1 MARK

1. Three primes, p, q and r satisfy p + q = r and 1 < p < q. The p equals

(1) 2 (2) 3 (3) 7 (4) 13

2. Let n be the smallest non-prime integer greater than 1 with no prime factor less than 10. Then

(1) $100 < n \leq 110$ (2) $110 < n \leq 120$

(3) $120 < n \leq 130$ (4) $130 < n \leq 140$

3. The largest integer n for which $n^{200} < 5^{300}$ is

(1) 8 (2) 9 (3) 10 (4) 11

4. The number of digits in $4^{16}5^{25}$ (when written in the usual base 10 form) is

(1) 31 (2) 30 (3) 29 (4) 28

5. Let S be the statement " If the sum of the digits of the whole number n is divisible by 6 , then n is divisible 6". A value of n which shows S to be false is

(1) 30 (2) 33 (3) 40 (4) 42

6. How many primes less than 100 have 7 as the ones digit ? (Assume the usual base 10 representation)

(1) 4 (2) 5 (3) 6 (4) 7

7. How many different values of n are there such that n is a natural number and $n^2 - 440$ is a perfect square ?

(1) 1 (2) 2

(3) 3 (4) 4

8. The stronger Goldbach conjecture states that any even integer greater than 7 can be written as the sum of two different prime numbers. For such representations of the even number 126, the largest possible difference between the two primes is -

(1) 112 (2) 100 (3) 92 (4) 88

9. If $p \geq 5$ is a prime number, then 24 divides p^2-1 without remainder -

(1) never (2) sometimes only

(3) always (4) only if p = 5

10. In the following equation, each of the letters represents uniquely a different digit in base ten: (YE). (ME) = TTT

The sum E + M + T + Y equals -

(1) 19 (2) 20

(3) 21 (4) 22

11. What is the smallest prime number dividing the sum $3^{11} + 5$ -

(1) 2 (2) 3

(3) 5 (4) $3^{11}+5^{13}$

12. The integers greater than one are arranged in five columns as follows :

2	3	4	5
9	8	7	6
10	11	12	13
17	16	15	14

(Four consecutive integers appear in each row; in the first, third and other odd numbered rows, the integers appear in the last four columns and increase from left to right; in the second. Fourth and other even numbered rows, the integers appear in the first four columns and increase from right to left.) In which column will the number 1,000 fall ?

(1) first (2) second

(3) third (4) fourth

13. How many integers greater than ten and less than one hundred, written in basc ten notation. are increased by nine when their digits are reversed ?

(1) 0 (2) 1 (3) 8 (4) 9

14. If r is the remainder when each of the numbers 1059. 1417 and 2312 is divided by d. where d is an integer greater than one, then d - r equals-

(1) 1 (2) 15 (3) 179 (4) d-15

15. Al's age is 16 more than the sum of Bob's age and Carl's age, and the square of Al's age is 1632 more than the square of the sum of Bob's age and Carl's age. The sum of the ages of Al, Bob and Carl is

(1) 64 (2) 94 (3) 96 (4) 102

16. How many pairs (m,n) of integers satisfy the equation m + n = mn ?

(1) 1 (2) 2 (3) 3 (4) 4

17. There is more than one integer greater than 1 which, when divided by any integer k such that 2 < k < 11 has a remainder of 1. What is the difference between the two smallest such integers-

(1) 2310 (2) 2311

(3) 2520 (4) 2720

18. Find the sum of the digits of the largest even three digit number (in base ten representation) which is not changed when its units and hundreds digits are interchanged.

(1) 22 (2) 23 (3) 24 (4) 25

19. The square of an integer is called a perfect square. If x is a perfect square, the next larger perfect square is :-

(1) $x + 1$ (2) $x^2 + 1$

(3) $x^2 + 2x + 1$ (4) $x+2\sqrt{x}+1$

20. Find the number of pairs (m, n) of integers which satisfy the equation

$m^3 + 6m^2 + 5m = 27n^3 + 9n^2 + 9n + 1$.

