UNIT 2: Algebra

CHAPTER

2

POLYNOMIALS

Syllabus

- Zeroes of a polynomial. Relationship between zeroes and coefficients of quadratic polynomials. Statement and simple problems on division algorithm of polynomials with real coefficients.

Trend Analysis

List of Concepts	2018		2019		2020	
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Zeroes of a Polynomial and coefficients of Quadratic Polynomials	1 Q (1 M)		1 Q (3 M)	1 Q (3 M)	2 Q (1 M) 2 Q (3 M)	1 Q (1 M) 1 Q (2 M)
Finding zeroes with Division Method	1 Q (3 M)		2 Q (3 M)	1 Q (3 M)	1 Q (3 M)	

TOPIC - 1

Zeroes of a Polynomial and Coefficients of Quadratic Polynomials

Revision Notes

- **Polynomial:** An algebraic expression in the form of $a_nx^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$, (where n is a whole number and $a_0, a_1, a_2, ..., a_n$ are real numbers) is called a polynomial in one variable x of degree n.
- **Value of a Polynomial at a given point :** If $p(x)$ is a polynomial in x and 'α' is any real number, then the value obtained by putting $x = \alpha$ in $p(x)$, is called the value of $p(x)$ at $x = \alpha$.
- **Zero of a Polynomial:** A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$.

  Geometrically, the zeroes of a polynomial $p(x)$ are precisely the X-co-ordinates of the points, where the graph of $y = p(x)$ intersects the X-axis.

  (i) A linear polynomial has one and only one zero.
  (ii) A quadratic polynomial has at most two zeroes.

(iii) A cubic polynomial has at most three zeroes.
(iv) In general, a polynomial of degree n has at most n zeroes.

- **Graphs of Different types of Polynomials:**
  - **Linear Polynomial:** The graph of a linear polynomial $p(x) = ax + b$ is a straight line that intersects X-axis at one point only.
  - **Quadratic Polynomial: (i)** Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens upwards, if $a > 0$ and intersects X-axis at a maximum of two distinct points.

    (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens downwards, if $a < 0$ and intersects X-axis at a maximum of two distinct points.
  - **Cubic polynomial:** Graph of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ intersects X-axis at a maximum of three distinct points.

- **Relationship between the Zeroes and the Coefficients of a Polynomial :**

(i)
$$\text{Zero of a linear polynomial} = \frac{(-1)^1 \text{Constant term}}{\text{Coefficient of } x}$$

If $ax + b$ is a given linear polynomial, then zero of linear polynomial is $\frac{-b}{a}$

(ii) In a quadratic polynomial,

$$\text{Sum of zeroes of a quadratic polynomial} = \frac{(-1)^1 \text{ Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes of a quadratic polynomial} = \frac{(-1)^2 \text{ Constant term}}{\text{Coefficient of } x^2}$$

$\therefore$ If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

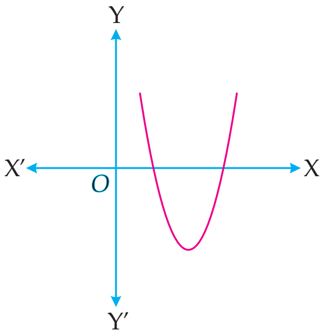
$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

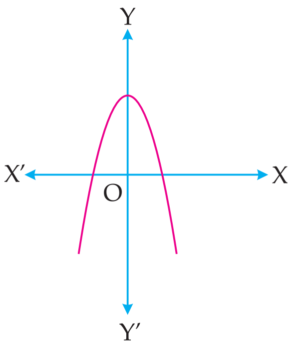
(iii) If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = (-1)^1 \frac{b}{a} = -\frac{b}{a},\ \alpha\beta + \beta\gamma + \gamma\alpha = (-1)^2 \frac{c}{a} = \frac{c}{a} \text{ and } \alpha\beta\gamma = (-1)^3 \frac{d}{a} = -\frac{d}{a}$$

- **Discriminant of a Quadratic Polynomial:** For $f(x) = ax^2 + bx + c$, where $a \neq 0$, $b^2 - 4ac$ is called its discriminant D. The discriminant D determines the nature of roots/zeroes of a quadratic polynomial.

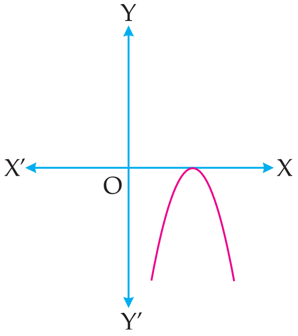
Case I : If D > 0, graph of $f(x) = ax^2 + bx + c$ will intersect the X-axis at two distinct points, x-co-ordinates of points of intersection with X-axis is known as 'zeroes' of $f(x)$.

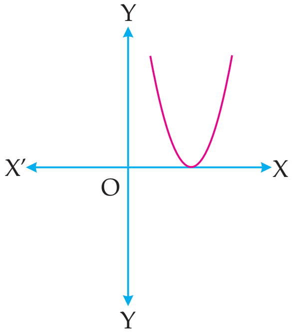




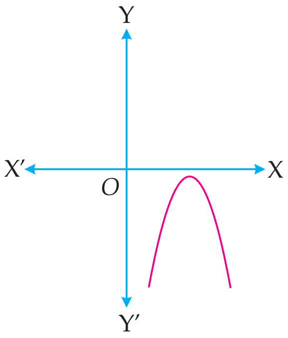
$\therefore f(x)$ will have two zeroes and we can say that roots/zeroes of the two given polynomials are real and unequal.

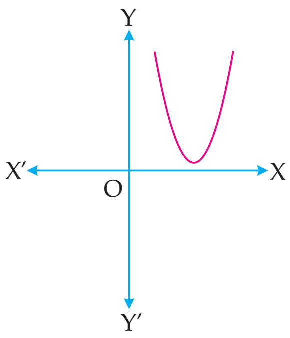
Case II : If $D = 0$, graph of $f(x) = ax^2 + bx + c$ will touch the X-axis at one point only.





$\therefore$ $f(x)$ will have only one 'zero' and we can say that roots/zeroes of the given polynomial are real and equal.
Case III: If $D < 0$, graph of $f(x) = ax^2 + bx + c$ will neither touch nor intersect the X-axis.





$\therefore$ $f(x)$ will not have any real zero.

Know the Formulae

Relationship between the zeroes and the coefficients of a Polynomial :

S. No.	Type of polynomial	General form	Maximum Number of zeroes	Relationship between zeroes and coefficients
1.	Linear	$ax + b$, where $a \neq 0$	1	$k = -\frac{b}{a}$, i.e., $k = \frac{-\text{Constant term}}{\text{Coefficient of } x}$
2.	Quadratic	$ax^2 + bx + c$, where $a \neq 0$	2	Sum of zeroes, $(\alpha + \beta) = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$ Product of zeroes, $(\alpha\beta) = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$
3.	Cubic	$ax^3 + bx^2 + cx + d$, where $a \neq 0$	3	Sum of zeroes, $(\alpha + \beta + \gamma) = \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{b}{a}$ Product of sum of zeroes taken two at a time, $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$ Product of zeroes, $(\alpha\beta\gamma) = \frac{-\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{d}{a}$

Mnemonics

Concept: Formula → $\alpha.\beta = \frac{c}{a}$

Amitabh **B**achchan went **C**anada **by** **a**eroplane.

Interpretation:

Amitabh's A ⇒ **Alpha (α)**
Bachchan's B ⇒ **Beta (β)**
Canada's C ⇒ **Constant (c)**
By for Divide by and aeroplane's a ⇒ **Variable.**

How is it done on the GREENBOARD?

Q.1. If α and β are the roots of equation $x^2 - 5x + 6 = 0$, then find the value of $\alpha - \beta$.

Solution

Step I: Compare $x^2 - 5x + 6 = 0$ with $ax^2 + bx + c = 0$ to get $a = 1$, $b = -5$ and $c = 6$.

Step II: Sum of roots, $\alpha + \beta = \frac{-b}{a}$

$$= \frac{-(-5)}{1}$$

or, $$\alpha + \beta = 5 \qquad \text{...(i)}$$

Step III: Product of roots

$$\alpha\beta = \frac{c}{a} = \frac{6}{1}$$

or, $$\alpha\beta = 6 \qquad \text{...(ii)}$$

Step IV: $$\alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

or, $$\alpha - \beta = \pm\sqrt{(5)^2 - 4 \times 6}$$

$$= \pm\sqrt{25 - 24}$$

$$= \pm\sqrt{1} = \pm 1$$

$$= 1 \text{ or } -1$$

Very Short Answer Type Questions

1 mark each

Q. 1. If the sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

C + U [CBSE SQP, 2020-21]

Sol. Let the roots of the given quadratic equation be α and β

So we have,

$$\alpha + \beta = \frac{k}{3} \qquad ½$$

$$3 = \frac{k}{3}$$

$$k = 9 \qquad ½$$

[CBSE Marking Scheme, 2020-21]

Q. 2. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of k.

C + U [CBSE Delhi Set-I, 2020]

Sol. Let $p(x) = x^2 + 3x + k$

$\because$ 2 is a zero of $p(x)$, then

$$p(2) = 0 \qquad ½$$

$\therefore$ $(2)^2 + 3(2) + k = 0$

$\Rightarrow$ $4 + 6 + k = 0$

$\Rightarrow$ $10 + k = 0$

$\Rightarrow$ $k = -10$ ½

[CBSE Marking Scheme, 2020]

Q. 3. It the sum of the zeroes of a quadratic polynomial is – 5 and their product is 6, then find the equation of the quadratic polynomial.

