Chapter 7
Probability Distributions

Ex 7.1

Question 1.
Define Binomial distribution.

Solution:
A random variable X is said to follow a binomial distribution with parameter 'n' and 'p' if it assumes only non-negative value and its probability mass function is given by

$$P(X = x) = p(x) = \begin{cases} {}^nC_x p^x q^{n-x}, x = 0, 1, 2, ...n, q = 1 - p \\ 0, \quad \text{otherwise} \end{cases}$$

Question 2.
Define Bernoulli trials.

Solution:
A random experiment whose outcomes are of two types namely success S and failure F, occurring with probabilities p and q, is called a Bernoulli trial.
Example 1, Tossing of a coin (Head or Tail)
Example 2, Writing an exam (Pass or Fail)

Question 3.
Derive the mean and variance of binomial distribution.

Solution:
Let X be a random variable with the Binomial distribution.
The probability function of X is

$$p(x) = \binom{n}{x} p^x q^{n-x}. \text{ Then } E(X) = \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x q^{n-x}$$

(Since the $x = 0$ term vanishes)

$$E(X) = \sum_{x=1}^{n} \frac{n(n-1)!}{(x-1)!(n-x)!}(p)p^{x-1}q^{n-x}$$

$$= np\sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1}q^{n-x}$$

$$= np\sum_{x=1}^{n} \frac{(n-1)!}{(x-1)![(n-1)-(x-1)]!} p^{x-1}q^{n-x}$$

$$= np\sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1}q^{n-x}$$

$$= np\left[{}^{n-1}C_0 q^{n-1} + {}^{n-1}C_1 pq^{n-2} + {}^{n-1}C_2 p^2 q^{n-3} + ... + {}^{n-1}C_{n-1} p^{n-1}\right]$$

$= np$ [This is a binomial expansion of $(p + q)^{n-1}$]

$= np\,[p + q]^{n-1}$

But we know that $p + q = 1$

So $E(X) = np\,(1)^{n-1} = np$

Thus the mean of binomial distribution is np.

Now $Var(X) = E(X^2) - [E(X)]^2$

$$E(X^2) = \sum_{x=0}^{n} x^2 \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} [x(x-1) + x]\binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} [x(x-1)]\binom{n}{x} p^x q^{n-x} + \sum_{x=0}^{n} x\binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=2}^{n} \frac{x(x-1)n!}{(n-x)!x(x-1)(x-2)!} p^x q^{n-x} + np$$

$$= n(n-1)p^2\sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2}q^{n-x} + np$$

$$= n(n-1)p^2\sum_{x=2}^{n} \frac{(n-2)!}{(x-2)![(n-2)-(x-2)]!} p^{x-2}q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \binom{n-2}{x-2} p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^2 \left[{}^{n-2}C_0 q^{n-2} + {}^{n-2}C_1 p q^{n-3} + \ldots + {}^{n-2}C_{n-2} p^{n-2} \right] + np$$

$$= n(n-1)\, p^2\, [(p+q)^{n-2}] + np$$

Since $p + q = 1$, we have

$E(X^2) = n(n-1)\, p^2 + np$

Using this,

$$\text{Var}(X) = n(n-1)\, p^2 + np - (np)^2$$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np\,(1-p) = npq$$

Hence the variance of binomial distribution is npq.

Question 4.

Write down the conditions for which the binomial distribution can be used.

Solution:

The binomial distribution can be used under the following conditions:

- The number of trials (or) observations 'n' is fixed (finite).
- Each observation is independent of each other.
- In every trial, there are only two possible outcomes – success or failure.
- The probability of success 'p' is the same for each outcome.

Question 5.

Mention the properties of the binomial distribution.

Solution:

Property 1:

The binomial distribution is symmetrical when the probability of success 'p' is 0.5 (or) when a number of trials 'n' is very large. In other words, if $p = q = 1/2$, the distribution is symmetric about the median. If $p \neq q$, then it is skewed distribution, ($p < 0.5 \rightarrow$ positively skewed, $p > 0.5 \rightarrow$ negatively skewed)

Property 2:

The variance is less than mean (i,e,) $npq < np$

Question 6.

If 5% of the items produced turn out to be defective, then find out the probability that out of 10 items selected at random there are

(i) exactly three defectives
(ii) at least two defectives
(iii) exactly 4 defectives
(iv) find the mean and variance

Solution:
Let p be the probability of a defective item.
Given that, p = 5% = $\frac{5}{100}$ = 0.05
So q = 1 – p = 1 – 0.05 = 0.95. Also n = 10.
Let X be the random variable which follows the binomial distribution. Then X ~ B (10, 0.05)

(i) P(exactly three defectives) = P(X = 3)

$$P(X = x) = {}^{10}C_x (0.05)^x (0.95)^{10-x}$$
$$P(X = 3) = {}^{10}C_3 (0.05)^3 (0.95)^7$$
$$= \frac{10 \times 9 \times 8}{3 \times 2} (0.05)^3 (0.95)^7$$
$$= (120)(0.000125)(0.6983)$$
$$P(X = 3) = 0.0105$$

(ii) P(atleast two defectives) = P (X ≥ 2) = 1 – P (X < 2)
= 1 – [P(X = 1) + P(X = 0)]

$$= 1 - [{}^{10}C_1 (0.05)^1 (0.95)^9 + {}^{10}C_0 (0.05)^0 (0.95)^{10}]$$
$$= 1 - [10 (0.05)(0.95)^9 + (0.95)^{10}]$$
$$= 1 - [(0.5)(0.6302) + 0.5987]$$
$$= 1 - [0.3151 + 0.5987]$$
$$= 0.0862$$

(iii) P(exactly 4 defectives) = P(X = 4)

$$= {}^{10}C_4 (0.05)^4 (0.95)^6$$
$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2} (0.05)^4 (0.95)^6$$
$$= (210)(625 \times 10^{-8})(0.7351)$$
$$= 0.00096$$

(iv) We know that mean = np = (10) (0.05) = 0.5
Variance = npq = (10) (0.05) (0.95) = 0.475

Question 7.
In a particular university, 40% of the students are having newspaper reading habit. Nine

university students are selected to find their views on reading habit. Find the probability that
(i) none of those selected has newspaper reading habit
(ii) all those selected have newspaper reading habit
(iii) at least two-third have newspaper reading habit.

Solution:
Let X be the binomial random variable which denotes the number of students having newspaper reading habit.
It is given that 40% of students have reading habit.
$p = \frac{40}{100} = 0.4$ and $q = 1 - 0.4 = 0.6$

(i) P(none of selected have newspaper reading habit) = P(X = 0)
Now X ~ B (9, 0.4)
The p.m.f is given by $P(X = x) = p(x) = 9C_x(0.4)^x(0.6)^{9-x}$
$P(X = 0) = {}^9C_0(0.4)^0(0.6)^9 = (0.6)^9 = 0.01008$ (using calculator)

(ii) P (all selected have newspaper reading habit)
= P (X = 9)
$= 9C_9(0.4)^9(0.6)^0$
$= (0.4)^9$
= 0.000262 (using calculator)

(iii) P (at least two third have newspaper reading habit) = P (X ≥ 6)
{9 students are selected. Two third of them means $\frac{2}{3}(9) = 6$}
Now P (X ≥ 6) = P (X = 6) + P (X = 7) + P (X = 8) + P (X = 9)
$$= {}^9C_6(0.4)^6(0.6)^3 + {}^9C_7(0.4)^7(0.6)^2 + {}^9C_8(0.4)^8(0.6) + {}^9C_9(0.4)^9$$
$$= \frac{9\times8\times7}{3\times2}(0.4)^6(0.6)^3 + \frac{9\times8}{2}(0.4)^7(0.6)^2 + 9(0.4)^8(0.6) + (0.4)^9$$
= (84) (0.004096) (0.216) + 36 (0.0016384) (0.36) + 9 (0.00065536) (0.6) + 0.000262
= 0.074318 + 0.021234 + 0.003539 + 0.000262
= 0.099353

Question 8.
In a family of 3 children, what is the probability that there will be exactly 2 girls?

Solution:
Let X denote the binomial variable which denotes the number of girls.
Given that n = 3 and $p = q = \frac{1}{2}$

The p.m.f is $P(X = x) = {}^3C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{3-x}$

$\Rightarrow \quad P(X = x) = {}^3C_x \left(\frac{1}{2}\right)^3$

$= {}^3C_x \left(\frac{1}{8}\right)$

We want $P(X = 2) = {}^3C_2 \left(\frac{1}{8}\right) = \frac{3}{8} = 0.375$

Hence the probability that there will be exactly 2 girls is. 0.375.

Question 9.
Defects in yarn manufactured by a local mill can be approximated by a distribution with a mean of 1.2 defects for every 6 metres of length. If lengths of 6 metres are to be inspected, find the probability of fewer than 2 defects.

Solution:
Given mean np = 1.2 and n = 6
$p = \frac{1.2}{6} = 0.2, q = 1 - 0.2 = 0.8$
Let X be a binomial variable denoting the number of defects, (i.e,) $X \sim B(6, 0.2)$
p.m.f is given by $P(X = x) = 6Cx(0.2)x(0.8)6-x$
We want $P(X < 2) = P(X = 0) + P(X = 1)$
$= {}^6C_0(0.2)^0(0.8)^6 + {}^6C1(0.2)^1(0.8)^5$
$= (0.8)^6 + 6(0.2)(0.8)^5$
$= 0.262144 + 0.393216$
$= 0.65536$
Thus if lengths of 6 metres are to be inspected, the probability of less than 2 defects is 0.65536.

Question 10.
If 18% of the bolts produced by a machine are defective, determine the probability that out of the 4 bolts chosen at random
(i) exactly one will be defective
(ii) none will be defective
(iii) at most 2 will be defective

Solution:
Let X be the random variable denoting the number of defective bolts.
The probability of defective bolts $p = \frac{18}{100} = 0.18 \Rightarrow q = 0.82$.
Also n = 4
The p.m.f is $P(X = x) = {}^4C_x(0.18)^x(0.82)^{4-x}$

(i) P (exactly one defective) = P(X = 1)
$= {}^4C_1(0.18)^1(0.82)^3$
$= 4\ (0.18)\ (0.82)^3$
= 0.3969

(ii) P (no defective) = P(X = 0)
$= {}^4C_0(0.18)^0(0.82)^4$
$= (0.82)^4$
= 0.45212

(iii) P (atmost 2 defective) = P(X ≤ 2)
= P(X = 2) + P(X = 1) + P(X = 0)
$= {}^4C_2\ (0.18)^2\ (0.82)^2 + 0.3969 + 0.45212$
= 0.1307 + 0.3969 + 0.45212
= 0.97972

Question 11.
If the probability of success is 0.09, how many trials are needed to have a probability of at least one success as 1/3 or more?

Solution:
Given p = 0.09 (success)
q = 0.91 (failure)
We have to find number of trials 'n.'
According to the problem,
$P(X \geq 1) > \frac{1}{3}$
(We must have atleast one success)
$1 - P(X < 1) > \frac{1}{3}$
$1 - P(X = 0) > \frac{1}{3}$
(or) $P(X = 0) < \frac{2}{3}$
Using p.m.f, we have,
${}^nC_0(0.09)^0(0.91)^n < \frac{2}{3}$
(0.91)n < 23
we can use log tables to calculate (or) by trial method try for n = 1, 2,...... using calculator.
We observe that $(0.91)5 < \frac{2}{3}$. Thus we need minimum 5 trial or more.

Question 12.
Among 28 professors of a certain department, 18 drive foreign cars and 10 drive locally made cars. If 5 of these professors are selected at random, what is the probability that at least 3 of them drive foreign cars?

Solution:

Here n = 5, p = $\frac{18}{28} = \frac{9}{14}$, q = $\frac{10}{28} = \frac{5}{14}$

(i.e.) the probability of professors driving foreign cars p = $\frac{9}{14}$, and those who drive local cars q = $\frac{5}{14}$.