(1) 0 (2) 1 (3) 3 (4) 9

21. The largest whole number such that seven times the number is less than 100 is :-

(1) 12 (2) 13 (3) 14 (4) 15

22. A store prices an item in rupees and paise so that when 4% sales tax is added no rounding is necessary because the result is exactly n rupees, where n is a positive interger. The smallest value of n is :-

(1) 1 (2) 13 (3) 25 (4) 26

23. If three times the largest of two numbers is four times the smaller and the difference between the numbers is 8. then the largest of the two numbers is :-

(1) 16 (2) 24 (3) 32 (4) 44

24. How many of the first one hundred positive integers are divisible by all of the numbers 2, 3, 4, 5 ?

(1) 0 (2) 1 (3) 3 (4) 4

25. If a number eight times as large as x is increased by two. then one fourth of the result equals

(1) $2x + \frac{1}{2}$ (2) $x + \frac{1}{2}$

(3) $2x + 2$ (4) $2x + 4$

26. The last digit of the value of the number $2003^{2002} + 2001^{2002}$ is :-

(1) 9 (2) 1 (3) 4 (4) 0

27. Which of the following numbers can be written both as a sum of two consecutive natural numbers and as a sum of three consecutive natural numbers?

(1) 20,002 (2) 30,004

(3) 40,005 (4) 50,006

28. The sum of digits of every possible 8 digit is noted. Which sum occurs most often ?

(1) Both 27, 28 (2) 41

(3) 32 (4) 36

29. The number of positive integers less than 1849 and prime to 43 is :-

(1) 1764 (2) 1806 (3) 86 (4) 1681

30. Which of the following is closest to $\sqrt{65}-\sqrt{63}$?

(1) .12 (2) .13 (3) .14 (4) .15

31. Determine the number of integer n for which $n^2 + 19n + 92$ is a square.

(1) 1 (2) 2 (3) 3 (4) 4

32. Find all the number of unordered pairs of natural numbers, the difference of whose square is 45.

(1) 2 (2) 3 (3) 4 (4) 5

33. Find the number of elements of the set

$$S = \left\{ x \in \mathbb{Z} \left| \frac{x^3 - 3x + 2}{2x+1} \in \mathbb{Z} \right. \right\}.$$

(1) 6 (2) 8 (3) 7 (4) 9

34. Consider two positive integer a and b. Find the least possible value of the product ab if $a^b b^a$ is divisible by 2000

(1) 10 (2) 20 (3) 40 (4) 80

2 MARK

1. Let x = .123456789101112 ...998999, Where the digits are obtained by writing the integers 1 through 999 in order. The 1983rd digit to the right of the decimal point is

(1) 2 (2) 3 (3) 5 (4) 7

2. The number of distinct pairs of integers (x, y) such that $0 < x < y$ and $\sqrt{1984} = \sqrt{x} + \sqrt{y}$ is

(1) 0 (2) 1 (3) 3 (4) 4

3. The odd positive integers, 1,3,5, are arranged in five columns continuing with the pattern shown on the right counting from the left, the column in which 1985 appears is the

	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	33	35	37	39
47	45	43	41	
	49	51	53	55
•	•	•	•	
	•	•	•	•
•	•	•	•	
	•	•	•	•

(1) first (2) second (3) third (4) fourth

4. Let p,q and r be distinct prime numbers, where 1 is not considered a prime. Which of the following is the smallest positive perfect cube having $n = pq^2r^4$ as a divisor ?

(1) $p^8q^8r^8$ (2) $(pq^2r^2)^3$
(3) $(p^2q^2r^2)^3$ (4) $(pqr^2)^3$

5. Six bags of marbles contain 18, 19, 21, 23, 25 and 34 marbles respectively. One bag contains chipped marbles only. The other 5 bags contain no chipped marbles. Jane takes three of the bags and George takes two of the other. Only the bag of chipped marbles remains. If Jane gets twice as many marbles as George, how many chipped marbles are there ?

(1) 18 (2) 19 (3) 21 (4) 23

6. How many ordered triples (a,b,c) of non-zero real numbers have the property that each number is the product of the other two ?

(1) 1 (2) 2 (3) 3 (4) 4

7. Each integer 1 through 9 is written on a separate slip of paper and all nine slips are put into a hat. Jack picks one of these slips at random and puts it back. Then Jill picks a slip at random. Which digit is most likely to be the units digit of the sum of Jack's integer and Jill's integer ?

(1) 0 (2) 1 (3) 8 (4) 9

8. How many ten-digits numbers (written in base ten) have this property : For $1 \leq k \leq 10$, the kth digit from the left is equal to the number of times the digit k-1 appears in the number.

