Some Applications of Trigonometry

Objective Section (1 mark each)

Fill in the Blanks

Q. 1. In Fig. 3, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are ...., ... [CBSE OD, Set 1, 2020]

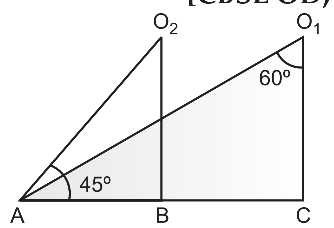
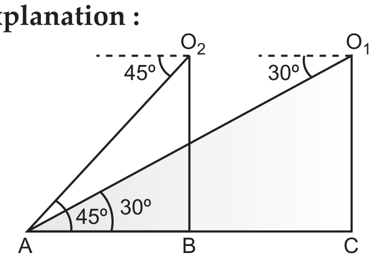


Fig. - 3

Ans. 30°, 45°

Explanation :



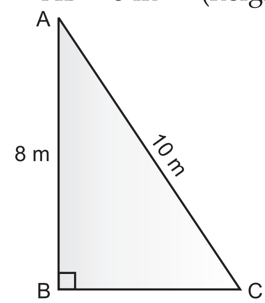
Q. 2. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is .... m. [CBSE Delhi, Set 3, 2020]

Ans. 6

Explanation : In ΔABC,

$$AC = 10 \text{ m} \quad \text{(length of ladder)}$$

$$AB = 8 \text{ m} \quad \text{(height of window)}$$



Now $AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow (10)^2 = (8)^2 + BC^2$

$\Rightarrow 100 - 64 = BC^2$

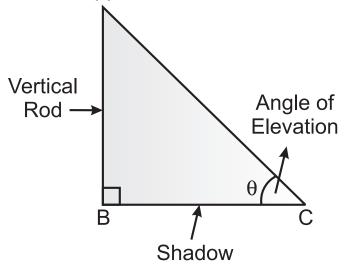
$\Rightarrow BC^2 = 36$

$\Rightarrow BC = 6 \text{ m}$

Very Short Answer Type Questions (1 mark each)

Q. 1. The ratio of the length of a vertical rod and the length of its shadow is $1:\sqrt{3}$. Find the angle of elevation of the sun at that moment ? [CBSE Delhi, Set 1, 2020]

Ans. Let the angle of elevation be θ.



We know that,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan\theta = \frac{AB}{BC}$$

$$= \frac{1}{\sqrt{3}}$$

$$= \tan 30°$$

$\therefore \tan\theta = \tan 30°$

$\Rightarrow \theta = 30°$ **Ans.**

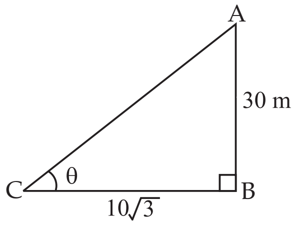
Q. 2. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun?

[CBSE OD, Term 2, Set 1, 2017]

Ans.

In Δ *ABC*,

$$\tan\theta = \frac{AB}{BC}$$



$$\tan\theta = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

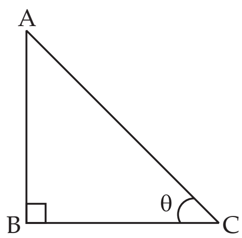
$$\tan\theta = \tan 60^\circ \quad \Rightarrow \quad \theta = 60^\circ$$

Hence angle of elevation is 60°.

Q. 3. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}$: 1. What is the angle of elevation of the sun?

[CBSE Delhi, Term 2, Set 1, 2017]

Ans. Given, $\frac{AB}{BC} = \frac{\sqrt{3}}{1}$



In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{\sqrt{3}}{1}$$

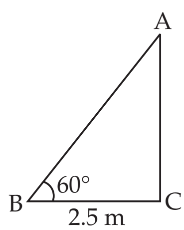
$$\Rightarrow \qquad = 60^\circ$$

Hence, the angle of elevation is 60°.

Q. 4. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

[CBSE OD, Term 2, Set 1, 2016]

Ans. Let AB be the ladder leaning against a wall AC.



Then, $\cos 60^\circ = \frac{BC}{AB}$

$$\Rightarrow \qquad \frac{1}{2} = \frac{2.5}{AB}$$

$$\Rightarrow \qquad AB = 2.5 \times 2 = 5 \text{ m}$$

$\therefore$ Length of ladder is 5 m.

Q. 5. In Fig. 1, AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If $AD = 2.54$ m, find the length of the ladder. (use $\sqrt{3} = 1.73$)

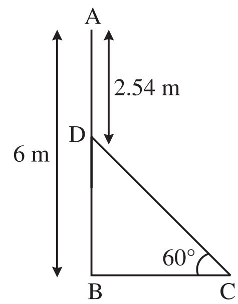


Fig. 1

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Given, $AB = 6$ m and $AD = 2.54$ m.

$\therefore$ $DB = (6 - 2.54)$ m $= 3.46$ m

In $\Delta\, DBC$,

$$\sin 60^\circ = \frac{DB}{DC}$$

$$\Rightarrow \qquad \frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

$$\Rightarrow \qquad DC = \frac{3.46 \times 2}{1.73} = 4$$

$\therefore$ The length of the ladder is 4 m.

Q. 6. In figure 1, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the sun's altitude.

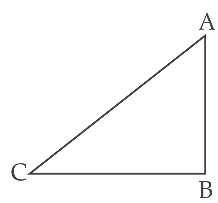
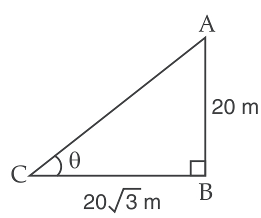


Figure 1

[CBSE OD, Term 2, Set 1, 2015]

Ans. Given AB is the tower and BC is its shadow.



$\therefore \quad \tan\theta = \frac{AB}{BC} \quad \left[\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$

$\Rightarrow \quad \tan\theta = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$

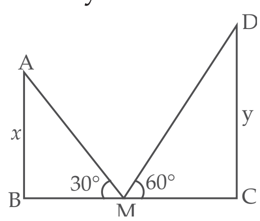
$\Rightarrow \quad \tan\theta = \tan 30° \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$

$\Rightarrow \quad \theta = 30°$

Q. 7. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. Let AB and CD be two towers of height x and y respectively.



M is the mid-point of BC *i.e.*, $BM = MC$

In Δ *ABM*, we have

$$\frac{AB}{BM} = \tan 30°$$

$$\Rightarrow \quad BM = \frac{x}{\tan 30°} \quad \text{...(i)}$$

In Δ *CDM*, we have

$$\frac{DC}{MC} = \tan 60°$$

$$\Rightarrow \quad \frac{y}{MC} = \tan 60°$$

$$MC = \frac{y}{\tan 60°} \quad \text{...(ii)}$$

From *eq.* (i) and (ii), we get

$$\frac{x}{\tan 30°} = \frac{y}{\tan 60°}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{\tan 30°}{\tan 60°}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

$$\therefore \quad x : y = 1 : 3.$$

Short Answer Type Questions-II

(3 marks each)

Q. 1. The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in Fig. 4. If AC = 1.5 m long and CD = 3 m, find (i) tan θ (ii) sec θ + cosec θ

[CBSE Delhi, Set 1, 2020]

