KEY Concepts

1. Limit of a function f(x) is said to exist as, $x \to a$ when

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \text{finite quantity.}$$

2. FUNDAMENTAL THEOREMS ON LIMITS:

Let $\lim_{x\to a} f(x) = l$ & $\lim_{x\to a} g(x) = m$. If l & m exists then :

(i) $\lim_{x\to a} f(x) \pm g(x) = l \pm m$

(ii) $\lim_{x\to a} f(x) \cdot g(x) = l . m$

(iii) $\lim_{x\to a} \dfrac{f(x)}{g(g)} = \dfrac{\ell}{m}$, provided $m \neq 0$

(iv) $\lim_{x\to a} k f(x) = k \lim_{x\to a} f(x)$; where k is a constant.

(v) $\lim_{x\to a} f[g(x)] = f\left(\lim_{x\to a} g(x)\right) = f(m)$; provided f is continuous at $g(x) = m$.

For example $\lim_{x\to a} l n(f(x) = l n \left[\lim_{x\to a} f(x)\right] l n\, l$ $(l > 0)$.

3. STANDARD LIMITS :

(a) $\lim_{x\to 0} \dfrac{\sin x}{x} = 1 = \lim_{x\to 0} \dfrac{\tan x}{x} = \lim_{x\to 0} \dfrac{\tan^{-1} x}{x} = \lim_{x\to 0} \dfrac{\sin^{-1} x}{x}$

[Where x is measured in radians]

(b) $\lim_{x\to 0} (1 + x)^{1/x} = e = \lim_{x\to \infty} \left(1 + \dfrac{1}{x}\right)^x$ note

however there $\lim_{\substack{h\to 0\\ n\to\infty}} (1 - h)^n = 0$

and $\lim_{\substack{h\to 0\\ n\to\infty}} (1 + h)^n \to \infty$

(c) If $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} \phi(x) = \infty$, then ;

$$\lim_{x\to a} [f(x)]^{\phi(x)} = e^{\lim_{x\to a} \phi(x)[f(x)-1]}$$

(d) If $\lim_{x\to a} f(x) = A > 0$ & $\lim_{x\to a} \phi(x) = B$ (a finite quantity) then ;

$\lim_{x\to a} [f(x)]^{\phi(x)} = e^z$ where $z = \lim_{x\to a} \phi(x). \ln[f(x)]$ $= e^{B\ln A} = A^B$

(e) $\lim_{x\to 0} \dfrac{a^x - 1}{x} = l n\, a \ (a > 0)$. In particular $\lim_{x\to 0} \dfrac{e^x - 1}{x} = 1$

(f) $\lim_{x\to a} \dfrac{x^n - a^n}{x - a} = n a^{n-1}$

4. SQUEEZE PLAY THEOREM :

If $f(x) \le g(x) \le h(x) \ \forall\ x$ & $\text{Limit}_{x\to a} f(x) = l = \text{Limit}_{x\to a} h(x)$ then $\text{Limit}_{x\to a} g(x) = l$.

5. INDETERMINANT FORMS :

$\dfrac{0}{0}, \dfrac{\infty}{\infty}, 0 \times \infty, 0^\circ, \infty^\circ, \infty - \infty$ and 1^∞

Note :

(i) We cannot plot ∞ on the paper. Infinity (∞) is a symbol & not a number. It does not obey the laws of elementry algebra.

(ii) $\infty + \infty = \infty$

(iii) $\infty \times \infty = \infty$

(iv) $(a/\infty) = 0$ if a is finite

(v) $\dfrac{a}{0}$ is not defined , if $a \neq 0$.

(vi) $a b = 0$, if & only if $a = 0$ or $b = 0$ and a & b are finite.