A [CBSE Delhi Set-I, 2020]

Sol. Let α and β be the zeroes of the quadratic polynomial, then

$$\alpha + \beta = -5$$

and $$\alpha\beta = 6$$

So, required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-5)x + 6$$

$$= x^2 + 5x + 6. \qquad 1$$

[CBSE Marking Scheme, 2020]

Q. 4. Find the zeroes of the polynomial $x^2 - 3x - m(m + 3)$.

U [CBSE OD Set-I, 2020]

Sol. Given, $x^2 - 3x - m(m + 3)$

putting $x = -m$, we get

$$= (-m)^2 - 3(-m) - m(m + 3)$$

$$= m^2 + 3m - m^2 - 3m = 0,$$

putting $x = m + 3$, we get

$$= (m + 3)^2 - 3(m + 3) - m(m + 3)$$

$$= (m + 3)[m + 3 - 3 - m]$$

$$= (m + 3)[0] = 0.$$

Hence, $-m$ and $m + 3$ are the zeroes of given polynomial. 1

[CBSE Marking Scheme, 2020]

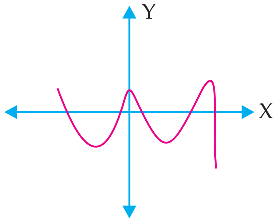
COMMONLY MADE ERROR

- Students often make mistakes in analyzing the zeroes as they get confused with the different terms.

ANSWERING TIP

- Understand the different cases for zeroes.

AI **Q. 5. The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows :**



Find the number of zeroes of $p(x)$.

R [CBSE SQP, 2020]

Sol. Since the graph touches the X-axis 5 times.

Hence, the number of zeroes of $p(x)$ is 5. 1

[CBSE SQP Marking Scheme, 2020]

Short Answer Type Questions-I

2 marks each

AI **Q. 1. Find a quadratic polynomial where zeroes are $5 - 3\sqrt{2}$ and $5 + 3\sqrt{2}$**

A [CBSE SQP, 2020-21]

Sol. Sum of zeroes $= 5 - 3\sqrt{2} + 5 + 3\sqrt{2} = 10$ ½

Product of zeroes $= (5 - 3\sqrt{2})(5 + 3\sqrt{2})$ 1

$= 25 - 18 = 7$

Polynomial is given by

$x^2 - (\text{sum of zeroes})x + (\text{product of zeroes}) = 0$

$p(x) = x^2 - 10x + 7$ ½

[CBSE Marking Scheme, 2020-21]

COMMONLY MADE ERROR

- Students often commit errors in finding a quadratic polynomial. Some students find sum of zeroes and product of zeroes but not find a complete polynomial.

ANSWERING TIP

- Students should read the question properly and solved step by step.

Q. 2. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students :

$2x + 3$, $3x^2 + 7x + 2$, $4x^3 + 3x^2 + 2$, $x^3 + \sqrt{3x} + 7$,

$7x + \sqrt{7}$, $5x^3 - 7x + 2$, $2x^2 + 3 - \frac{5}{x}$, $5x - \frac{1}{2}$,

$ax^3 + bx^2 + cx + d$, $x + \frac{1}{x}$.

Answer the following question :

(i) How many of the above ten, are not polynomials ?

(ii) How many of the above ten, are quadratic polynomials ?

A + C [CBSE OD Set-I/II/III, 2020]

Sol. (i) $x^3 + \sqrt{3x} + 7$, $2x^2 + 3 - \frac{5}{x}$ and $x + \frac{1}{x}$ are not polynomials. 1

(ii) $3x^2 + 7x + 2$ is only one quadratic polynomial. 1

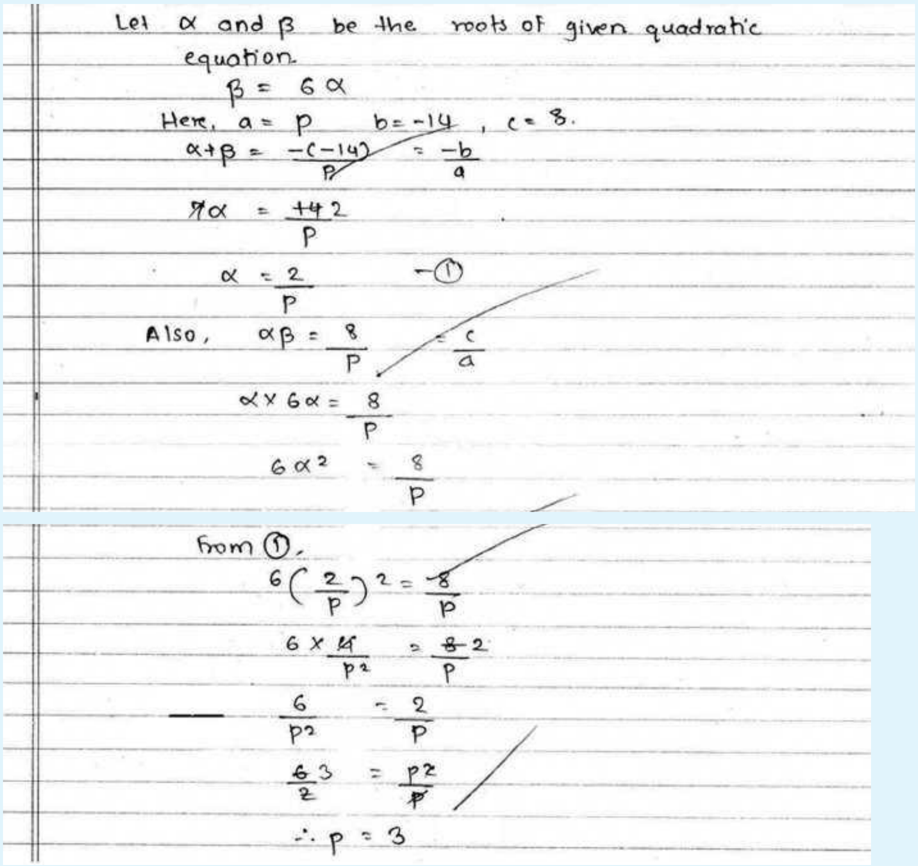
[CBSE Marking Scheme, 2020]

AI **Q. 3. Find the value of p, for which one root of the quadratic polynomial $px^2 - 14x + 8 = 0$ is 6 times the other.**

C + A [CBSE OD Set III, 2017]

Topper Answer, 2017

Sol.



Let α and β be the roots of given quadratic equation

$\beta = 6\alpha$

Here, $a = p$ $b = -14$, $c = 8$.

$\alpha + \beta = \frac{-(-14)}{p} = \frac{-b}{a}$

$\cancel{7}\alpha = \frac{+\cancel{14}\,2}{p}$

$\alpha = \frac{2}{p}$ —①

Also, $\alpha\beta = \frac{8}{p} = \frac{c}{a}$

$\alpha \times 6\alpha = \frac{8}{p}$

$6\alpha^2 = \frac{8}{p}$

from ①,

$6\left(\frac{2}{p}\right)^2 = \frac{8}{p}$

$6 \times \frac{\cancel{4}}{p^2} = \frac{\cancel{8}\,2}{p}$

$\frac{6}{p^2} = \frac{2}{p}$

$\frac{\cancel{6}\,3}{\cancel{2}} = \frac{p^2}{\cancel{p}}$

$\therefore p = 3$

2

Q. 4. If zeroes of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$, then find the value of a.

C + U [Board Term, 2016]

Sol. Product of (zeroes) roots $= \frac{c}{a}$

$$= \frac{2a}{1} = \alpha . \frac{2}{\alpha} \qquad 1$$

or, $2a = 2$

∴ $a = 1$ 1

[CBSE Marking Scheme, 2016]

AI **Q. 5. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.**

A [Board Term-1, 2016]

Sol. Given,

Sum of zeroes $= \frac{21}{8}$

and Product of zeroes $= \frac{5}{16}$ 1

So, quadratic polynomial

$$= x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$$

$$= x^2 - \left(\frac{21}{8}\right)x + \frac{5}{16}$$

$$= \frac{1}{16}(16x^2 - 42x + 5) \qquad 1$$

Q. 6. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

C + A [Board Term-1, 2015]

Sol. Let, $x^2 - 4\sqrt{3}x + 3 = 0$

If α and β are the zeroes of $x^2 - 4\sqrt{3}x + 3$

Then, $\alpha + \beta = -\frac{b}{a}$

⇒ $\alpha + \beta = -\frac{(-4\sqrt{3})}{1}$

⇒ $\alpha + \beta = 4\sqrt{3}$ 1

and $\alpha\beta = \frac{c}{a}$

$$\alpha\beta = \frac{3}{1}$$

$$\Rightarrow \quad \alpha\beta = 3$$

$$\therefore \quad \alpha + \beta - \alpha\beta = 4\sqrt{3} - 3. \qquad 1$$

Q. 7. If α and β are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, find the value of k, such that $\alpha^2 + \beta^2 = 40$. C + A [Board Term-1, 2015]

Sol.
$$\alpha + \beta = -\frac{b}{a}$$

$$= \frac{-(-6)}{1} = 6$$

and
$$\alpha\beta = \frac{c}{a} = \frac{k}{1} = k \qquad 1$$

Given,
$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$$

$$\Rightarrow \quad (6)^2 - 2k = 40$$

$$\Rightarrow \quad 36 - 2k = 40$$

$$\Rightarrow \quad -2k = 4$$

$$\therefore \quad k = -2 \qquad 1$$

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II

3 marks each

AI **Q. 1. If one root of the quadratic equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, then find the value of p and the other root of the equation.** C + A [CBSE SQP, 2020-21]

Sol.
$$3x^2 + px + 4 = 0$$

$\because$ $\frac{2}{3}$ is a root so it must satisfy the given equation

$$3\left(\frac{2}{3}\right)^2 + p\left(\frac{2}{3}\right) + 4 = 0$$

$$\frac{4}{3} + \frac{2p}{3} + 4 = 0 \qquad \tfrac{1}{2}$$

On solving we get

$$p = -8 \qquad \tfrac{1}{2}$$

$$3x^2 - 8x + 4 = 0$$

$$3x^2 - 6x - 2x + 4 = 0 \qquad \tfrac{1}{2}$$

$$3x(x - 2) - 2(x - 2) = 0 \qquad \tfrac{1}{2}$$

$$x = \frac{2}{3} \text{ or } x = 2 \qquad \tfrac{1}{2}$$

Hence, $x = 2$ ½

So the other root is 2.