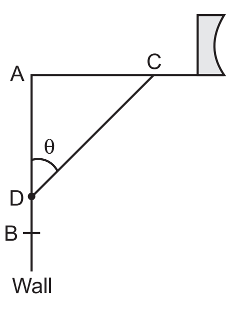


Fig. 4

Solution : Given : AC = 1.5 m and CD = 3 m

In ΔADC,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{AC}{CD}$$

$$= \frac{1.5}{3} = \frac{1}{2}$$

$$\therefore \quad \sin\theta = \frac{1}{2}$$

$$\Rightarrow \quad \sin\theta = \sin 30°$$

$$\Rightarrow \quad \theta = 30°$$

(i) $\tan\theta = \tan 30°$

$$= \frac{1}{\sqrt{3}} \quad \textbf{Ans.}$$

(ii) $\sec\theta + \operatorname{cosec}\theta = \sec 30° + \operatorname{cosec} 30°$

$$\Rightarrow \quad = \frac{2}{\sqrt{3}} + \frac{2}{1}$$

$$= \frac{2 + 2\sqrt{3}}{\sqrt{3}} \quad \textbf{Ans.}$$

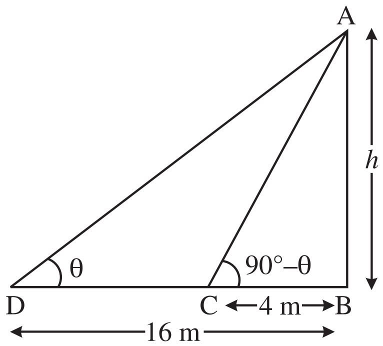
Q. 2. On a straight line passing through the foot of a tower, two points *C* and *D* are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from *C* and *D* of the top of the tower are

complementary, then find the height of the tower.

[CBSE OD, Term 2, Set 1, 2017]

Ans.

Let height AB of tower = h m.



In ΔABC,

$$\frac{AB}{BC} = \tan(90° - \theta)$$

$$\frac{h}{4} = \cot\theta \qquad \text{....(i)}$$

In ΔABD,

$$\frac{AB}{BD} = \tan\theta$$

$$\frac{h}{16} = \tan\theta \qquad \text{...(ii)}$$

Multiply eq. (i) and (ii),

$$\frac{h}{4}\times\frac{h}{16} = \cot\theta \times \tan\theta$$

$$\frac{h^2}{64} = 1$$

$$\Rightarrow \qquad h^2 = 64 \Rightarrow h = 8 \text{ m}$$

$\therefore$ Height of tower = 8 m.

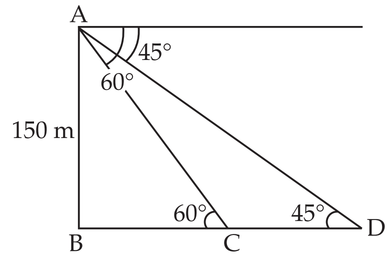
Q. 3. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h.

[CBSE Delhi, Term 2, Set 1, 2017]

Ans. From ΔABC, $\frac{AB}{BC} = \tan 60°$

or $$BC = \frac{AB}{\tan 60°}$$

$$BC = \frac{150}{\sqrt{3}} \text{ m}$$



From ΔABD, $\frac{AB}{BD} = \tan 45° \Rightarrow AB = BD$

$[\because \tan 45° = 1]$

$$\Rightarrow \qquad BD = 150 \text{ m}$$

Distance covered in 2 min. = $BD - BC$

$$= 150 - \frac{150}{\sqrt{3}} = \frac{150\sqrt{3} - 150}{\sqrt{3}}$$

So, distance covered in 1 hour (*ie.*, speed)

$$= \frac{150(\sqrt{3}-1)}{\sqrt{3}\times 2} \times 60$$

$$\text{Speed} = \frac{4500(\sqrt{3}-1)}{\sqrt{3}}$$

$$= 4500 - 1500\sqrt{3}$$

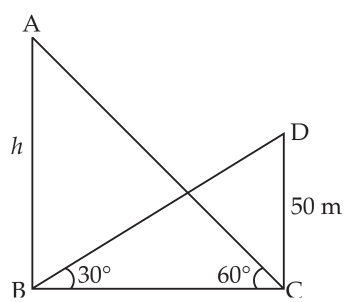
$$= 4500 - 2598 = 1902 \text{ m/hr}$$

Hence, the speed of boat is 1902 m/hr.

Q. 4. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If height of the tower is 50 m, find the height of the hill.

[CBSE Delhi, Term 2, Set 3, 2017]

Ans. Let AB be hill and DC be tower.



From ΔABC, $\frac{AB}{BC} = \tan 60°$

$$\Rightarrow \qquad h = BC\tan 60° = \sqrt{3}\ BC \qquad \text{...(i)}$$

From ΔDBC, $\frac{DC}{BC} = \tan 30° = \frac{1}{\sqrt{3}}$

$$\Rightarrow \qquad BC = \sqrt{3}\ DC = 50\sqrt{3} \qquad \text{...(ii)}$$

Now, $h = 50\sqrt{3} \times \sqrt{3}$

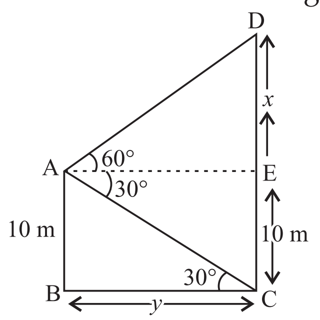
(from equations (i) & (ii))

$= 50 \times 3$

$= 150$ m

Q. 5. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill. [CBSE OD, Term 2, Set 1, 2016]

Ans. Let *AB* be the height of deck of ship from the water level and *CD* be the height of hill.



Then,

In Δ *ABC*,

$$\tan 30° = \frac{10}{y}$$

$$\Rightarrow \quad y = 10\sqrt{3} \quad ...(i)$$

In Δ *ADE*,

$$\tan 60° = \frac{x}{y}$$

$$\Rightarrow \quad y = \frac{x}{\sqrt{3}} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{x}{\sqrt{3}} = 10\sqrt{3}$$

$$x = 10 \times 3 = 30 \text{ m}$$

∴ Distance of the hill from the ship is $10\sqrt{3}$ m and the height of the hill is $30 + 10 = 40$ m.

Q. 6. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (use $\sqrt{3} = 1.73$) [CBSE Delhi, Term 2, Set 1, 2016]

Ans. Let *AB* and *CD* be the tower and high building, respectively.

Given, $CD = 50$ m

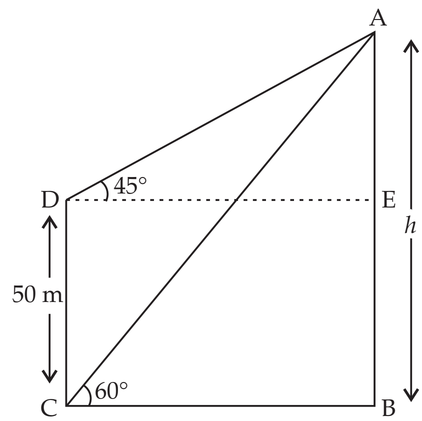
Let, $AB = h$ m

Then, in Δ *ADE*,

$$\tan 45° = \frac{AE}{DE}$$

$$\Rightarrow \quad 1 = \frac{h-50}{DE}$$

$$\Rightarrow \quad DE = h - 50 \quad ...(i)$$



and, in Δ *ACB*,

$$\tan 60° = \frac{AB}{CB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{CB}$$