6. The following strategies should be born in mind for evaluating the limits:

(a) Factorisation

(b) Rationalisation or double rationalisation

(c) Use of trigonometric transformation ; appropriate substitution and using standard limits

(d) Expansion of function like Binomial expansion, exponential & logarithmic expansion, expansion of sinx , cosx , tanx should be remembered by heart & are given below :

$$a^x = 1 + \frac{x\,ln\,a}{1!} + \frac{x^2\,ln^2 a}{2!} + \frac{x^3 ln^3 a}{3!} + ...a > 0$$

(ii) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\ x \in R$

(iii) $ln(1+x)$

$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... \text{for} -1 < x \le 1$

(iv) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(v) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...x$

$\in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(vi) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + ...x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(vii) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$

1. $\lim_{x\to 1}(1-x+[x-1]+[1-x])$ is

(where [] denotes greatest integer function)

(A) 0 (B) 1

(C) –1 (D) does not exist

2. $\lim_{x\to 0}\left[\frac{\sin[x-3]}{[x-3]}\right]$

(where [ ] denotes greatest integer function)

(A) 0 (B) 1

(C) does not exist (D) sin 1

3. $\lim_{x\to 0}\frac{\sin[\cos x]}{1+[\cos x]}$ (where [ ] denotes greatest integer function)

(A) equal to 1 (B) sin 1

(C) equal to zero (D) non existent

4. $\lim_{x\to 0^+}\frac{\left(\frac{\pi}{2}-\cot^{-1}\{x\}\right)x}{\sin(x)-\cos x}$ is (where [ ] denotes fractional part function)

(A) 2 (B) 1

(C) 0 (D) does not exist

5. $\lim_{x\to\infty}\sec^{-1}\left(\frac{x}{x+1}\right)=$

(A) 0 (B) π

(C) $\pi/2$ (D) does not exist

6. If $f(x)=\begin{cases}x-1 & x\ge 1\\ 2x^2-2 & x<1\end{cases}, g(x)=\begin{cases}x+1 & x>0\\ -x^2+1, & x\le 0\end{cases}$

and h(x) = |x| then find $\lim_{x\to 0} f(g(h(x)))$

(A) 1 (B) 0

(C) –1 (D) Does not exists

7. $\lim_{n\to\infty}\frac{-3n+(-1)^n}{4n-(-1)^n}$ is

(A) $-\frac{3}{4}$

(B) $-\frac{3}{4}$ if n is even ; $\frac{3}{4}$ if n is odd

(C) not exist if n is even ;$-\frac{3}{4}$ if n is odd

(D) + 1 if n is even ; does not exist if n is odd

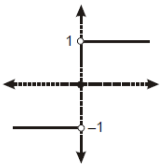
8. $\lim_{x\to\infty}\frac{\log_x n-[x]}{[x]}, n\in N$

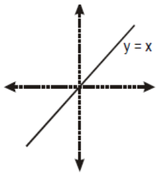
(where [ ] denotes greatest integer function)

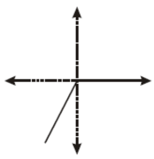
(A) has value – 1 (B) has value 0

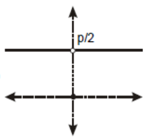
(C) has value 1 (D) does not exists

9. The graph of the function $f(x)=\lim_{t\to 0}\left(\frac{2x}{\pi}\cot^{-1}\frac{x}{t^2}\right)$, is

(A)


(B)


(C)


(D)


10. Let $f(x)=\frac{n(x^2+e^x)}{n(n^4+e^{2x})}$ If $\lim_{x\to -\infty} f(x)=\ell$ and $\lim_{x\to -\infty} f(x)=m$ then

(A) $\ell = m$ (B) $\ell = 2m$

(C) $2\ell = m$ (D) $\ell + m = 0$

11. Evaluate

$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+2}}+.....+\frac{1}{\sqrt{n^2+2n}}\right)$

(A) 1 (B) 1/2

(C) 0 (D) 2

12. $\lim_{x\to\infty} n\cos\left(\frac{\pi}{4n}\right)\sin\left(\frac{n}{4n}\right)$ has the value equal to

(A) $\pi/3$ (B) $\pi/4$

(C) 1/6 (D) None of these

13. $\lim_{x\to\pi/2}\frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$ is