[CBSE Marking Scheme, 2020-21]

AI **Q. 2. The roots α and β of the quadratic equation $x^2 - 5x + 3(k - 1) = 0$ are such that $\alpha - \beta = 1$. Find the value of k.** U [CBSE SQP, 2020-21]

Sol. We have
$$\alpha + \beta = 5 \qquad \text{...(i)} \ \tfrac{1}{2}$$

$$\alpha - \beta = 1 \qquad \text{...(ii)} \ \tfrac{1}{2}$$

Solving (i) and (ii), we get

$$\alpha = 3 \text{ and } \beta = 2 \qquad \tfrac{1}{2}$$

also
$$\alpha\beta = 6 \qquad \tfrac{1}{2}$$

or
$$3(k - 1) = 6 \qquad \tfrac{1}{2}$$

$$k - 1 = 2$$

$$k = 3 \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2020-21]

Q. 3. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$. A [CBSE Delhi Set-I, 2020]

Sol. Let α and β be zero of the given polynomial $ax^2 + bx + c$

$$\therefore \quad \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a} \qquad 1$$

$$s = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c} \qquad \tfrac{1}{2}$$

$$p = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{a}{c} \qquad \tfrac{1}{2}$$

Quadratic polynomial

$$= x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c}$$

$$p(x) = \frac{1}{c}(cx^2 + bx + a) \qquad 1$$

[CBSE Marking Scheme, 2020]

Q. 4. If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes. A + E [CBSE Delhi Set-I, 2020]

Sol. $x^3 - 3x - 10x + 24$

Let α, β and γ be the zeroes of given polynomial

$$\therefore \quad \alpha + \beta + \gamma = 3 \qquad \text{...(i)}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -10 \qquad \text{...(ii)}$$

$$\alpha\beta\gamma = -24 \qquad \text{...(iii)} \ 1$$

Given : $\alpha = 4$

from eqn. (i) $\beta + \gamma = -1$

from eqn (ii) $\beta\gamma = -6$

$$(\beta - \gamma)^2 = (\beta + \gamma)^2 - 4\beta\gamma$$

$$= (-1)^2 - 4(-6)$$

$$= 25$$

$$\therefore \quad \beta - \gamma = \pm 5$$

$$\beta - \gamma = 5 \qquad \text{...(iv)} \ 1$$

$$\beta + \gamma = -1$$

$$2\beta = 4 \Rightarrow \beta = 2$$

and $\gamma = -3$

Hence zeroes are – 3, 2 and 4. **1**

[CBSE Marking Scheme, 2020]

Q. 5. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeroes equal to half of their product.

R [CBSE Delhi Set-I, 2019]

Sol. Sum of zeroes $= k + 6$ **1**

Product of zeroes $= 2(2k - 1)$ **1**

Hence $k + 6 = \frac{1}{2} \times 2\,(2k - 1)$

$\Rightarrow$ $k = 7$ **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let α and β be the roots of given quadratic equation

$$x^2 - (k + 6)x + 2(2k - 1) = 0$$

Now, sum of roots, $\alpha + \beta = -\frac{\{-(k+6)\}}{1} = k+6$ ½

product of roots, $\alpha\beta = \frac{2(2k-1)}{1} = 2(2k - 1)$ ½

According to given condition,

Sum of roots (zeroes) $= \frac{1}{2} \times$ product of roots (zeroes)

1

$\Rightarrow$ $k + 6 = \frac{1}{2}[2(2k - 1)]$

$\Rightarrow$ $k + 6 = 2k - 1$

$\Rightarrow$ $k = 7$

Hence, the value of k is 7. **1**

Q. 6. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

A [CBSE Outside Delhi-III, 2019]

Sol. $p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3} = \frac{1}{3}\,(21y^2 - 11y - 2)$

$= \frac{1}{3}[(7y + 1)(3y - 2)]$ **1**

$\therefore$ Zeroes are $\frac{2}{3}, \frac{-1}{7}$ ½

Sum of zeroes $= \frac{2}{3} - \frac{1}{7} = \frac{11}{21}$

$\frac{-b}{a} = \frac{11}{21}$ $\left|\ \therefore \text{sum of zeroes} = \frac{-b}{a}\ \right|$ **1**

Product of zeroes $= \left(\frac{2}{3}\right)\left(-\frac{1}{7}\right) = -\frac{2}{21}$

$\frac{c}{a} = -\frac{2}{3}\left(\frac{1}{7}\right) = -\frac{2}{21}$ $\left|\ \therefore \text{Product} = \frac{c}{a}\ \right|$

½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, polynomial is $7y^2 - \frac{11}{3}y - \frac{2}{3}$ ...(i)

$\Rightarrow$ $\frac{1}{3}(21y^2 - 11y - 2)$ ½

$\Rightarrow$ $\frac{1}{3}(21y^2 - 14y + 3y - 2)$

$\Rightarrow$ $\frac{1}{3}[(7y(3y - 2) + 1(3y - 2)]$

$\Rightarrow$ $\frac{1}{3}(3y - 2)(7y + 1)$

$\Rightarrow$ $y = \frac{2}{3}$ or $y = \frac{-1}{7}$ **1**

Hence, zeros of given polynomial are

$$y = \frac{2}{3} \text{ and } y = \frac{-1}{7}$$

On comparing eq (i) with $ax^2 + bx + c = 0$, we get $a = 21, b = -11$ and $c = -2$

Now, sum of zeroes $= \frac{2}{3} + \left(-\frac{1}{7}\right)$

$= \frac{2}{3} - \frac{1}{7}$

$= \frac{14 - 3}{21}$

$= \frac{11}{21}$

$= -\frac{b}{a}$ **Hence verified. 1**

and product of roots $= \frac{2}{3} \times \left(-\frac{1}{7}\right)$

$= \frac{-2}{21}$

$= \frac{c}{a}$ **Hence verified. ½**

Q. 7. Find the zeroes of the following polynomial :

$$5\sqrt{5}x^2 + 30x + 8\sqrt{5}$$

U [CBSE SQP, 2018]

Sol. $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$ **1**

$= 5\sqrt{5}x^2 + 20x + 10x + 8\sqrt{5}$

$= 5x(\sqrt{5}x + 4) + 2\sqrt{5}(\sqrt{5}x + 4)$

$= (\sqrt{5}x + 4)(5x + 2\sqrt{5})$ **1**

Thus, zeroes are $\frac{-4}{\sqrt{5}} = \frac{-4\sqrt{5}}{5}$ and $\frac{-2\sqrt{5}}{5}$ **1**

[CBSE Marking Scheme, 2018]

COMMONLY MADE ERROR

- Candidates commit error in simplifying the equation $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$.

ANSWERING TIP

- Adequate Practice is necessary for factorization problems.

Q. 8. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β. A [Board Term-2, 2015]

Sol. If α and β are the zeroes of $2x^2 - 3x + 1$, then

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad \alpha + \beta = \frac{3}{2}$$

and
$$\alpha\beta = \frac{c}{a}$$

$$\Rightarrow \quad \alpha\beta = \frac{1}{2} \qquad 1$$

New quadratic polynomial whose zeroes are 3α and 3β is :

$x^2 -$ (Sum of the roots) $x +$ Product of the roots 1

$$= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$$
$$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$
$$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$$
$$= x^2 - \frac{9}{2}x + \frac{9}{2}$$
$$= \frac{1}{2}(2x^2 - 9x + 9)$$

Hence, required quadratic polynomial is

$$\frac{1}{2}(2x^2 - 9x + 9). \qquad 1$$

Q. 9. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients of the polynomial. U [Board Term-1, 2015]

Sol. Let $\quad f(x) = 4x^2 + 4x - 3$

Thus,
$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) - 3$$
$$= 1 + 2 - 3$$
$$= 0$$

and
$$f\left(-\frac{3}{2}\right) = 4\left(\frac{9}{4}\right) + 4\left(-\frac{3}{2}\right) - 3$$
$$= 9 - 6 - 3$$
$$= 0$$

$\therefore \frac{1}{2}$ and $-\frac{3}{2}$ are zeroes of polynomial $4x^2 + 4x - 3$. 1

$$\text{Sum of zeroes} = \frac{1}{2} - \frac{3}{2} = -1 \Rightarrow \frac{-4}{4}$$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \qquad 1$$

$$\text{Product of zeroes} = \left(\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{-3}{4}$$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$ **Hence verified. 1**