Let X be the Binomial random variable denoting persons who drive foreign cars.

Then the p.m.f of X is given by P (X = x) = ${}^5C_x\left(\frac{9}{14}\right)^x\left(\frac{5}{14}\right)^{5-x}$

We want P (X ≥ 3) = P (X = 3) + P (X = 4) + P (X = 5)

$$= {}^5C_3\left(\frac{9}{14}\right)^3\left(\frac{5}{14}\right)^2 + {}^5C_4\left(\frac{9}{14}\right)^4\left(\frac{5}{14}\right) + {}^5C_5\left(\frac{9}{14}\right)^5$$

$$= \frac{10(729)(25)}{(14)^5} + \frac{5(6561)5}{(14)^5} + \frac{59049}{(14)^5}$$

$$= \frac{182250 + 164025 + 59049}{537824}$$

$$= \frac{405324}{537824} = 0.7536$$

Question 13.

Out of 750 families with 4 children each, how many families would be expected to have

(i) at least one boy

(ii) at most 2 girls

(iii) and children of both sexes?

Assume equal probabilities for boys and girls.

Solution:

Given that 750 families are considered each with 4 children. We will find the probabilities for one particular family and then multiply by 750.

In other words, n = 4, p = q = $\frac{1}{2}$ (since boy and girl child have equal probability).

Let X denote the binomial random variable which denotes the number of boys in the family.

Then X ~B $\left(4, \frac{1}{2}\right)$. The p.m.f is given by P (X = x) = ${}^4C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{4-x}$

$\Rightarrow$ P (X = x) = ${}^4C_x\left(\frac{1}{2}\right)^4$

(i) P(at least one boy) = P (X ≥ 1) = 1 − P (X < 1)

= 1 − P (X = 0)

$$= 1 - {}^4C_0\left(\frac{1}{2}\right)^4 = 1 - \left(\frac{1}{2}\right)^4$$

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

So out of 750 families the number of families would be expected to have atleast one boy is $\frac{15}{16} \times 750 = 703$

(ii) P(atmost 2 girls) = P(2G, 2B) + P(1G, 3B) + P(0G, 4B)
= P(X = 2) + P(X = 3) + P(X = 4)

$$= {}^4C_2\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4 + {}^4C_4\left(\frac{1}{2}\right)^4$$

$$= \frac{1}{16}[6 + 4 + 1] = \frac{11}{16}$$

$$= 750 \times \frac{11}{16} \simeq 516$$

Thus out of 750 families, 516 families would be expected to have atmost 2 girls.

(iii) P(children of both sexes) = P(both boys and girls)
Out of 4 children the sample space is given by {BGGG, BBGG, BBBG}and each case in any order.
So we require P(1B, 3G) + P(2B, 2G) + P(3B, 1G)
(i.e,) P(X = 1) + P(X = 2) + P(X = 3)

$$= {}^4C_1\left(\frac{1}{2}\right)^4 + {}^4C_2\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4$$

$$= \frac{1}{16}[4 + 6 + 4] = \frac{14}{16} = \frac{7}{8}$$

$$= 750 \times \frac{7}{8} \simeq 656$$

Thus out of 750 families, 656 families would be expected to have children of both sexes.

Question 14.
Forty percent of business travellers carry a laptop. In a sample of 15 business travelers
(i) what is the probability that 3 will have a laptop?
(ii) what is the probability that 12 of the travelers will not have a laptop?
(iii) what is the probability that atleast three of the travelers have a laptop?

Solution:
Let X be the binomial variables which denotes the number of business travellers having a laptop.
Given that n = 15 and P = 40% = 0.4. So q = 1 – 0.4 = 0.6. Thus $X \sim B(15, 0.4)$.
The p.m.f of X is given by $P(X = x) = {}^{15}C_x(0.4)^x(0.6)^{15-x}$

(i) P(3 travellers will have a laptop) = P (X = 3)

$$= {}^{15}C_3 (0.4)^3 (0.6)^{12}$$

$$= \frac{15 \times 14 \times 13}{3 \times 2} (0.064)(0.6)^{12}$$

$$= 455 (0.064) (0.6)^{12}$$

Now $(0.6)^{12} = [(0.6)^6]^2 = [0.046656]^2$

$$= 0.002177$$

Note: The calculation can be done by method of logarithms also.
P(X = 3) = 455 (0.064) (0.002177) = 0.0634

(ii) P(12 of the travellers will not have a laptop)
= P(15 – 12 = 3 will have a laptop)
= P(X = 3) = 0.0634 (from the previous subdivision)

(iii) P(atleast three of the travellers have a laptop)

$$P(X \geq 3) = 1 - P(X < 3)$$

$$= 1 - [P(X = 2) + P(X = 1) + P(X = 0)]$$

$$= 1 - \left[{}^{15}C_2 (0.4)^2 (0.6)^{13} + {}^{15}C_1 (0.4)^1 (0.6)^{14} + {}^{15}C_0 (0.4)^0 (0.6)^{15}\right]$$

$$= 1 - [105 (0.16)(0.6)^{13} + 15(0.4)(0.6)^{14} + (0.6)^{15}]$$

$$= 1 - (0.6)^{12} [105(0.16)(0.6) + 15(0.4)(0.6)^2 + (0.6)^3]$$

Using $(0.6)^{12} = 0.002177$ from the previous subdivision, we have
= 1 – (0.002177) [10.08 + 2.16 + 0.216]
= 1 – (0.002177) (12.456)
= 1 – 0.02712
= 0.9729

Question 15.
A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of 2 successes.

Solution:
Let p be the probability of getting a doublet, (i.e,) probability of success. When we throw a pair of dice there are 36 possibilities. The number of doublets is 6 [(1, 1) (2, 2), (3, 3) (4, 4) (5, 5) (6, 6)].

So $p = \frac{6}{36} = \frac{1}{6}$

$q = 1 - \frac{1}{6} = \frac{5}{6}$

Let X be the random variable denoting the number of doublet in 4 throws.
Then X ~ B (4, \(\frac{1}{6}[/latex)]

The p.m.f is given by $P(X = x) = {}^4C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{4-x}$

We want $P(X = 2) = {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$

$= 6\left(\frac{1}{36}\right)\left(\frac{25}{36}\right) = \frac{25}{216}$

Hence the probability of 2 successes is [latex]\frac{25}{216}\)

Question 16.
The mean of a binomial distribution is 5 and the standard deviation is 2. Determine the distribution.

Solution:
Given mean = 5 and standard deviation = 2
(i.e,) np = 5 and $\sqrt{npq} = 2 \Rightarrow npq = 4$
$5q = 4 \Rightarrow q = \frac{4}{5}, p = 1 - \frac{4}{5} = \frac{1}{5}$
Again np = 5 gives $\frac{n}{5} = 5 \Rightarrow n = 25$

So the p.m.f of the distribution is given by $P(X = x) = \binom{25}{x} \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{25-x}$

Question 17.
Determine the binomial distribution for which the mean is 4 and variance 3. Also find P(X = 15)

Solution:
Given mean = 4 and variance is 3.
(i.e,) np = 4 and npq = 3

$4q = 3$, (or) $q = \frac{3}{4}$; $p = 1 - \frac{3}{4} = \frac{1}{4}$

$n\left(\frac{1}{4}\right) = 4 \Rightarrow n = 16$

Hence the p.m.f of the binomial distribution is given by

$P(X = x) = \binom{16}{x}\left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{16-x}$

$P(X = 15) = \binom{16}{15}\left(\frac{1}{4}\right)^{15} \left(\frac{3}{4}\right)^1$

$= (16)\frac{(3)}{(4)^{16}} = \frac{3}{4^{14}}$

Question 18.
Assume that a drug causes a serious side effect at a rate of three patients per one hundred. What is the probability that at least one person will have side effects in a random sample of ten patients taking the drug?

Solution:
According to the problem, n = 10, p = $\frac{3}{100}$ = 0.03 where p is the probability that a drug causes side effect. Now X ~ B (10, 0.03). The p.m.f is given by

$$P(X = x) = \binom{10}{x}(0.03)^x (0.97)^{10-x}$$

We want $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$$= 1 - {}^{10}C_0 (0.03)^0 (0.97)^{10-0}$$
$$= 1 - (0.97)^{10}$$
$$= 1 - [(0.97)^5]^2$$
$$= 1 - [0.8587]^2$$
$$= 1 - 0.7374 = 0.2626$$

Thus the probability that at least one person will have side effects is 0.2626.

Question 19.
Consider five mice from the same litter, all suffering from Vitamin A deficiency. They are fed a certain dose of carrots. The positive reaction means recovery from the disease. Assume that the probability of recovery is 0.73. What is the probability that at least 3 of the 5 mice recover?

Solution:
Given n = 5 and the probability of recovery p = 0.73.
So q = 1 – 0.73 = 0.27. X ~ B (5, 0.73).
The p.m.f of X is given by

$$P(X = x) = \binom{5}{x}(0.73)^x (0.27)^{5-x}$$

We want P (atleast 3 mice recover) = $P(X \geq 3)$

$$= P(X = 3) + P(X = 4) + P(X = 5)$$
$$= {}^5C_3 (0.73)^3 (0.27)^2 + {}^5C_4 (0.73)^4 (0.27)^1 + {}^5C_5 (0.73)^5 (0.27)^0$$
$$= 10(0.73)^3 (0.27)^2 + 5(0.73)^4 (0.27) + (0.73)^5$$
$$= (0.73)^3 + [10(0.27)^2 + 5(0.73)(0.27) + (0.73)^2]$$
$$= (0.389)[0.729 + 0.9855 + 0.5329]$$
$$= (0.389)(2.2474) = 0.8743$$

Thus the probability that at least 3 of the 5 mice recover is 0.8743.

Question 20.
An experiment succeeds twice as often as it fails, what is the probability that in the next five trials there will be
(i) three successes and
(ii) at least three successes.

Solution:
Given a number of trials n = 5.
Let P be the probability of success and q be the probability of failure. It is given that p = 2q.

We know that $p + q = 1 \Rightarrow 2q + q = 1 \Rightarrow q = \frac{1}{3}$ and $p = \frac{2}{3}$. So $X \sim B\left(5, \frac{2}{3}\right)$

The p.m.f is $\quad P(X = x) = \binom{5}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{5-x}$

(*i*) P (three successes)

$$= P(X=3)$$
$$= {}^5C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2$$
$$= 10\left(\frac{8}{27}\right)\left(\frac{1}{9}\right) = \frac{80}{243}$$

(*ii*) P (atleast three successes)

$$= P(X \geq 3) = 1 - P(X < 3)$$
$$= 1 - [P(X = 2) + P(X = 1) + P(X = 0)]$$
$$= 1 - \left[{}^5C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 + {}^5C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^4 + {}^5C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^5\right]$$
$$= 1 - \left[10\left(\frac{4}{9}\right)\left(\frac{1}{27}\right) + 5\left(\frac{2}{3}\right)\left(\frac{1}{81}\right) + \frac{1}{243}\right]$$
$$= 1 - \left[\frac{40 + 10 + 1}{243}\right] = 1 - \frac{51}{243} = \frac{192}{243}$$

Thus the probability of atleast three successes in five trials will be $\frac{192}{243}$

Ex 7.2

Question 1.
Define Poisson distribution.

Solution:
Poisson distribution is a discrete frequency distribution which gives the probability of a number of independent events occurring in a fixed time. It is useful for characterizing events with very low probabilities of occurrence within some definite time or space.

Question 2.
Write any 2 examples for Poisson distribution.

Solution:
Examples of Poisson distribution are given by

- The number of printing mistakes per page in a textbook.
- A number of lightning per second.
- The number of bacteria in one cubic centimetre.

Question 3.
Write the conditions for which the Poisson distribution is a limiting case of the binomial distribution.