(A) 1/16 (B) –1/16

(C) 1/32 (D) –1/32

14. $\lim_{x\to 0}\left[(1-e^x)\frac{\sin x}{|x|}\right]$ is (where [ ] denotes greatest integer function)

(A) –1 (B) 1

(C) 0 (D) does not exist

15. $\lim_{x\to 0}\frac{2\left(\sqrt{3}\sin\left(\frac{\pi}{6}+x\right)-\cos\left(\frac{\pi}{6}+x\right)\right)}{x\sqrt{3}(\sqrt{3}\cos x-\sin x)}$

(A) –1/3 (B) 2/3
(C) 4/3 (D) –4/3

16. $\lim_{x\to\pi/2}\frac{2^{-\cos x}-1}{(x-\pi/2)}=$

(A) $\frac{2\ n2}{\pi}$ (B) ℓ n2 (C) $\frac{2}{\pi}$ (D) Done

17. $\lim_{x\to 0}\frac{(4^x-1)^3}{\sin\left(\frac{x}{p}\right)\ln\left(1+\frac{x^2}{3}\right)}=$

(A) 9 p (log 4) (B) 3 p $(\log 4)^3$
(C) 12 p $(\log 4)^3$ (D) 27 p $(\log 4)^2$

18. The limit $\lim_{x\to a}\left(2-\frac{a}{x}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ is equal to

(A) $e^{-a/\pi}$ (B) $e^{-2a/\pi}$ (C) $e^{-2/\pi}$ (D) 1

19. The value of $\lim_{x\to\pi/4}\left(1+[x]\right)^{1/\ln(\tan x)}$ is equal to (where [] denotes greatest integer function)

(A) 0 (B) 1 (C) e (D) e^{-1}

20. $\lim_{x\to\infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^x$ is

(A) e (B) e^2
(C) 1/e (D) does not exist

21. $\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) None of these

22 $\lim_{x\to\infty}\left(\left(\frac{n}{n+1}\right)^{\alpha}+\sin\frac{1}{n}\right)^n$ when $\alpha\in Q$ is equal to

(A) $e^{-\alpha}$ (B) $-\alpha$
(C) $e^{1-\alpha}$ (D) $e^{1+\alpha}$

23. If $\lim_{x\to 0}\frac{(\sin nx)[(a-n)nx-2\tan x]}{x^2}=0$ then value of a =

(A) $\frac{1}{n}$ (B) $n-\frac{1}{n}$

(C) $n+\frac{1}{n}$ (D) None of these

24. $\lim_{x\to\infty}\frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}}=$

(A) 5 (B) 3
(C) 1 (D) zero

25. The value of $\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to

(A) 1/5 (B) 1/6
(C) 1/4 (D) 1/2

26. $\lim_{x\to 2}\frac{\sin\left(e^{x-2}-1\right)}{\ln(x-1)}=$

(A) 0 (B) –1
(C) 2 (D) 1

27. If $A_j=\frac{x-a_j}{|x-a_j|}$, j = 1, 2, ... n and $a_1<a_2<a_3<$ ....$<a_n$ $\lim_{x\to a_m}(A_1.A_2.\ ......A_n)$, $1\le m\le n$

(A) is equal to $(-1)^{n-m+1}$
(B) is equal to $(-1)^{n-m}$
(C) is equal to $(-1)^{m}$
(D) does not exist

28. $\lim_{x\to a^-}\frac{|x|^3}{a}-\left[\frac{x}{a}\right]^3$ (a > 0), where [x] donotes the greatest integer less than or equal to x is :

(A) a^2-3 (B) a^2-1
(C) a^2 (D) none

29. $\lim_{x\to\infty}\frac{e^x\left(\left(2^{x^n}\right)^{\frac{1}{e^x}}-\left(3^{x^n}\right)^{\frac{1}{e^x}}\right)}{x^n}$, $n\in N$ is equal to

(A) 0 (B) ln (2/3)
(C) ln(3/2)z (D) none

30. $\lim_{x\to\infty} x\left\{\tan^{-1}\left(\frac{x+1}{x+2}\right)-\pi/4\right\} =$