Long Answer Type Questions

5 marks each

Q. 1. Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the value of p and q. A [Board Term-1, 2015]

Sol. Factors of $x^2 + 7x + 12$:

$$x^2 + 7x + 12 = 0$$
$$\Rightarrow \quad x^2 + 4x + 3x + 12 = 0$$
$$\Rightarrow \quad x(x + 4) + 3(x + 4) = 0$$
$$\Rightarrow \quad (x + 4)(x + 3) = 0$$
$$\Rightarrow \quad x = -4 \text{ or } -3 \qquad \text{...(i) } 1$$

Since, $\quad p'(x) = x^4 + 7x^3 + 7x^2 + px + q$

If $p'(x)$ is exactly divisible by $x^2 + 7x + 12$, then $x = -4$ and $x = -3$ are its zeroes. So, putting $x = -4$ and $x = -3$.

$$p'(-4) = (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q$$

but $\quad p'(-4) = 0$

$$\therefore \quad 0 = 256 - 448 + 112 - 4p + q$$
$$0 = -4p + q - 80$$
$$\Rightarrow \quad 4p - q = -80 \qquad \text{...(i) } 1$$

and $\quad p'(-3) = (-3)^4 + 7(-3)^3 + 7(-3)^2 + p(-3) + q$

but $\quad p'(-3) = 0$

$$\therefore \quad 0 = 81 - 189 + 63 - 3p + q$$
$$0 = -3p + q - 45 \qquad 1$$
$$\Rightarrow \quad 3p - q = -45 \qquad \text{...(ii)}$$

Subtracting equation (ii) from equation (i),

$$\begin{array}{r} 4p - q = -80 \\ 3p - q = -45 \\ - \quad + \qquad + \\ \hline p = -35 \end{array}$$

On putting the value of p in eq. (i), 1

$$4(-35) - q = -80$$
$$-140 - q = -80$$
$$-q = 140 - 80$$
$$\Rightarrow \quad -q = 60$$
$$\therefore \quad q = -60$$

Hence, $\quad p = -35$ and $q = -60$ 1

Q. 2. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k. C + A

Sol. Given, $\quad p(x) = 2x^2 + 5x + k$

Then, $\quad \text{sum of zeroes} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} \qquad ½$

$$\Rightarrow \quad \alpha + \beta = \frac{-5}{2} \qquad 1$$

and product of zeroes $= \dfrac{\text{constant term}}{\text{coefficient of } x^2}$ ½

$\Rightarrow \qquad \alpha\beta = \dfrac{k}{2}$ 1

According to equation,

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$

or, $(\alpha + \beta)^2 - 2\alpha\beta + \alpha\beta = \dfrac{21}{4}$ 1

$$\Rightarrow \quad \left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \quad \frac{k}{2} = \frac{25}{4} - \frac{21}{4} = \frac{4}{4} = 1$$

Hence, $\qquad k = 2.$ 1

Q. 3. If α and β are zeroes of the quadratic polynomial $x^2 - 6x + a$; find the value of 'a' if $3\alpha + 2\beta = 20$.

Sol. We have quadratic polynomial $= x^2 - 6x + a$

$\therefore \qquad \alpha + \beta = \dfrac{-(-6)}{1} = 6$ 1

and $\qquad \alpha\beta = \dfrac{a}{1} = a$ 1

It is given that; $3\alpha + 2\beta = 20$ ...(i)

and $\qquad \alpha + \beta = 6$ ...(ii)

Multiplying by 2 in eq. (ii), we get

$2\alpha + 2\beta = 12$ ...(iii) ½

Subtracting eq. (iii) from eq. (i), we get

$\alpha = 8$ ½

Substituting $\alpha = 8$ in eq. (ii), we get

$8 + \beta = 6$

$\Rightarrow \qquad \beta = 6 - 8 = -2$ 1

$\because \qquad \alpha\beta = a$

Then, $\qquad a = 8(-2) = -16.$ 1

TOPIC - 2

Problems on Polynomials

Revision Notes

- **Degree of a Polynomial :** The exponent of the highest degree term in a polynomial is known as its degree. In other words, the highest power of x in a polynomial $f(x)$ is called the degree of the polynomial $f(x)$. *e.g.*,

  (i) $f(x) = 5x + \dfrac{1}{3}$ is a polynomial in variable x of degree 1.

  (ii) $g(y) = 3y^2 - \dfrac{5}{2}y + 7$ is a polynomial in variable y of degree 2.

- **Division Algorithm for Polynomials :** If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$, such that

  $$p(x) = g(x) \times q(x) + r(x),$$

  where, degree of $r(x) <$ degree of $g(x)$ and $r(x)$ is denoted for remainder.

  Note :

  (i) If $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.

  (ii) Dividend = Divisor × Quotient + Remainder.

- **To Divide Quadratic Polynomial by Linear Polynomial :**

  Let, $\qquad p(x) = ax^2 + bx + c$ and $g(x) = mx + n$

$$\frac{a}{m}x + \frac{1}{m}\left(b - \frac{an}{m}\right) \longrightarrow \text{Quotient}$$

$$\text{Divisor} \longrightarrow mx + n\overline{)\,ax^2 + bx + c} \longrightarrow \text{Dividend}$$

$$+ax^2 + \frac{an}{m}x$$

$$- \qquad -$$

$$\left(b - \frac{an}{m}\right)x + c$$

$$\left(b - \frac{an}{m}\right)x + \frac{n}{m}\left(b - \frac{an}{m}\right)$$

$$- \qquad\qquad -$$

$$c - \frac{n}{m}\left(b - \frac{an}{m}\right)$$

Remainder = Constant term

Step I : To obtain the first term of the quotient, divide the highest degree term of the dividend (*i.e.*, ax^2) by the highest degree term of the divisor (*i.e.*, mx). *i.e.*, $\frac{a}{m}x$. Then, carry out the division process.

What remains is $\left(b-\frac{an}{m}\right)x+c$.

Step II : Now, to obtain the second term of the quotient, divide the highest degree term of the new dividend $\left\{i.e.,\left(b-\frac{an}{m}x\right)\right\}$ by the highest degree term of the divisor (*i.e.*, mx). *i.e.*, $\frac{1}{m}\left(b-\frac{an}{m}\right)$. Then, carry on the division process.

What remain as remainder, $c-\frac{n}{m}\left(b-\frac{an}{m}\right)$ which is a constant term.

Know the Formulae

Division Algorithm for polynomials :

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$p(x) = g(x) \times q(x) + r(x)$$

Where, degree of $r(x)$ < degree of $g(x)$ and $r(x)$ is denoted for remainder.

How is it done on the GREENBOARD?

Q.1. What should be added to $x^2 - 8x + 10$ to make it divisible by $x - 3$:

Solution

Step I: Let k be added to $x^2 - 8x + 10$ to make it divisible by $x - 3$.

$$\Rightarrow \quad p(x) = x^2 - 8x + 10 + k$$

Step II: Applying Long division

$$x-3)\overline{x^2-8x+10+k}(\,x-5$$

$$\underline{\pm x^2 \mp 3x}$$

$$\underline{-5x+10+k}$$

$$\underline{\mp 5x \pm 15}$$

$$-5+k$$

Step III: Here, remainder is $-5+k$ which must be zero.

Then, $\quad -5+k = 0$

$\therefore \quad k = 5$

Very Short Answer Type Questions

1 mark each

Q. 1. If divisor $= 2x^2 - 3x + 2$, quotient $= 2x - 1$ and remainder $= 0$, then find dividend. $\boxed{U}$

Sol. By using division algorithm for polynomial,

Dividend = Divisor × Quotient + Remainder

$= (2x^2 - 3x + 2) \times (2x - 1) + 0$ ½

$= (2x - 1)\,(2x^2 - 3x + 2)$

$= 2x(2x^2 - 3x + 2) - 1(2x^2 - 3x + 2)$

$= 4x^3 - 6x^2 + 4x - 2x^2 + 3x - 2$

$= 4x^3 - 8x^2 + 7x - 2.$ ½

Q. 2. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be $2x + 3$, which is equal to $px + q$, then find the values of p and q. $\boxed{R}$

Sol. Remainder $= 2x + 3$

Comparing its with $px + q$, we get

$p = 2$ and $q = 3.$ 1

Q. 3. On dividing $(x^3 - 3x^2 + x + 2)$ by a polynomial $(x^2 - x + 1)$, the quotient was $(x - 2)$. Find the remainder. $\boxed{U}$

Sol. We have, dividend $= x^3 - 3x^2 + x + 2$

Divisor $= x^2 - x + 1$ and quotient $= x - 2$

By using division algorithm,

$(x^3 - 3x^2 + x + 2) = (x^2 - x + 1)(x - 2) + \text{Remainder}$

$\Rightarrow x^3 - 3x^2 + x + 2 = x^3 - x^2 + x - 2x^2 + 2x - 2 + \text{Remainder}$

Hence, remainder $= -2x + 4$ 1

Short Answer Type Questions-I

2 marks each

AI **Q. 1. What should be added to the polynomial $x^3 - 3x^2 + 6x - 15$ so that it is completely divisible by $x - 3$.**

A [Board Term-1, 2015, 2016]