$$\Rightarrow \quad CB = \frac{h}{\sqrt{3}} \quad ...(ii)$$

Now, $CB = DE$

then, from eq. (i) and (ii), we get

$$h - 50 = \frac{h}{\sqrt{3}}$$

$$\Rightarrow \quad h - \frac{h}{\sqrt{3}} = 50$$

$$\Rightarrow \quad \frac{(\sqrt{3}-1)}{\sqrt{3}} h = 50$$

$$\Rightarrow \quad h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \quad h = \frac{50\sqrt{3}(\sqrt{3}+1)}{3-1}$$

$$\Rightarrow \quad h = \frac{150 + 50\sqrt{3}}{2}$$

$\Rightarrow \quad h = 75 + 25\sqrt{3}$

$\Rightarrow \quad h = 75 + 25\ (1.73)$

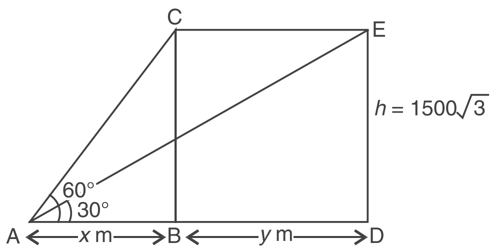
$= 118.25$ m

Hence, the height of the tower is 118.25 m and the horizontal distance between the tower and the building is 68.25 m.

Q. 7. The angle of elevation of an aeroplane from point *A* on the ground is 60°. After flight of 15 seconds, the angle of elevation change to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.

[CBSE OD, Term 2, Set 1, 2015]

Ans.



Let *BC* be the height at which the aeroplane flying.

Then, $\quad BC = 1500\sqrt{3}$ m

In 15 seconds, the aeroplane moves from *C* to *E* and makes angle of elevation 30°.

Let $AB = x$ m, $BD = y$ m

So, $\quad AD = (x + y)$ m

In $\Delta\ ABC$,

$$\tan 60^\circ = \frac{BC}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{1500\sqrt{3}}{x}$$

$[\because \tan 60^\circ = \sqrt{3}\,]$

$\Rightarrow \quad x = 1500$ m $\quad \ldots$(i)

In $\Delta\ EAD$

$$\tan 30^\circ = \frac{ED}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{x+y} \left[\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$$

$\Rightarrow \quad x + y = 1500 \times 3$

$\Rightarrow \quad y = 4500 - 1500 = 3000$ m

[Using equation (i)]

$$\text{Speed of aeroplane} = \frac{\text{Distance}}{\text{time}} = \frac{3000}{15}$$

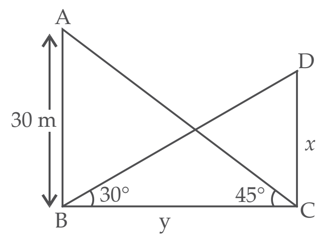
$= 200$ m/s or 720 km/hr

Q. 8. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. Let *AB* be the tower and *CD* be a building of height 30 m and *x* m respectively.

Let the distance between the two be *y* m.



Then, in ΔABC

$$\frac{30}{y} = \tan 45^\circ$$

$$\frac{30}{y} = 1 \Rightarrow y = 30$$

And, in ΔBDC

$$\frac{x}{y} = \tan 30^\circ$$

$$x = y \tan 30^\circ$$

$$x = 30 \times \frac{1}{\sqrt{3}} = 10\sqrt{3}$$

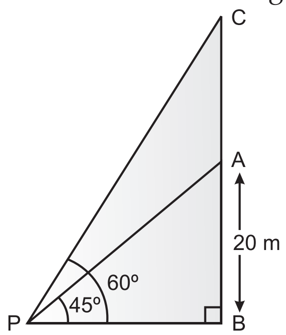
Hence, the height of the building is $10\sqrt{3}$ m.

Long Answer Type Questions (4 marks each)

Q. 1. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

[CBSE OD, Set 1, 2020]

Ans. Let the building be AB, tower be CA and the point of observation on the ground be P.



$\therefore$ AB = 20 m, $\angle APB = 45°$ and $\angle CAB = 60°$

Now, in right triangle APB,

$$\tan 45° = \frac{AB}{PB}$$

$$\Rightarrow \quad 1 = \frac{20}{PB}$$

$$\Rightarrow \quad PB = 20$$

Similarly, in right triangle CPB

$$\tan 60° = \frac{BC}{PB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{BC}{20}$$

$$\Rightarrow \quad BC = 20\sqrt{3}$$

Now, $AC = BC - AB = 20\sqrt{3} - 20$

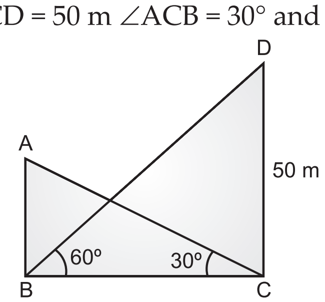
$$= 20(\sqrt{3} - 1)$$

Hence, the height of the tower is $20(\sqrt{3} - 1)$ m. **Ans.**

Q. 2. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of a tower from the foot of the building is 60°. If the tower is 50 m high, then find the height of the building. [CBSE OD, Set 2, 2020]

Ans. Let AB be the building and CD be the tower.

$\therefore$ CD = 50 m $\angle ACB = 30°$ and $\angle DBC = 60°$.



Now, in a right angled triangle DBC,

$$\tan 60° = \frac{DC}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{50}{BC}$$

$$\Rightarrow \quad BC = \frac{50}{\sqrt{3}}$$

And, in a right angled triangle ABC

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{50/\sqrt{3}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = AB \times \frac{\sqrt{3}}{50}$$

$$\Rightarrow \quad AB = \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}} = \frac{50}{3}$$

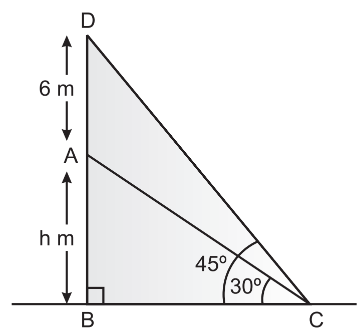
$$= 16.67 \text{ m.}$$

Hence, the height of the building is 16.67 m. **Ans.**

Q. 3. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3}$ = 1.73) [CBSE Delhi, Set 1, 2020]

Ans. Let the height of the tower AB = h.

and height of the flag-staff AD = 6 m.



Now, In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{BC}$$

$$\therefore \quad BC = h\sqrt{3} \text{ m} \qquad \text{...(i)}$$

In ΔDBC,

$$\tan 45° = \frac{DB}{BC}$$

$$\Rightarrow \quad 1 = \frac{AD + AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{(6+h)}{BC}$$

$$\Rightarrow \quad BC = (6+h)\text{m} \qquad ...(ii)$$

From (i) and (ii), we get

$$h\sqrt{3} = (6+h)$$

$$h\sqrt{3} - h = 6$$

$$h(\sqrt{3}-1) = 6$$

$$h = \frac{6}{\sqrt{3}-1}$$

$$= \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{6(\sqrt{3}+1)}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{6(\sqrt{3}+1)}{3-1} = \frac{6(\sqrt{3}+1)}{2}$$

$$= 3(\sqrt{3}+1)$$

= 3 (1.73 + 1)

= 3(2.73)

= 8.19 m

∴ 8.19 m is the height of the tower. **Ans.**

Q. 4. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

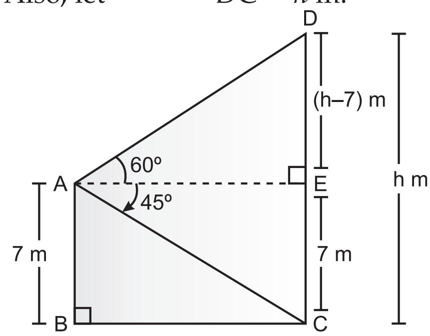
[CBSE Delhi, Term 2, Set 2, 2017]

[CBSE Delhi, Set 3, 2020]

Ans. Let AB be the building and CD be the tower.