Solution:
Poisson distribution is a limiting case of binomial distribution under the following conditions:

- the number of trials 'n' is indefinitely large i.e, $\to \infty$
- the probability of success 'p' in each trial is very small, i.e, $p \to 0$
- $np = \lambda$ is finite. Thus $p = \frac{\lambda}{n}$ and $q = 1 - \frac{\lambda}{n}, \lambda > 0$

Question 4.
Derive the mean and variance of the Poisson distribution.

Solution:
Let X be a Poisson random variable with parameter λ. The p.m.f is given by

$$P(x, \lambda) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\text{Mean } E(X) = \sum_{x=0}^{\infty} x\, p(x,\lambda)$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \lambda e^{-\lambda} \left\{ \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \right\}$$

$$= \lambda e^{-\lambda} \left\{ 1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots \right\}$$

$$= \lambda e^{-\lambda} (e^{\lambda}) = \lambda$$

$$\text{Variance } (X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_{x=0}^{\infty} x^2 p(x,\lambda)$$

$$= \sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} [x(x-1) + x] \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-2)!} + E(X)$$

$$= e^{-\lambda}\lambda^2 \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= e^{-\lambda}\lambda^2 \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \ldots\ldots \right] + \lambda$$

$$= e^{-\lambda}\lambda^2 (e^{\lambda}) + \lambda$$

$$= \lambda^2 + \lambda$$

$$\text{Var}(X) = \lambda^2 + \lambda - \lambda^2 = \lambda$$

Thus the mean and variance of Poisson distribution are both equal to λ.

Question 5.
Mention the properties of Poisson distribution.

Solution:

1. Poisson distribution is the only distribution in which the mean and variance are equal.
2. The probability that an event occurs in a given time, distance, area or volume is the same.

Question 6.
The mortality rate for a certain disease is 7 in 1000. What is the probability for just 2 deaths on account of this disease in a group of 400? [Given $e^{-2.8} = 0.06$]

Solution:
Let X denote the number of deaths due to the disease
$P(\text{death}) = \frac{7}{1000} = 0.007 \Rightarrow p = 0.007$ and $n = 400$
The value of mean $\lambda = np = (0.007)(400) = 2.8$
Hence X follows a Poisson distribution with

$$P(X = x) = \frac{e^{-2.8}(2.8)^x}{x!}$$

We want $$P(X = 2) = \frac{e^{-2.8}(2.8)^2}{2!}$$

$$= \frac{(0.06)(2.8)^2}{2} = 0.2352$$

So the probability of just 2 deaths on account of this disease in a group of 400 is 0.2352.

Question 7.
It is given that 5% of the electric bulbs manufactured by a company are defective. Using Poisson distribution find the probability that a sample of 120 bulbs will contain no defective bulb.

Solution:
Given $p = \frac{5}{100} = 0.05$ and $n = 120$
$\Rightarrow \lambda = np = (0.05)(120) = 6$

Thus X is a Poisson random variable with P (X = x) = $\frac{e^{-6}6^x}{x!}$

We want P (no defective bulb) = P (X = 0)

= $\frac{e^{-6}6^0}{0!}$

= e^{-6}

= 0.0025 (Using exponent table)

Thus the probability that a sample of 120 bulbs will not contain any defective bulb is 0.0025.

Question 8.
A car hiring firm has two cars. The demand for cars on each day is distributed as a Poisson variate, with mean 1.5. Calculate the proportion of days on which
(i) Neither car is used
(ii) Some demand is refused.

Solution:
Let X be the Poisson variable denoting the demand for the cars.

It is given that mean is 1.5 ⇒ $\lambda = 1.5$

(i) P (Neither car is used) = P (X = 0) = $\frac{e^{-1.5}(1.5)^0}{0!} = e^{-1.5} = 0.2231$

(ii) Some demand is refused when demand is more than 2 since the firm has only 2 cars. So we want P (X > 2)

Now P (X > 2) = 1 – P (X ≤ 2)

= 1 – [P(X = 2) + P(X = 1) + P(X = 0)]

$$= 1 - \left[\frac{e^{-1.5}(1.5)^2}{2!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^0}{0!}\right]$$

$$= 1 - e^{-1.5}\left[\frac{2.25}{2} + 1.5 + 1\right]$$

$$= 1 - e^{-1.5}(3.625)$$

$$= 1 - (0.2231)(3.625)$$

$$= 1 - 0.8087$$

$$= 0.1913$$

Question 9.
The average number of phone calls per minute into the switchboard of a company between 10.00 am and 2.30 pm is 2.5. Find the probability that during one particular minute there will be
(i) no phone at all
(ii) exactly 3 calls

(iii) at least 5 calls.

Solution:
Let X be the Poisson variable denoting the number of phone calls per minute.

Given that mean $= \lambda = 2.5$. The p.m.f is $p(X = x) = \frac{e^{-2.5}(2.5)^x}{x!}$

(i) P (no phone call) $= P(X = 0)$

$$= \frac{e^{-2.5}(2.5)^0}{0!}$$

$= e^{-2.5} = 0.08208$ (use tables)

(ii) P (exactly 3 calls) $= P(X = 3)$

$$= \frac{e^{-2.5}(2.5)^3}{3!} = \frac{(0.08208)(15.625)}{6}$$

$= 0.2138$

(iii) P (atleast 5 calls) $= P(X \geq 5)$

$= 1 - P(X < 5)$

$= 1 - [P(X = 4) + P(X = 3) + P(X = 2) + P(X = 1) + P(X = 0)]$

$$P(X = 4) = \frac{e^{-2.5}(2.5)^4}{4!} = \frac{(0.08208)(39.0625)}{24}$$

$= 0.1336$

$$P(X = 2) = \frac{e^{-2.5}(2.5)^2}{2!} = \frac{(0.08208)(6.25)}{2}$$

$= 0.2565$

$P(X = 1) = \frac{e^{-2.5}(2.5)}{1!} = (0.08208)(2.5) = 0.2052$

Using the above values and P (X = 0) and P (X = 3) from the previous subdivisions in (A) we get,

$P(X \geq 5) = 1 - [0.1336 + 0.2138 + 0.2565 + 0.2052 + 0.08208]$

$= 1 - 0.89118$

$= 0.10882$

Question 10.
The distribution of the number of road accidents per day in a city is Poisson with mean 4.
Find the number of days out of 100 days when there will be
(i) no accident
(ii) at least 2 accidents and
(iii) at most 3 accidents.

Solution:

Let X be the Poisson variable denoting the number of accidents per day.

Given that mean is 4 (i.e,) λ = 4. The p.m.f is given by P(X = x) = $\frac{e^{-4}4^x}{x!}$

(i) P (no accident) = P(X = 0) = e^{-4} = 0.0183

For 100 days we have 100 × 0.0183 = 1.83 ~ 2

Hence out of 100 days there will be no accident for 2 days.

(ii) P (atleast 2 accidents) = P (X ≥ 2)
= 1 – P (X < 2)
= 1 – [P(X = 1) + P(X = 0)]
= 1 – [e^{-4} (4) + e^{-4}]
= 1 – (0.0183) (5)
= 1 – 0.0915
= 0.9085
For 100 days we have 100 × 0.9085 ~ 91
Hence out of 100 days there will be at least 2 accidents for 91 days.

(iii) P (atmost 3 accidents) = P (X ≤ 3)
= P (X = 0) + P (X = 1) + P (X = 2) + P (X = 3)
= $e{-4}[1+\frac{4}{1}+\frac{16}{2}+\frac{64}{6}]$
= (0.0183) [23.6667]
= 0.4331
For 100 days we have 100 × 0.4331 ~ 43
Hence out of 100 days, there will be atmost 3 accidents for 43 days.

Question 11.
Assuming that a fatal accident in a factory during the year is 1/1200, calculate the probability that in a factory employing 300 workers there will be at least two fatal accidents in a year, (given $e^{-0.25}$ = 0.7788).

Solution:
Let X denote the number of accidents.
Given that probability of accidents 'p' is 1/1200 and n = 300

$$\Rightarrow \lambda = np = 300\left(\frac{1}{1200}\right) = 1/4 = 0.25$$

So X is poisson variable with mean 0.25

The p.m.f is $\quad P(X = x) = \frac{e^{-0.25}(0.25)^x}{x!}$

We want $\quad P(X \geq 2) = 1 - P(X < 2)$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$= 1 - [e^{-0.25} + e^{-0.25} (0.25)]$
$= 1 - e^{-0.25} (1.25)$
$= 1 - (0.7788) (1.25)$
$= 0.0265$
Thus the probability that there will be atleast two fatal accidents in a year is 0.0265.

Question 12.
The average number of customers, who appear in a counter of a certain bank per minute is two. Find the probability that during a given minute
(i) No customer appears
(ii) three or more customers appear.

Solution:
Let X denote the number of customers.
Given $\lambda = 2$
(i) P (no customer) = P (X = 0)
$$= \frac{e^{-2}(2)^0}{0!}$$
$= e^{-2}$
$= 0.1353$

(ii) P (3 or more customers) = P $(X \geq 3)$
$= 1 - P(X < 3)$
$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$
$$= 1 - \left[e^{-2} + e^{-2}(2) + \frac{e^{-2}(2)^2}{2!}\right]$$
$= 1 - e^{-2} [1 + 2 + 2]$
$= 1 - (0.1353) (5) = 0.3235$
Thus during a given minute, the probability that three or more customers appear is 0.3235.

Ex 7.3

Question 1.
Define Normal distribution.

Solution:
A random variable X is said to follow a normal distribution with parameters μ (mean) and σ^2 (variance) if its probability density function is given by

$$f(x : \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \quad \begin{matrix} -\infty < x < \infty, \\ -\infty < \mu < \infty, \\ \sigma > 0 \end{matrix}$$

Question 2.
Define Standard normal variate.

Solution:
A random variable $Z = \frac{X-\mu}{\sigma}$ is called a standard normal variate with mean 0 and standard deviation 1 (i.e.) Z ~ N (0, 1). Its probability density function is given by

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} \qquad -\infty < z < \infty$$

Question 3.
Write down the conditions in which the Normal distribution is a limiting case of the binomial distribution.

Solution:
The Normal distribution is a limiting case of Binomial distribution under the following conditions:

- n, the number of trials is infinitely large, i.e. $n \to \infty$
- neither p (or q) is very small.

Question 4.
Write down any five chief characteristics of Normal probability curve.

Solution:
Chief Characteristics of the Normal Probability Curve are as follows:

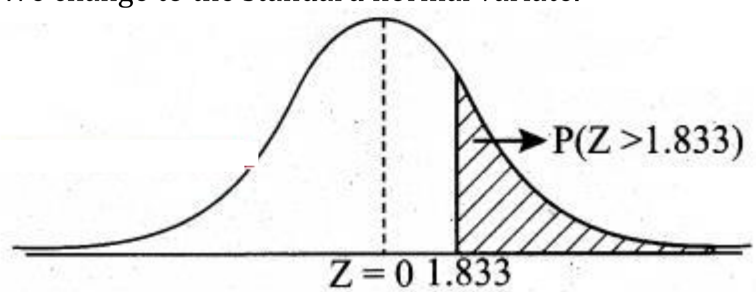
- The curve is bell-shaped and symmetrical about the line x = μ.
- Mean, median and mode of the distribution coincide.
- The total area under the normal curve is equal to unity.
- For a given μ and σ, there is only one normal distribution.

- The Points of inflexion are given by $x = \mu \pm \sigma$

Question 5.
In a test on 2,000 electric bulbs, it was found that bulbs of a particular make, was normally distributed with an average life of 2,040 hours and a standard deviation of 60 hours. Estimate the number of bulbs likely to burn for
(i) more than 2,150 hours
(ii) less than 1,950 hours
(iii) more 1,920 hours but less than 2,100 hours.