Sol. $x - 3\ \overline{)\ x^3 - 3x^2 + 6x - 15\ (}\ x^2 + 6$

$x^3 - 3x^2$

$-\quad +$

$6x - 15$ 1

$+ 6x - 18$

$-\quad +$

$+ 3$

Remainder = 3

Hence, – 3 must be added. 1

[CBSE Marking Scheme, 2016]

Detailed Solution:

$$p(x) = x^3 - 3x^2 + 6x - 15$$

When $p(x)$ is divided by $(x - 3)$ then

$$\begin{aligned} \text{Remainder} &= p(3) \\ &= (3)^3 - 3(3)^2 + 6(3) - 15 \\ &= 27 - 27 + 18 - 15 \\ &= 3 \end{aligned}$$

Hence –3 be added so that it is completely divided by $(x - 3)$

Q. 2. On dividing $x^3 - 5x^2 + 6x + 4$ by a polynomial $g(x)$, the quotient and the remainder were $x - 3$ and 4 respectively. Find $g(x)$. U [Board Term-1, 2016]

Sol. Given, $x^3 - 5x^2 + 6x + 4 = g(x)(x - 3) + 4$

$$g(x) = \frac{x^3 - 5x^2 + 6x + 4 - 4}{x - 3}$$

$$\Rightarrow \quad g(x) = \frac{x^3 - 5x^2 + 6x}{x - 3}$$ 1

$x - 3\ \overline{)\ x^3 - 5x^2 + 6x\ (}\ x^2 - 2x$

$+ x^3 - 3x^2$

$-\quad +$

$- 2x^2 + 6x$

$- 2x^2 + 6x$

$+\quad -$

$\times$

Hence, $g(x) = x^2 - 2x$. 1

[CBSE Marking Scheme, 2016]

AI **Q. 3. Find the quotient and remainder on dividing $p(x)$ by $g(x)$:**
$p(x) = 4x^3 + 8x^2 + 8x + 7$ and $g(x) = 2x^2 - x + 1$

U [Board Term-1, 2015]

Sol.

$2x + 5$

$2x^2 - x + 1)\ 4x^3 + 8x^2 + 8x + 7$

$4x^3 - 2x^2 + 2x$

$-\quad +\quad -$

$+ 10x^2 + 6x + 7$

$+ 10x^2 - 5x + 5$

$-\quad +\quad -$

$+ 11x + 2$

Thus, quotient = $2x + 5$ 1

and remainder = $11x + 2$ 1

Q. 4. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find the values of a and b.

U [Board Term-1, 2015]

Sol. $3x^2 + 4x + 1)\ \overline{6x^4 + 8x^3 + 17x^2 + 21x + 7\ (}\ 2x^2 + 5$

$6x^4 + 8x^3 + 2x^2$

$-\quad -\quad -$

$15x^2 + 21x + 7$

$15x^2 + 20x + 5$

$-\quad -\quad -$

$x + 2$

$\therefore \quad ax + b = x + 2$

On comparing both the sides, we get

$a = 1$ and $b = 2$. [CBSE Marking Scheme, 2015] 2

Short Answer Type Questions-II

3 marks each

AI **Q. 1. Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.** A [CBSE Delhi Set-I, 2020]

Sol.

$x - 2$

$-x^2 + x - 1 \overline{)\ -x^3 + 3x^2 - 3x + 5}$

$-x^3 + x^2 - x$

$(+)\ (-)\ (+)$

$2x^2 - 2x + 5$

$2x^2 - 2x + 2$

$(-)\ (+)\ (-)$

3

Thus, quotient = $x - 2$ and remainder = 3 1½

Then,

$$\begin{aligned} f(x) &= g(x) \times q(x) + r(x) \\ &= (-x^2 + x - 1)(x - 2) + 3 \\ &= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3 \\ &= -x^3 + 3x^2 - 3x + 5 \end{aligned}$$

Hence Verified. 1½

[CBSE Marking Scheme, 2020]

AI **Q. 2. Obtain all the zeroes of the polynomial $x^4 + 4x^3 - 2x^2 - 20x - 15$, if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.** [CBSE SQP, 2020]

Sol. $p(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$

$x^2 - 5$ is factor of $p(x)$

$\therefore \quad p(x) = (x^2 - 5)(x^2 + 4x + 3)$ **2**

or, $\quad p(x) = (x^2 - 5)(x + 3)(x + 1)$ **1**

So, all the zeroes of $p(x)$ are $\sqrt{5}$, $-\sqrt{5}$, -3 and -1

U **[CBSE SQP Marking Scheme, 2020]**

Detailed Solution:

The given polynomial is

$$p(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$$

It is given that $\sqrt{5}$ and $-\sqrt{5}$ are two of its zeroes. It means $(x - \sqrt{5})$ and $(x + \sqrt{5})$ are factors of given polynomial

$\therefore$ $(x - \sqrt{5})(x + \sqrt{5}) = x^2 - 5$ is a factor of polynomial

On dividing the given polynomial $p(x)$ by $x^2 - 5$,
Using long division method, we get

$$\begin{array}{r|l} & x^2 + 4x + 3 \\ \hline x^2 - 5 & x^4 + 4x^3 - 2x^2 - 20x - 15 \\ & x^4 \qquad\quad - 5x^2 \\ & - \qquad\quad + \\ \hline & \quad 4x^3 + 3x^2 - 20x - 15 \\ & \quad 4x^3 \qquad - 20x \\ & \quad - \qquad\quad + \\ \hline & \qquad 3x^2 \qquad - 15 \\ & \qquad 3x^2 \qquad - 15 \\ & \qquad - \qquad\quad + \\ \hline & \qquad\qquad 0 \end{array}$$

(marks: **1** after the $4x^3 - 20x$ step; **1** after the remainder 0)

Equate the quotient equal to 0, to find the remaining zeroes.

$$x^2 + 4x + 3 = 0$$
$$x^2 + 3x + x + 3 = 0$$
$$x(x + 3) + 1(x + 3) = 0$$
$$(x + 3)(x + 1) = 0$$

$\because \quad x + 3 = 0 \Rightarrow x = -3$

$\because \quad x + 1 = 0 \Rightarrow x = -1$

Hence, all the zeores are $\sqrt{5}$, $-\sqrt{5}$, -3 and -1. **1**

Q. 3. Find all zeroes of the polynomial $3x^3 + 10x^2 - 9x - 4$ if one of its zero is 1.

R + U **[CBSE Delhi Set-III, 2019]**

Sol. Let $p(x) = 3x^3 + 10x^2 - 9x - 4$.

One of the zeroes is 1, therefore dividing $p(x)$ by $(x - 1)$

$$p(x) = (x - 1)(3x^2 + 13x + 4) \quad \mathbf{1\tfrac{1}{2}}$$
$$= (x - 1)(x + 4)(3x + 1) \quad \mathbf{1}$$

All zeroes are $x = 1$, $x = -4$ and $x = \frac{-1}{3}$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

$$p(x) = 3x^3 + 10x^2 - 9x - 4$$

Let α, β and γ be the zero of the polynomial $p(x)$

$$\therefore \quad \alpha + \beta + \gamma = -\frac{10}{3}$$

$$\beta + \gamma = -\frac{10}{3} - 1 \quad (\alpha = 1 \text{ Given})$$

$$\beta + \gamma = -\frac{13}{3} \quad \text{...(i)}$$

Again,
$$\alpha\beta\gamma = +\frac{4}{3}$$

$$\beta\gamma = +\frac{4}{3} \quad \text{...(ii)}$$

Now,
$$(\beta - \gamma) = (\beta + \gamma)^2 - 4\beta\gamma$$
$$= \left(-\frac{13}{3}\right)^2 - 4\left(+\frac{4}{3}\right)$$
$$= \frac{169}{9} - \frac{16}{3}$$
$$= \frac{169 - 48}{9} = \frac{121}{9}$$
$$\beta - \gamma = \pm\sqrt{\frac{121}{9}} = \pm\frac{11}{3}$$
$$\beta - \gamma = \frac{11}{3} \quad \text{...(iii)}$$

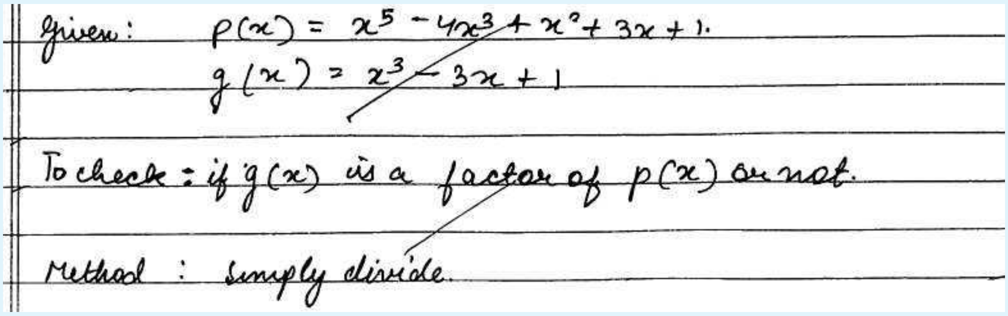
On solving equations (i) & (iii) $\beta = -\frac{1}{3}$ and $\gamma = -4$

$\therefore$ Zeroes of the polynomial are $-\frac{1}{3}$ and -4

Q. 4. Verify $g(x) = x^3 - 3x + 1$ is a factor of $p(x) = x^5 - 4x^3 + x^2 + 3x + 1$ or not. C + A **[CBSE Board term, 2019]**

