ASSERTION & REASONING QUESTIONS

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

1. Assertion: If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$ then the value of $x + y = 3$.
Reason : For any value of θ, $\sin^2\theta + \cos^2\theta = 1$

Ans: We know that for any value of θ, $\sin^2\theta + \cos^2\theta = 1$
So, Reason is correct.
For assertion: We have $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$
Then, $x + y = 2\sin^2\theta + 2\cos^2\theta + 1$
$= 2(\sin^2\theta + \cos^2\theta) + 1$
$= 2 \times 1 + 1$ [$\because$ $\sin^2\theta + \cos^2\theta = 1$]
$= 2 + 1 = 3$.
Hence, Assertion is also correct.
Correct option is (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

2. Assertion: Sin A is the product of Sin and A.
Reason : The value of sinθ increases as θ increases.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Ans: For assertion: Sin A is not the product of Sin and A.
It is the Sine of ∠A.
∴ Assertion is not correct.
For reason: The value of sinθ increases as θ increases in interval of $0° < \theta < 90°$
So, Reason is correct.
Correct option is (d) Assertion (A) is false but reason (R) is true.

3. Assertion: ($\cos^4 A - \sin^4 A$) is equal to $2\cos^2 A - 1$.
Reason : The value of $\cos\theta$ decreases as θ increases.

Ans: For reason: The value of $\cos\theta$ decreases as θ increases in interval of $0° < \theta < 90°$
So, Reason is also correct.
We have, $(\cos^4 A - \sin^4 A) = (\cos^2 A)^2 - (\sin^2 A)^2$
$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$
$= (\cos^2 A - \sin^2 A)(1)$
$= \cos^2 A - (1 - \cos^2 A)$
$= 2\cos^2 A - 1$
So, Assertion is also correct.
But reason (R) is not the correct explanation of assertion (A)
Correction option is (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

4. Assertion: In a right ΔABC, right-angled at B, if tanA = 1, then 2sinA·cosA = 1.
Reason : CosecA is the abbreviation used for Cosecant of angle A.

Ans: We know that CosecA is the abbreviation used for Cosecant of angle A.
So, Reason is correct
For Assertion, we have, tanA = 1
$\Rightarrow \frac{\sin A}{\cos A} = 1$
$\Rightarrow \sin A = \cos A$
$\Rightarrow \sin A - \cos A = 0$
Squaring both sides, we get $(\sin A - \cos A)^2 = 0$
$\Rightarrow \sin^2 A + \cos^2 A - 2\sin A \cdot \cos A = 0$
$\Rightarrow 1 - 2\sin A \cdot \cos A = 0 \Rightarrow 2\sin A \cdot \cos A = 1$
So, Assertion is also correct.
But reason (R) is not the correct explanation of assertion (A)
Correction option is (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

5. Assertion: In a right ΔABC, right-angled at B, if tanA = $\frac{12}{5}$, then secA = $\frac{13}{5}$.

Reason : CotA is the product of Cot and A.

Ans: We know that CotA is the abbreviation used for Cotangent of angle A.

So, Reason is not correct

For Assertion, we have, tanA = $\frac{12}{5}$

We know that $sec^2A = 1 + tan^2A$

$\Rightarrow sec^2A = 1 + \left(\frac{12}{5}\right)^2 \Rightarrow sec^2A = 1 + \frac{144}{25} \Rightarrow sec^2A = \frac{169}{25}$

$\Rightarrow secA = \frac{13}{5}$

So, Assertion is correct

Correction option is (c) Assertion (A) is true but reason (R) is false.

6. Assertion : If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ and $x \sin\theta = y \cos\theta$, then $x^2 + y^2 = 1$
Reason : For any value of θ, $\sin^2\theta + \cos^2\theta = 1$

Ans: We know that for any value of θ, $\sin^2\theta + \cos^2\theta = 1$
So, Reason is correct.
For assertion: We have $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ and $x \sin\theta = y \cos\theta$
$x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$
$\Rightarrow x \sin \theta (\sin^2 \theta) + y \cos \theta (\cos^2 \theta) = \sin \theta \cos \theta$
$\Rightarrow x \sin \theta (\sin^2 \theta) + x \sin \theta (\cos^2 \theta) = \sin \theta \cos \theta$
$\Rightarrow x \sin \theta (\sin^2 \theta + \cos^2 \theta) = \sin \theta \cos \theta \Rightarrow x \sin \theta = \sin \theta \cos \theta \Rightarrow x = \cos \theta$
Now, $x \sin \theta = y \cos \theta \Rightarrow \cos \theta \sin \theta = y \cos \theta \Rightarrow y = \sin \theta$
Hence, $x^2 + y^2 = \cos^2 \theta + \sin^2 \theta = 1$
So, Assertion is also correct.
Correct option is (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

7. Assertion : sin (A + B) = sin A + sin B.
Reason : For any value of θ, $1 + \tan^2\theta = \sec^2\theta$

Ans: We know that for any value of θ, $1 + \tan^2\theta = \sec^2\theta$
So, Reason is correct.
For assertion: Let $A = 30^0$ and $B = 60^0$
LHS = sin (A + B) = sin $(30^0 + 60^0)$
= sin 900 = 1
RHS = sinA + sinB = = $\sin 30^0 + \sin 60^0$
$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$
⇒ LHS ≠ RHS
So, Assertion is not correct.