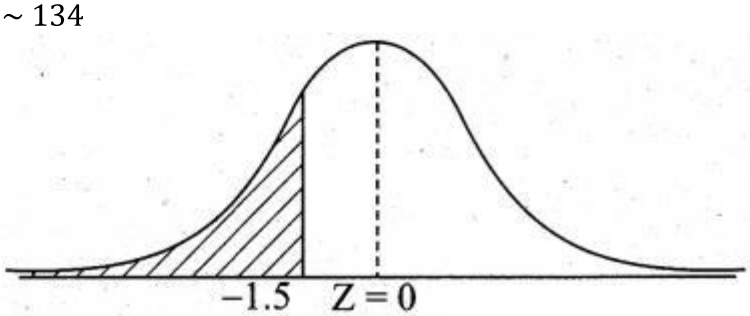
Solution:
Let X be the numbers of hours for which the bulbs are in use. It is given that X is normally distributed with mean 2040 hours and a standard deviation of 60 hours, (i.e) $X \sim N(2040, 60^2)$

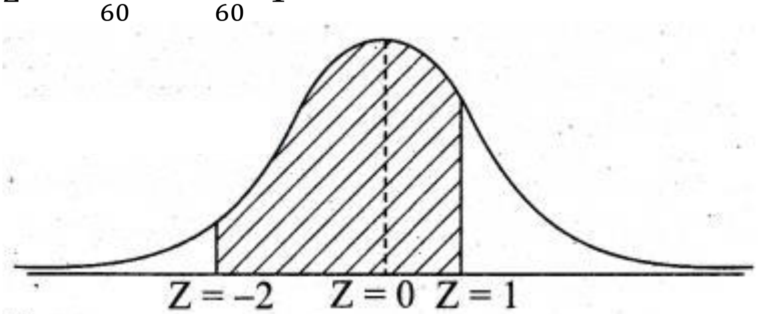
(i) $P(X > 2150)$
We change to the standard normal variate.



The total area to the right of Z = 0 is 0.5.
The area between Z = 0 and 1.833 is 0.4664 (from tables)
So $P(Z > 1.833) = 0.5 - 0.4664 = 0.0336$
The number of bulbs likely to bum for more than 2150 hours is $2000 \times 0.0336 = 67.2 \sim 67$

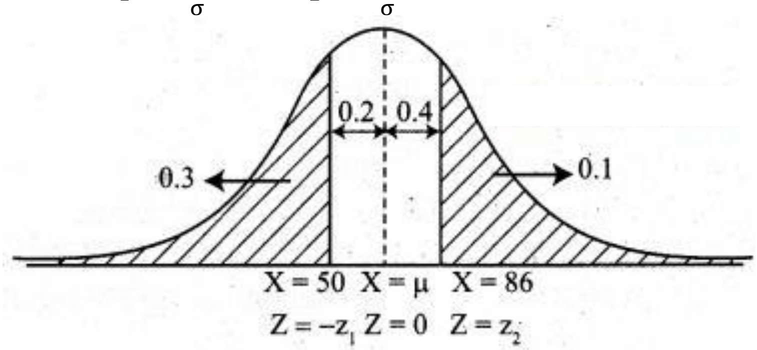
(ii) We want $P(X < 1950)$
$= P(\frac{X-\mu}{\sigma} < \frac{1950-2040}{60})$
$= P(Z < -1.5)$
The area between Z = -1.5 and Z = 0 is same as area between Z = 0 and Z = 1.5.
From the tables, area between Z = 0 and Z = 1.5 is 0.4332
$P(Z < -1.5) = 0.5 - 0.4332 = 0.0668$
Hence the number of bulbs likely to bum for less than 1950 hours is $2000 \times 0.0668 = 133.6 \sim 134$



(iii) We want $P(1920 < X < 2100)$
When X = 1920,
$Z = X - \mu\sigma = \frac{1920-2040}{60} = -2$
When X = 2100,
$Z = \frac{2100-2040}{60} = \frac{60}{60} = 1$


So $P(1920 < X < 2100) = P(-2 < Z < 1)$
$= P(-2 < Z < 0) + P(0 < Z < 1)$
$= P(0 < Z < 2) + P(0 < Z < 1)$
$= 0.4772 + 0.3413$
$= 0.8185$
Hence the number of bulbs likely to bum for more than 1920 hours but less than 2100 hours is $2000 \times 0.8185 = 1637$.

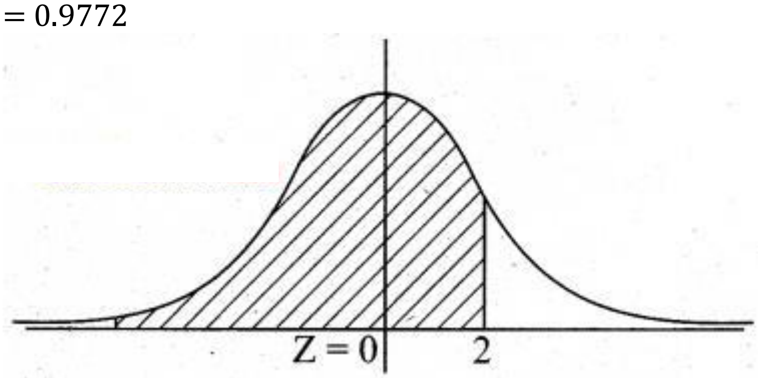
Question 6.
In a distribution, 30% of the items are under 50 and 10% are over 86. Find the mean and standard deviation of the distribution.

Solution:
Let X be the normal random variable denoting the number of items in the distribution.
It is given that 30% of items are under 50
$\Rightarrow P(X < 50) = 30\% = 0.3$.
Also, 10% are over 86
$\Rightarrow P(X > 86) = 10\% = 0.1$.
We have to find μ and σ.
Representing the given data diagrammatically,
Where $Z_1 = \frac{50-\mu}{\sigma}$ and $Z_2 = \frac{86-\mu}{\sigma}$


From the diagram,

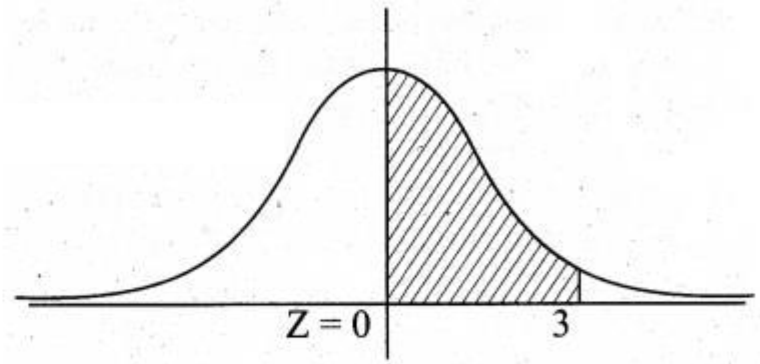
$P(-Z_1 < Z < 0) = 0.2$
By symmetry $P(0 < Z < Z_1) = 0.2$
$Z_1 = 0.525$ (from the normal table)
Hence $-0.525 = \frac{50-\mu}{\sigma}$
(i.e.) $50 - \mu = -0.525\sigma$ …….. (1)
Again $P(0 < Z < Z_2) = 0.4$
$Z_2 = 1.28$
Hence $\frac{86-\mu}{\sigma} = 1.28$
(or) $86 - \mu = 1.28\sigma$ ……. (2)
Solving (1) and (2)
$50 - \mu = -0.525\sigma$
$86 - \mu = 1.28\sigma$
Subtracting,
$36 = 1.28\sigma + 0.525\sigma$
$36 = 1.805\sigma$
$\sigma = 19.94$
Using this in (2),
$86 - \mu = 1.28\ (19.94)$
$86 - \mu = 25.52$
$\mu = 60.48$
Hence the mean of the distribution is 60.48 and standard deviation is 19.94.

Question 7.
X is normally distributed with mean 12 and SD 4. Find $P(X \leq 20)$ and $P(0 \leq X \leq 12)$.
Solution:
Given X ~ N (12, 42), (i.e) mean (μ) = 12 and s.d (σ) = 4.
$P(X \leq 20)$
$= P(\frac{X-\mu}{\sigma} \leq \frac{20-12}{4})$
$= P(X \leq 2)$
Now $P(Z \leq 2) = P(Z \leq 0) + P(0 \leq Z \leq 2)$
$= 0.5 + 0.4772$
$= 0.9772$



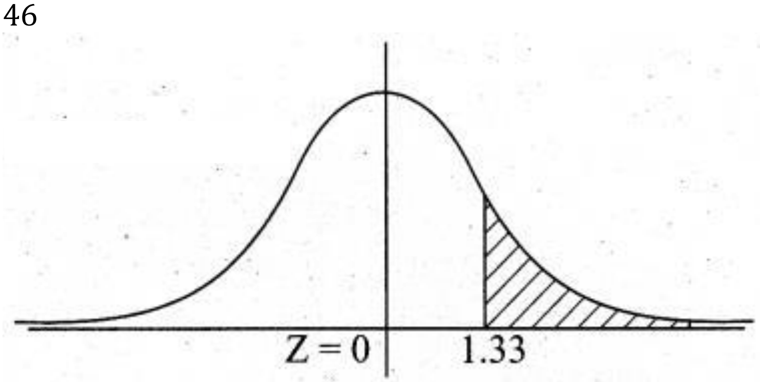
$P(0 \leq X \leq 12)$
$= P(\frac{0-12}{4} \leq \frac{X-\mu}{\sigma} \leq \frac{12-12}{4})$

= P $(-3 \leq Z \leq 0)$
= P$(0 \leq Z \leq 3)$
P$(0 \leq Z \leq 3)$ = 0.49865 (from normal tables)



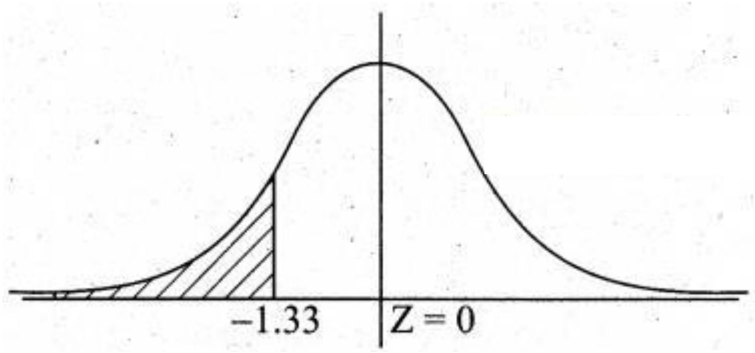
Question 8.
If the heights of 500 students are normally distributed with mean 68.0 inches and standard deviation of 3.0 inches, how many students have height
(a) greater than 72 inches
(b) less than or equal to 64 inches
(c) between 65 and 71 inches.