AB = 7 m

Also, let DC = h m.



Now, in ΔAEC, we have

$$\tan 45° = \frac{EC}{AE}$$

$$\Rightarrow \quad 1 = \frac{7}{AE}$$

$$\Rightarrow \quad AE = 7$$

In ΔADE, we have

$$\tan 60° = \frac{DE}{AE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h-7}{7}$$

$$\Rightarrow \quad 7\sqrt{3} = h - 7$$

$$\Rightarrow \quad h = 7\sqrt{3} + 7$$

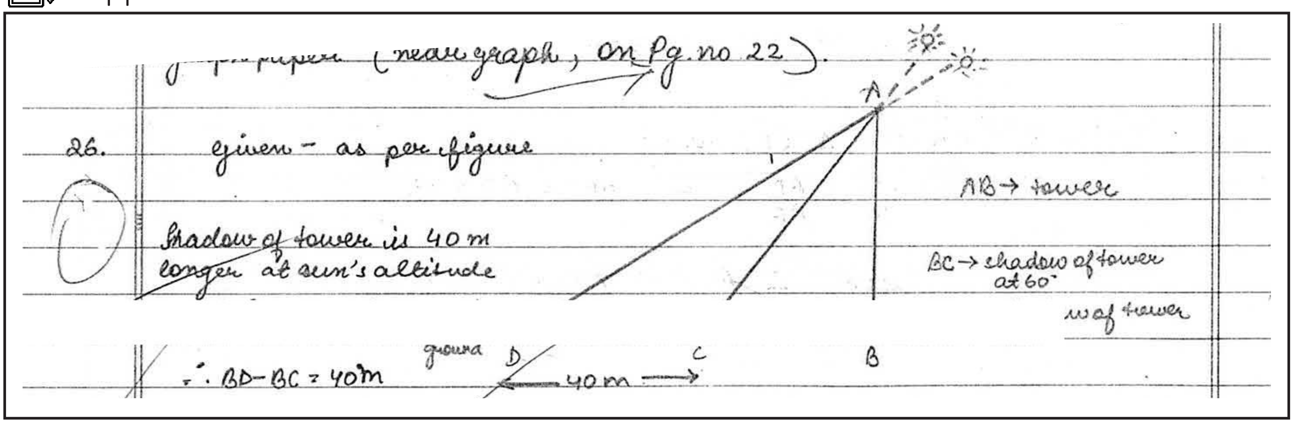
$$\Rightarrow \quad h = 7(\sqrt{3}+1)$$

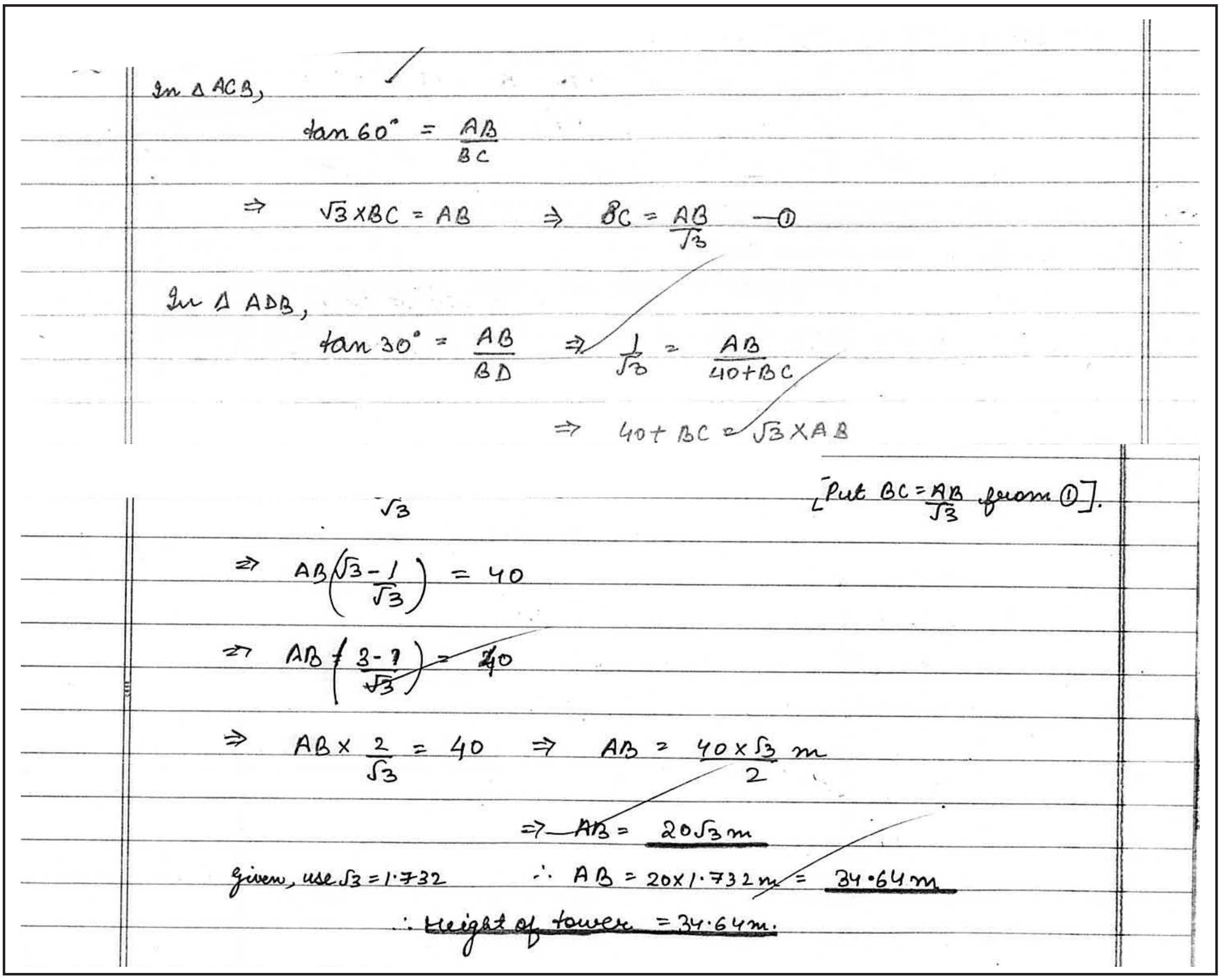
∴ The height of the tower is $7(\sqrt{3}+1)$ meters. **Ans.**

Q. 5. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower. (Given $\sqrt{3}$ = 1.732)

[CBSE, 2019]

Topper's Answers





In Δ ACB,

$\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} \times BC = AB \Rightarrow BC = \frac{AB}{\sqrt{3}}$ —(1)

In Δ ADB,

$\tan 30^\circ = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{40+BC}$

$\Rightarrow 40 + BC = \sqrt{3} \times AB$

$\sqrt{3}$ [Put $BC = \frac{AB}{\sqrt{3}}$ from (1)].