(1) 0 (2) 1 (3) 2 (4) 3

9. Let [x] be the greatest integer $\leq$ x. Let r be a random number strictly between 0 and 1. Then which expression below gives an integer from 0 to 10, inclusive, with each integer equally likely ?

(1) [10r] (2) 10[r]
(3) [10r + .5] (4) [11r]

10. The number of sets of two or more consecutive positive integers whose sum is 100 is -

(1) 1 (2) 2 (3) 3 (4) 4

11. What is the smallest positive odd integer n such that the product $2^{1/7}2^{3/7}......2^{(2n+1)/7}$ is greater than 1000 ? (In the product the denominators of the exponents are all sevens, and the numerators are the successive odd integers from 1 to 2n + 1)

(1) 7 (2) 9 (3) 11 (4) 17

12. A six digit number (base 10) is squarish if it satisfies the following conditions :-

(i) none of its digits is zero
(ii) it is a perfect square; and
(iii) the first two digits, the middle two digits and the last two digits of the number are all perfect squares when considered as twodigit numbers.

How many squarish numbers are there ?

(1) 0 (2) 2 (3) 3 (4) 8

13. If an integer of 2 digits is k times the sum of its digits, then the value of the number got by interchanging the digits is the sum of the digits multiplied by :-

(1) (9 – k) (2) (10 – k)
(3) (11 – k) (4) (k – 1)

14. Which of the following statements is true ?

(1) The sum of 3 consecutive integers is 2005
(2) The sum of 4 consecutive integers is 2005
(3) The sum of 5 consecutive integers is 2005
(4) The product of 3 consecutive integers is 2005

15. Find the number of positive integer n for which (i) $n \leq 1991$ (ii) 6 is a factor of $n^2 + 3n + 2$.

(1) 1328 (2) 1024
(3) 628 (4) 1724

16. Find all the number of non negative integers n such that there are integers a and b with the property $n^2 = a + b$ and $n^3 = a^2 + b^2$.

(1) 3 (2) 5 (3) 4 (4) 2

17. When the tens digit of a three digit number abc is deleted, a two digit number bc is formed. How many numbers abc are there such that abc = 9ac + 4c.

(1) 4 (2) 5

(3) 6 (4) 8

18. Let S(M) denote the sum of the digits of a positive integer M written is base 10. Let N be the smallest positive integer such that S(N) = 2017. The value of S(5N + 2017)

(1) 4 (2) 5 (3) 6 (4) 7

19. For how many natural numbers n between 1 and 2014 (both inclusive) is $\frac{8n}{9999-n}$ an integer ?

(1) 2 (2) 3 (3) 1 (4) 4

20. Positive integers a and b are such that $a + b = \frac{a}{b} + \frac{b}{a}$. Value of $(a^2 + b^2)$ equals

(1) 2 (2) 3 (3) 4 (4) 5

21. What is the greatest possible perimeter of a right angled triangle with integer side lengths if one of the sides has length 12 ?

(1) 72 (2) 56 (3) 42 (4) 84

22. Let n be the largest integer that is the product of exactly 3 distinct prime numbers x, y and 10x + y where x and y are digits. Sum of digits of n equals

(1) 14 (2) 16 (3) 12 (4) 18

23. The digits of a positive integer n are four consecutive integers in decreasing order when read from left to right. What is the sum of the possible remainders when n is divided by 37?

(1) 217 (2) 197 (3) 327 (4) 177

24. Find the number of triplets of (p, q, r) of primes such that pq = r + 1 and $2(p^2 + q^2) = r^2 + 1$

(1) 4 (2) 2 (3) 1 (4) 3

1 MARK

1. q cannot be 2
r can not be 2
$p = r - q$
r and q are odd prime numbers
$\therefore$ r – q must be even
only even prime no. = 2
$\therefore$ p = 2

2. n must have factors as 11 or greater than 11
n is not prime also.

For smallest n

$n = 11 \times 11 = 121$

Here, Option (3)

3. $n < (5)^{300/200}$
$\Rightarrow n < 5^{3/2}$
$\Rightarrow n < 5\sqrt{5}$
$\Rightarrow n^2 < 125$
$\therefore$ Largest n = 11

4. Let $x = 4^{16}.5^{25}$
$x = 2^{32}.5^{25}$
$x = 2^7.(10)^{25}$
$x = 128 \times 10^{25}$
$\therefore$ No. of digits = 28

5. n = 30 then 3 + 0 = 3
Here, sum of digits $\neq$ 6
n = 33 then 3 + 3 = 6
Sum of digits = 6
But n is not divisible by 6
Hence, option (2)
Check for other options in same way.