(A) –1 (B) –1/2
(C) 1 (D) 0

31. $\lim_{n\to\infty} (0.2)^{\log_{\sqrt{5}}(1/4+1/8+1/16+\ldots\ldots \text{n terms})}$ is equal to :

(A) 2 (B) 4
(C) 8 (D) None

32. $\lim_{x\to 0} \frac{(\cos x)^{1/2}-(\cos x)^{1/3}}{\sin^2 x}$ is :

(A) 1/6 (B) –1/12
(C) 2/3 (D) 1/3

33. $\lim_{n\to\infty} \frac{1.n+2(n-1)+3(n-2)+\ldots\ldots+n.1}{1^2+2^2+3^3+\ldots\ldots+n^2}$ has the value

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

34. If $\lim_{x\to 0} \frac{x^3}{\sqrt{a+x}\,(bx-\sin x)} = 1$ then the constants 'a' and 'b' are (where a > 0)

(A) b = 1, a = 36 (B) a = 1, b = 6
(C) a = 1, b = 36 (D) b = 1, a = 6

35. If $\lim_{x\to\infty} f(x)$ exist and is finite & non zero and if $\lim_{x\to\infty}\left(f(x)+\frac{3f(x)-1}{f^2(x)}\right)$ then the velue of $\lim_{x\to\infty}$ is

(A) 1 (B) –1
(C) 2 (D) none of these

36. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is **[AIEEE 2005]**

(A) $\frac{a^2}{2}(\alpha-\beta)^2$ (B) 0

(C) $\frac{-a^2}{2}(\alpha-\beta)^2$ (D) $\frac{1}{2}(\alpha-\beta)^2$

37. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ **[AIEEE 2011]**

(A) does not exist (B) equals $\sqrt{2}$

(C) equals $-\sqrt{2}$ (D) equals $\frac{1}{\sqrt{2}}$

38. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to :

(A) 1 (B) 2

(C) $-\frac{1}{4}$ (D) $\frac{1}{2}$ **[AIEEE 2015]**

39. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to : **[AIEEE 2014]**

(A) $\frac{\pi}{2}$ (B) 1

(C) –π (D) π

40. Let p = $\lim_{x\to 0+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to :

(A) 1 (B) 1/2
(C) 1/4 (D) 2 **[AIEEE 2017]**

41. $\lim_{n\to\infty} \frac{1^2+2^2+3^2+\ldots\ldots\ldots+n^2}{n^3}$ is equal to -

(A) ∞ (B) 0
(C) 1 / 2 (D) 1 / 3

42. $\lim_{n\to\infty} \frac{1}{1.3}+\frac{1}{3.5}+\ldots\ldots+$ to n terms is equal to -

(A) 1/4 (B) 1/2
(C) 1 (D) 2

43 $\lim_{x\to 1} \frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$ is equal to -

(A) $\frac{1}{9}$ (B) $\frac{1}{6}$

(C) $\frac{1}{3}$ (D) none of these

44. $\lim_{x\to\infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$ is equal to -

(A) 0 (B) 1 / 2
(C) log 2 (D) e^4

45. If $\lim_{x\to a}$ [f(x) + g(x)]=2 and $\lim_{x\to a}$ [f(x) – g(x)]=1, then $\lim_{x\to a}$ f(x)g(x) -

(A) need not exist (B) exist and is $\frac{3}{4}$

(C) exists and is $-\frac{3}{4}$ (D) exists and is $\frac{4}{3}$

46. $\underset{x\to 0}{\text{Lim}}\dfrac{\tan x-\sin x}{x^3}$ is equal to -

(A) $1/2$ (B) -1
(C) 2 (D) -2

47. Let α and β be the distinct roots of $ax^2+bx+c=0$ then $\underset{x\to\alpha}{\text{Lim}}\dfrac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to -

(A) $\dfrac{1}{2}(\alpha-\beta)^2$ (B) $\dfrac{-a^2}{2}(\alpha-\beta)^2$
(C) 0 (D) $\dfrac{a^2}{2}(\alpha-\beta)^2$

