$

Buoyancy

When a body is partially or fully immersed in a fluid, an upward force acts on it, which is called buoyant force, the phenomena is called buoyancy.

The buoyant force acts at the centre of gravity of the liquid displaced by the immersed part of the body and this point is called the centre of buoyancy. The magnitude of buoyant force, $F = v\rho g$.

Pascal's Law

The increase in pressure at a point in the enclosed liquid in equilibrium is transmitted equally in all directions in liquid and to the walls of the container.

The working of hydraulic lift and hydraulic brakes arebased on Pascal's law.

Archimedes' Principle

When a body is partially or fully immersed in a liquid, it loses some of its weight and it is equal to the weight of the liquid displaced by the immersed part of the body. If a is loss of weight of a body in water and b is loss of weight in another liquid, then

$$\frac{a}{b} = \frac{w_{\text{air}} - w_{\text{liquid}}}{w_{\text{air}} - w_{\text{water}}}$$

If T is the observed weight of a body of density σ when it is fully immersed in a liquid of density ρ, then real weight of the body

$$w = \frac{T}{\left(1 - \frac{\rho}{\sigma}\right)}$$

If w_1 = weight of body in air, w_2 = weight of body in liquid,
V_i = immersed of volume of liquid,
ρ_L = density of liquid and g = acceleration due to gravity

$$\Rightarrow \qquad V_i = \frac{w_1 - w_2}{\rho_L g}$$

Laws of Floatation

A body will float in a liquid, if the weight of the body is equal to the weight of the liquid displaced by the immersed part of the body.

If W is the weight of the body and w is the buoyant force, then

(a) If $W > w$, then body will sink to the bottom of the liquid.

(b) If $W < w$, then body will float partially submerged in the liquid.

(c) If $W = w$, then body will float in liquid if its whole volume is just immersed in the liquid.

The floating body will be in stable equilibrium, if meta-centre (centre of buoyancy) lies vertically above the centre of gravity of the body.

The floating body will be in unstable equilibrium, if meta-centre (centre of buoyancy) lies vertically below the centre of gravity of the body. The floating body will be in neutral equilibrium, if meta-centre (centre of buoyancy) coincides with the centre of gravity of the body.

Fraction of volume of a floating body outside the liquid

$$\left(\frac{V_{\text{out}}}{V}\right) = \left[1 - \frac{\rho}{\sigma}\right]$$

where, ρ = density of body and σ = density of liquid

If two different bodies A and B are floating in the same liquid, then

$$\frac{\rho_A}{\rho_B} = \frac{(v_{\text{in}})_A}{(v_{\text{in}})_B}$$

If the same body is made to float in different liquids of densities σ_A and σ_B respectively, then

$$\frac{\sigma_A}{\sigma_B} = \frac{(V_{\text{in}})_B}{(V_{\text{in}})_A}$$

Density and Relative Density

Density of a substance is defined as the ratio of its mass to its volume.

$$\text{Density of a liquid} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Density of water} = 1\ \text{g/cm}^3 \text{ or } 10^3\ \text{kg/m}^3$$

In case of homogneous (isotropic) substance it has no directional properties, so it is scalar quantity and its dimensional formula is $[ML^{-3}]$.

Relative density of a substance is defined as the ratio of its density to the density of water at 4°C.

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at } 4°\text{C}}$$

$$= \frac{\text{Weight of substance in air}}{\text{Loss of weight in water}}$$

Relative density also known as specific gravity has no unit, no dimensions.

For a solid body, density of body = density of substance.

While for a hollow body, density of body is lesser than that of substance.

When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.

Density of a Mixture of Substances

- When two liquids of masses m_1 and m_2 having densities ρ_1 and ρ_2 are mixed together, then density of mixture is

$$\rho = \frac{m_1 + m_2}{\left(\frac{m_1}{\rho_1}\right) + \left(\frac{m_2}{\rho_2}\right)} = \frac{\rho_1\rho_2\,(m_1 + m_2)}{(m_1\rho_2 + m_2\rho_1)}$$

- When two liquids of same mass m but of different densities ρ_1 and ρ_2 are mixed together, then density of mixture is $\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$.
- When two liquids of same volume V but of different densities ρ_1 and ρ_2 are mixed together, then density of mixture is $\rho = \frac{\rho_1 + \rho_2}{2}$.

  Density of a liquid varies with pressure, $\rho = \rho_0 \left[1 + \frac{\Delta p}{K}\right]$

  where, ρ_0 = initial density of the liquid, K = bulk modulus of elasticity of the liquid and Δp = change in pressure.
- With rise in temperature (ΔT) due to thermal expansion of a given body, volume will increase while mass will remains constant, so density will decrease $\rho = \frac{\rho_0}{(1 + \gamma \cdot \Delta T)} \simeq \rho_0 (1 - \gamma \cdot \Delta T)$; where γ is volumetric expansion.