6. Check for 7, 17, 27, 37, 47, 57, 67, 77, 87, 97
Out of above nos. 7, 17, 37, 47, 67, 97 are prime nos. Hence, total 6 nos.

7. $n^2 - 440 = k^2$
$\Rightarrow n^2 - k^2 = 440$
$\Rightarrow (n + k)(n - k) = 440$
Here, both n and k must be even
or both n and k must be odd.
$\therefore$ (n + k) and (n – k) will be even
Now, $(n + k)(n - k) = 440 \times 1$
$= 220 \times 2$
$= 110 \times 4$
$= 55 \times 8$
$= 88 \times 5$
$= 40 \times 10$
$= 44 \times 10$
$= 22 \times 20$
n + k will be greater than n – k out of above for factor only 220 × 2, 110 × 4, 44 × 10 and 22 × 20 are possible because both factors must be even.

8. One prime no. must be as close to no. 126 as possible.
And that is 113
126 = 113 + 13
difference = 113 – 13 = 100

9. Consider three consecutive numbers (p – 1), p, (p + 1)
Now, one of them must be divisible by 3.
P is prime and neighbours of P are consecutive even numbers.
$\therefore$ One of them must be divisible by 2 and other by 4.
$\therefore$ (p – 1).p.(p + 1) is div. by 3 × 2 × 4 = 24
Here, P is prime
$\therefore$ Actually (p – 1).(p + 1) or $(p^2 - 1)$ is div. by 24.

10. TTT has all three digits same.
111 = 3 × 37
222 = 6 × 37
333 = 9 × 37
444 = 12 × 37
555 = 15 × 37
666 = 18 × 37
777 = 21 × 37
888 = 24 × 37
<u>999 = 27 × 37</u>
TTT must have two factor (both factors of two digits and unit digit in both factors are same)
$\therefore$ Only possible case = 999
27 × 37 = 999
$\therefore$ Y = 2, E = 7, M = 3, T = 9
E + M + T + Y = 21

11. 3^{11} is odd
$\therefore$ $3^{11} + 5$ is even
$\therefore$ It is divisible by 2 (prime no.)

12. 1000 will be in the series 8, 16, 24,..... that is 2^{nd} column.

13. $12 \rightarrow 21$

$23 \rightarrow 32$

$34 \rightarrow 43$

$45 \rightarrow 54$

$56 \rightarrow 65$

$67 \rightarrow 76$

$78 \rightarrow 87$

$89 \rightarrow 98$

Total 8 cases.

14. $2312 = a \times d + r$ .... (i)

$1417 = b \times d + r$ .... (ii)

$1059 = c \times d + r$ .... (iii)

(i) - (ii) $\Rightarrow 895 = (a - b) \times d$

$5 \times 179 = (a - b) \times d$ ..... (iv)

(ii) - (iii) $\Rightarrow 358 = (b - c) \times d$

$2 \times 17d = (b - c) \times d$ ..... (v)

from (iv) & (v)

$d = 179$

from (iii)

$1059 = 5 \times 179 + 164$

$\therefore r = 164$

$d - r = 179 - 164 = 15$

15. Let age of Al's, Bob's and carls are a, b, c is

$a = b + c + 16$

$a^2 = (b + c)^2 + 1632$

$\therefore a^2 = (a - 16)^2 + 1632$

$\Rightarrow 0 = -32a + 256 + 1632$

$\Rightarrow a = 59$

$\therefore b + c = 43$

$a + b + c = 59 + 43 = 102$

16. $m - mn + n = 0$

$\Rightarrow m(1 - n) + n - 1 = -1$

$\Rightarrow m(1 - n) - (1 - n) = -1$

$\Rightarrow (m - 1)(1 - n) = -1$

$\Rightarrow (m - 1)(n - 1) = +1$

$$\begin{cases} m-1=1 \\ n-1=1 \end{cases} \text{ or } \begin{cases} m-1=-1 \\ n-1=-1 \end{cases}$$

$\Rightarrow m = 2, n = 2$ or $m = 0, n = 0$

17. K = 3, 4, 5, 6, 7, 8, 9, 10

LCM of (3, 4, 5, 6, 7, 8, 9, 10) = 2520

Smallest such number = 2520 + 1

Next smallest such number = $2520 \times 2 + 1$

Difference = $(2520 \times 2 + 1) - (2520 + 1)$

= 2520

18. Unit place will have largest even no. that is 8.

Hundreds place no. must also be 8

For largest no., thes digit must be 9.