48. $\underset{x\to 0}{\text{L im}}\dfrac{\left[\dfrac{x}{\sin x}\right]-\left[\dfrac{\sin x}{x}\right]}{\left[\dfrac{\tan x}{x}\right]}$ is equal to ([.] represents the greatest integer function) -

(A) 1 (B) -1
(C) 0 (D) does not exists

49. $\underset{x\to\pi/2}{\text{Lim}}\dfrac{[1-\tan x/2]\,[1-\sin x]}{[1+\tan x/2]\,[\pi-2x]^3}$ is equal to -

(A) 0 (B) 1/32
(C) ∞ (D) 1/8

50. $\underset{x\to\infty}{\text{Lim}}\left(\sin\sqrt{x+1}-\sin\sqrt{x}\right)$ is equal to -

(A) 1 (B) –1
(C) 0 (D) none of these

51. $\underset{x\to 0}{\text{Lim}}\dfrac{\log_e\cos x}{x^2}$ is equal to -

(A) $-\dfrac{1}{2}$ (B) $\dfrac{1}{2}$
(C) 0 (D) none of these

52. $\underset{x\to 0}{\text{Lim}}\dfrac{10^x-2^x-5^x+1}{x\tan x}$ is equal to -

(A) ln(2 + 5) (B) ln5 . ln2
(C) ln 10 (D) $\ln\left(\dfrac{5}{2}\right)$

53. $\underset{x\to\infty}{\text{L im}}\left[\dfrac{2x^2+3}{2x^2+5}\right]^{8x^2+3}$ is equal to -

(A) –8 (B) 1
(C) e^8 (D) e^{-8}

54 If $\underset{x\to\infty}{\text{Lim}}\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x}=e^2$, then the values of *a* and *b* are -

(A) $a\in R$, $b=2$ (B) $a=1$, $b\in R$
(C) $a\in R$, $b\in R$ (D) $a=1$ and $b=2$

55. $\underset{x\to 0}{\text{Lim}}\dfrac{\sqrt[3]{1+3x}-1-x}{(1+x)^{101}-1-101x}$ has the value equal to -

(A) $-\dfrac{3}{5050}$ (B) $-\dfrac{1}{5050}$
(C) $\dfrac{1}{5051}$ (D) $\dfrac{1}{4950}$

56. $\underset{x\to 0}{\text{Lim}}\dfrac{e^{x^3}-\tan x+\sin x-1}{x^n}$ exists and is non-zero, then the value of n is -

(A) 1 (B) 3
(C) 2 (D) 0

57. $\underset{x\to 0}{\text{Lim}}\dfrac{\left(\sqrt{(1-\cos x)+\sqrt{(1-\cos x)+\sqrt{(1-\cos x)+\ldots\ldots\ldots\infty}}}\right)-1}{x^2}$ equals -

(A) 0 (B) $\dfrac{1}{2}$
(C) 1 (D) 2

58. $\underset{n\to\infty}{\text{Lim}}\left(\left(\dfrac{n}{n}\right)^n+\left(\dfrac{n-1}{n}\right)^n+\ldots\ldots+\left(\dfrac{1}{n}\right)^n\right)$ equals -

(A) $\dfrac{1}{e-1}$ (B) $\dfrac{1}{1+e}$
(C) $\dfrac{1}{1-e}$ (D) $\dfrac{e}{e-1}$

59. $\underset{n\to\infty}{\text{Lim}}\cos\left(\pi\sqrt{n^2+n}\right)$ when n is an integer -

(A) is equal to 1 (B) is equal to –1
(C) is equal to zero (D) does not exist

60. The value of $\underset{x\to 0}{\lim}\dfrac{\sqrt[3]{1+\sin x}-\sqrt[3]{1-\sin x}}{x}$ is -

(A) $\dfrac{2}{3}$ (B) $-\dfrac{2}{3}$
(C) $\dfrac{3}{2}$ (D) $-\dfrac{3}{2}$