Solution:
Given X is the normal random variables denoting the height of the students with mean μ = 68 and s.d (σ) = 3.
(a) To find $P(X > 72)$
$= P(Z > \frac{72-68}{3})$
$= P(Z > \frac{4}{3})$
$= P(Z > 1.33)$
Now $P(Z > 1.33) = 0.5 – P(0 < Z < 1.33)$
= 0.5 – 0.4082
= 0.0918
Hence number of students whose height is greater than 72 inches is $500 \times 0.0918 = 45.9 \sim 46$

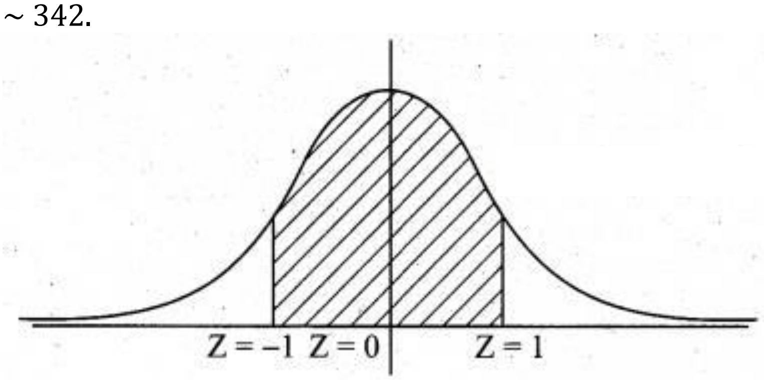


(b) To find $P(X \leq 64)$
$= P(Z \leq \frac{64-68}{3})$
$= P(Z \leq -1.33)$

By Symmetry,
P(Z ≤ -1.33) = P(Z ≥ 1.33) = 0.0918 (see before section)
Hence number of students whose heights are less than or equal to 64 inches is 0.0918 × 500 = 46



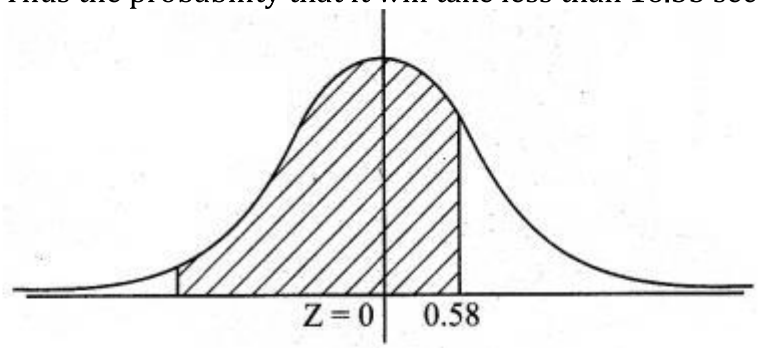
(c) To find P (65 < X < 71)
$= P(\frac{65-68}{3} < Z < \frac{71-68}{3})$
= P(-1 < Z < 1)
= P(-1 < Z < 0) + P(0 < Z < 1)
= 2P(0 < Z < 1) {By symmetry}
= 2 (0.3413)
= 0.6826
Hence number of students with height between 65 and 71 inches is (500) (0.6826) = 341.3 ~ 342.



Question 9.
In a photographic process, the developing time of prints may be looked upon as a random variable having the normal distribution with a mean of 16.28 seconds and a standard deviation of 0.12 second. Find the probability that it will take less than 16.35 seconds to develop prints.

Solution:
Let X be the normal random variable denoting the developing time of the prints.
Given that mean μ = 16.28 and s.d σ = 0.12
To find P(X < 16.35)
When X = 16.35, $Z = \frac{X-\mu}{\sigma} = \frac{16.35-16.28}{0.12}$
Z = 0.58
So P(X < 16.35) = P(Z < 0.58)

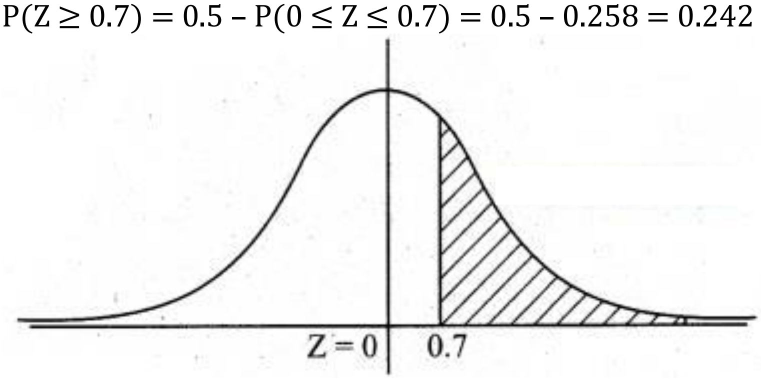
Now $P(Z < 0.58) = 0.5 + P(0 < Z < 0.58)$
$= 0.5 + 0.2190$
$= 0.7190$
Thus the probability that it will take less than 16.35 seconds to develop prints is 0.719.



Question 10.
Time taken by a construction company to construct a flyover is a normal variate with mean 400 labour days and a standard deviation of 100 labour days. If the company promises to construct the flyover in 450 days or less and agree to pay a penalty of ₹ 10,000 for each labour day spent in excess of 450. What is the probability that
(i) the company pays a penalty of at least ₹ 2,00,000?
(ii) the company takes at most 500 days to complete the flyover?

Solution:
Let X be the normal variate denoting the number of labour days.
Given mean $\mu = 400$ and s.d $\sigma = 100$

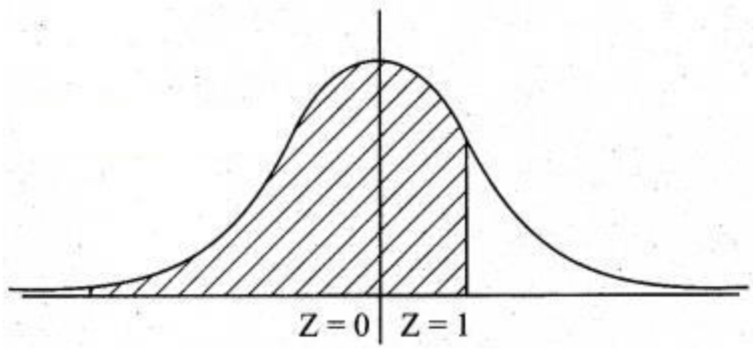
(i) The company pays penalty of ₹ 2,00,000 at the rate of ₹ 10,000 per each extra labour day.
No. of extra days $= \frac{2,00,000}{10,000} = 20$
So the probability that company pays a penalty of atleast ₹ 2,00,000 is probability that labour days should be atleast $450 + 20 = 470$ days, (i.e.) $P(X \geq 470)$
Now $P(X \geq 470) = P(Z \geq \frac{470-400}{100}) = P(Z \geq 0.7)$
$P(Z \geq 0.7) = 0.5 - P(0 \leq Z \leq 0.7) = 0.5 - 0.258 = 0.242$



(ii) $P(X \leq 500) = P(Z \leq \frac{500-40}{100}) = P(Z \leq 1)$

$P(Z \leq 1) = 0.5 + P\ (0 \leq Z \leq 1) = 0.5 + 0.3413 = 0.8413$

Thus the probability that the company takes at most 500 days to complete the flyover is 0.8413.



Ex 7.4

Choose the correct answer:

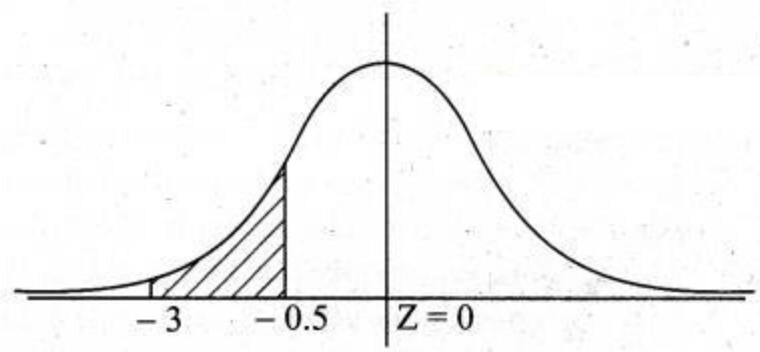
Question 1.
Normal distribution was invented by ______
(a) Laplace
(b) De-Moivre
(c) Gauss
(d) all the above

Answer:
(b) De-Moivre

Question 2.
If X ~ N (9, 81) the standard normal variate Z will be ______
(a) $Z = \frac{x-81}{9}$
(b) $Z = \frac{X-9}{81}$
(c) $Z = \frac{X-9}{9}$
(d) $Z = \frac{9-x}{9}$

Answer:
(c) $Z = \frac{X-9}{9}$
Hint:
$\mu = 9, \sigma = 9$
$Z = \frac{X-9}{9}$

Question 3.
If Z is a standard normal variate, the proportion of items lying between Z = -0.5 and Z = -3.0 is ________
(a) 0.4987
(b) 0.1915
(c) 0.3072
(d) 0.3098
Answer:
(c) 0.3072
Hint:



P(-0.5 < Z < -3)
By symmetry we want P (0.5 < Z < 3)
= P(0 < Z < 3) – P(0 < Z < 0.5)
= 0.49865 – 0.1915
= 0.30715 ~ 0.3072

Question 4.
If X ~ N(μ, σ^2), the maximum probability at the point of inflexion of normal distribution is ______
(a) $\left(\frac{1}{\sqrt{2\pi}}\right) e^{\frac{1}{2}}$
(b) $\left(\frac{1}{\sqrt{2\pi}}\right) e^{\left(-\frac{1}{2}\right)}$
(c) $\left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{\left(-\frac{1}{2}\right)}$
(d) $\left(\frac{1}{\sqrt{2\pi}}\right)$

Answer:
(c) $\left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{\left(-\frac{1}{2}\right)}$

Hint:
p.d.f is $f(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{\left(-\frac{1}{2}\right)\left(\frac{x-\mu}{\sigma}\right)^2}$

The points of inflexion of the curve are $x = \mu \pm \sigma$. Using this,

$$f(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{\left(-\frac{1}{2}\right)\left(\frac{\mu\pm\sigma-\mu}{\sigma}\right)^2} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{\left(-\frac{1}{2}\right)}$$

Question 5.
In a parametric distribution the mean is equal to variance is ________
(a) binomial
(b) normal

(c) Poisson
(d) all of the above
Answer:
(c) Poisson

Question 6.
In turning out certain toys in a manufacturing company, the average number of defectives is 1%. The probability that the sample of 100 toys there will be 3 defectives is _________
(a) 0.0613
(b) 0.613
(c) 0.00613
(d) 0.3913
Answer:
(a) 0.0613
Hint:

$$n = 100, p = 1\% = \frac{1}{100}$$

$$\lambda = np = (100)\left(\frac{1}{100}\right) = 1$$

$$P(X = 3) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-1}(1)^3}{3!} = \frac{e^{-1}}{6} = 0.0613$$

Question 7.
The parameters of the normal distribution f(x) = $\frac{1}{\sqrt{72\pi}} \frac{e^{(-x-10)^2}}{72} \quad -\infty < x < \infty$

(a) (10, 6)
(b) (10, 36)
(c) (6, 10)
(d) (36, 10)
Answer:
(b) (10, 36)
Hint:
Comparing f(x) with p.d.f of normal distribution, μ = 10,

$$\sigma\sqrt{2\pi} = \sqrt{72\pi} = \sqrt{36 \times 2\pi} = 6\sqrt{2\pi}$$

σ = 6

Question 8.
A manufacturer produces switches and experiences that 2 per cent switches are defective. The probability that in a box of 50 switches, there are at most two defective is:
(a) 2.5 e^{-1}

(b) e^{-1}
(c) $2e^{-1}$
(d) none of the above
Answer:
(a) 2.5 e^{-1}
Hint:

$$n = 50, p = \frac{2}{100} = 0.02, \qquad \lambda = np = 1$$

We want P (X ≤ 2) = P (X = 0) + P (X = 1) + P (X = 2)

$$= e^{-1}\left[1 + \frac{1}{11} + \frac{1^2}{2!}\right] = e^{-1}\left[\frac{5}{2}\right]$$

$$= 2.5\, e^{-1}$$

Question 9.
An experiment succeeds twice as often as it fails. The chance that in the next six trials, there shall be at least four successes is _______

(a) $\frac{240}{729}$
(b) $\frac{489}{729}$
(c) $\frac{496}{729}$
(d) $\frac{251}{729}$

Answer:

(c) $\frac{496}{729}$

Hint:
Let X be the binomial random variable. Given p = 2q
From p + q = 1 ,we get p = 23, q = 13
We want P(X ≥ 4) = P(X = 4) + P(X = 5) + P(X = 6)

$$= {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 + {}^6C_6\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0$$

$$= 15\left(\frac{16}{729}\right) + 6\left(\frac{32}{729}\right) + \left(\frac{64}{729}\right) = \frac{496}{729}$$

Question 10.
If for a binomial distribution b(n, p) mean = 4 and variance = 43, the probability, P(X ≥ 5) is equal to _______
(a) $(2/3)^6$