$\therefore$ No. is 898

Sum of digits = 8 + 9 + 8 = 25

19. $(\sqrt{x}+1)^2 = x + 2\sqrt{x} + 1$

Hence, option (4)

20. LHS = $m^3 + 6m^2 + 5m$

$= m(m + 1)(m + 5)$

$\because m(m + 1)(m + 2)$ is div. by 3

Hence, LHS is div. by 3

RHS $\equiv 27n^3 + 9n^2 + 9n + 1$

For any integer 'n' when RHS is divided by 3 than remainder = 1

$\therefore$ RHS is not div. by 3

Hence, LHS $\neq$ RHS

for any pair (m, n)

21. $100 = 14 \times 7 + 2$

$\therefore$ Largest number = 14

22. Let initial amount = x

And final amount = n

$$n = x + \frac{4}{100} \times x$$

$$\Rightarrow n = \frac{104x}{100}$$

$$\Rightarrow x = \left(\frac{100}{104}\right)n = \left(\frac{25}{26}\right)n$$

Now, n must be a factor of 26.

$\therefore$ Possible values of n = 1, 2, 13, 26

But x is terminating decimal.

$\therefore$ n = 13

23. $a - b = 8$ .... (i)

$3a = 4b$ .... (ii)

$\therefore 3(8 + b) = 4b$

$\Rightarrow b = 24$

$\therefore a = 32$

24. LCM of (2, 3, 4, 5) = 60

$\therefore$ Number must be multiple of 60 so, only one number.

25. $\frac{8x+2}{4}$

$= 2x + \frac{1}{2}$

26. Unit digit of 2003^{2002}

= Unit digit of 3^{2002}

= Unit digit of 3^2

= 9

Unit digit of 2001^{2002}

= Unit digit of 1^{2002}

= 1

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$

Frequency = 4

$\therefore$ Last digit = Last digit of (9 + 1)

= 10

$\therefore$ Last digit = 0

27. $\because$ Number can be written as two consecutive natural nos

$\therefore$ it must be odd.

$\because$ Number can be written as three consecutive natural nos.

$\therefore$ it must be multiple of 3

Hence, No. = 40,005

28. Maximum sum = 8 × 9 = 72

Minimum sum = 8 × 0 = 0

Average of two $= \frac{72+0}{2} = 36$

Sum 36 will occur most often.

29. 1849 = 43 × 43

There are 43 numbers from 1 to 1849 which are not prime to 43.

$\therefore$ Numbers which are prime to 43 from 1 to 1849 = 1849 – 43 = 1806

30. $\sqrt{65} = 8 + x$

$\sqrt{63} = 8 - y$

x < y

$\left(\sqrt{65}-\sqrt{63}\right)\left(\sqrt{65}+\sqrt{63}\right) = 2$

$\sqrt{65}-\sqrt{63} = \frac{2}{(8+x)+(8-y)}$

$\therefore$ $\sqrt{65}-\sqrt{63}$ is slightly greater than $\frac{2}{16}$ or 0.125

$\therefore$ $\sqrt{65}-\sqrt{63}$ is close to 0.13

31. $n^2 + 19n + 92 = k^2$

$\Rightarrow n^2 + 19n + 92 - k^2 = 0$ .... (i)

$\because$ Eq^n (i) has integral roots $\therefore$ it D must be perfect square.

$\therefore (19)^2 - 4(1)(92 - k^2) = m^2$

$\Rightarrow 4k^2 - m^2 = 7$

$\Rightarrow (2k)^2 - (m)^2 = 7$

$\Rightarrow (2k + m)(2k - m) = 7 \times 1$

$2k + m = 7$

$\underline{2k - m = 1}$

$4k = 8 \Rightarrow k = 2$

From (i)

$n^2 + 19n + 92 - 4 = 0$

$\Rightarrow n^2 + 19n + 88 = 0$

$\Rightarrow (n + 11)(n + 8) = 0$

$\Rightarrow (n = -11, -8$

32. $a^2 - b^2 = 45$

$(a - b)(a + b) = 45 \times 1$
$= 15 \times 3$
$= 9 \times 5$

$\begin{cases} a-b=1 \\ a+b=45 \end{cases}$ $\begin{cases} a-b=3 \\ a+b=15 \end{cases}$ $\begin{cases} a-b=5 \\ a+b=9 \end{cases}$

a = 23 a = 9 a = 7

b = 22 b = 6 b = 2

$\therefore$ three unordered pairs.