Topper Answer, 2019

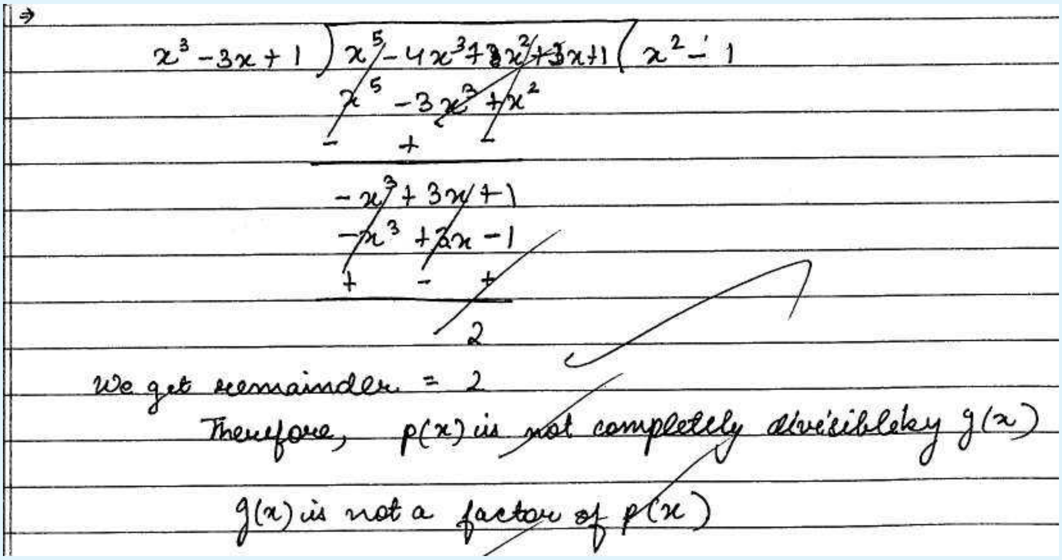
Sol.



Given: $p(x) = x^5 - 4x^3 + x^2 + 3x + 1.$
$g(x) = x^3 - 3x + 1$

To check: if $g(x)$ is a factor of $p(x)$ or not.

Method : simply divide.



$x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1} (x^2 - 1$

$x^5 - 3x^3 + x^2$

$-$ $+$ $-$

$-x^3 + 3x + 1$

$-x^3 + 3x - 1$

$+$ $-$ $+$

2

We get remainder = 2

Therefore, $p(x)$ is not completely divisible by $g(x)$

$g(x)$ is not a factor of $p(x)$

3

Q. 5. For what value of k, is the polynomial :

$$f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$$

is completely divisible by $3x^2 - 5$.

A [CBSE OD-I, 2019] [Board Term-I, 2015]

Sol. $3x^2 - 5) 3x^4 - 9x^3 + x^2 + 15x + k (x^2 - 3x + 2$

$3x^4 \quad - 5x^2$

$- \quad +$

$-9x^3 + 6x^2 + 15x + k$

$-9x^3 \quad + 15x$

$+ \quad -$

$6x^2 + k$

$6x^2 - 10$

$- \quad +$

$k + 10$ 2

$\therefore \quad k + 10 = 0 \Rightarrow k = -10$ 1

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, the polynomial

$f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$ is completely divisible by $3x^2 - 5$.

$x^2 - 3x + 2$

$3x^2 - 5) 3x^4 - 9x^3 + x^2 + 15x + k$

$3x^4 \quad - 5x^2$

$- \quad +$

$-9x^3 + 6x^2 + 15x + k$

$-9x^3 \quad + 15x$

$+ \quad -$

$6x^2 \quad + k$

$6x^2 \quad - 10$

$- \quad +$

$k + 10$

Since, $3x^2 - 5$ is completely divide $f(x)$.

Therefore, remainder is zero

i.e., $\quad k + 10 = 0$

$\Rightarrow \quad k = -10$

Hence, value of k is -10.

COMMONLY MADE ERROR

- Students commit errors in division method as they get confused in degrees of coefficients.

ANSWERING TIP

- Be careful with the degree and the terms.

Q. 6. Find all zeroes of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$.

A [CBSE Delhi, OD, 2018]

Sol. Let $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

and $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of $p(x)$.

$\therefore \quad p(x) = (x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) \times g(x)$

$= (x^2 - 4x + 1)\, g(x)$ 1

$(2x^4 - 9x^3 + 5x^2 + 3x - 1) \div (x^2 - 4x + 1) = 2x^2 - x - 1$

$\therefore \quad g(x) = 2x^2 - x - 1$

$= (2x + 1)(x - 1)$ 1

Therefore, other zeroes are $x = -\frac{1}{2}$ and $x = 1$ 1

$\therefore$ Therefore, all zeroes are $2 + \sqrt{3}, 2 - \sqrt{3}, \frac{-1}{2}$ and 1

[CBSE Marking Scheme, 2018]

Detailed Solution:

Since, $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two zeroes of given polynomial.

So, $(x-2-\sqrt{3})$, $(x-2+\sqrt{3})$ will be its two factors.

$\therefore\ (x-2-\sqrt{3})$, and $(x-2+\sqrt{3})\ = x^2-4x+1$

is a factor of the given polynomial.

Now, dividing it by $2x^2-4x+1$ **1**

$x^2-4x+1\,\overline{)\,2x^4-9x^3+5x^2+3x-1}\,(2x^2-x-1$

$2x^4-8x^3+2x^2$

$-\quad+\quad-$

$-x^3+3x^2+3x-1$

$-x^3+4x^2-x$

$+\quad-\quad+$

$-x^2+4x-1$

$-x^2+4x-1$

$+\quad-\quad+$

0 **1**

$2x^2-x-1=(2x+1)(x-1)$

Two other zeroes $=-\dfrac{1}{2}$ and 1

Therefore, all zeroes are

$(2+\sqrt{3}), (2-\sqrt{3}), -\dfrac{1}{2}$ and 1. **1**

COMMONLY MADE ERROR

- Many candidates makes mistake in dividing the polynomial. A few candidates do not write all four zeroes.

ANSWERING TIP

- Adequate practice is necessary for division of polynomials.

Q. 7. Ram's mother has given him money to buy some boxes from the market at the rate of $4x^2+3x-2$. The total amount of money is represented by $8x^4+14x^3-2x^2+7x-8$. Out of this money he donated some amount to a child who was studying in the light of street lamp. Find how much amount of money he donated and purchased how many boxes from the market ? C [Board Term-1, 2015]

Sol. $p(x)\ = 8x^4+14x^3-2x^2+7x-8$

$g(x)\ =$ rate of the each box $= 4x^2+3x-2$

$q(x)\ =$ number of boxes

$r(x)\ =$ amount of money he donated to child

and by using long division method

$2x^2+2x-1$

$4x^2+3x-2)\,\overline{8x^4+14x^3-2x^2+7x-8}$

$8x^4+6x^3-4x^2$

$-\quad-\quad+$

$8x^3+2x^2+7x-8$

$8x^3+6x^2-4x$

$-\quad-\quad+$

$-4x^2+11x-8$

$-4x^2-3x+2$

$+\quad+\quad-$

$14x-10$

$\because\ q(x) = 2x^2+2x-1 =$ number of Boxes and $r(x) = 14x-10$ [CBSE Marking Scheme, 2015] **3**

Long Answer Type Questions 5 marks each

AI **Q. 1. Obtain all zeroes of $3x^4-15x^3+13x^2+25x-30$, if two of its zeroes are $\sqrt{\dfrac{5}{3}}$ and $-\sqrt{\dfrac{5}{3}}$.**

[CBSE Compt Set- I/II/III, 2018]
[Board Term-I, 2015, 2017] [SQP 2017]

Sol. Let, $p(x)\ = 3x^2-15x^3+13x^2+25x-30$ and

$x-\sqrt{\dfrac{5}{3}}$ and $x+\sqrt{\dfrac{5}{3}}$ are factors of $p(x)$.

$\Rightarrow\ x^2-\dfrac{5}{3}$ or $\dfrac{(3x^2-5)}{3}$ is also a factor of $p(x)$. **1½**

$$\Rightarrow\qquad p(x)\ = \frac{(3x^2-5)}{3}(x^2-5x+6)$$ **2**

$$= \frac{1}{3}(3x^2-5)(x-3)(x-2)$$

$\therefore$ Zeroes of $p(x)$ are $\sqrt{\dfrac{5}{3}}, -\sqrt{\dfrac{5}{3}}$, 2 and 3. **1½**

[CBSE Marking Scheme, 2018]

Detailed Solution:

Since, $\sqrt{\dfrac{5}{3}}$ and $-\sqrt{\dfrac{5}{3}}$ are two zeroes of the given polynomial.

So, $\left(x-\sqrt{\dfrac{5}{3}}\right)$ and $\left(x+\sqrt{\dfrac{5}{3}}\right)$ will be its two factors. **½**

$$\therefore\ \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right) = \frac{1}{3}(3x^2-5)$$

is also a factor of given polynomial. **1**

Now dividing the given polynomial by $3x^2-5$.