$\Rightarrow AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 40$

$\Rightarrow AB\left(\frac{3-1}{\sqrt{3}}\right) = 40$

$\Rightarrow AB \times \frac{2}{\sqrt{3}} = 40 \Rightarrow AB = \frac{40 \times \sqrt{3}}{2}$ m

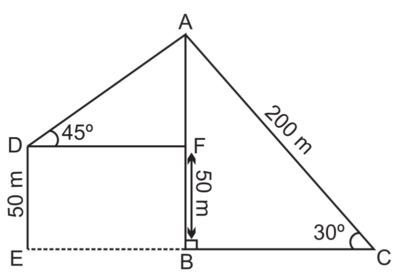
$\Rightarrow AB = 20\sqrt{3}$ m

given, use $\sqrt{3} = 1.732$ ∴ $AB = 20 \times 1.732$ m $= 34.64$ m

∴ Height of tower $= 34.64$ m.

Q. 6. Amit, standing on a horizontal plane, finds a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, finds the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak. [CBSE OD, Set 1, 2019]

Ans. Let Amit be at *C* point and bird is at *A* point, such that $\angle ACB = 30^\circ$, *AB* is the height of bird from point *B* on ground and Deepak is at *D* point, *DE* is the building of height 50 m.



Now, In right triangle *ABC*, we have

$$\sin 30^\circ = \frac{P}{H} = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{200}$$

$$AB = 100 \text{ m}$$

In right Δ*AFD*, we have

$$\sin 45^\circ = \frac{P}{H} = \frac{AF}{AD}$$

$$\frac{1}{\sqrt{2}} = \frac{50}{AD}$$

$$\begin{bmatrix} \because AB = AF + BF \\ \Rightarrow 100 = AF + 50 \\ \Rightarrow AF = 50 \end{bmatrix}$$

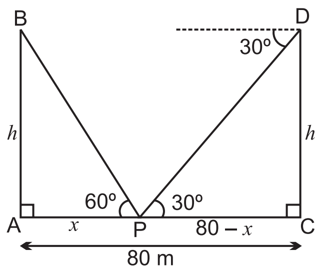
$$AD = 50\sqrt{2} \text{ m}$$

Hence, the distance of bird from Deepak is $50\sqrt{2}$ m.

Q. 7. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point *P* between them on the road, the angle of elevation of the top of a pole is 60° and the angle of depression from the top of the other pole of point *P* is 30°. Find the heights of the poles and the distance of the point *P* from the poles.

[CBSE OD, Set 2, 2019]

Ans. Let *AC* is road of 80 m width, *P* is the point on road *AC* and height of poles *AB* and *CD* is *h* m.



From right Δ*PAB*, we have

$$\frac{AB}{AP} = \tan 60° = \sqrt{3}$$

$$\frac{h}{x} = \sqrt{3} \qquad (\because AP = x)$$

$$h = \sqrt{3}\,x \qquad ...(i)$$

From right Δ*DCP*, we have

$$\frac{CD}{PC} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\frac{h}{80-x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{80-x}{\sqrt{3}} \qquad ...(ii)$$

Equating the values of *h* from equation (i) and (ii) we get

$$\Rightarrow \quad x\sqrt{3} = \frac{80-x}{\sqrt{3}}$$

$$\Rightarrow \quad 3x = 80 - x$$

$$\Rightarrow \quad 4x = 80$$

$$\Rightarrow \quad x = 20 \text{ m}$$

On putting $x = 20$ in equation (i), we get

$$h = \sqrt{3} \times 20 = 20\sqrt{3}$$

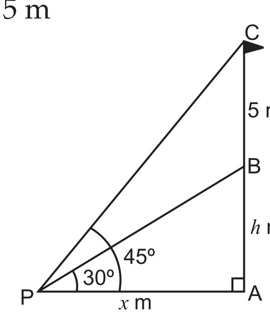
Thus, height of poles is $20\sqrt{3}$ m and point P is at a distance of 20 m from left pole and (80 – 20) i.e., 60 m from right pole.

Q. 8. From a point *P* on the ground, the angle of elevation of the top of a tower is 30° and that of the top of the flag-staff fixed on the top of the tower is 45°. If the length of the flag-staff is 5 m, find the height of the tower. (Use $\sqrt{3} = 1.732$)

[CBSE OD, Set 3, 2019]

Ans. Let *AB* be the tower and *BC* be the flag-staff.

So, *BC* = 5 m



Let *P* be a point on the ground such that $\angle APB = 30°$ and $\angle APC = 45°$, *BC* = 5 m.

Let *AB* = *h* m and *PA* = *x* metres

From right Δ*PAB*, we have

$$\cot 30° = \frac{x}{h} = \frac{PA}{AB}$$

$$\sqrt{3} = \frac{x}{h}$$

$$\Rightarrow \quad x = \sqrt{3}\,h \qquad ...(i)$$

From right Δ*PAC*, we have

$$\cot 45° = \frac{PA}{AC} = \frac{x}{h+5}$$

$$x = h + 5 \qquad ...(ii)$$

Equating the values of *x* from equations (i) and (ii), we get

$$\sqrt{3}\,h = h + 5$$

$$\sqrt{3}\,h - h = 5$$

$$h(\sqrt{3} - 1) = 5$$

$$h = \frac{5}{\sqrt{3}-1} = \frac{5}{1.732-1}$$

$$= \frac{5}{0.732}$$

$$= \frac{5000}{732}$$

$$= 6.83 \text{ m}$$

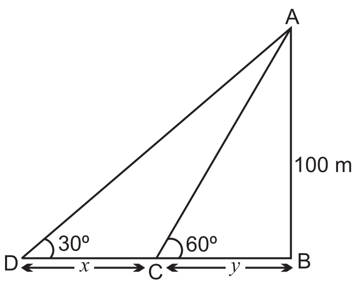
Hence, the height of tower is 6.83 m.

Q. 9. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$]

[CBSE Delhi, Set 1, 2019]

Ans. Let AB be the light house, C and D be the two positions of the boat, such that,

$$CD = x \text{ m and } BC = y \text{ m}$$



Now, In ΔABC

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{100}{y}$$

$$y = \frac{100}{\sqrt{3}} \quad \text{...(i)}$$

In ΔABD

$$\tan 30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{x+y}$$

$$\Rightarrow \quad x + y = 100\sqrt{3}$$

$$\text{or} \quad y = 100\sqrt{3} - x \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{100}{\sqrt{3}} = 100\sqrt{3} - x$$

$$\Rightarrow \quad x = 100\sqrt{3} - \frac{100}{\sqrt{3}}$$

$$= 100\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$= 100 \times \frac{2}{\sqrt{3}} = \frac{200}{\sqrt{3}}$$

$$= 115.48 \text{ metres}$$

$\because$ Time taken to cover 115.48 m = 2 min

$\therefore$ Speed of boat $= \frac{115{\cdot}48}{2} = 57.74$ m/min

Hence, speed of boat = 57.74 m/min

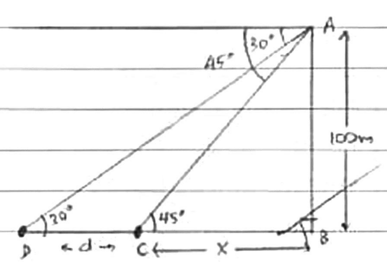
Q. 10. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

[CBSE, 2018]

Ans.

Topper's Answers

29) Diagram:



AB→ lighthouse = 100m high

C→ boat 1

D→ boat 2.

To find: $\overline{CD}$ or d.