(b) $(2/3)^5 (1/3)$
(c) $(1/3)^6$
(d) $4(2/3)^6$
Answer:
(d) $4(2/3)^6$
Hint:

$$np = 4, npq = \frac{4}{3} \Rightarrow q = \frac{1}{3} \text{ and } p = \frac{2}{3}$$

$$n = \frac{4}{p} = \frac{4}{\frac{2}{3}} = 6$$

$$P(X \geq 5) = P(X = 5) + P(X = 6)$$

$$= {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + {}^6C_6 \left(\frac{2}{3}\right)^6$$

$$= 6\left(\frac{2^5}{3^6}\right) + \frac{2^6}{3^6} = 3\left(\frac{2^6}{3^6}\right) + \frac{2^6}{3^6} = 4\left(\frac{2}{3}\right)^6$$

Question 11.
The average percentage of failure in a certain examination is 40. The probability that out of a group of 6 candidates atleast 4 passed in the examination are _______
(a) 0.5443
(b) 0.4543
(c) 0.5543
(d) 0.4573
Answer:
(a) 0.5443
Hint:
The percentage of success $p = 0.6 \Rightarrow q = 0.4$
$P (X \geq 4) = P (X = 4) + P (X = 5) + P (X = 6)$

$$= {}^6C_4 (0.6)^4 (0.4)^2 + {}^6C_5 (0.6)^5 (0.4) + (0.6)^6$$

$$= 15 (0.6)^4 (0.16) + 6 (0.6)^5 (0.4) + (0.6)^6$$

$= 0.31104 + 0.186624 + 0.046656$
$= 0.54432$

Question 12.
Forty per cent of the passengers who fly on a certain route do not check in any luggage. The planes on this route seat 15 passengers. For a full flight, what is the mean of the number of passengers who do not check in any luggage?
(a) 6.00
(b) 6.45

(c) 7.20
(d) 7.50
Answer:
(a) 6.00
Hint:
n = 15, p = 0.4 ⇒ mean (np) = 6

Question 13.
Which of the following statements is/are true regarding the normal distribution curve?
(a) it is a symmetrical and bell-shaped curve
(b) it is asymptotic in that each end approaches the horizontal axis but never reaches it
(c) its mean, median and mode are located at the same point
(d) all of the above statements are true
Answer:
(d) all of the above statements are true

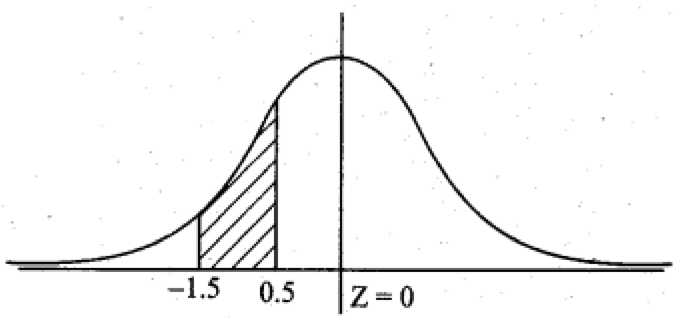
Question 14.
Which of the following cannot generate a Poisson distribution?
(a) The number of telephone calls received in a ten-minute interval
(b) The number of customers arriving at a petrol station
(c) The number of bacteria found in a cubic foot of soil
(d) The number of misprints per page
Answer:
(b) The number of customers arriving at a petrol station

Question 15.
The random variable X is normally distributed with a mean of 70 and a standard deviation of 10. What is the probability that X is between 72 and 84?
(a) 0.683
(b) 0.954
(c) 0.271
(d) 0.340
Answer:
(d) 0.340
Hint:
$\mu = 70, \sigma = 10$

$$P(72 < X < 84) = P\left(\frac{72-70}{10} < Z < \frac{84-70}{10}\right)$$

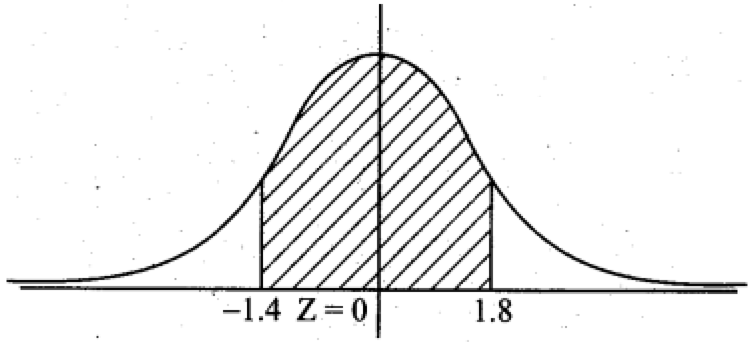
$= P(0.2 < Z < 1.4)$
$= P(0 < Z < 1.4) - P(0 < Z < 0.2)$
$= 0.4192 - 0.0793$
$= 0.3399$
$= 0.340$

Question 16.
The starting annual salaries of newly qualified chartered accountants (CA's) in South Africa follow a normal distribution with a mean of ₹ 180,000 and a standard deviation of ₹ 10,000. What is the probability that a randomly selected newly qualified CA will earn between ₹ 165,000 and ₹ 175,000.
(a) 0.819
(b) 0.242
(c) 0.286
(d) 0.533
Answer:
(b) 0.242
Hint:



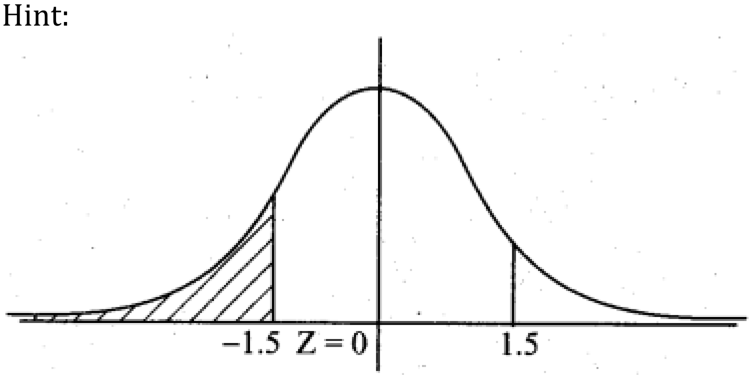
$\mu = 180{,}000$, $\sigma = 10{,}000$
$P(165{,}000 < X < 175{,}000)$
$= P(\frac{165{,}000-180{,}000}{10{,}000} < Z < \frac{175{,}000-180{,}000}{10{,}000})$
$= P(-1.5 < Z < -0.5)$
By symmetry of the normal curve,
$= P\ (0.5 < Z < 1.5)$
$= P\ (0 < Z < 1.5) - P(0 < Z < 0.5)$
$= 0.4332 - 0.1915$
$= 0.2417$
$= 0.242$

Question 17.
In a large statistics class the heights of the students are normally distributed with a mean of 172 cm and a variance of 25 cm. What proportion of students are between 165cm and 181 cm in height?
(a) 0.954
(b) 0.601
(c) 0.718
(d) 0.883
Answer:
(d) 0.883
Hint:



$\mu = 172, \sigma^2 = 25 \Rightarrow \sigma = 5$
P (165 < X < 181)
$= P(\frac{165-172}{5} < Z < \frac{181-172}{5})$
= P(-1.4 < Z < 1.8)
= P (-1.4 < Z < 0) + P(0 < Z < 1.8)
= P(0 < Z < 1.4) + P(0 < Z < 1.8)
= 0.4192 + 0.4641
= 0.8833

Question 18.
A statistical analysis of long-distance telephone calls indicates that the length of these calls is normally distributed with a mean of 240 seconds and a standard deviation of 40 seconds. What proportion of calls lasts less than 180 seconds?
(a) 0.214
(b) 0.094
(c) 0.933
(d) 0.067
Answer:
(d) 0.067
Hint:



$\mu = 240, \sigma = 40$
P(X < 180)
$= P(Z < \frac{180-240}{40})$
= P(Z < -1.5) = P(Z > 1.5)
= 0.5 – P (0 < Z < 1.5)
= 0.5 – 0.4332

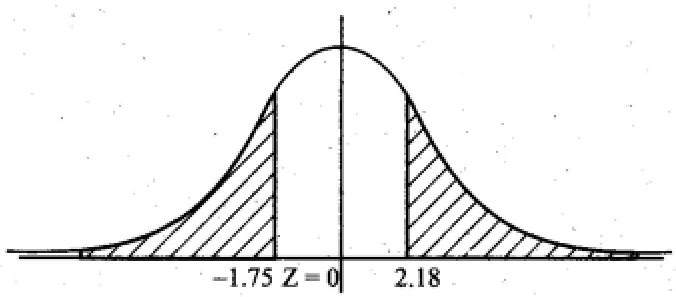
= 0.0668
= 0.067

Question 19.
Cape town is estimated to have 21% of homes whose owners subscribe to the satellite service, DSTV. If a random sample of your home in taken, what is the probability that all four home subscribe to DSTV?
(a) 0.2100
(b) 0.5000
(c) 0.8791
(d) 0.0019
Answer:
(d) 0.0019
Hint:

$$p = \frac{21}{100} = 0.21, q = 0.79, n = 4$$

$$P(X = 4) = {}^4C_4 \left(\frac{21}{100}\right)^4 (0.79)^0 = (0.21)^4 = 0.0019$$

Question 20.
Using the standard normal table, the sum of the probabilities to the right of z = 2.18 and to the left of z = -1.75 is:
(a) 0.4854
(b) 0.4599
(c) 0.0146
(d) 0.0547
Answer:
(d) 0.0547
Hint:



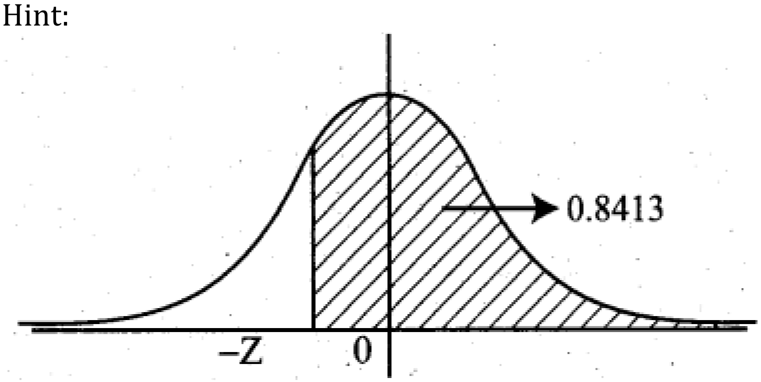
P(Z > 2.18) + P(Z < -1.75)
= 0.5 – P(0 < Z < 2.18) + P (Z > 1.75)
= 0.5 – P(0 < Z < 2.18) + 0.5 – P(0 < Z < 1.75)
= 0.5 – 0.4854 + 0.5 – 0.4599
= 0.0547

Question 21.
The time until first failure of a brand of inkjet printers is normally distributed with a mean of 1,500 hours and a standard deviation of 200 hours. What proportion of printers fails before 1000 hours?
(a) 0.0062
(b) 0.0668
(c) 0.8413
(d) 0.0228
Answer:
(a) 0.0062
Hint:
$\mu = 1500, \sigma = 200$
$P(X < 1000)$
$= P(Z < \frac{1000-1500}{200})$
$= P(Z < -2.5) = P(Z > 2.5)$
$= 0.5 - P(0 < Z < 2.5)$
$= 0.5 - 0.4938$
$= 0.0062$

Question 22.
The weights of newborn human babies are normally distributed with a mean of 3.2 kg and a standard deviation of 1.1 kg. What is the probability that a randomly selected newborn baby weight less than 2.0 kg?
(a) 0.138
(b) 0.428
(c) 0.766
(d) 0.262
Answer:
(a) 0.138
Hint:
$\mu = 3.2, \sigma = 1.1$
$P(X < 2)$
$= P(Z < \frac{2-3.2}{1.1})$
$= P(Z < -1.09) = P(Z > 1.09)$
$= 0.5 - P(0 < Z < 1.09)$
$= 0.5 - 0.3621$
$= 0.138$