33. Let $2x + 1 = t \Rightarrow x = \frac{t-1}{2}$

$\therefore\ y = \frac{\left(\frac{t-1}{2}\right)^3 - 3\left(\frac{t-1}{2}\right) + 2}{t}$

$\Rightarrow y = \frac{t^3 - 3t^2 + 3t - 1 - 12t + 12 + 16}{8t} \in I$

$\Rightarrow \frac{t^3 - 3t^2 - 9t + 27}{t} \in I$

$\Rightarrow t^3 - 3t^2 - 9 + \frac{27}{t} \in I$

$\frac{27}{t}$ must be int.

$\therefore t = \pm1, \pm3, \pm9, \pm27$

Hence, 8 values of $t \in I$

or 8 values of $x \in I$

34. Number of the form $a^b.b^a$ and divisible by 2000 and having smallest a & b is

$5^4 \times 4^5$

$\therefore$ a = 5, b = 4

a × b = 5 × 4 = 20

2 MARK

1. Single digits = $1 \times 9 = 9$

double digits = $2 \times 90 = 180$

$1983 - 9 - 180 = 1794$

$\therefore$ 1794^{th} digit starting from 100 onwards

$= \frac{1794}{3} = 598$

So, Number will be 598 + (100 – 1)

= 597

0.123 .... .596.597 ....

↑

1983th digit

2. $\sqrt{1984} = \sqrt{x} + \sqrt{y}$

$8\sqrt{31} = \sqrt{x} + \sqrt{y}$

$\therefore \sqrt{x} = a \times \sqrt{31}$

$\sqrt{y} = b \times \sqrt{31}$

Such that $a + b = 8$ & $(a < b)$

$\therefore$ a = 1, b = 7

a = 2, b = 6

a = 3, b = 5

Hence, three pairs.

3. Differential series are

15, 31, 47, ...

1, 17, 33, ...

13, 29, 45, ...

Common difference in each = 16

$\frac{1985}{16} = 124\frac{1}{16}$ Remainder = 1

$\because$ 1 is in 2^{nd} column

$\therefore$ Ans. $\Rightarrow$ Second coloumn

4. $p^8.q^8.r^8$ is not a perfect cube.

Remaining three options are perfect cube and all are divisible by $pq^2.r^4$. Smallest of those is $(pqr^2)^3$

Hence, option(4)

5. $\left.\begin{array}{l}\text{Let George get x marbles} \\ \therefore \text{Jane gets 2x marbles.}\end{array}\right\} x + 2x = 3x$

$\therefore$ Marbles of 5 begs must be divisible by 3

$\therefore$ 5 bags can be of 18, 19, 21, 25, 34 marbles

18 + 19 + 21 + 25 + 34 = 117

$\therefore$ George gets $\frac{17}{3} = 39$ Marbles

39 = 18 + 21

Jane get 78 marbles

78 = 19 + 25 + 34

$\therefore$ No. of chipped marbles = 23

6. a = bc, b = ca, c = ab

Multiply $abc = a^2b^2c^2$

$\Rightarrow abc = 1$

$\Rightarrow a = \pm 1$

Similary $b = \pm 1$, $c = \pm 1$

Now, possible order triplets

a = 1, b = 1, c = 1

a = 1, b = –1, c = –1

a = –1, b = 1, c = –1

a = –1, b = –1, c = –1

7. 1, 2, 3, 4, 5, 6, 7, 8, 9

1, 2, 3, 4, 5, 6, 7, 8, 9

Sum of digits can be 2, 3, 4, 5, ...,17, 18

And their no. of ways are

1,2,3,4, ... 8, 9, 8, ... 2, 1 respective

Sum of digit 10 can occure 9 ways

Which are maximum no. of ways.

$\therefore$ Unit digit (most likely) = 0

8. $\lfloor 12 + \lfloor 13 + \lfloor 14$

$= \lfloor 12 \, (1 + 13 + 14 \times 13)$

$= \lfloor 12 (14)^2$

It is divided by prime numbers 2,3,5,7,11

$\therefore$ 5 distinct prime numbers.