$3x^2-5\,\overline{)\,3x^4-15x^3+13x^2+25x-30}\,(x^2-5x+6$

$3x^4\qquad-5x^2$

$-\qquad+$

$-15x^3+18x^2+25x-30$ **2**

$-15x^3\qquad+25x$

$+\qquad-$

$18x^2-30$

$18x^2 - 30$

$- \quad +$

0

On factorising the quotient, we get

$$x^2 - 5x + 6 = (x-2)(x-3) \qquad \tfrac{1}{2}$$

Thus, two other zeroes are 2 and 3

Hence, all the zeroes of given polynomial are

$\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, 2$ and 3. **1**

Q. 2. If two zeroes of a polynomial $x^3 + 5x^2 + 7x + 3$ are -1 and -3, then find the third zero.

A [Board Term-1, 2016]

Sol. Given, $x = -1$ and $x = -3$ are zeroes. **1**

$$(x+1)(x+3) = x^2 + 4x + 3$$

$x^2 + 4x + 3\,)\,x^3 + 5x^2 + 7x + 3\,(\,x + 1$

$x^3 + 4x^2 + 3x$ **3**

$- \quad - \quad -$

$x^2 + 4x + 3$

$x^2 + 4x + 3$

$- \quad - \quad -$

0

Since, remainder is 0.

$\therefore \qquad x + 1 = 0$

$\Rightarrow \qquad x = -1$

$\therefore$ The third zero is -1. **1**

[CBSE Marking Scheme, 2016]

Alternate Method:

$$p(x) = x^3 + 5x^2 + 7x + 3$$

Let the zeroes of the polynomial be α, β and γ

$\therefore \qquad \alpha + \beta + \gamma = -5$

$-1 - 3 + \gamma = -5$

[Given $\alpha = 1$ & $\beta = -3$]

$\gamma = -5 + 4 = -1$

$\therefore$ The third zero is -1

Q. 3. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $(x^2 - 2x + k)$, the remainder comes out to be $x + a$, find k and a. A

Sol. $\qquad x^2 - 4x + (8-k)$

$x^2 - 2x + k\,)\,x^4 - 6x^3 + 16x^2 - 25x + 10$

$x^4 - 2x^3 + kx^2$

$- \quad + \quad -$

$-4x^3 + (16-k)x^2 - 25x + 10$

$-4x^3 + 8x^2 - 4kx$ **1**

$+ \quad - \quad +$

$(8-k)x^2 - (25-4k)x + 10$

$(8-k)x^2 - (16-2k)x + (8k - k^2)$

$- \quad + \quad -$

$(2k-9)x + (10 - 8k + k^2)$ **1**

Given, remainder $= x + a$

On comparing the multiples of x we get

$$(2k-9)x = 1 \times x \qquad \mathbf{1}$$

$$\Rightarrow \qquad 2k - 9 = 1 \text{ or } k = \frac{10}{2} \Rightarrow 5 \qquad \mathbf{1}$$

On putting this value of k into other portion of remainder, we get

$$a = 10 - 8k + k^2 \Rightarrow 10 - 40 + 25$$

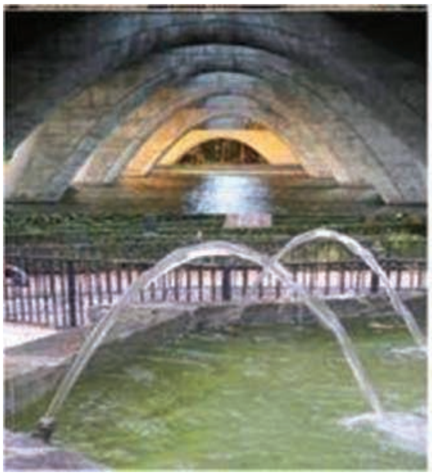
$$= -5 \qquad \mathbf{1}$$

Visual Case Based Questions

4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark

Q. 1. The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms. [CBSE QB, 2021]



(i) In the standard form of quadratic polynomial, $ax^2 + bx, c, a, b$ and c are

(a) **All are real numbers.**

(b) **All are rational numbers.**

(c) **'a' is a non zero real number and b and c are any real numbers.**

(d) **All are integers.**

Sol. Correct option: **(c).**

(ii) If the roots of the quadratic polynomial are equal, where the discriminant $D = b^2 - 4ac$, then

(a) $D > 0$ (b) $D < 0$

(c) D (d) $D = 0$

Sol. Correct option: **(d).**

Explanation: If the roots of the quadratic polynomial are equal, then discriminant is equal to zero

$$D = b^2 - 4ac = 0$$

(iii) If α are $\frac{1}{\alpha}$ the zeroes of the quadratic polynomial $2x^2 - x + 8k$, then k is

(a) 4 (b) $\frac{1}{4}$

(c) $\frac{-1}{4}$ (d) 2

Sol. Correct option: **(b).**

Explanation: Given equation, $2x^2 - x + 8k$

$$\text{Sum of zeroes} = \alpha + \frac{1}{\alpha}$$

$$\text{Product of zeroes} = \alpha . \frac{1}{\alpha} = 1$$

$$\text{Product of zeroes} = \frac{c}{a} = \frac{8k}{2}$$

$$\text{So,} \quad \frac{8k}{2} = 1$$

$$k = \frac{2}{8}$$

$$k = \frac{1}{4}$$

(iv) The graph of $x^2 + 1 = 0$

(a) **Intersects x-axis at two distinct points.**

(b) **Touches x-axis at a point.**

(c) **Neither touches nor intersects x-axis.**

(d) **Either touches or intersects x-axis.**

Sol. Correct option: **(c).**

(v) If the sum of the roots is $-p$ and product of the roots is $-\frac{1}{p}$, then the quadratic polynomial is

(a) $k\left(-px^2 + \frac{x}{p} + 1\right)$ (b) $k\left(px^2 - \frac{x}{p} - 1\right)$

(c) $k\left(x^2 + px - \frac{1}{p}\right)$ (d) $k\left(x^2 + px + \frac{1}{p}\right)$

Sol. Correct option: **(c).**

Q. 2. An asana is a body posture, originally and still a general term for a sitting meditation pose, and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting, and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial. [CBSE QB, 2021]

(i) The shape of the poses shown is

(a) Spiral (b) Ellipse

(c) Linear (d) Parabola

Sol. Correct option: **(d).**

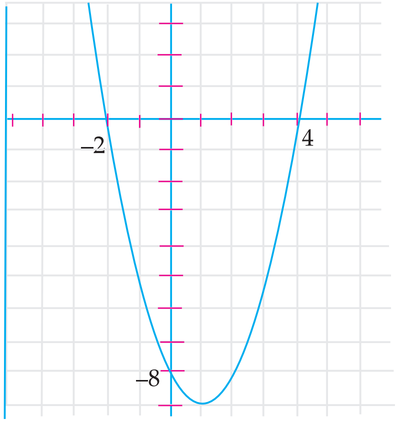
(ii) The graph of parabola opens downwards, if _______

(a) $a \geq 0$ (b) $a = 0$

(c) $a < 0$ (d) $a > 0$

Sol. Correct option: **(c).**

(iii) In the graph, how many zeroes are there for the polynomial?



(a) 0 (b) 1

(c) 2 (d) 3

Sol. Correct option: **(c).**

(iv) The two zeroes in the above shown graph are

(a) 2, 4 (b) – 2, 4

(c) – 8, 4 (d) 2, – 8

Sol. Correct option: **(b).**

(v) The zeroes of the quadratic polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are

(a) $\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ (b) $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

(c) $\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$ (d) $-\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$

Sol. Correct option: **(b).**

Explanation: $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ (given)

$$= 4\sqrt{3}x^2 + (8-3)x - 2\sqrt{3}$$

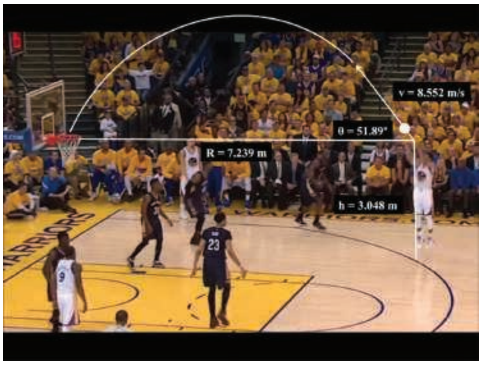
$$= 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$$

$$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$$

$$= (\sqrt{3}x + 2)(4x - \sqrt{3})$$

Hence, zeroes of polynomial $= \frac{-2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

Q. 3. Basketball and soccer are played with a spherical ball. Even though an athlete dribbles the ball in both sports, a basketball player uses his hands and a soccer player uses his feet. Usually, soccer is played outdoors on a large field and basketball is played indoor on a court made out of wood. The projectile (path traced) of soccer ball and basketball are in the form of parabola representing quadratic polynomial. [CBSE QB, 2021]



(i) The shape of the path traced shown is

(a) Spiral (b) Ellipse

(c) Linear (d) Parabola

Sol. Correct option: **(d).**

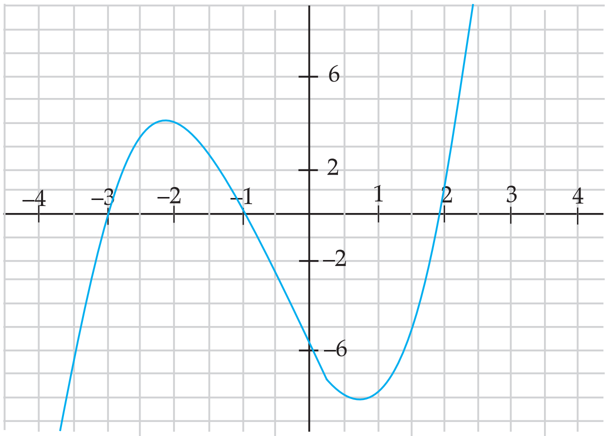
(ii) The graph of parabola opens upwards, if _______

(a) $a = 0$ (b) $a < 0$

(c) $a > 0$ (d) $a \geq 0$

Sol. Correct option: **(c).**

(iii) Observe the following graph and answer



In the above graph, how many zeroes are there for the polynomial?