(distance b/w ships)

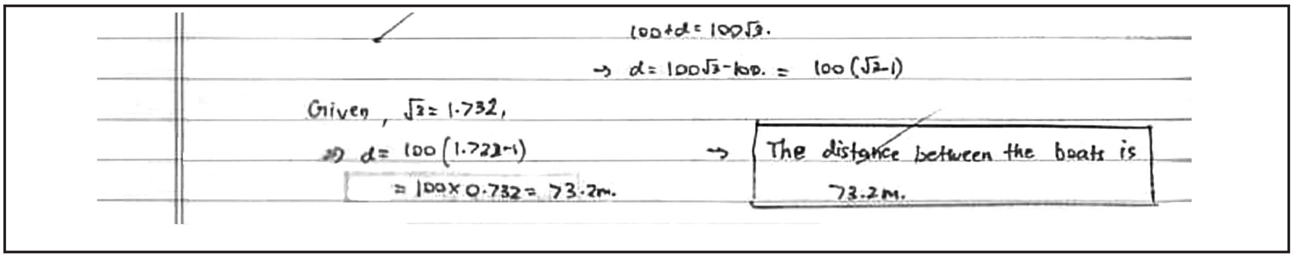
We know,

$\tan\angle ACB = \frac{\text{Opp.}}{\text{adj.}} = \frac{AB}{BC}$. $\quad$ $\tan\angle ADB = \frac{\text{opp.}}{\text{adj.}} = \frac{AB}{BD}$.

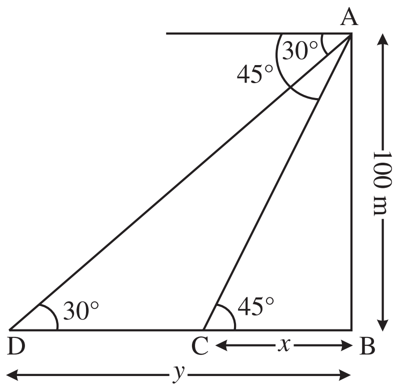
$\rightarrow \tan 45° = \frac{100}{x}$ $\quad$ $\rightarrow \tan 30 = \frac{100}{x+d}$

$1 = \frac{100}{x}$ $\quad$ $\frac{1}{\sqrt{3}} = \frac{100}{d+100}$ [x ....]

$\Rightarrow x = 100$ m.


100+d = 100√3.
⟹ d = 100√3 − 100. = 100 (√3 − 1)
Given, √3 = 1.732,
⟹ d = 100 (1.732 − 1)
= 100 × 0.732 = 73.2m.
⟹ The distance between the boats is 73.2m.

Let AB be the light house and two ships be at C and D.



In ΔABC,

$$\frac{BC}{AB} = \cot 45°$$

$$\Rightarrow \quad \frac{x}{100} = 1$$

$$\Rightarrow \quad x = 100 \quad \text{...(i)}$$

Similarly, in ΔABD,

$$\frac{BD}{AB} = \cot 30°$$

$$\Rightarrow \quad \frac{y}{100} = \sqrt{3}$$

$$\Rightarrow \quad y = 100\sqrt{3} \quad \text{...(ii)}$$

Distance between two ships $= y - x$

$= 100\sqrt{3} - 100$

[from equation (i) and (ii)]

$= 100(\sqrt{3} - 1)$

$= 100\,(1.732 - 1)$

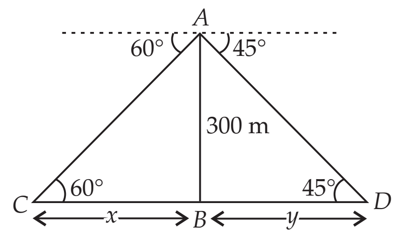
$= 100\,(0.732)$

$= 73.2$ m

Q. 11. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use $\sqrt{3} = 1.732$]

[CBSE OD, Term 2, Set 1, 2017]

Ans. Let aeroplane is at A, 300 m high from a river, C and D are opposite banks of river.



In right ΔABC,

$$\frac{BC}{AB} = \cot 60°$$

$$\Rightarrow \quad \frac{x}{300} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$= 100\sqrt{3}$ m

$= 100 \times 1.732 = 173.2$ m

In right ΔABD,

$$\frac{BD}{AB} = \cot 45°$$

$$\Rightarrow \quad \frac{y}{300} = 1 \Rightarrow y = 300 \text{ m}$$

Width of river $= x + y$

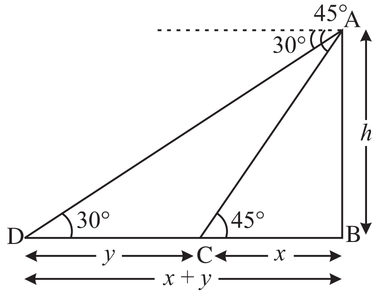
$= 173.2 + 300$

$= 473.2$ m

Q. 12. A man observes a car from the top of a tower, which is moving towards the tower with a uniform speed. If the angle of depression of the car changes from 30° to 45° in 12 minutes, find the time taken by the car now to reach the tower.

[CBSE OD, Term 2, Set 2, 2017]

Ans. Let AB is a tower, car is at point D at 30° and goes to C at 45° in 12 minutes.



In Δ ABC,

$$\frac{AB}{BC} = \tan 45°$$

$$\Rightarrow \quad \frac{h}{x} = 1 \Rightarrow h = x \quad \text{...(i)}$$

In Δ ABD,

$$\frac{AB}{BD} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{x+y} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{x+y}{\sqrt{3}} \quad \text{...(ii)}$$

From eq. (i) & (ii), we get

$$x = \frac{x+y}{\sqrt{3}} \Rightarrow \sqrt{3}x = x + y$$

$$\Rightarrow \quad (\sqrt{3}-1)\,x = y$$

Car covers the distance y in time = 12 min
So $(\sqrt{3}-1)$ x distance covers in 12 min

Distance x covers in time $= \dfrac{12}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$$= \frac{12(\sqrt{3}+1)}{3-1}$$

$$= 6\,(\sqrt{3}+1)$$

$$= 6 \times 2.732$$

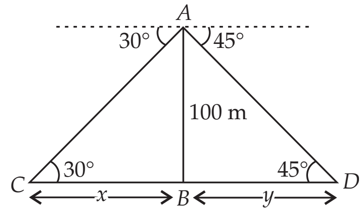
$$= 16.39$$

Now, car reaches to tower in 16.39 minutes.

Q. 13. From the top of a tower, 100 m high, a man observes two cars on the opposite sides of the tower and in same straight line with its base, with angles of depression 30° and 45°. Find the distance between the cars. [Take $\sqrt{3}$ = 1.732]

[CBSE OD, Term 2, Set 3, 2017]

Ans. Let AB is a tower, cars are at point C and D respectively



In Δ ABC,

$$\frac{AB}{BC} = \tan 30°$$

$$\frac{100}{x} = \frac{1}{\sqrt{3}}$$

$$x = 100\sqrt{3}$$

$$= 100 \times 1.732$$

$$= 173.2 \text{ m}$$

In Δ ABD,

$$\frac{AB}{BD} = \tan 45°$$

$$\frac{100}{y} = 1$$

$$y = 100 \text{ m}$$

Distance between two cars = $x + y$

$$= 173.2 + 100$$

$$= 273.2 \text{ m}$$

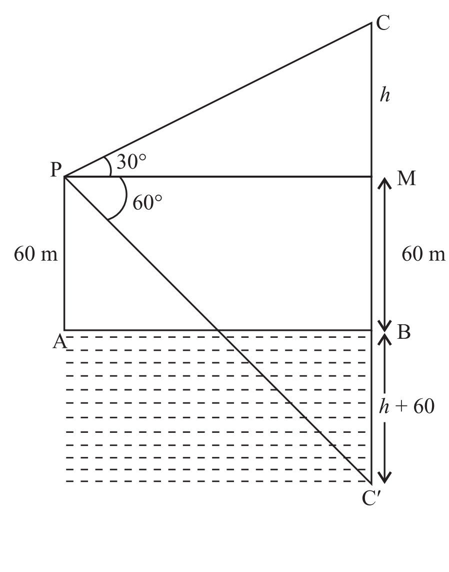
Q. 14. The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water.