9. $0 \le [10\,r\,] \le 9$

$10[r] = 0$

$0 \le [10r + 0.5] \le 10$

But 0 to 10 will not be equally likely.

$0 \le [11r] \le 10$ In this case 0 to 10 will come equally likely.

10. $\left.\begin{array}{ll}18,19,20,21,22 & \text{first set} \\ 9,10,11,12,13,14,15,16 & 2^{nd} \text{ set}\end{array}\right\}$

For 8 numbers

n + (n + 1) + (n + 2) + (n + 3) + (n + 4)

+ 4(n + 5) + (n + 6) + (n + 7) = 100

$\Rightarrow 8n + 28 = 100$

$\Rightarrow n = 9$

Similarly make cases for 9, 10, 11, ...

numbers. etc.

11. $2^{\left(\frac{1}{7}+\frac{3}{7}+......+\frac{2n+1}{7}\right)}{}_{>1000}$

$\Rightarrow\ 2^{n^{2/7}} > 1000$ .... (i)

$\Rightarrow\ 2^{n^{2/7}} > 2^{9.........}$

$\frac{n^2}{7} > 9$ ..... $\Rightarrow n^2 > 63$ ....

But, n = 8 not satisfies eq^n (i)

& n = 9 satisfies eq^n (i)

$\therefore$ min. values of n = 9

12. $N = a^2 \times 10^4 + b^2 \times 10^2 + c^2 \times 1$

& $N = (a^2 \times 100 + c)^2$

$\therefore\ b^2 = 2 \times a \times c$

but $4 \le a,b,c \le 9$

$\therefore$ Possible cases $a = 2 \times 2 \times 2$, $C = 2 \times 2$

& $b = 2 \times 2 \times 2$

or $a = 2 \times 2$, $c = 2 \times 2 \times 2$ & $b = 2 \times 2 \times 2$

$\therefore$ (a,b,c) $\equiv$ (4, 8, 8), (8 , 8, 4)

N = 166464, 646416

13. ab = k(a + b)

$\Rightarrow$ 10a + b = k(a + b) ..... (i)

ba = k_1(a + b)

$\Rightarrow$ 10b + a = k_1(a + b) ..... (ii)

(i) + (ii)

11a + 11b = k(a + b) + k_1(a + b)

$\Rightarrow$ 11 = k + k_1

$\Rightarrow$ k_1 = 11 – k

$\therefore$ Option (3)

14. Option (1) k + k + 1 + k + 2 = 2005

$\Rightarrow k = \frac{2002}{3} \notin I$

$\therefore$ option (1) is wrong

Option (2) k + k + 1 + k + 2 + k + 3 = 2005

$\Rightarrow\ 4k = 1999 \Rightarrow k = \frac{1999}{4} \notin I$

$\therefore$ Option (2) is wrong

Option (3)

k + k + 1 + k + 2 + k + 3 + k + 4 = 2005

$\Rightarrow$ 5k = 1995 $\Rightarrow$ k = 399 $\in$I

$\therefore$ option (3) is correct

Option (4)

(k) (k + 1) (k + 2) = 2005 = 401 × 5 × 1

$\therefore$ option (4) is wrong.

15. $n^2 + 3n + 2 = \underbrace{(n + 1)(n + 2)}$

Prod. consecutive integers are always div. by 2.
Now, one of (n + 1) or (n + 2) must be multiple of 3

n + 1 = 3k or n + 2 = 3k

$\therefore$ n = 3k – 1 or n = 3k – 2

So, n cannot be multiple of 3.

Numbers which are multiple of 3 and less than or equal to 1991

$= \left[\frac{1991}{3}\right] = 663$ nos.

$\therefore$ Possible values = 1991 – 663

= 1328

16. $n^2 = a + b$

$\therefore\ n^4 = a^2 + b^2 + 2ab$

$\Rightarrow n^3(n - 1) = 2ab$

$\Rightarrow (a^2 + b^2)(n - 1) = 2ab$

$\Rightarrow n - 1 = \frac{2ab}{a^2 + b^2}$

but, $\frac{2ab}{a^2+b^2} \le 1$

$\therefore$ (n – 1) $\le$ 1 $\Rightarrow$ n $\le$ 2

n = 0, 1, 2

For n = 0, a = 0, b = 0

For n = 1, a = 0, b = 1 or a = 1, b = 0

For n = 2, a = 2, b = 2

$\therefore$ 3 cases

17. abc = 9ac + 4c

$\Rightarrow$ 100a + 10b + c = 90a + 9c + 4c

$\Rightarrow$ 10a + 10 b = 12c

$\Rightarrow$ 5(a + b) = 6c

$\therefore$ c = 5

5(a + b) = 6 × 5

$\Rightarrow$ a + b = 6

(a, b) $\equiv$ (6, 0), (5, 1), (4, 2), (3, 3), (2, 4), (1, 5)

abc $\Rightarrow$ 605, 515, 425, 335, 245, 155

Total 6 case.