Correct option is (d) Assertion (A) is false but reason (R) is true.

8. Assertion: The value of sin 60° cos 30° + sin 30° cos 60° is 1.
Reason : sin 90° = 1 and cos 90° = 0.

Ans: We know that sin 90° = 1 and cos 90° = 0.
So, Reason is correct
For Assertion, we have, sin 60° cos 30° + sin 30° cos 60°

$$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$=\frac{3}{4}+\frac{1}{4}=\frac{4}{4}=1$$

So, Assertion is also correct.
But reason (R) is not the correct explanation of assertion (A)
Correction option is (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

9. Assertion: The value of $2\tan^2 45° + \cos^2 30° - \sin^2 60°$ is 2.

Reason : Value of $\tan 45° = 1$, $\cos 30° = \frac{\sqrt{3}}{2}$ and $\sin 60° = \frac{\sqrt{3}}{2}$.

Ans: We know that $\tan 45° = 1$, $\cos 30° = \frac{\sqrt{3}}{2}$ and $\sin 60° = \frac{\sqrt{3}}{2}$.

So, Reason is correct

For Assertion, we have, $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2.$$

So, Assertion is also correct.

Correction option is (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

10. Assertion : If $\cos A + \cos^2 A = 1$ then $\sin^2 A + \sin^4 A = 1$.
Reason : $\sin^2 A + \cos^2 A = 1$, for any value of A.

Ans: For Reason, we know that $\sin^2 A + \cos^2 A = 1$, for any value of A.
So, Reason is correct
For Assertion, we have $\cos A + \cos^2 A = 1$
$\Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$ $(\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \cos^2 A = \sin^2 A)$
Now, $\sin^2 A + \sin^4 A$
$= \cos A + \cos^2 A = 1$
So, Assertion is also correct
Correction option is (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

11. Assertion : The value of $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}$ + sinθcosθ is 1

Reason : $\sin^2A + \cos^2A = 1$, for any value of A.

Ans: For Reason, we know that $\sin^2A + \cos^2A = 1$, for any value of A.
So, Reason is correct

For Assertion, we have $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}$ + sinθcosθ

$$= \frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)}{(\sin\theta+\cos\theta)} + \sin\theta\cos\theta \quad [\because (a^3+b^3) = (a+b)(a^2+b^2-ab)]$$

$$= (1-\sin\theta\cos\theta) + \sin\theta\cos\theta \quad (\because \sin^2A + \cos^2A = 1)$$

$$= 1$$

So, Assertion is also correct
Correction option is (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

12. Assertion : The value of $\frac{2\tan 300}{1+\tan^2 30^0}$ is $\frac{\sqrt{3}}{2}$

Reason : cot A is not defined for A = 0°.

Ans: For Reason, we know that cot A is not defined for A = 0°.
So, Reason is correct

For Assertion, we have $\frac{2\tan 300}{1+\tan^2 30^0} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$

So, Assertion is also correct
But reason (R) is not the correct explanation of assertion (A)
Correction option is (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

13. Assertion : The value of sec A (1 – sin A)(sec A + tan A) is 1.
Reason : $1 + \tan^2 A = \sec^2 A$, for any value of A.

Ans: For Reason, we know that $1 + \tan^2 A = \sec^2 A$, for any value of A.
So, Reason is correct
For Assertion, we have sec A (1 – sin A)(sec A + tan A)

$$= \left(\frac{1}{\cos A}\right)(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) = \frac{(1 - \sin A)(1 + \sin A)}{\cos^2 A}$$

$$= \frac{1 - \sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1 \quad (\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A)$$

So, Assertion is also correct
But reason (R) is not the correct explanation of assertion (A)
Correction option is (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

13. Assertion : The value of $\sec A (1 - \sin A)(\sec A + \tan A)$ is 1.
Reason : $1 + \tan^2 A = \sec^2 A$, for any value of A.

Ans: For Reason, we know that $1 + \tan^2 A = \sec^2 A$, for any value of A.
So, Reason is correct
For Assertion, we have $\sec A (1 - \sin A)(\sec A + \tan A)$
$= (\sec A - \sec A.\sin A)(\sec A + \tan A)$
$= (\sec A - \tan A)(\sec A + \tan A)$
$= \sec^2 A - \tan^2 A$
$= 1$
So, Assertion is also correct
Correction option is (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)