[CBSE Delhi, Term 2, Set 1, 2017]

Ans. In Δ CMP,

$$\tan 30° = \frac{CM}{PM}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{PM} \text{ or } PM = \sqrt{3}h \quad \text{...(i)}$$



In ΔPMC,

$$\tan 60° = \frac{C'M}{PM}$$

$$\sqrt{3} = \frac{h+60+60}{PM}$$

or $$PM = \frac{h+120}{\sqrt{3}} \quad ...(ii)$$

From equation (i) and (ii)

$$\sqrt{3}\,h = \frac{h+120}{\sqrt{3}}$$

$\Rightarrow$ $3h = h + 120$

$\Rightarrow$ $2h = 120$

$\Rightarrow$ $h = 60$ m

Height of cloud from surface of water

$= h + 60$

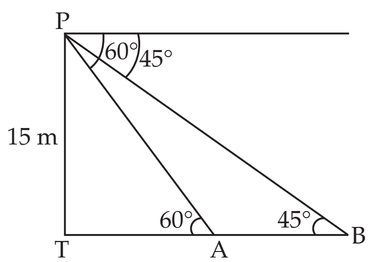
$= 60 + 60$

$= 120$ m.

Q. 15. Two points *A* and *B* are on the same side of a tower and in the same straight line with its base. The angles of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

[CBSE Delhi, Term 2, Set 2, 2017]

Ans. Let *PT* be tower



From ΔPTA,

$$\tan 60° = \frac{PT}{TA} \Rightarrow TA = \frac{15}{\sqrt{3}}$$

From ΔPTB,

$$\tan 45° = \frac{PT}{TB} \Rightarrow TB = PT = 15 \text{ m}$$

Distance between two points

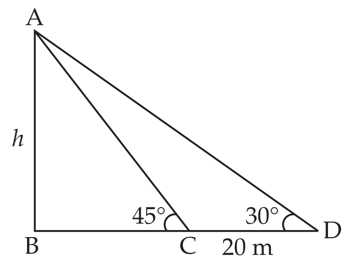
$$AB = TB - TA$$

$$= 15 - \frac{15}{\sqrt{3}} = \frac{15(\sqrt{3}-1)}{\sqrt{3}} \text{ m}$$

Q. 16. An observer finds the angle of elevation of the top of the tower from a certain point on the ground as 30°. If the observer moves 20 m towards the base of the tower, the angle of elevation of the top increases by 15°, find the height of the tower.

[CBSE Delhi, Term 2, Set 3, 2017]

Ans. Let *AB* be tower of height *h*.



From ΔABC, $\frac{AB}{BC} = \tan 45°$

$\Rightarrow$ $h = BC$

From ΔABD, $\frac{AB}{BD} = \tan 30°$

$\Rightarrow$ $h = \frac{BD}{\sqrt{3}}$ or $BD = \sqrt{3}\,h$

Now, $CD = BD - BC$

$= \sqrt{3}h - h = (\sqrt{3}-1)h$

$\Rightarrow$ $20 = (\sqrt{3}-1)h$

$\Rightarrow$ $h = \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$$= \frac{20(\sqrt{3}+1)}{(\sqrt{3})^2-(1)^2}$$

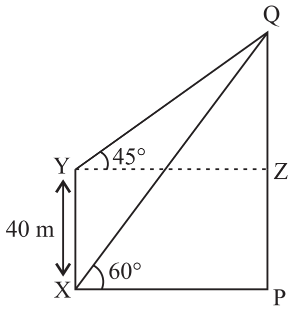
$$= \frac{20(\sqrt{3}+1)}{2}$$

$$= 10(\sqrt{3}+1) \text{ m}$$

Q. 17. The angle of elevation of the top *Q* of a vertical tower *PQ* from a point *X* on the ground is 60°. From a point *Y*, 40 m vertically above *X*, the angle of elevation of the top *Q* of tower is 45°. Find the height of the tower *PQ* and the distance *PX*. (Use $\sqrt{3} = 1.73$)

[CBSE OD, Term 2, Set 1, 2016]

Ans. We have, PQ as a vertical tower



In ΔYZQ,

$$\tan 45° = \frac{QZ}{YZ}$$

$$\Rightarrow \quad \frac{QZ}{YZ} = 1$$

$$\Rightarrow \quad QZ = YZ \quad ...(i)$$

And, in $\Delta\ XPQ$,

$$\tan 60° = \frac{QP}{XP}$$

$$\Rightarrow \quad \sqrt{3} = \frac{QZ + 40}{XP}$$

$$\Rightarrow \quad \sqrt{3} = \frac{QZ + 40}{YZ} \quad (\because XP = YZ)$$

$$\Rightarrow \quad \sqrt{3}\,QZ = QZ + 40 \quad \text{[Using (i)]}$$

$$\Rightarrow \quad \sqrt{3}\,QZ - QZ = 40$$

$$\Rightarrow \quad QZ\ (\sqrt{3} - 1) = 40$$

$$\Rightarrow \quad QZ = \frac{40}{\sqrt{3}-1} = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 20\ (\sqrt{3} + 1)$$

$$= 20\ (2.73)$$

$$= 54.60 \text{ m}$$

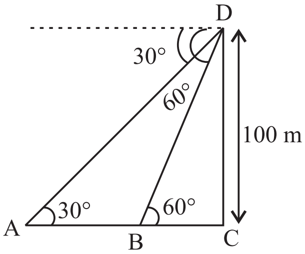
$$\therefore \quad PX = 54.6 \text{ m}$$

And $PQ = (54.6 + 40)$ m $= 94.6$ m.

Q. 18. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3} = 1.73$)

[CBSE OD, Term 2, Set 2, 2016]

Ans. Let CD be a light house of length 100 m and A & B be the positions of ship sailing towards it.



Then, in $\Delta\ CBD$,

$$\tan 60° = \frac{CD}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{100}{BC}$$

$$\Rightarrow \quad BC = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3} \quad ...(i)$$

And, in $\Delta\ CAD$,

$$\tan 30° = \frac{CD}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{AC}$$

$$\Rightarrow \quad AC = 100\sqrt{3} \quad ...(ii)$$

$\therefore$ Distance travelled by the ship (AB)

$$= AC - BC$$

$$= 100\sqrt{3} - \frac{100\sqrt{3}}{3}$$

[from equation (i) & (ii)]

$$= 100\sqrt{3}\left(\frac{3-1}{3}\right)$$

$$= \frac{200 \times 1.73}{3}$$

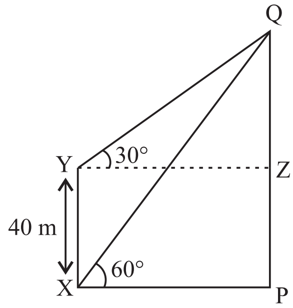
$$= 115.33 \text{ m}$$

Q. 19. From a point on the ground, the angle of elevation of the top of a tower is observed to be 60°. From a point 40 m vertically above the first point of observation, the angle of elevation of the top of the tower is 30°. Find the height of the tower and its horizontal distance from the point of observation.