18. $N = 1\underbrace{999.....9}_{224\text{ times}}$

$\Rightarrow N = 2\underbrace{000.....0}_{225\text{times}} - 1$

$N = 1\underbrace{000.....0}_{226\text{times}} - 5 + 2017$

$N = 1\underbrace{000.....0}_{226\text{times}} + 2012$

$\therefore$ S(5N + 2017) = 1 + 2 + 1 + 2 = 6

19. Let $\frac{8N}{9999-n} = k$ $(K \in I)$

$\Rightarrow 8n = 9999 \times k - nk$

$\Rightarrow n(8 + k) = 9999.k$

$\Rightarrow n = \frac{9999 \times k}{8+k}$

For k = 1 $n = \frac{9999 \times 1}{9} = 1111 < 2014$

$\boxed{\therefore n = 1111}$

For k = 2 $n = \frac{9999 \times 2}{10} \notin I$

For k = 3 $n = \frac{9999 \times 3}{11} = \underset{\text{(rejected)}}{2727 > 2014}$

If we increase value of k then value of n will also increase. And will be greater then 2014.

$\therefore$ 1 value of n (n = 1111)

20. $a + b = \frac{a^2+b^2}{ab}$

$\Rightarrow a^2b + ab^2 = a^2 + b^2$

$\Rightarrow a^2b + ab^2 - a^2 - b^2 = 0$

$\Rightarrow a^2(b - 1) + b^2 (a - 1) = 0$

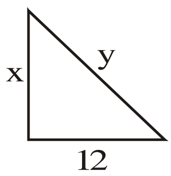
$\because$ a & b are +ve int.

$\therefore a - 1 = 0$ & $b - 1 = 0$

a = 1, b = 1

$\therefore a^2 + b^2 = 1^2 + 1^2 = 2$

21. $y^2 = x^2 + 12^2$



$\Rightarrow y^2 - x^2 = 144$

$\Rightarrow (y + x)(y - x) = 144 \times 1$
$= 72 \times 2$

$\left.\begin{matrix} x + y = 144 \\ y - x = 1 \end{matrix}\right\}$ not possible as $2y = 145 \Rightarrow y \notin I$

$\therefore x + y = 72$ $\quad y - x = 22$

$\Rightarrow 2y = 74 \Rightarrow y = 37$ $\quad \Rightarrow x = 35$ } For max. perimeta

Max. perimeter = x + y + 12

= 72 + 12

= 84

22. For max. prod., x = 7 y = 3

$10x + y = 10 \times 7 + 3 = 73$ (y cannot be 5)

$n = x \times y\,(10x + y)$

$= 7 \times 3 \times (10 \times 7 + 3)$

$= 21 \times 73 = 1533$

Sum of digits = 1 + 5 + 3 + 3 = 12

23. Possible N are 9876, 8765, 7654, 6543, 5432, 4321, 3210

Now, $3210 = (86 \times 37) + 28$

rem. = 28

Now, difference of consecuitiove values of N is 1111

& $1111 = 30 \times 37 + 1$

$\therefore$ Remainder will be

28, 29, 30, 31, 32, 33, 34

Sum = $31 \times 7 = 217$

24. $2(p^2 + q^2) = (pq - 1)^2 + 1$

$\Rightarrow 2(p^2 + q^2 + pq - 1) = p^2q^2$

LHS is even $\therefore$ RHS must be even

Hence, one of P and q must be 2 Let P = 2

then $2(p^2+q^2+pq-1) = p^2.q^2$

$\Rightarrow q^2 + 2q - 3 = 0$

$\Rightarrow q = -1, 3$

but q = – 1 is rejected $\therefore$ q = 3

$\therefore r = 2 \times 3 - 1 = 5$

$(p, q, r) \equiv (2, 3, 5), (3, 2, 5)$