Question 23.
Monthly expenditure on their credit cards, by credit card holders from a certain bank, follows a normal distribution with a mean of ₹ 1,295.00 and a standard deviation of ₹ 750.00. What proportion of credit card holders spend more than ₹ 1,500.00 on their credit cards per month?
(a) 0.487
(b) 0.394

(c) 0.500
(d) 0.791
Answer:
(b) 0.394
Hint:
$\mu = 1295, \sigma = 750$
$P(X > 1500)$
$= P(Z > \frac{1500-1295}{750})$
$= P(Z > 0.27)$
$= 0.5 - P(0 < Z < 0.27)$
$= 0.5 - 0.1064$
$= 0.3936 \sim 0.394$

Question 24.
Let z be a standard normal variable. If the area to the right of z is 0.8413, then the value of z must be:
(a) 1.00
(b) -1.00
(c) 0.00
(d) -0.41
Answer:
(b) -1.00
Hint:


$P(Z > -Z) = 0.8413$
$\Rightarrow P(-z < Z < 0) + 0.5 = 0.8413$
$P(0 < Z < z) = 0.8413 - 0.5 = 0.3413$
from normal tables, z = 1.
The required value is $-z = -1$

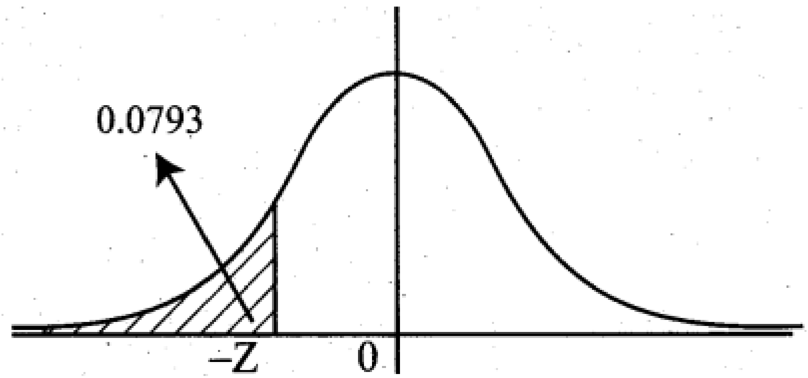
Question 25.
If the area to the left of a value of z (z has a standard normal distribution) is 0.0793, what is the value of z?
(a) -1.41
(b) 1.41.
(c) -2.25

(d) 2.25

Answer:

(a) -1.41

Hint:



P(Z < -z) = 0.0793 By symmetry, P (Z > z) = 0.0793

(i.e) 0.5 – P (0 < Z < z) = 0.0793

P(0 < Z < z) = 0.4207 from normal tables, z = 1.41

Thus the required value is -1.41 (since it is left of Z = 0)

Question 26.

If P(Z > z) = 0.8508 what is the value of z (z has a standard normal distribution)?

(a) -0.48

(b) 0.48

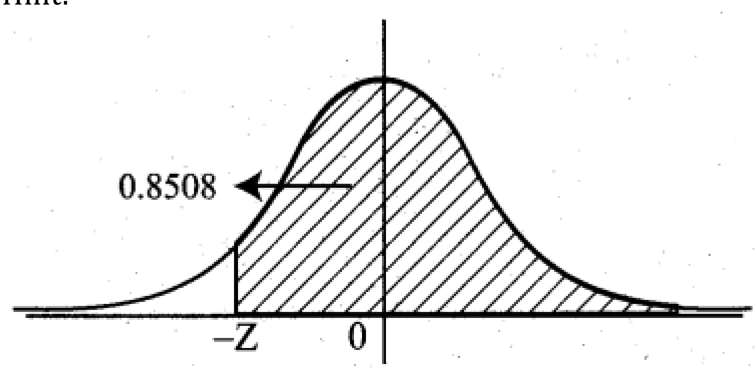
(c) -1.04

(d) -0.21

Answer:

(c) -1.04

Hint:



P (Z > z) = 0.8508

Since the given value is more than we take z to the left of Z = 0 axis.

P (Z > -z) = 0.8508

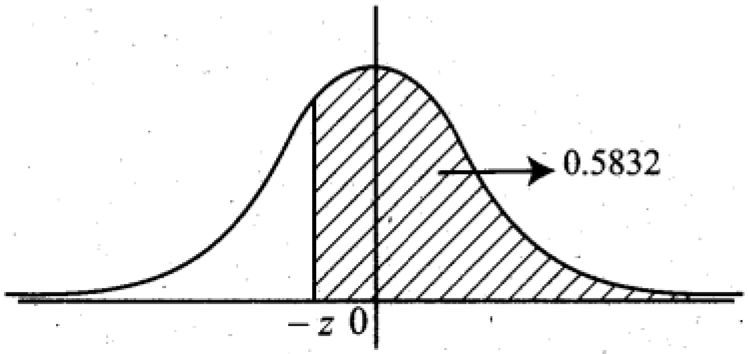
P(-z < Z < 0) + 0.5 = 0.8508

P(0 < Z < z) = 0.3508

From normal tables, z = 1.04

So required value is -1.04

Question 27.
If $P(Z > z) = 0.5832$ what is the value of z (z has a standard normal distribution)?
(a) -0.48
(b) 0.48
(c) 1.04
(d) -0.21
Answer:
(d) -0.21
Hint:



$P(0 < Z < z) = 0.5832 – 0.5 = 0.0832$
From tables, z = 0.21
Since z is to the left of Z = 0, the required value is -0.21

Question 28.
In a binomial distribution, the probability of success is twice as that of failure. Then out of 4 trials, the probability of no success is _______

(a) $\frac{16}{81}$
(b) $\frac{1}{16}$
(c) $\frac{2}{27}$
(d) $\frac{1}{81}$

Answer:
(d) $\frac{1}{81}$
Hint:

$$p = 2q \Rightarrow p + q = 1 \Rightarrow q = \frac{1}{3}, p = \frac{2}{3}$$

$$P(X = 0) = {}^4C_0 p^0 q^4 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$$

Miscellaneous Problems

Question 1.
A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain
(a) no more than 2 rejects?
(b) at least 2 rejects?

Solution:
Let X be the binomial random variable denoting the number of metal pistons.
Let p be the probability of rejections.
Given that $p = 12\% = \frac{12}{100} = 0.12$, $q = 0.88$, $n = 10$.
So $X \sim B(0.12, 10)$. Hence the p.m.f of X is given by

$$P(X = x) = {}^{10}C_x (0.12)^x (0.88)^{10-x}$$

(a)
$$\begin{aligned} P(X \le 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\ &= {}^{10}C_0 (0.12)^0 (0.88)^{10} + {}^{10}C_1 (0.12)^1 (0.88)^9 + {}^{10}C_2 (0.12)^2 (0.88)^8 \\ &= (0.88)^{10} + 10(0.12)(0.88)^9 + 45(0.12)^2 (0.88)^8 \\ &= (0.88)^8 + [(0.88)^2 + (1.2)(0.88) + 45(0.12)^2] \\ &= (0.35963)[0.7744 + 1.056 + 0.648] \\ &= (0.35963)(2.4784) \\ &= 0.89131 \end{aligned}$$

Thus out of a batch of 10 pistons, the probability of no more than 2 rejects is 0.89131

(b)
$$\begin{aligned} P(2 \le X) &= 1 - P(X < 2) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - [(0.88)^{10} + 10(0.12)(0.88)^9] \\ &= 1 - [(0.88)^9 [0.88 + 1.2]] \\ &= 1 - [(0.88)^9 (2.08)] \end{aligned}$$

To find $(0.88)^9$ using calculator.

$$\begin{aligned} (0.88)^9 &= [(0.88)^3]^3 = [0.681472]^3 \\ &= 0.316478 \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad P(2 \le X) &= 1 - [(0.316478)(2.08)] \\ &= 1 - 0.65827 = 0.34173 \end{aligned}$$

Thus out of 10 pistons, the probability that at least 2 will be rejected is 0.34173

Question 2.
Hospital records show that of patients suffering from a certain disease 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover?

Solution:
Let X be the binomial random variable denoting the number of patients.
Let p be the probability that the patient will recover and q be the probability that patient will die.
According to the problem, q = 75% = 0.75 and p = 25% = 0.25 and n = 6

The p.m.f is $P(X = x) = {}^6C_x (0.25)^x (0.75)^{6-x}$

We want $P(X = 4) = {}^6C_4 (0.25)^4 (0.75)^2$

$= 15 (0.25)^4 (0.75)^2$

$= 0.03295$

Hence the probability that 4 patients will recover out of 6 patients is 0.03295

Question 3.
If electricity power failures occur according to a Poisson distribution with an average of 3 failures every twenty weeks, calculate the probability that there will not be more than one failure during a particular week.

Solution:
Let X be the poisson random variable. It is given that mean $\lambda = \frac{3}{20} = 0.15$
The Poisson probability law, giving x failures per week is given by,

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.15} (0.15)x}{x!}, x = 0,1,2,3,.....$$

Hence probability that there will not be more than one failure is given by $P(X \leq 1)$
$= P(X = 0) + P(X = 1)$
$= e^{-0.15} [1 + 0.15]$
$= e^{-0.15} (1.15)$
$= (0.8607) (1.15)$
$= 0.98981$

Question 4.
Vehicles pass through a junction on a busy road at an average rate of 300 per hour.
(a) Find the probability that none passes in a given minute.
(b) What is the expected number passing in two minutes?

Solution:
Let X be the Poisson random variable. It is given that mean
λ = 300/hour = 300/60 minutes = 5 per minute.
(a) The Poisson law giving x vehicles passing on a road in one minute is given by

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-5} (5)^x}{x!} x = 0,1,2,3,.....$$

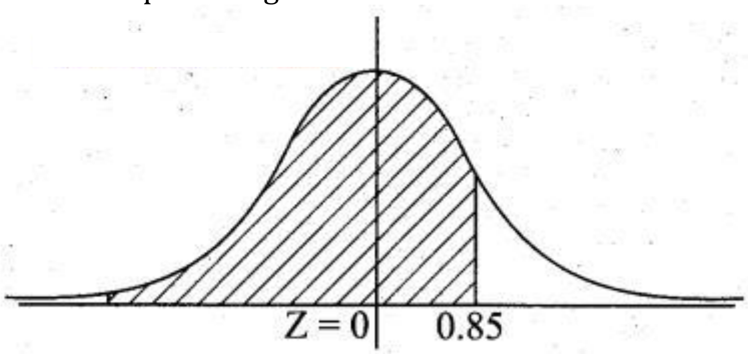
Now the probability that no vehicles pass in a given minute is given by,

$$P(X=0) = \frac{e^{-5}5^0}{0!} = e^{-5} = 0.0067379$$

(b) E(X) = λ (i.e) No of vehicles passing per minute, since λ = 5, the expected number passing in two minutes is 10.

Question 5.
Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Raghul wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Raghul takes the test and scores 585. Will he be admitted to this university?

Solution:
Let X be the normal random variable denoting the scores of the students. Given that mean μ = 500 and s.d σ = 100. The total area under the normal curve represents the total number of students who took the test. If we multiply the values of the areas under the curve by 100, we obtain percentages.



When X = 585, $Z = \frac{585-500}{100} = 0.85$
The proportion of students who scored below 585 is given by P[area to the left of Z = 0.85]
(i.e.) $P(Z < 0.85) = 0.5 + P(0 < Z < 0.85)$
$= 0.5 + 0.3023$.
$= 0.8023$
$= 80.23\%$
Raghul scored better than 80.23% of the students who took the test and he will be admitted to this university.

Question 6.
The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assembled at this plant in a period of time?
(a) less than 19.5 hours?
(b) between 20 and 22 hours?
Solution:
Let X be the normal random variable denoting the time taken to assemble a car.