(a) **0** **(b)** **1**

(c) **2** **(d)** **3**

Sol. Correct option: **(d).**

Explanation: The number of zeroes of polynomial is the number of times the curve intersects the x-axis, *i.e.* attains the value 0.

Here, the polynomial meets the x-axis at 3 points.

So, number of zeroes = 3.

(iv) The three zeroes in the above shown graph are

(a) 2, 3, –1 **(b)** **–2, 3, 1**

(c) **–3, –1, 2** **(d)** **–2, –3, –1**

Sol. Correct option: **(c).**

(v) What will be the expression of the polynomial?

(a) $x^3 + 2x^2 - 5x - 6$ **(b)** $x^3 + 2x^2 - 5x - 6$

(c) $x^3 + 2x^2 + 5x - 6$ **(d)** $x^3 + 2x^2 + 5x + 6$

Sol. Correct option: **(a).**

Explanation: Since, the three zeroes = – 3, – 1, 2

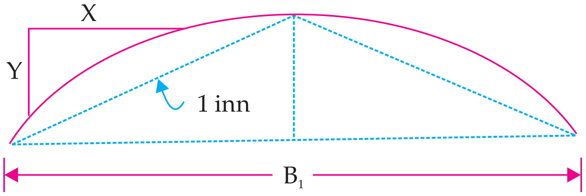
Hence, the expression is $(x + 3)(x + 1)(x - 2)$

$$= [x^2 + x + 3x + 3](x - 2)$$
$$= x^3 + 4x^2 + 3x - 2x^2 - 8x - 6$$
$$= x^3 + 2x^2 - 5x - 6$$

AI **Q. 4. Applications of Parabolas: Highway Overpasses/ Underpasses**

A highway underpass is parabolic in shape.

Shape of Cross Slope :



a. Paraboilc camber

$y = 2x^2/nw$

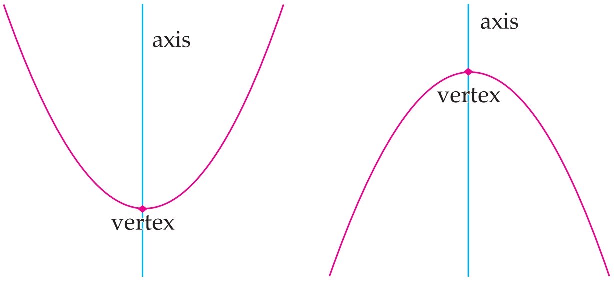
Parabola

A parabola is the graph that results from

$$p(x) = ax^2 + bx + c.$$

Parabolas are symmetric about a vertical line known as the **Axis of Symmetry.**

The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the vertex.



C + AE [CBSE SQP, 2020-21]

(i) If the highway overpass is represented by $x^2 - 2x - 8$. Then its zeroes are

(a) **(2, – 4)** **(b)** **(4, – 2)**

(c) **(– 2, – 2)** **(d)** **(– 4, – 4)**

Sol. Correct option: **(b)**

(4, – 2) [CBSE Marking Scheme, 2020]

Detailed Solution:

$$x^2 - 2x - 8 = 0$$

or, $x^2 - 4x + 2x - 8 = 0$

or, $x(x - 4) + 2(x - 4) = 0$

or, $(x - 4)(x + 2) = 0$

or, $x = 4, x = -2$ 1

(ii) The highway overpass is represented graphically. Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial:

(a) **Intersects X-axis**

(b) **Intersects Y-axis**

(c) **Intersects Y-axis or X-axis**

(d) **None of the above**

Sol. Correct option: **(a)**

Explanation: We know that the number of zeroes of polynomial is equal to number of points where the graph of polynomial intersects X-axis. 1

(iii) Graph of a quadratic polynomial is a:

(a) **straight line** **(b)** **circle**

(c) **parabola** **(d)** **ellipse**

Sol. Correct option: **(c)**

Explanation: Here, the given graph of a quadratic polynomial is a parabola. 1

(iv) The representation of Highway Underpass whose one zero is 6 and sum of the zeroes is 0, is:

(a) $x^2 - 6x + 2$ **(b)** $x^2 - 36$

(c) $x^2 - 6$ **(d)** $x^2 - 3$

Sol. Correct option: **(b)**

$x^2 - 36$ [CBSE Marking Scheme, 2020]

Detailed Solution:

$x^2 - 36 = 0$ $\Rightarrow x^2 = 36$

$\Rightarrow$ $x = 6, -6.$ $\Rightarrow x = \pm\sqrt{36}$

1

(v) The number of zeroes that polynomial $f(x) = (x-2)^2 + 4$ can have is:

(a) 1 **(b)** 2

(c) 0 **(d)** 3

Sol. Correct option: **(c)**

Explanation: We have,

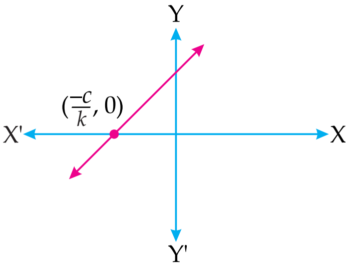
$$f(x) = (x-2)^2 + 4$$
$$= x^2 + 4 - 4x + 4$$
$$= x^2 - 4x + 8.$$

i.e., It has no factorisation.

Hence no real value of x is possible, *i.e.*, no zero. **1**

Q. 5. For a linear polynomial $kx + c$, $k \neq 0$, the graph of $y = kx + c$ is a straight line which intersects the X-axis at exactly one point, namely, $\left(\frac{-c}{k}, 0\right)$,

Therefore, the linear polynomial $kx + c$, $k \neq 0$, has exactly one zero, namely, the X-coordinate of the point where the graph of $y = kx + c$ intersects the X-axis. $\boxed{C}$ + $\boxed{AE}$



Give answer the following questions:

(i) If a linear polynomial is $2x + 3$, then the zero of $2x + 3$ is:

(a) $\frac{3}{2}$ **(b)** $-\frac{3}{2}$

(c) $\frac{2}{3}$ **(d)** $-\frac{2}{3}$

Sol. Correct option: **(b)**

Explanation: Given, polynomial = $2x + 3$

Let $p(x) = 2x + 3$

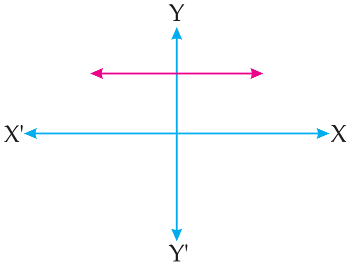
For a zero of $p(x)$,

$$2x + 3 = 0$$
$$\Rightarrow \quad 2x = -3$$
$$\Rightarrow \quad x = -\frac{3}{2}.$$ **1**

(ii) The graph of $y = p(x)$ is given in figure below for some polynomial $p(x)$. The number of zero/zeroes of $p(x)$ is/are:

(a) 1 **(b)** 2

(c) 3 **(d)** 0



Sol. Correct option: **(d)**

Explanation: Since the graph does not intersect the X-axis, therefore it has no zero. **1**

(iii) If α and β are the zeroes of the quadratic polynomial $x^2 - 5x + k$ such that $\alpha - \beta = 1$, then the value of k is:

(a) 4 **(b)** 5

(c) 6 **(d)** 3

Sol. Correct option: **(c)**

Explanation: $\because \; p(x) = x^2 - 5x + k$

Then $\alpha - \beta = \frac{-(-5)}{1} = 5$

and $\alpha\beta = \frac{k}{1} = k$ ½

Also given, $\alpha - \beta = 1$

$$\Rightarrow \quad \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 1$$
$$\Rightarrow \quad \sqrt{(5)^2 - 4k} = 1$$
$$\Rightarrow \quad 25 - 4k = 1 \quad \text{(Squaring both sides)}$$
$$\Rightarrow \quad -4k = 1 - 25 = -24$$
$$\Rightarrow \quad k = 6.$$ ½

(iv) If α and β are the zeroes of the quadratic polynomial $p(x) = 4x^2 + 5x + 1$, then the product of zeroes is:

(a) -1 **(b)** $\frac{1}{4}$

(c) -2 **(d)** $-\frac{5}{4}$

Sol. Correct option: **(b)**

Explanation: We have, $p(x) = 4x^2 + 5x + 1$

$$\therefore \quad \alpha\beta = \frac{c}{a} = \frac{1}{4}.$$ **1**

(v) If the product of the zeroes of the quadratic polynomial $p(x) = ax^2 - 6x - 6$ is 4, then the value of a is:

(a) $-\frac{3}{2}$ **(b)** $\frac{3}{2}$

(c) $\frac{2}{3}$ **(d)** $-\frac{2}{3}$

Sol. Correct option: **(a)**

Explanation: We have,

$$p(x) = ax^2 - 6x - 6$$

Let α and β be the zeroes of the given polynomial, then

$$\alpha\beta = \frac{c}{a}$$

i.e., $$4 = \frac{-6}{a}$$

$$\Rightarrow \quad 4a = -6$$

$$\Rightarrow \quad a = -\frac{6}{4} = -\frac{3}{2}$$ **1**