[CBSE OD, Term 2, Set 3, 2016]

Ans. We have, PQ as a vertical tower.

Now, in ΔYZQ



$$\tan 30° = \frac{QZ}{YZ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{QZ}{YZ}$$

$$\Rightarrow \quad YZ = QZ\sqrt{3} \quad ...(i)$$

And, in $\Delta\ XPQ$,

$$\tan 60° = \frac{QP}{XP}$$

$$\Rightarrow \quad \sqrt{3} = \frac{QZ + 40}{XP}$$

$$\Rightarrow \quad YZ\sqrt{3} = QZ + 40$$

$(\because XP = YZ)$

$$\Rightarrow \quad QZ\sqrt{3}(\sqrt{3}) = QZ + 40$$

[using equation (i)]

$$\Rightarrow \quad 3QZ = QZ + 40$$

$$\Rightarrow \quad 2QZ = 40$$

$$\therefore \quad QZ = 20$$

$\therefore$ Height of tower $= (40 + 20)$ m $= 60$ m

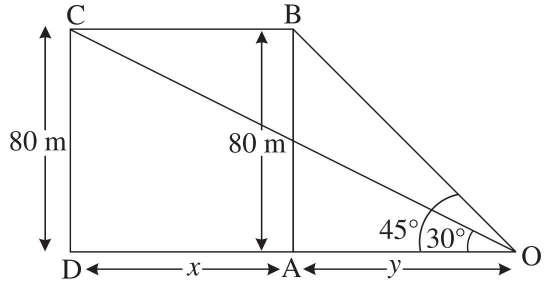
and Horizontal distance $= QZ\sqrt{3}$

$= 20\sqrt{3}$ m

Q. 20. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3} = 1.732$)

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Let *B* be the initial position of bird sitting on top of tree of length 80 m.



After 2 sec, the position of bird becomes *C*. Let the distance travel by bird from *B* to *C* is *x* m.

Now, in $\Delta\ ABO$

$$\tan 45° = \frac{AB}{AO} = \frac{80}{y}$$

$$\Rightarrow \quad y = 80 \text{ m} \quad ...(i)$$

And, in $\Delta\ DCO$,

$$\tan 30° = \frac{CD}{DO} = \frac{80}{x+y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{x+80}$$

[Using equation (i)]

$$\Rightarrow \quad x + 80 = 80\sqrt{3}$$

$$\Rightarrow \quad x = 80\,(\sqrt{3} - 1)$$

$$= 80 \times 0.732$$

$$\therefore \quad x = 58.56 \text{ m}$$

Hence, speed of flying of the bird $= \frac{58.56}{2}$

$$\left(\because \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right)$$

$$= 29.28 \text{ m/s}$$

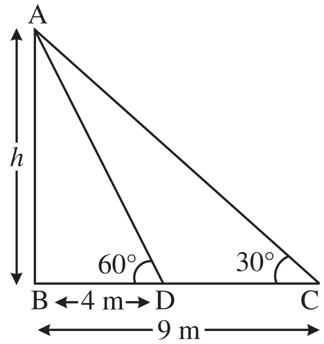
Q. 21. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are 60° and 30° respectively. Find the height of the tower.

[CBSE Delhi, Term 2, Set 2, 2016]

Ans. Let length of tower is *h*

In $\Delta\ ABD$,

$$\tan 60° = \frac{h}{4} \quad ...(i)$$



In Δ ABC,

$$\tan 30° = \frac{h}{9}$$

$$\Rightarrow \quad \cot(90° - 30°) = \frac{h}{9}$$

$$\Rightarrow \quad \cot 60° = \frac{h}{9} \quad \text{...(ii)}$$

Multiplying equation (i) and (ii), we get

$$\tan 60° \cdot \cot 60° = \frac{h}{4} \times \frac{h}{9}$$

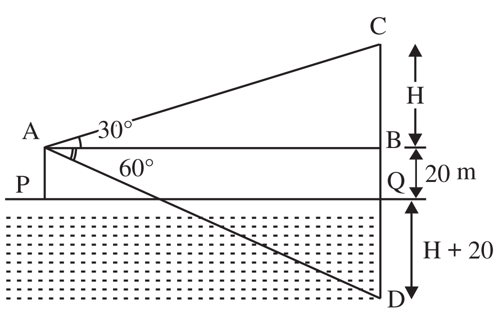
$$\Rightarrow \quad 1 = \frac{h^2}{36}$$

$$\therefore \quad h = 6 \text{ m}$$

Note: In this question, it has not been specified whether two points from tower are taken in same or opposite side we have taken these points on the same side of tower.

Q. 22. At a point *A*, 20 m above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at *A* is 60°. Find the distance of the cloud from *A*.
[CBSE OD, Term 2, Set 1, 2015]

Ans.



Let PQ be the surface of the lake, A is the point vertically above P such at $AP = 20$m.

Let C be the position of the cloud and D be its reflection in the lake.

Let $\quad BC = H$ metres

Now, In Δ ABD,

$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{H + 20 + 20}{AB}$$

$$\Rightarrow \quad \sqrt{3}.AB = H + 40$$

$$\Rightarrow \quad AB = \frac{H + 40}{\sqrt{3}} \quad \text{...(i)}$$

And in Δ ABC,

$$\tan 60° = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{H}{AB}$$

$$\Rightarrow \quad AB = \sqrt{3}H \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{H + 40}{\sqrt{3}} = \sqrt{3}H$$

$$\Rightarrow \quad 3H = H + 40$$

$$\Rightarrow \quad 2H = 40 \Rightarrow H = 20$$

Putting the value of H in equation (ii), we get

$$AB = 20\sqrt{3}$$

Again, in Δ ABC

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= (20\sqrt{3}^2) + (20)^2$$

$$= 1200 + 400$$

$$= 1600$$

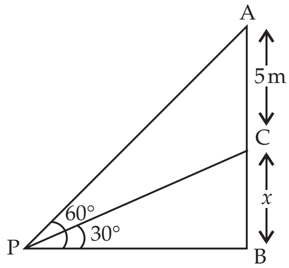
$$\Rightarrow \quad AC = \sqrt{1600} = 40$$

Hence, the distance of cloud from A is 40 m.

Q. 23. From a point *P* on the ground the angle of elevation of the top of a tower is 30° and that of the top of a flag staff fixed on the top of the tower, is 60°. If the length of the flag staff is 5 m, find the height of the tower.
[CBSE Delhi, Term 2, Set 1, 2015]

Ans. Let CB be the tower of x m and AC be the flag staff of 5 m.

Then, in $\Delta\, CPB$



$$\tan 30° = \frac{x}{PB}$$

$$\Rightarrow \quad PB = \frac{x}{\tan 30°} = \sqrt{3}x \quad \text{...(i)}$$

In $\Delta\, APB$,

$$\tan 60° = \frac{5+x}{PB}$$

$$\Rightarrow \quad PB = \frac{5+x}{\sqrt{3}} \quad \text{...(ii)}$$

From eq. (i) and (ii),

$$\sqrt{3}x = \frac{x+5}{\sqrt{3}}$$

$$\Rightarrow \quad 3x = x + 5$$

$$\Rightarrow \quad 3x - x = 5$$

$$\Rightarrow \quad 2x = 5$$

$$\Rightarrow \quad x = 5/2 = 2.5$$

$\because$ Height of the tower is 2.5 m.