Given that mean $\mu = 20$ and s.d $\sigma = 2$

(a) Probability that car is assembled in less than 19.5 hours

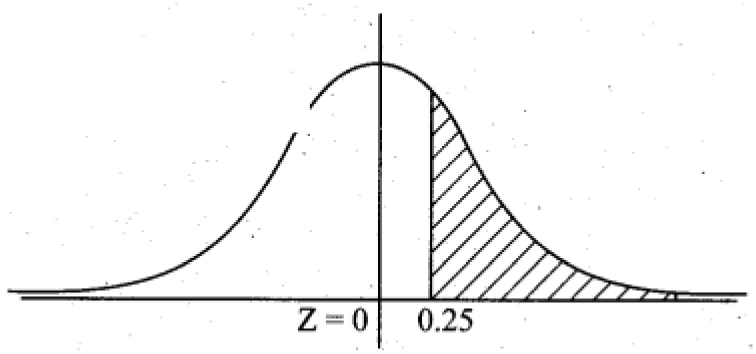
$P(X < 19.5) = P(Z < \frac{19.5-20}{2})$

$P(Z < -0.25) = P(Z > 0.25)$

$= 0.5 - P(0 < Z < 0.25)$

$= 0.5 - 0.0987$

$= 0.4013$



(b) The probability that a car can be assembled between 20 and 22 hours is given by

$P(20 < X < 22) = P(\frac{20-20}{2} < Z < \frac{22-20}{2})$

$P(0 < Z < 1) = 0.3413$

Question 7.

The annual salaries of employees in a large company are approximately normally distributed with a mean of \$50,000 and a standard deviation of \$20,000.

(a) What per cent of people earn less than \$ 40,000?

(b) What per cent of people earn between \$ 45,000 and \$65,000?

(c) What per cent of people*earn more than \$ 70,000?

Solution:

Let X be the normal variable denoting the annual salaries of employees.

Given the mean $\mu = 50,000$ and s.d $\sigma = 20,000$

(a) Probability of people ehming less than \$ 40,000 is given by P $(X < 40,000)$

$= P(X < 40,000)$

$= P(Z < \frac{40000-50000}{20,000})$

$= P(Z < -0.5)$

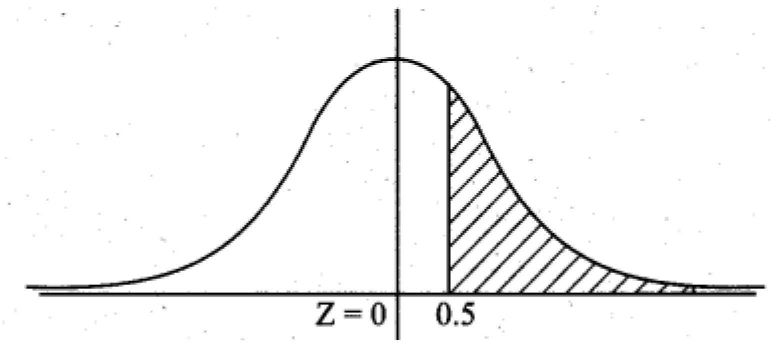
$= P(0.5 < Z)$ (By symmetry)

$= 0.5 - P(0 < Z < 0.5)$

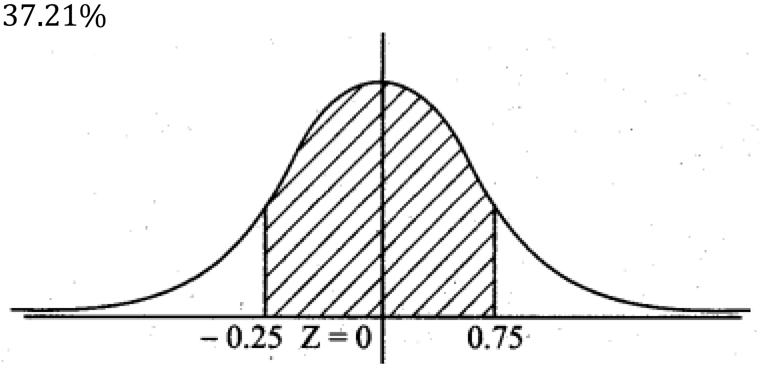
$= 0.5 - 0.1915$

$= 0.3085$

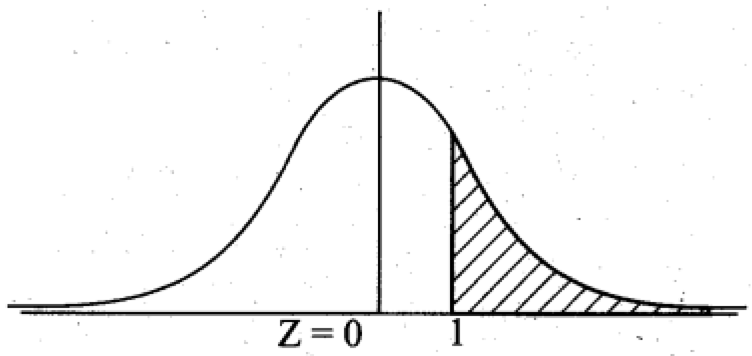
Hence people who earn less than \$40,000 is $0.3085 \times 100 = 30.85\%$



(b) Probability of people earning between \$45,000 and \$65,000 is $P(45000 < X < 65000)$
$= P(\frac{45000-50000}{20000} < Z < \frac{65000-50000}{20000})$
$= P\ (-0.25 < Z < 0.75)$
$= P(-0.25 < Z < 0) + P\ (0 < Z < 0.75)$
$= P(0 < Z < 0.25) + P\ (0 < Z < 0.75)$
$= 0.0987 + 0.2734$
$= 0.3721$
Hence percent of people who earn between \$45,000 and \$65,000 is $0.3721 \times 100 =$ 37.21%

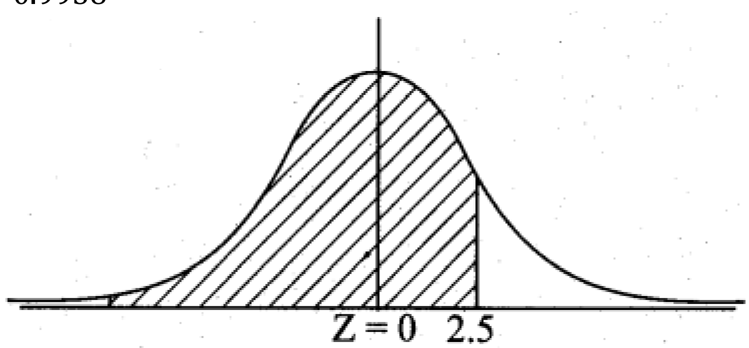


(c) Probability of people earning more than \$ 70,000 is $P(X > 70,000)$
$= P(Z > \frac{70000-50000}{20000})$
$= P(Z > 1)$
$= 0.5 - P(0 < Z < 1)$
$= 0.5 - 0.3413$
$= 0.1587$
Hence percent of people who earn more than \$70,000 is $0.1587 \times 100 = 15.87\%$

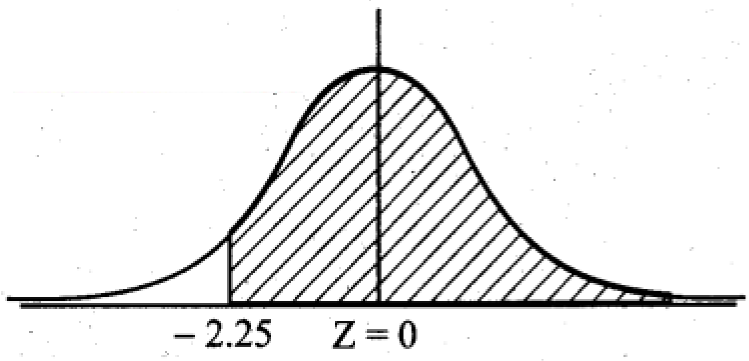


Question 8.
X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find
(a) $P(X < 40)$ (b) $P(X > 21)$
(c) $P(30 < X < 35)$.

Solution:
Given $X \sim N(\mu, \sigma^2)$
$\mu = 30, \sigma = 4$
(a) $P(X < 40) = P(Z < \frac{40-30}{4})$
$= P(Z < 2.5)$
$= 0.5 + P(0 < Z < 2.5)$
$= 0.5 + 0.4938$
$= 0.9938$



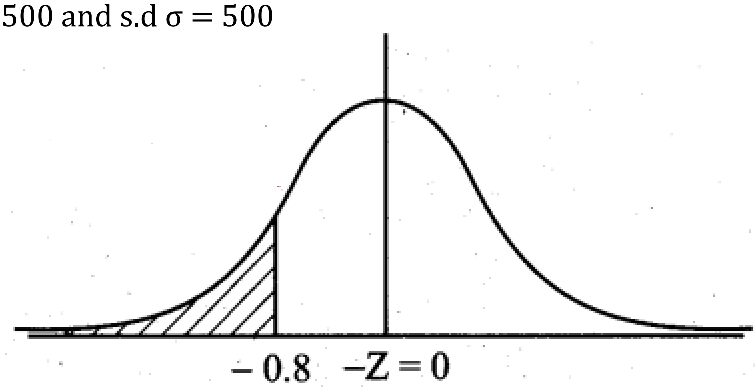
(b) $P(X > 21) = P(Z > \frac{21-30}{4})$
$= P(Z > -2.25)$
$= 0.5 + P(-2.25 < Z < 0)$
$= 0\ 5 + P(0 < Z < 2.25)$
$= 0.5 + 0.4878$
$= 0.9878$



(c) P(30 < X < 35)
$= P(\frac{30-30}{4} < Z < \frac{35-30}{4})$
= P(0 < Z < 1.25)
= 0.3944

Question 9.
The birth weight of babies is normally distributed with mean 3,500g and standard deviation 500g, What is the probability that a baby is born that weight less than 3,100g?

Solution:
Let X be the normal variable denoting the birth weights of babies. Given that the mean μ = 3500 and s.d σ = 500

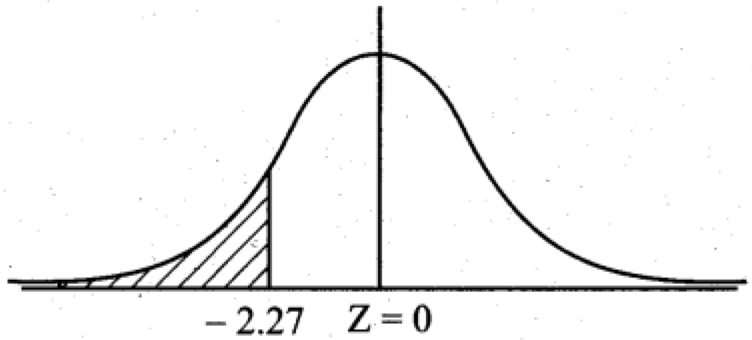


Probability that a baby is bom with weight less than 3100 g = P (X < 3100)
$= P(Z < \frac{3100-3500}{500})$
= P (Z < -0.8) = P (Z > 0.8) (by symmetry)
= 0.5 – P(0 < Z < 0.8)
= 0.5 – 0.2881
= 0.2119

Question 10.
People's monthly electric bills in Chennai are normally distributed with a mean of ₹ 225 and a standard deviation of ₹ 55. Those people spend a lot of time online. In a group of 500 customers, how many would we expect to have a bill that is ₹ 100 or less?

Solution:

Let X be the normal variable denoting the monthly bills in rupees.



Given mean $\mu = 225$ and s.d $\sigma = 55$

Now the probability that the bill will be ₹100 or less is $P(X \leq 100)$

$= P(Z \leq \frac{100-225}{55})$

$= P(Z \leq -2.27)$

$= 0.5 - P(-2.27 < Z < 0)$

$= 0.5 - P(0 < Z < 2.27)$

$= 0.5 - 0.4884$

$= 0.0116$

Thus, in a group of 500 customers, we expect to have $500 \times 0.0116 = 5.8 \sim 6$ customers whose electric bills will be ₹ 100 or less.